ERRATA

1. VOLUME I: Page 222 (Fig. 209) is missing
(Email JC Turner to get this page).

2. VOLUME IV. At the end of Appendix 2009 (see heading CWH KNOTS) there are three pages numbered 197, i and ii.
Disgard these three pages, as they do not belong in the book.

THE BRAIDER

VOLUME II

ALBERTUS GEORG SCHAAKE

THE BRAIDER

VOLUME II : Issue Numbers 16–30

ALBERTUS GEORG SCHAAKE

— November 1998 to May 2002

PREFACE

This is the second of four volumes on the Theory and Practice of Braiding. It brings together the second fifteen issues of a Journal called simply *THE BRAIDER*.

Albertus Georg Schaake was born in Holland in 1933. He studied Engineering at Delft Technical University, before emigrating to New Zealand in 1962. His career has been spent first as a Public Works Engineer, and then as a Tutor in Engineering subjects at the Waikato Polytechnic in Hamilton.

For the last thirty years he has devoted countless hours to the study and practice of braiding : to the creation of decorative braids in particular. Towards the end of the 1980s, he began a project to publish his discoveries, occasionally collaborating with other authors, to produce a stream of books, pamphlets, articles and research papers on a variety of topics in braiding theory. These publications range from books on Regular Knots, Fiador Knots and Herringbone Knots through to extensive pamphlets on the braiding of Wheelknots. For a lengthy account of the progress and philosophy of this work up to 1995, the reader is referred to chapter twelve of *History and Science of Knots* edited by J C Turner and P van de Griend, and published by World Scientific (1995).

In 1995, such was the large output of Schaake's ideas on the subject, that he decided to produce a quarterly Journal which would summarize and extend the description of his work. He called the Journal *The Braider*; and in the next eleven years, virtually single-handedly, he produced 60 issues. Such was his industry and application, that when he stopped his researches, due to illness in 2006, he had already prepared quarterly issues which would take the series up to November 2009.

In total, some 1600 pages of ideas, philosophies, theories, and instruction on design and production of decorative braids, have flowed from his computer and have been included in the issues of *The Braider*. In all the issues there are numerous diagrams which illustrate the theories and techniques. All of these were produced by Schaake by means of a computer-aided design package.

Now, in 2007, it is my privilege to bring together all sixty of the issues, unchanged and unedited, into a collection of four spirally-bound Volumes. They will be available for future generations of braiders to study and further the great body of work that Schaake has initiated.

Georg Schaake may truly be said to be the father of Modern Braiding Theory; very little of consequence existed before he began his researches.

The four-volume collection may be taken as a text for all braiders, from beginners to the most advanced. It is essential reading for all serious decorative braiders.

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A quarterly publication
for
the braiding artisan

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Solution to the Question in Issue No. 15

Question on pp. 336 – 337.

Gaucho-coding : $P = 2m(\alpha + 1) + 1$

We have seen in **The Braider**, No. 11, pg. 234, that a **Gaucho-coding** is a **Column-coding** which consists of $2m$ sets of adjacent intersection columns. The number of intersection-columns in each set is the same. The intersection-columns in a set all have the same coding for their intersections (crossings), while the coding of the intersections in an odd-numbered set is opposite to the coding of the intersections in an even-numbered set. Hence when a Gaucho-coded Regular Knot with α intersection-columns in each set evolves to a Gaucho-coded Regular Knot with $(\alpha + 1)$ intersection-columns in each set, the number of intersection-columns in each set must increase by 1. Let this additional intersection-column in each set occupy the same relative position in each set, then we have only two possibilities for this evolution process:

(1). The evolution process associated with a Method I enlargement. See leftmost diagram of Fig. 301.

(2). The evolution process associated with a Method II enlargement. See rightmost diagram of Fig. 301.

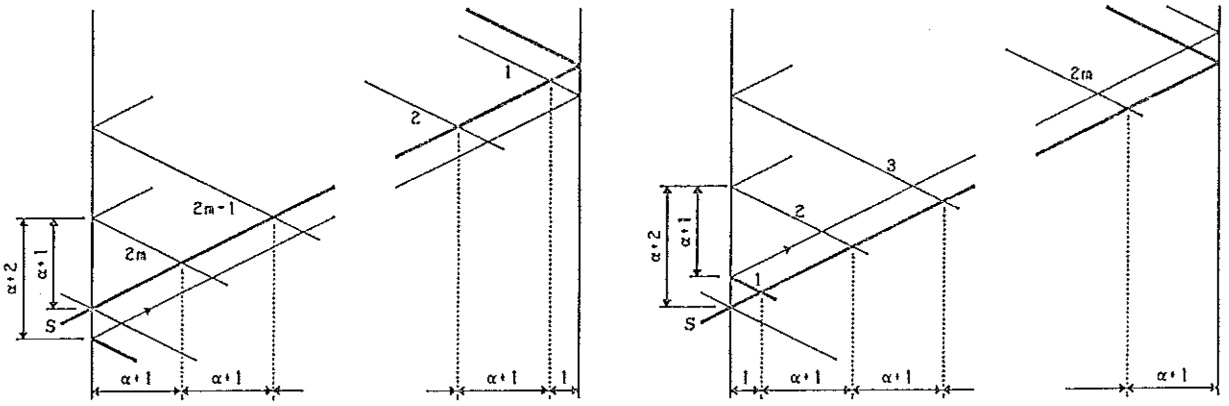


Fig. 301 — Method I evolution left-hand side, Method II evolution right-hand side.

Thus during an evolution stage there are $2m$ half-cycles which intersect the **Standing End** half-cycle on an intersection-column, and 1 half-cycle which intersects the **Standing End** half-cycle on the bight-boundary. Since the additional intersection-column in each set occupies the same relative position in each set, the number of cycles between the end-points of any two consecutive additional half-cycles which intersect the **Standing End** half-cycle is equal to $|\Delta^*(\alpha + 1)|_B$ for a Method I evolution and equal to $|-\Delta^*(\alpha + 1)|_B$ for a Method II evolution.[†] Hence $|\Delta^*(\alpha + 1)|_B = N$ for evolution Method I, and $|-\Delta^*(\alpha + 1)|_B = N$ for evolution method II, where N is a positive integer. Let's call the number of cycles N a **braiding sequence.** Then a braiding sequence spans $NP = [B - (\alpha + 1)]$ bights in a Method I evolution, and spans $NP = [B + (\alpha + 1)]$ bights in a Method II evolution.[‡]

[†] This follows immediately from the construction of the algorithm diagram with Δ^*, and the respective diagrams in Fig. 301.

[‡] This follows immediately from the respective diagrams in Fig. 301.

Thus for a Method I evolution we obtain:

$$\begin{aligned} N\{2m(\alpha+1)+1\} &= B-(\alpha+1)\,, \quad \text{hence} \\ B &= \{2m(\alpha+1)+1\}N+(\alpha+1)\,. \\ |\Delta^*(\alpha+1)|_B &= N\,, \quad \text{hence} \\ \Delta^* &= \frac{nB+N}{\alpha+1} = \frac{n[\{2m(\alpha+1)+1\}N+(\alpha+1)]+N}{\alpha+1} \\ &= 2mnN+n+\frac{(n+1)N}{\alpha+1}\,, \\ \Delta^* &= \text{positive integer, hence } n=\alpha \text{ since } 0<\Delta^*<B.\ \text{Thus} \\ \Delta^* &= (2m\alpha+1)N+\alpha. \end{aligned}$$

For a Method II evolution we obtain:

$$\begin{aligned} N\{2m(\alpha+1)+1\} &= B+(\alpha+1)\,, \quad \text{hence} \\ B &= \{2m(\alpha+1)+1\}N-(\alpha+1)\,. \\ |-\Delta^*(\alpha+1)|_B &= N\,, \quad \text{hence} \\ \Delta^* &= \frac{nB-N}{\alpha+1} = \frac{n[\{2m(\alpha+1)+1\}N-(\alpha+1)]-N}{\alpha+1} \\ &= 2mnN-n+\frac{(n-1)N}{\alpha+1}\,, \\ \Delta^* &= \text{positive integer, hence } n=1 \text{ since } 0<\Delta^*<B.\ \text{Thus} \\ \Delta^* &= 2mN-1. \end{aligned}$$

Headhunter's-coding: $P=(2m+1)(\alpha+1)+1$

We have seen in **The Braider**, No. 12, pg. 257, that a **Headhunter's-coding** is a **Column-coding** which consists of $(2m+1)$ sets of adjacent intersection columns. The number of intersection-columns in each set is the same. The intersection-columns in a set all have the same coding for their intersections (crossings), while the coding of the intersections in an odd-numbered set is opposite to the coding of the intersections in an even-numbered set. Hence when a Headhunter's-coded Regular Knot with α intersection-columns in each set evolves to a Headhunter's-coded Regular Knot with $(\alpha+1)$ intersection-columns in each set, the number of intersection-columns in each set must increase by 1. Let this additional intersection-column in each set occupy the same relative position in each set, then we have only two possibilities for this evolution process:

(1). The evolution process associated with a Method I enlargement. See leftmost diagram of Fig. 302.

(2). The evolution process associated with a Method II enlargement. See rightmost diagram of Fig. 302.

Thus during an evolution stage there are $(2m+1)$ half-cycles which intersect the **Standing End** half-cycle on an intersection-column, and 1 half-cycle which intersects the **Standing End** half-cycle on the bight-boundary. Since the additional intersection-column in each set occupies the same relative position in each set, the number of cycles between the end-points of any two consecutive additional half-cycles which intersect the **Standing End** half-cycle is equal to $|\Delta^*(\alpha+1)|_B$ for a Method I evolution and equal to $|-\Delta^*(\alpha+1)|_B$ for a Method II evolution.[†] Hence $|\Delta^*(\alpha+1)|_B = N$ for evolution Method I, and $|-\Delta^*(\alpha+1)|_B = N$ for evolution method II, where N is a positive integer. The number of cycles N constitute a **braiding sequence**. Then a braiding sequence spans $NP = [B-(\alpha+1)]$ bights in a Method I evolution, and spans $NP = [B+(\alpha+1)]$ bights in a Method II evolution.[‡]

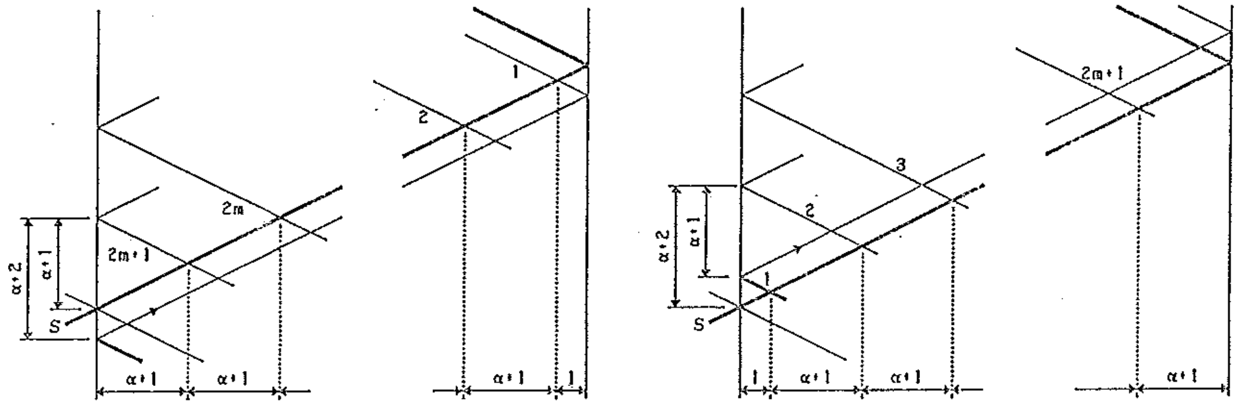


Fig. 302 — Method I evolution left-hand side, Method II evolution right-hand side.

Thus for a Method I evolution we obtain:

$$N\{(2m+1)(\alpha+1)+1\} = B-(\alpha+1)\,, \quad \text{hence}$$

$$B = \{(2m+1)(\alpha+1)+1\}N + (\alpha+1)\,.$$

$$|(\alpha+1)\Delta^*|_B = N\,, \quad \text{hence}$$

$$\Delta^* = \frac{nB+N}{\alpha+1} = \frac{n[\{(2m+1)(\alpha+1)+1\}N+(\alpha+1)]+N}{\alpha+1}$$

$$= (2m+1)nN + n + \frac{(n+1)N}{\alpha+1}\,,$$

$\Delta^* =$ positive integer, hence $n=\alpha$ since $0<\Delta^*<B$. Thus

$$\Delta^* = [(2m+1)\alpha+1]N+\alpha.$$

For a Method II evolution we obtain:

$$N\{(2m+1)(\alpha+1)+1\} = B+(\alpha+1)\,, \quad \text{hence}$$

$$B = \{(2m+1)(\alpha+1)+1\}N - (\alpha+1)\,.$$

$$|-\Delta^*(\alpha+1)|_B = N\,, \quad \text{hence}$$

$$\Delta^* = \frac{nB-N}{\alpha+1} = \frac{n[\{(2m+1)(\alpha+1)+1\}N-(\alpha+1)]-N}{\alpha+1}$$

$$= (2m+1)nN - n + \frac{(n-1)N}{\alpha+1}\,,$$

$\Delta^* =$ positive integer, hence $n=1$ since $0<\Delta^*<B$. Thus

$$\Delta^* = (2m+1)N-1.$$

[†] This follows immediately from the construction of the algorithm diagram with Δ^*, and the respective diagrams in Fig. 302.

[‡] This follows immediately from the respective diagrams in Fig. 302.

Fan-coding: $P = (2m+2)(\alpha+1) - 1$

We have seen in **The Braider**, No. 12, pg. 257, that a **Fan-coding** is a **Column-coding** consisting of $2m$ sets of adjacent intersection columns which have the same number of intersection-columns in each set, flanked on each side by a set of intersection-columns which have each one less intersection-column than each of the $2m$ adjacent sets. The intersection-columns in a set all have the same coding for their intersections (crossings). The coding of the intersections in an odd-numbered set is opposite to the coding of the intersections in an even-numbered set. Hence when a Fan-coded Regular Knot with α intersection-columns in each of the $2m$ adjacent sets evolves to a Fan-coded Regular Knot with $(\alpha+1)$ intersection-columns in each of these $2m$ adjacent sets, the number of intersection-columns in every set must increase by 1. Let this additional intersection-column in each set occupy the same relative position in each set, then we have only two possibilities for this evolution process:

(1). The evolution process associated with a Method I enlargement. See leftmost diagram of Fig. 303.

(2). The evolution process associated with a Method II enlargement. See rightmost diagram of Fig. 303.

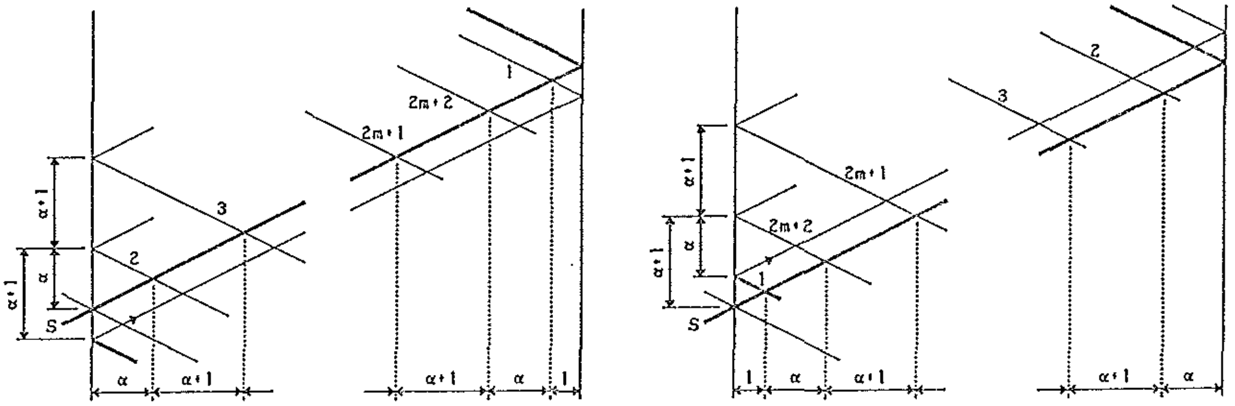


Fig. 303 — Method I evolution left-hand side, Method II evolution right-hand side.

Thus during an evolution stage there are $(2m+2)$ half-cycles which intersect the **Standing End** half-cycle on an intersection-column, and 1 half-cycle which intersects the **Standing End** half-cycle on the bight-boundary. Since the additional intersection-column in each set occupies the same relative position in each set, the number of cycles between the end-points of any two consecutive additional half-cycles, belonging to the $2m$ central sets, which intersect the **Standing End** half-cycle is equal to $|-\Delta^*(\alpha+1)|_B$ for a Method I evolution and equal to $|\Delta^*(\alpha+1)|_B$ for a Method II evolution.[†] Hence $|-\Delta^*(\alpha+1)|_B = N$ for evolution Method I, and $|\Delta^*(\alpha+1)|_B = N$ for evolution method II, where N is a positive integer. The number of cycles N constitute a **braiding sequence** associated with the central $2m$ sets of intersection-columns. Such a braiding sequence spans $NP = [B + (\alpha+1)]$ bights in a Method I evolution, and spans $NP = [B - (\alpha+1)]$ bights in a Method II evolution.[‡]

Thus for a Method I evolution we obtain:

[†] This follows immediately from the construction of the algorithm diagram with Δ^*, and the respective diagrams in Fig. 303.

[‡] This follows immediately from the respective diagrams in Fig. 303.

$$
\begin{aligned}
N\{(2m+2)(\alpha+1)-1\} &= B+(\alpha+1)\,, \quad \text{hence}\\
B &= \{(2m+2)(\alpha+1)-1\}N-(\alpha+1)\,.\\
|-\Delta^*(\alpha+1)|_B &= N\,, \quad \text{hence}\\
\Delta^* &= \frac{nB-N}{\alpha+1} = \frac{n[\{(2m+2)(\alpha+1)-1\}N-(\alpha+1)]-N}{\alpha+1}\\
&= (2m+2)nN-n-\frac{(n+1)N}{\alpha+1}\,,\\
\Delta^* &= \text{positive integer, hence } n=\alpha \text{ since } 0<\Delta^*<B. \text{ Thus}\\
\Delta^* &= [(2m+2)\alpha-1]N-\alpha.
\end{aligned}
$$

For a Method II evolution we obtain:

$$
\begin{aligned}
N\{(2m+2)(\alpha+1)-1\} &= B-(\alpha+1)\,, \quad \text{hence}\\
B &= \{(2m+2)(\alpha+1)-1\}N+(\alpha+1)\,.\\
|\Delta^*(\alpha+1)|_B &= N\,, \quad \text{hence}\\
\Delta^* &= \frac{nB+N}{\alpha+1} = \frac{n[\{(2m+2)(\alpha+1)-1\}N+(\alpha+1)]+N}{\alpha+1}\\
&= (2m+2)nN+n-\frac{(n-1)N}{\alpha+1}\,,\\
\Delta^* &= \text{positive integer, hence } n=1 \text{ since } 0<\Delta^*<B. \text{ Thus}\\
\Delta^* &= (2m+2)N+1.
\end{aligned}
$$

THE BRAIDER'S NOTEBOOK

We mentioned in *The Braider*, Issue No. 8, pg. 161, that experimental braidwork should be carried out with **flat** string or **at least initially never with round string**. The reason being that it is difficult to see twists in round string. The braider should be fully aware of the fact that one full twist (two half twists) in the string is equivalent to a **cylindrical loop**, and that one half twist as well as three half twists are equivalent to a **Möbius loop** (see Fig. 304: upper row for cylindrical band and loops; lower two rows for Möbius band and loops).

⋆⋆ *To what are the following number of half twists in a string equivalent?*

(1). *4 half twist with a left-hand helix.*
(2). *4 half twist with a right-hand helix.*
(3). *5 half twist with a left-hand helix.*
(4). *5 half twist with a right-hand helix.*
(5). *6 half twist with a left-hand helix.*
(6). *6 half twist with a right-hand helix.*
(7). *7 half twist with a left-hand helix.*
(8). *7 half twist with a right-hand helix.*
(9). *8 half twist with a left-hand helix.*

(10). 8 *half twist with a right-hand helix.*
(11). 9 *half twist with a left-hand helix.*
(12). 9 *half twist with a right-hand helix.*

What is the general relationship?

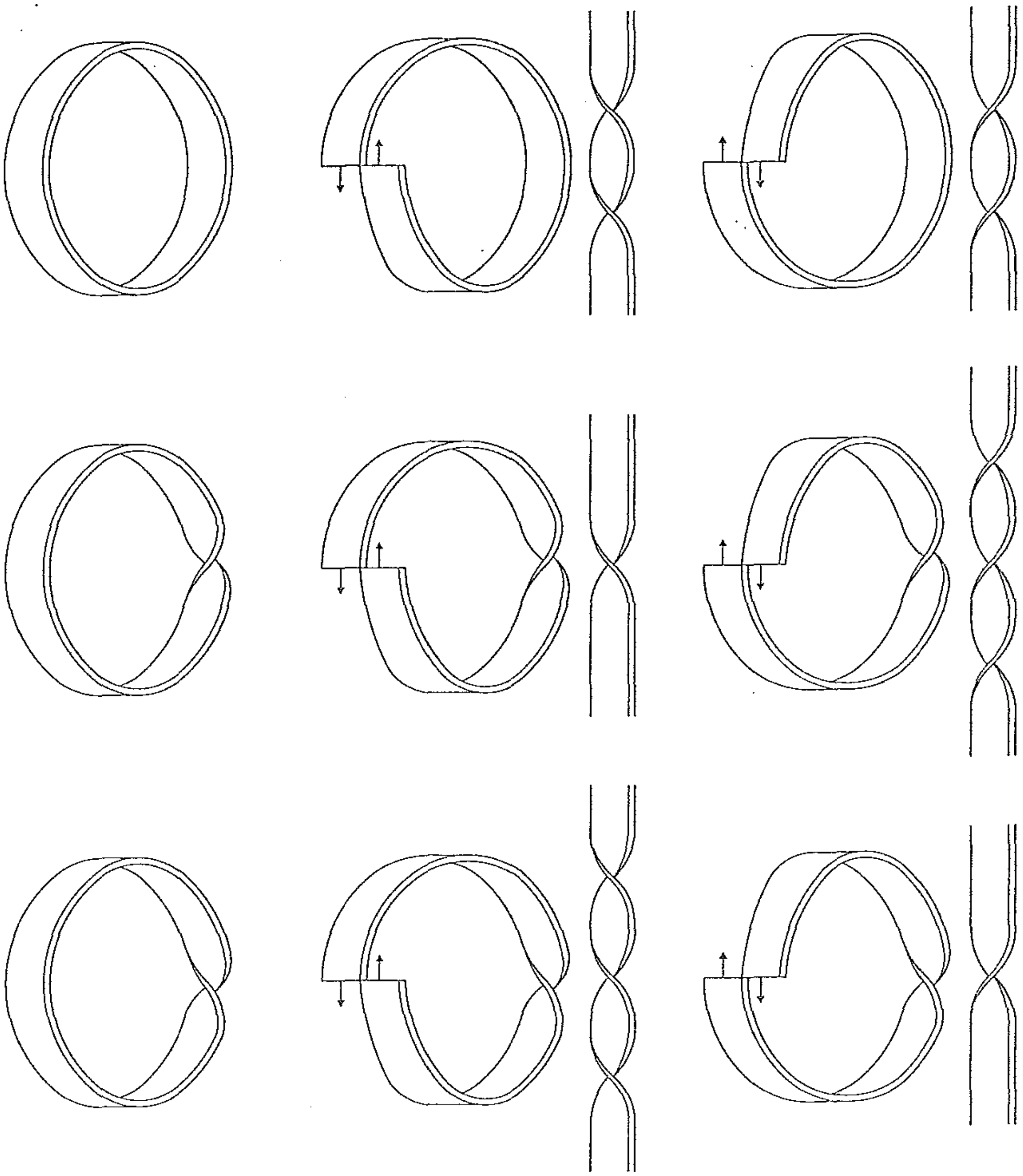


Fig. 304 — The equivalence between twists and loops.

The cylindrical band in Fig. 304 has an inner (cylindrical) surface and a separate outer (cylindrical) surface; furthermore it has a left end-surface and a separate right end-surface. In the Möbius bands of Fig. 304 the inner surface is also the outer surface; furthermore the left end-surface is also the right end-surface.

When we use round string, these surface properties are seemingly lost, but in reality the phenomenon is not lost. Draw for instance four lines along a length of round string: one north, one south, one east and one west. Then these compass-lines on the round string behave as the surfaces of the flat string. Consequently, in order to avoid erroneous conclusions, we use for any cross sectional shape a north-line, a south-line, an east-line and a west-line. The easiest way to see those lines is to represent them by separate surfaces, and flat string is the ideal braiding material to do this with.

For a cylindrical band the four compass-lines remain separated, while for a Möbius band the north and south-lines become one, and the east and west-lines become one.

When we give the "string" of a closed cylindrical band a half twist with a right-hand helix, then automatically we have also introduced a half twist with a left-hand helix, and vice versa.

The above very important fundamental properties of twists are often by many people overlooked, wich results in erroneous theories, conclusions and procedures.[†]

Note that we produce a half twist with a left-hand helix when we give the string half a turn in accordance with a right-hand screw thread, and that we produce a half twist with a right-hand helix when we give the string half a turn in accordance with a left-hand screw thread.

Let's first braid the braid depicted on the left-hand side of Fig. 305 (the reader should braid the indicated braids in order to fully understand what is going on). The easiest way to braid this Regular Cylindrical Braid is first to braid the multiple underhand knot with 17 crossings (a Column-coded Regular Knot with $p/b = 2/17$), then to continue with the following half-cycle algorithms:

(35). $L \longrightarrow R$: u.
(36). $R \longrightarrow L$: $u - o$.
(37). $L \longrightarrow R$: $o - u$.
(38). $R \longrightarrow L$: $u - o$.
(39). $L \longrightarrow R$: $o - u$.
(40). $R \longrightarrow L$: $u - o$.
(41). $L \longrightarrow R$: $o - u$.
(42). $R \longrightarrow L$: $u - o$.
(43). $L \longrightarrow R$: $o - u$.
(44). $R \longrightarrow L$: $2o$.
(45). $L \longrightarrow R$: $o - u$.
(46). $R \longrightarrow L$: $u - o$.
(47). $L \longrightarrow R$: $o - u$.
(48). $R \longrightarrow L$: $u - o$.
(49). $L \longrightarrow R$: $o - u$.
(50). $R \longrightarrow L$: $u - o$.
(51). $L \longrightarrow R$: $o - u$.
(52). $R \longrightarrow L$: $u - o$.
(53). $L \longrightarrow R$: $o - u$.
(54). $R \longrightarrow L$: $o - u - o$.
(55). $L \longrightarrow R$: $u - o - u$.
(56). $R \longrightarrow L$: $o - u - o$.
(57). $L \longrightarrow R$: $u - o - u$.
(58). $R \longrightarrow L$: $o - u - o$.
(59). $L \longrightarrow R$: $u - o - u$.
(60). $R \longrightarrow L$: $o - u - o$.
(61). $L \longrightarrow R$: $u - o - u$.
(62). $R \longrightarrow L$: $3o$.
(63). $L \longrightarrow R$: $u - o - u$.
(64). $R \longrightarrow L$: $o - u - o$.
(65). $L \longrightarrow R$: $u - o - u$.
(66). $R \longrightarrow L$: $o - u - o$.
(67). $L \longrightarrow R$: $u - o - u$.
(68). $R \longrightarrow L$: $o - u - o$.
(69). $L \longrightarrow R$: $u - o - u$.
(70). $R \longrightarrow L$: $o - u - o$.

[†] For example, in the topological knot theory any twists in the string are neglected. Hence in order for their theory to possess any real consistency, a Möbius band should for topologists be identical to a cylindrical band, but since a topologist makes a clear distinction between a Möbius band and a cylindrical band, their knot theory contains a serious inconsistency.

When we give this braided band half a turn in accordance with a left-hand helix, the string crossings between the lower two horizontal lines will fall away and the resulting slack can then be taken out by retightening the braid. Although the resulting band may look like a Möbius band, it is however still a cylindrical braid because the half turn which we gave the band generates a compensating half twist with a right-hand helix. This compensating half twist is by the braid transferred to its four parts by giving each part a half twist with a right-hand helix. Thus the four compass lines remain separated. Although it looks like that this braided band has only one bight-boundary with 66 bights, it has in fact two bight-boundaries with 35 bights along each boundary. It are the half twists in the string which confuse the issue since they not only divide the bights along two parallel boundaries but also represent additional bights and crossings.

In the topological knot theory this braided band will erroneously be seen as a Möbius band, hence we have here another case which clearly demonstrates that the topological knot theory is a mathematical fallacy.

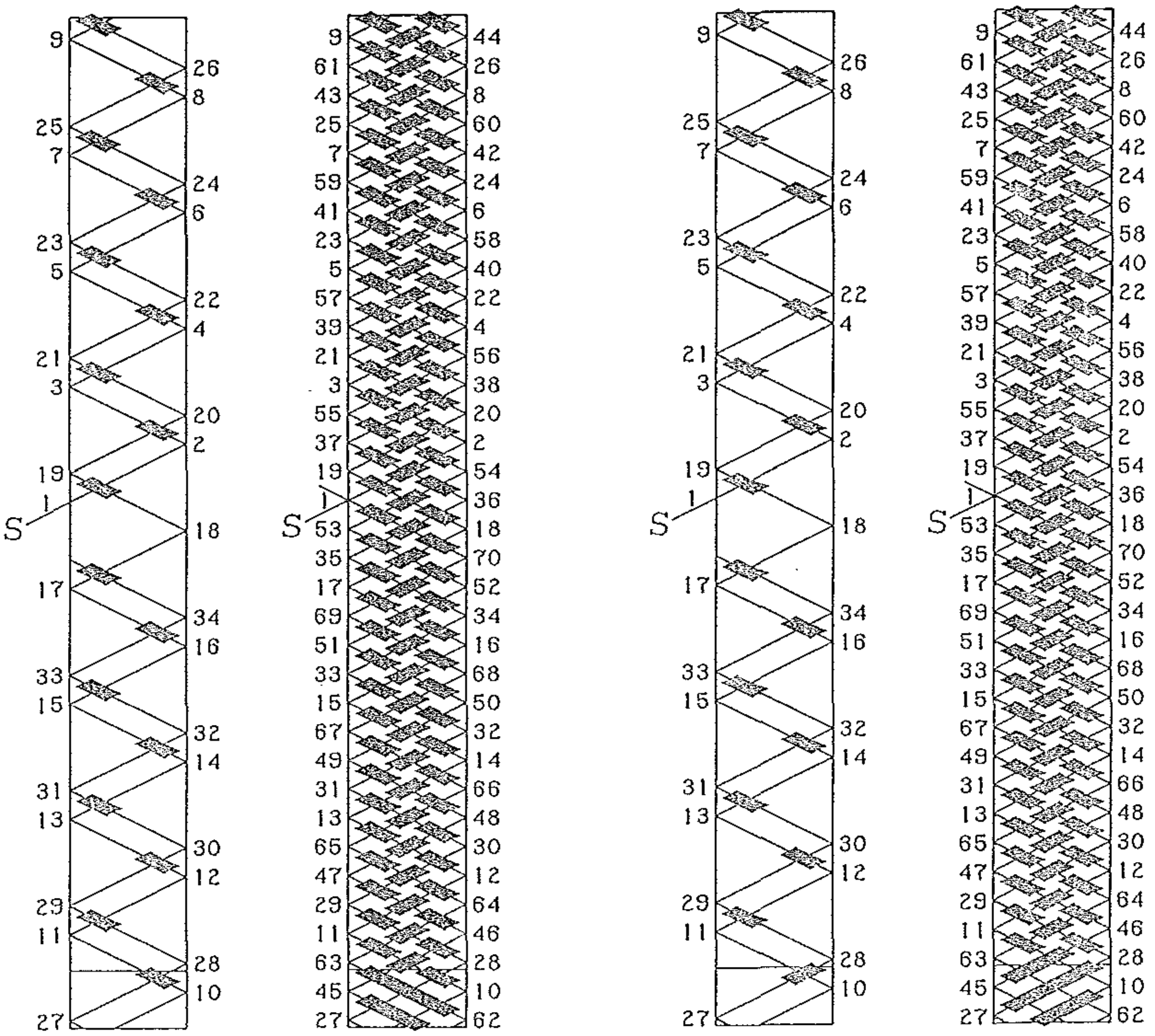


Fig. 305 — Regular Cylindrical braids with $p = 4$ and $b = 35$.

The braid on the right-hand side of Fig. 305 is similar but not identical. Furthermore, it is not quite as easy to braid as the previous one since the $p/b = 2/17$ Regular Cylindrical Braid is not column-coded (it is not a multiple underhand knot). We can give this braided band half a turn in accordance with a right-hand helix, the string crossings between the lower two horizontal lines will fall away and the resulting slack can again be taken out by retightening the braid. The resulting band looks here also like a Möbius band, but is in fact a cylindrical braid because the half turn which we gave the band generates a compensating half twist with a left-hand helix. This compensating half twist is by the braid transferred to its four parts by giving each part a half twist with a left-hand helix. Thus the four compass lines remain separated. The half twists in the string also here confuse the issue in a similar way as in the previous braid.

Let's braid the braid on the left-hand side of Fig. 305 again but during the braiding operation we give the half-cycles $9, 27, 44$ and 62 each a half twist with a left-hand helix. Finally we give the braided band half a turn in accordance with a left-hand helix. The string crossings between the lower two horizontal lines will again fall away and the resulting slack can then be taken out by retightening the braid. The resulting band is now a proper Möbius band, because the half turn which we gave the band generates a compensating half twist with a right-hand helix. This compensating half twist is by the braid transferred to its four parts by giving each part a half twist with a right-hand helix which cancels the half twist with the left-hand helix which we gave each part during the construction phase.

We can follow a similar process with the braid on the right-hand side of Fig. 305, by giving the half-cycles $9, 27, 44$ and 62 each a half twist with a right-hand helix. Finally we give the braided band half a turn in accordance with a right-hand helix. The string crossings between the lower two horizontal lines will again fall away and the resulting slack can then be taken out by retightening the braid. The resulting band is now a proper Möbius band, because the half turn which we gave the band generates a compensating half twist with a left-hand helix. This compensating half twist is by the braid transferred to its four parts by giving each part a half twist with a left-hand helix which cancels the half twist with the right-hand helix which we gave each part during the construction phase.

These examples should clearly show that in the final braid any twist in the string cannot be neglected!!!

Hence if we want to braid a Regular Möbius Braid via the braiding process for Regular Cylindrical Braids we must during the construction put the appropriate half twist in each of the p half-cycles which make up the Matthew Walker coded section that ultimately will fall away. Thus in this case we braid a Regular Möbius Braid as a virtual Regular Cylindrical Braid.

Let the Regular Möbius braid have p_m parts and b_m bights (since a regular Möbius Braid has only one bight-boundary, b_m is the total number of bights along this boundary). Its virtual Regular Cylindrical Braid has then $p = p_m$ parts and $b = \frac{p_m + b_m}{2}$ bights, and contains a Matthew Walker section with a total of $(2b - b_m) = p_m = p$ bights. From $b = \frac{p_m+b_m}{2}$ follows that $(p_m + b_m)$ must be divisible by 2, hence:

$$p_m = \text{odd} \quad \longleftrightarrow \quad b_m = \text{odd}.$$
$$p_m = \text{even} \quad \longleftrightarrow \quad b_m = \text{even}.$$

When the g.c.d. $(p_m, b_m) = 1$ (p_m and b_m both odd) $\longrightarrow$ Regular Möbius Knot (1 string).
When the g.c.d. $(p_m, b_m) = g$ (p_m and b_m both odd) $\longrightarrow$ Semi Regular Möbius Knot (g strings).
When the g.c.d. $(2p_m, [p_m + b_m]) = 2$ (p_m and b_m both even) $\longrightarrow$ Regular Möbius Knot (1 string).
When the g.c.d. $(2p_m, [p_m + b_m]) = 2g$ (p_m and b_m both even) $\longrightarrow$ Semi Regular Möbius Knot (g strings).

The enlargement processes associated with Regular Möbius Knots are similar to those associated with Regular knots. Their enlargement paths are depicted in the

Regular Möbius Knot Tree (RMKT), see Fig. 306.

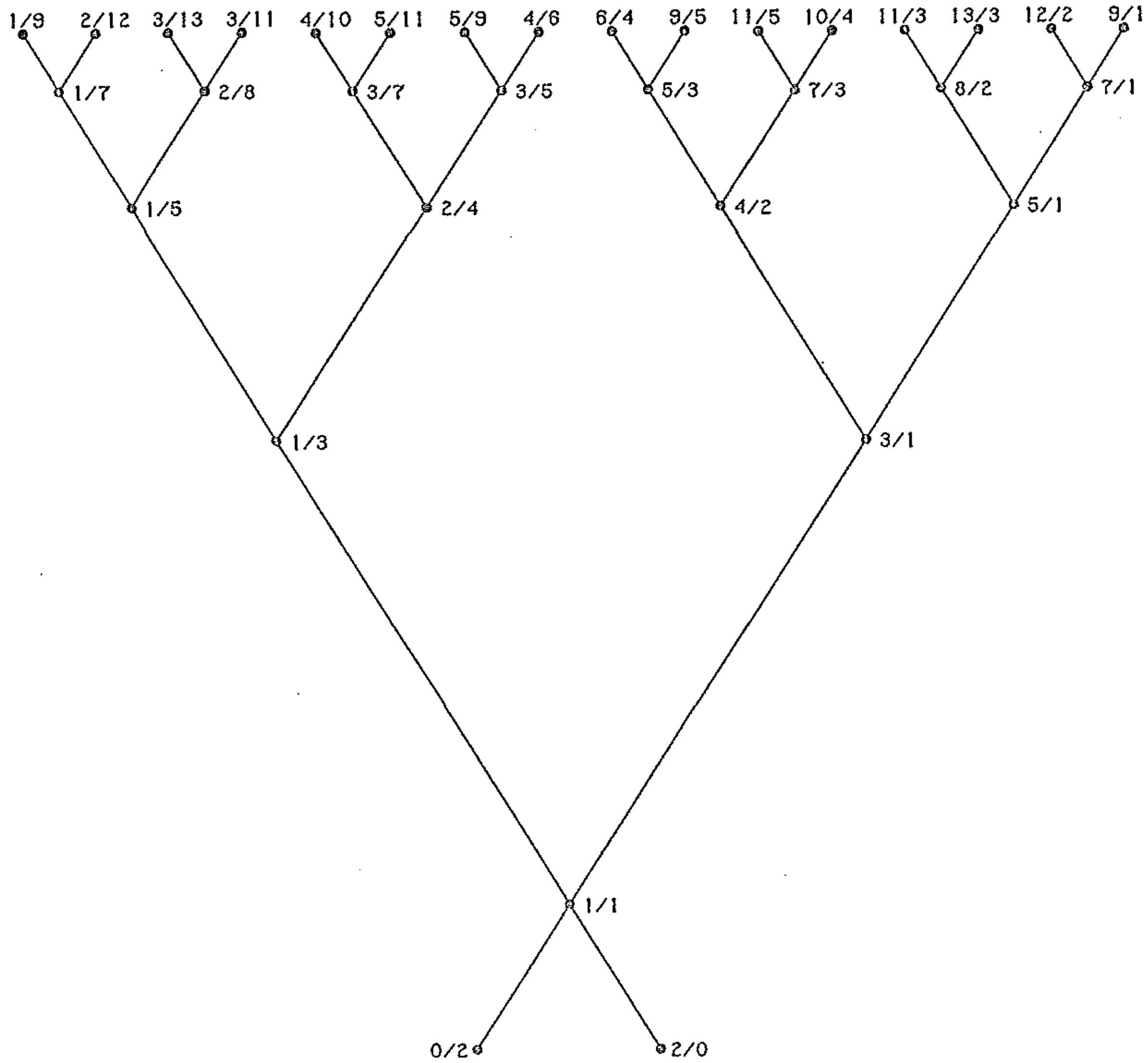


Fig. 306 — The Regular Möbius Knot Tree.

We may obtain the string-run diagram of a Regular Möbius Braid from the string-run diagram of its virtual Regular Cylindrical Braid by deleting the Matthew Walker section. Since p_m and b_m are both either odd or even, we have two types of string-run diagrams. The leftmost diagram in Fig. 307 depicts the type of string-run diagram (solid lines) for a Regular Möbius Braid with p_m and b_m both **odd**; the fourth diagram from the left depicts the type of string-run diagram (solid lines) for a Regular Möbius Braid with p_m and b_m both **even**. The uppermost solid horizontal line is the lowermost solid horizontal line after it has received a rotation of 180° about the vertical centre-line of the diagram as axis of rotation. The virtual Regular Cylindrical braid incorporates the dotted Matthew Walker section. This Matthew Walker Section has one of the two depicted codings.

A Regular Möbius Braid with an aesthetically acceptable weaving pattern requires a coding which has **Evert-Lateral** equivalency. In Fig. 308 two column-coded and two row-coded examples of such a coding are depicted. Besides the restriction that p_m and b_m must both have the same parity, row-coding demands the further condition that b_m must be a multiple of the number of rows in a coding-block. Hence for the leftmost bottom coding in Fig. 308 b_m must be a multiple of 4, and for the rightmost bottom coding in Fig. 308 b_m must be a multiple of 5.

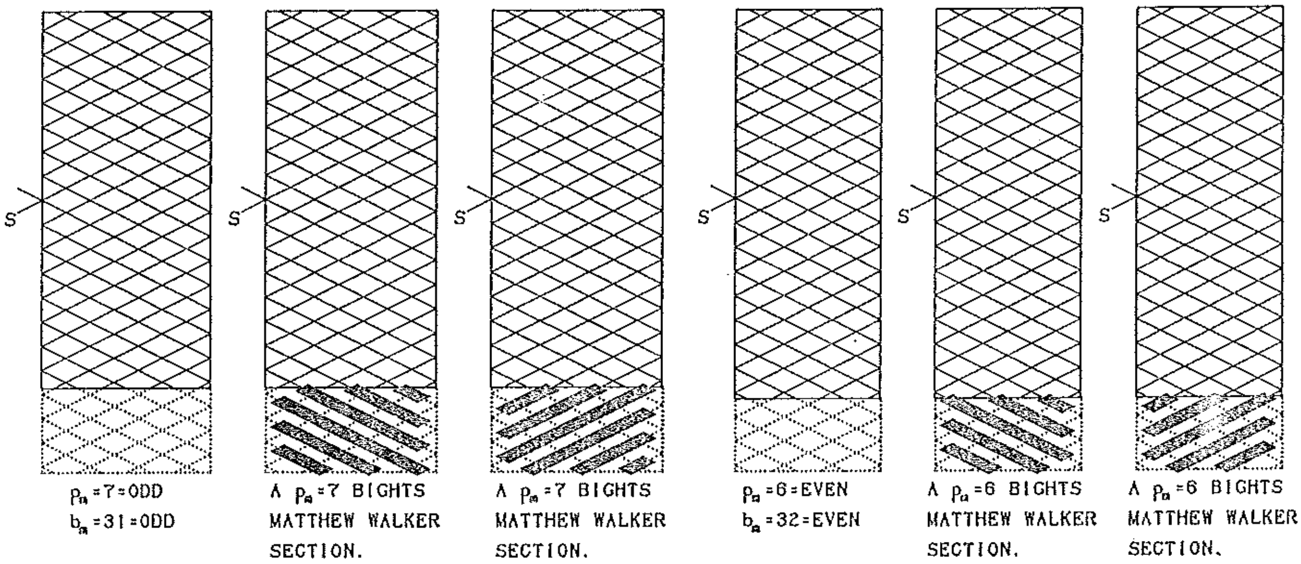
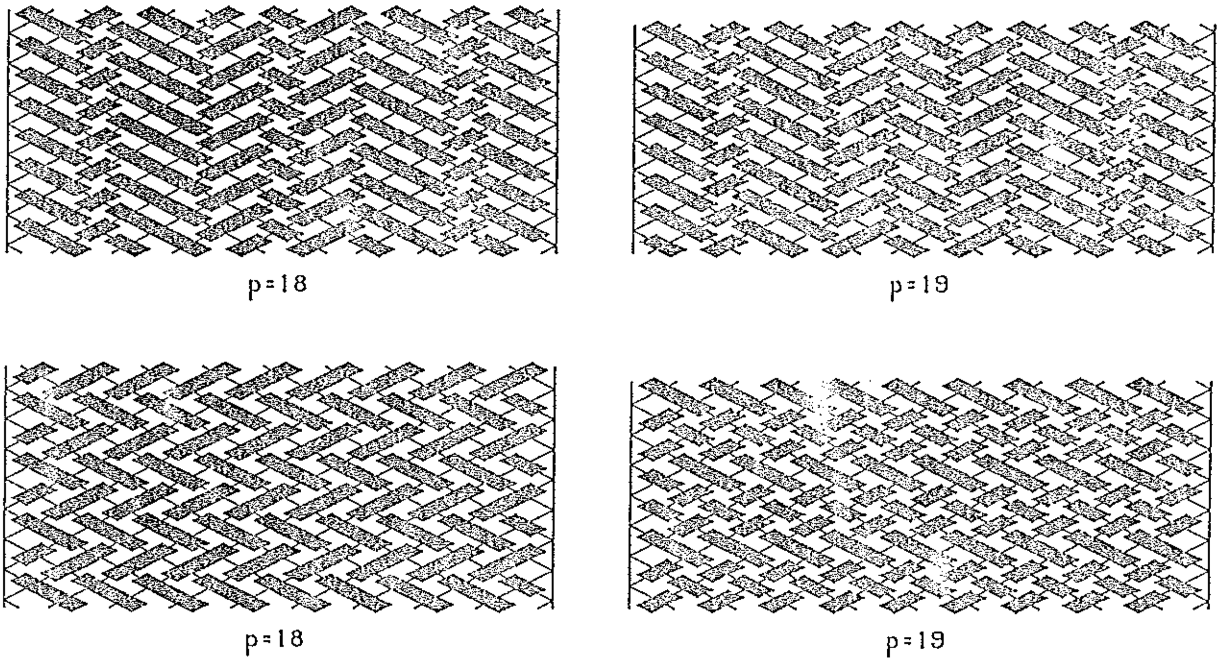


Fig. 307 — The string-run diagrams for p_m and b_m both odd, and both even.


p=18

p=19

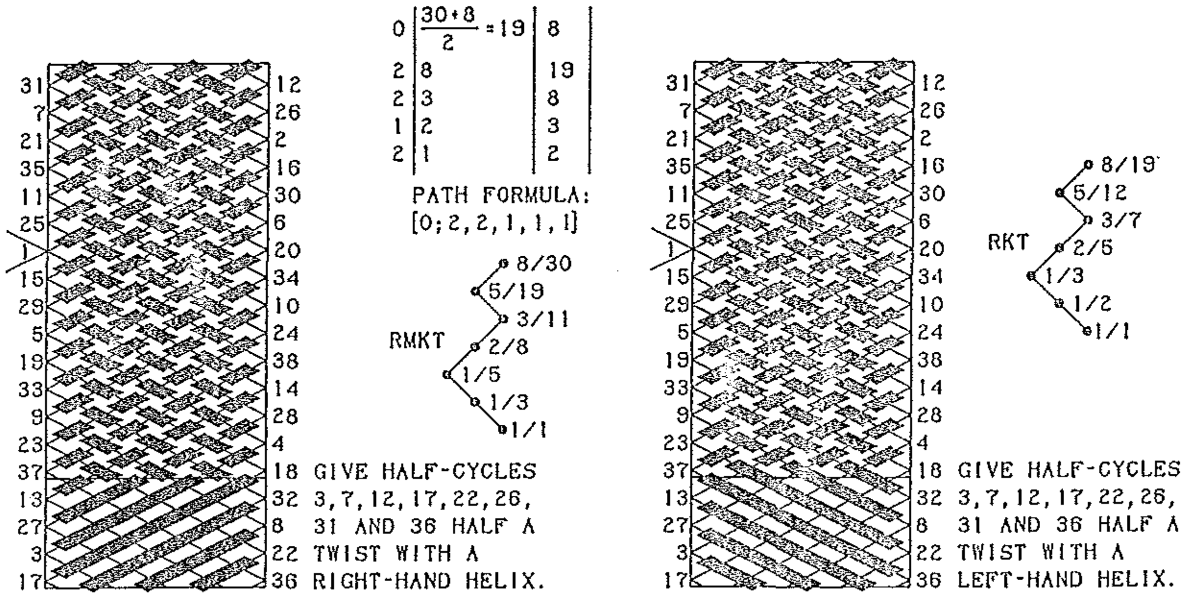
p=18

p=19

Fig. 308 — Examples of Evert-Lateral coding equivalency.

It will thus be evident that a Regular Möbius Braid cannot have a Gaucho-coding, nor can it have a Casa-coding when p_m is odd. It can, however, have a Headhunter's-coding, or when p_m is even a Casa-coding.

Fig. 309 shows the grid-diagrams with their associated instructions of the virtual Regular Cylindrical Braids which represent Regular Möbius Knots with $p_m = 8$ parts and $b_m = 30$ bights. After braiding the left-hand virtual Regular Cylindrical Braid, the band should be given a half twist with a right-hand helix, while the right-hand virtual Regular Cylindrical Braid should be given after braiding a half twist with a left-hand helix. Each braid can then be tightened to a correctly finished Regular Möbius Knot.



0	$\frac{30+8}{2}=19$	8
2	8	19
2	3	8
1	2	3
2	1	2

PATH FORMULA:
[0; 2, 2, 1, 1, 1]

18, 32, 8, 22, 36 GIVE HALF-CYCLES 3, 7, 12, 17, 22, 26, 31 AND 36 HALF A TWIST WITH A RIGHT-HAND HELIX.

18, 32, 8, 22, 36 GIVE HALF-CYCLES 3, 7, 12, 17, 22, 26, 31 AND 36 HALF A TWIST WITH A LEFT-HAND HELIX.

1) FREE RUN.
2) FREE RUN.
3) FREE RUN.
4) FREE RUN.
5) FREE RUN.
6) u.
7) o.
8) u.
9) o.
10) u-o.
11) o-u.
12) 2u.
13) o-u.
14) u-o.
15) o-u.
16) 2u-o.
17) 3o.
18) 2u-o.
19) 2o-u.
20) 2u-2o.
21) 2o-u-o.
22) 3u-o.
23) 2o-2u.
24) 2u-2o-u.
25) 2o-2u-o.
26) 5u.
27) 2o-2u-o.
28) 2u-2o-u.
29) 2o-2u-o.
30) u-o-u-2o-u.
31) 6o.
32) u-o-u-2o-u.
33) o-u-o-2u-o.
34) u-o-u-o-u-o-u.
35) o-u-o-u-o-u-o.
36) 7u.
37) o-u-o-u-o-u-o.
38) u-o-u-o-u-o-u.

1) FREE RUN.
2) FREE RUN.
3) FREE RUN.
4) FREE RUN.
5) FREE RUN.
6) u.
7) o.
8) o.
9) o.
10) u-o.
11) o-u.
12) 2o.
13) o-u.
14) u-o.
15) o-u.
16) 2u-o.
17) 3u.
18) 2u-o.
19) 2o-u.
20) 2u-2o.
21) 2o-2u.
22) 4o.
23) 2o-2u.
24) 2u-2o-u.
25) 2o-2u-o.
26) 2u-3o.
27) 4u-o.
28) 2u-2o-u.
29) 2o-2u-o.
30) u-o-u-2o-u.
31) o-5u.
32) 2o-u-2o-u.
33) o-u-o-2u-o.
34) u-o-u-o-u-o-u.
35) o-u-o-u-o-u-o.
36) 7o.
37) o-u-o-u-o-u-o.
38) u-o-u-o-u-o-u.

Fig. 309 — Casa-coded $p_m/b_m = 8/30$ Regular Möbius Knots.

Note that the procedure at the left in Fig. 309 leads through a virtual multi overhand knot (it has two half twists with a right-hand helix), whereas the procedure at the right in Fig. 309 does not lead through a virtual multi overhand knot.

Although we can braid any Regular Möbius Knot by means of a virtual Regular Cylindrical Knot, a procedure which has theoretical advantages with respect to its path in the RMKT, it is from the practical point of view not a good method. The reason being threefold:

(1). The half twists in the Matthew Walker section makes the braiding process more cumbersome, because we have to pay careful attention that they do not accidentally disappear.

(2). The Matthew Walker coding of the crossings in the Matthew Walker section makes the braiding process, through the half-cycles affected, more cumbersome.

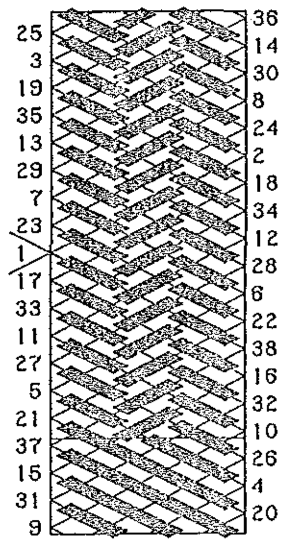
(3). At the end of the braiding process much slack will have to be taken out.

It will thus be obvious that, if possible, a much more practical braiding process should be used. A process which is direct rather than indirect through some virtual braid-form. Fortunately such a process does exist, and what is more, the procedures involved fit in beautifully with our braiding procedures for Regular Cylindrical Knots and Braids. This is of course not surprising since there is clearly some close relationship between Regular Möbius Braids and Regular Cylindrical Braids, a relationship well demonstrated by their respective evolution trees.

Let's take a closer look at the grid-diagram of a Regular Möbius Knot, and let's first look at the case where both p_m and b_m are **odd**.

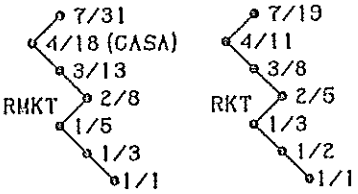
We shall start with the grid-diagram of the virtual Regular Cylindrical Knot having $p = p_m = 7$ and $b = \frac{p_m + b_m}{2} = 19$, which represents a 2-pass Headhunter's-coded Regular Möbius Knot with $p_m = 7$ and $b_m = 31$. This diagram with its braiding instructions is presented in Fig. 310.

1) FREE RUN.
2) FREE RUN.
3) FREE RUN.
4) FREE RUN.
5) FREE RUN.
6) o.
7) u.
8) o.
9) u.
10) o.
11) u.
12) u-o.
13) o-u.
14) 2o.
15) o-u.
16) u-o.
17) o-u.
18) o-u-o.
19) u-o-u.
20) 3o.
21) u-o-u.
22) o-u-2o.
23) u-o-2u.
24) o-u-2o.
25) 4u.
26) o-u-2o.
27) u-o-2u.
28) o-2u-2o.
29) u-2o-2u.
30) o-2u-2o.
31) 5u.
32) o-2u-2o.
33) u-2o-2u.
34) 2o-2u-2o.
35) 2u-2o-2u.
36) 6o.
37) 2u-2o-2u.
38) 2o-2u-2o.



0	$\frac{31+7}{2} = 19$	7
2	7	19
1	5	7
2	2	5
2	1	2

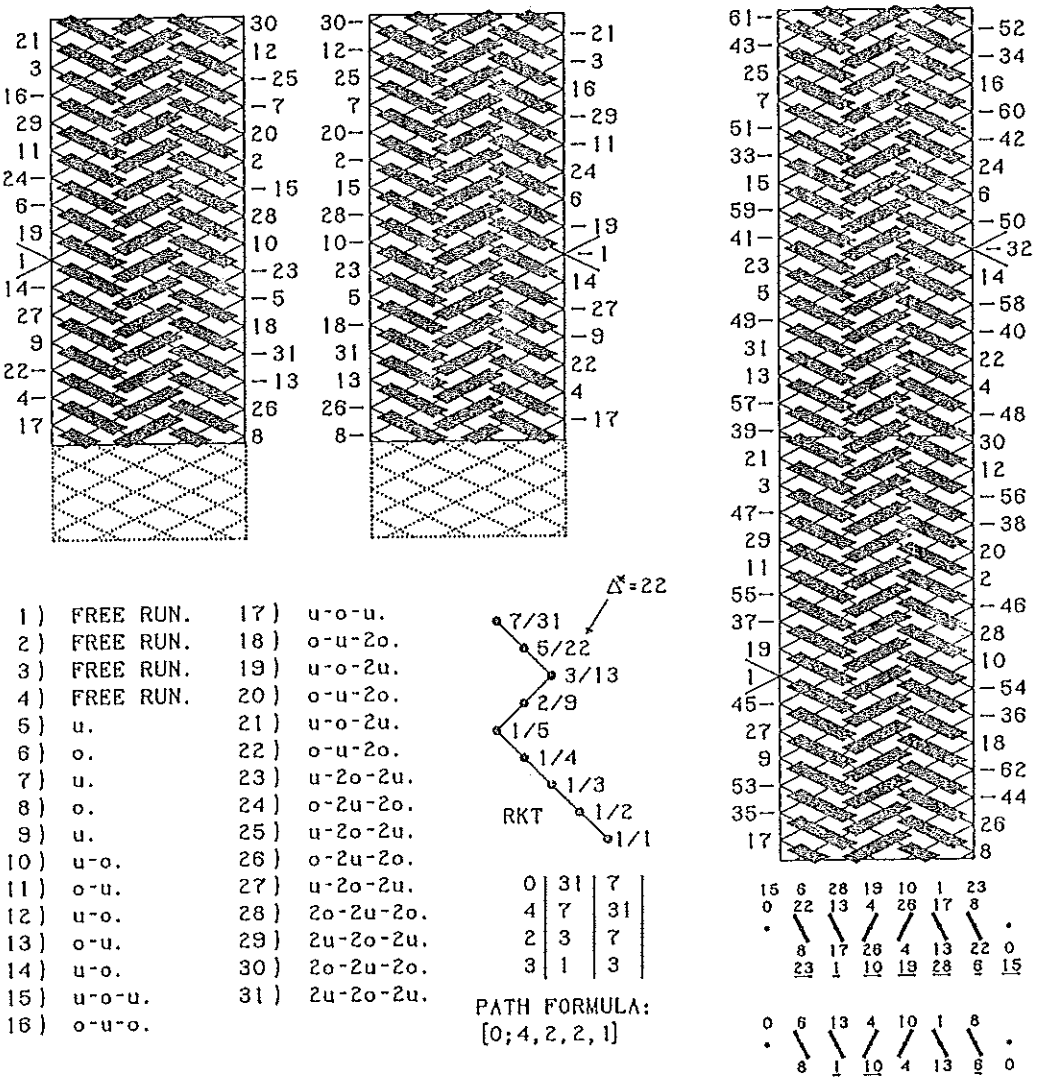
PATH FORMULA:
[0; 2, 1, 2, 1, 1]



GIVE HALF-CYCLES
4, 9, 14, 20, 25,
31 AND 36 HALF A
TWIST WITH A
LEFT-HAND HELIX.

Fig. 310 — The virtual Regular Knot $p/b = 7/19$ which represents a Regular Möbius Knot $p_m/b_m = 7/31$.

Let's redraw this grid-diagram with the string-run only (dotted lines) in the Matthew Walker section; see Fig. 311. Take the left-hand diagram and follow half-cycle # 1, then follow half-cycle # 2, then half-cycle # 3 till we reach the upper horizontal solid line. We have now to rotate this line through 180° as described on page 356, and continue, from the bottom horizontal solid line, with this half-cycle till we reach the bight-boundary. But in reality the braid **gradually** rotates through the 180°, and hence half-cycle 3 continues to the apparent bight-boundary on the right. In order to show this clearly in a grid-diagram, we rotate our left-hand grid-diagram through the 180° and obtain the second grid-diagram from the left. Thus half-cycle # 3 runs from the apparent left-hand bight-boundary of the leftmost grid-diagram to the upper horizontal solid line and continues from the bottom horizontal solid line of the second grid-diagram from the left to the apparent right-hand bight-boundary of this grid-diagram. Next in this second grid-diagram from the left, half-cycle # 4 runs from the apparent right-hand bight-boundary to the apparent left-hand bight-boundary. Then follows half-cycle # 5 in this diagram, next half-cycle # 6, then half-cycle # 7 till the upper horizontal line (all in the second grid-diagram from the left). This half-cycle continues from the bottom horizontal solid line of the leftmost grid-diagram to the apparent right-hand bight-boundary of this grid-diagram. Then follows half-cycle #8 which runs from the apparent right-hand bight-boundary (leftmost grid-diagram) to the apparent left-hand bight-boundary of this grid-diagram. And so on.



1) FREE RUN.
2) FREE RUN.
3) FREE RUN.
4) FREE RUN.
5) u.
6) o.
7) u.
8) o.
9) u.
10) u-o.
11) o-u.
12) u-o.
13) o-u.
14) u-o.
15) u-o-u.
16) o-u-o.
17) u-o-u.
18) o-u-2o.
19) u-o-2u.
20) o-u-2o.
21) u-o-2u.
22) o-u-2o.
23) u-2o-2u.
24) o-2u-2o.
25) u-2o-2u.
26) o-2u-2o.
27) u-2o-2u.
28) 2o-2u-2o.
29) 2u-2o-2u.
30) 2o-2u-2o.
31) 2u-2o-2u.

0	31	7
4	7	31
2	3	7
3	1	3

PATH FORMULA:
[0;4,2,2,1]

Fig. 311 — The Regular Möbius Knot $p_m/b_m = 7/31$ from Fig. 310.

It is as if two strings are being laid down simultaneously; one string lays down the half-cycles

#1 ; #2 ; #3 ; #4 ; #5 ; #6 ; #7 ; etc.

and the other string lays down simultaneously the half-cycles

# −1 ; #2− ; # −3 ; #4− ; # −5 ; #6− ; # −7 ; etc.

However the string which lays down the half-cycles #1 ; #2 ; etc. is **real**, but the string which lays down the half-cycles # −1 ; #2− ; etc. is **imaginary** since these half-cycles are in fact the half-cycles of the real string.

It will now be obvious that the string-run in the Regular Möbius Braid can much easier be followed in a grid-diagram which consists of the second grid-diagram from the left stacked on top of the leftmost grid-diagram. Such a diagram is depicted on the right-hand side of Fig. 311. It is the grid-diagram of a Regular Cylindrical Braid with $p/b = p_m/b_m$ in which two strings are being laid down simultaneously. One string starts with half-cycle #1 and the other string starts simultaneously with half-cycle

$\# -1 = 32$. The half-cycles $\#2$ and $\#2- = 33$ are being laid down simultaneously, as are the half-cycles $\#3$ and $\# -3 = 34$; $\#4$ and $\#4- = 35$; $\#5$ and $\# -5 = 36$; etc. The string which lays down the half-cycles $\#1$; $\#2$; etc. is **real**, but the string which lays down the half-cycles $\#32$; $\#33$; etc. is **imaginary**, nevertheless in the Regular Möbius Braid the half-cycles of the **real string-run** intersect in actual fact the **imaginary string-run**. In order to handle this situation in a simple way, we do **as if** the imaginary string-run starts at the left-hand bight-boundary with half-cycle $\#31^*$. This is thus an imaginary half-cycle of higher order, and consequently the *real string-run* does **not** intersect this particular half-cycle.

With Euclid's algorithm we can readily calculate the path formula for this $p/b = p_m/b_m = 7/31$ Regular Cylindrical Braid (a Regular Knot since g.c.d.$(7,31) = 1$), and hence we can readily find its associated Δ^*-value, which in this case is equal to 22. With this Δ^*-value we can construct the algorithm-diagram for the *real* string-run in the usual manner with its complementary cyclic bight-number scheme which has the i-value sequence $0, 22, 13, 4, 26, 17, 8, 30, 21, 12, 3, 25, 16, 7, 29, 20, 11, 2, 24, 15, 6, 28, 19, 10, 1, 23, 14, 5, 27, 18, 9$.
This gives in the algorithm diagram the i-value sequence $0, 22, 13, 4, 26, 17, 8$.

With this Δ^*-value, the complementary cyclic bight-number scheme associated with the *imaginary* string-run, including the imaginary half-cycle of higher order, has from the Standing End bight-point of the *real* string-run the i-value sequence $\frac{b_m-1}{2} = 15, 6, 28, 19, 10, 1, 23, 14, 5, 27, 18, 9, 0, 22, 13, 4, 26, 17, 8, 30, 21, 12, 3, 25, 16, 7, 29, 20, 11, 2, 24$.
This gives in the algorithm diagram the i-value sequence $15, 6, 28, 19, 10, 1, 23$.

Both these i-value sequences are set off in the algorithm diagram for the Möbius braid, but since a real half-cycle cannot intersect the imaginary half-cycle of higher order, we underline, for the half-cycles which run from lower right to upper left, the i-values associated with the imaginary string-run.

It is as if we lay down 1 imaginary half-cycle of higher order, $b = b_m$ real half-cycles, and $b = b_m$ imaginary half-cycles, hence a total of $(2b+1) = (2b_m+1)$ half-cycles. Consequently since $(2b+1) = (2b_m+1) = \text{odd}$, the maximum i-value in the i-value sequence is $\frac{(2b_m+1)-3}{2} = 15$. Thus the actual applicable complementary cyclic bight-number scheme has the i-value sequence $0, \underline{6}, 13, 4, \underline{10}, \underline{1}, 8, \underline{14}, \underline{5}, 12, 3, \underline{9}, \underline{0}, 7, \underline{13}, \underline{4}, 11, 2, \underline{8}, 15, 6, \underline{12}, \underline{3}, 10, 1, \underline{7}, 14, 5, \underline{11}, \underline{2}, 9$. The underlining of the values concerned is deleted for the half-cycles from lower left to upper right. Note that each value occurs twice, exept $\frac{(2b_m+1)-3}{2} = 15$ of course.

Thus the algorithm diagram for our case with $p = p_m = 7$ has the i-value sequence $0, \underline{6}, 13, 4, \underline{10}, \underline{1}, 8$, and after entering the coding for the intersection-columns, we can read off the half-cycle algorithms for the real string-run.

To braid the Möbius braid in accordance with the half-cycle algorithms obtained, we put in the first revolution (the circumference of the cylinder) of the string half a twist and then follow the same apparent surface of the string. This will ensure that we automatically obtain the required half twists in the string with as end result a correctly braided Regular Möbius Knot.

In order to get a good understanding of the process which determines the half-cycle algorithms, we shall give a few examples involving the algorithm diagram for the 2-pass Headhunter's-coded Regular Möbius Knot with $p_m/b_m = 7/31$, depicted in Fig. 311.

Half-cycle 1 :

Half-cycle 1 is always a free run. Half-cycle 1 runs from lower left to upper right.

Half-cycle 2 :

Half-cycle 2 is associated with bight-number $i = \frac{2-2}{2} = 0$. Hence from the algorithm diagram we have to read off, for half-cycle 2, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are equal to $i = 0$. Half-cycle 2 runs from lower right to upper left, hence in the algorithm diagram we read the lower line from right to left. This gives us: no crossings, hence: free run.

Half-cycle 3 :

Half-cycle 3 is associated with bight-number $i = \frac{3-3}{2} = 0$. Hence from the algorithm diagram we have to read off, for half-cycle 3, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are equal to $i = 0$. Half-cycle 3 runs from lower left to upper right, hence in the algorithm diagram we read the upper line from left to right. This gives us: no crossings, hence: free run.

Half-cycle 4 :

Half-cycle 4 is associated with bight-number $i = \frac{4-2}{2} = 1$. Hence from the algorithm diagram we have to read off, for half-cycle 4, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 1$. Half-cycle 4 runs from lower right to upper left, hence in the algorithm diagram we read the lower line from right to left. However, $i = 1$ is underlined, hence should be neglected for this half-cycle only (since it is a crossing of half-cycle 4 with the imaginary half-cycle of higher order). This gives us: no crossings, hence: free run.

Half-cycle 5 :

Half-cycle 5 is associated with bight-number $i = \frac{5-3}{2} = 1$. Hence from the algorithm diagram we have to read off, for half-cycle 5, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 1$. Half-cycle 5 runs from lower left to upper right, hence in the algorithm diagram we read the upper line from left to right. This gives us: u.

Half-cycle 6 :

Half-cycle 6 is associated with bight-number $i = \frac{6-2}{2} = 2$. Hence from the algorithm diagram we have to read off, for half-cycle 6, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 2$. Half-cycle 6 runs from lower right to upper left, hence in the algorithm diagram we read the lower line from right to left. This gives us: o.

Half-cycle 7 :

Half-cycle 7 is associated with bight-number $i = \frac{7-3}{2} = 2$. Hence from the algorithm diagram we have to read off, for half-cycle 7, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 2$. Half-cycle 7 runs from lower left to upper right, hence in the algorithm diagram we read the upper line from left to right. This gives us: u.

Half-cycle 8 :

Half-cycle 8 is associated with bight-number $i = \frac{8-2}{2} = 3$. Hence from the algorithm diagram we have to read off, for half-cycle 8, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 3$. Half-cycle 8 runs from lower right to upper left, hence in the algorithm diagram we read the lower line from right to left. This gives us: o.

Half-cycle 9:

Half-cycle 9 is associated with bight-number $i = \frac{9-3}{2} = 3$. Hence from the algorithm diagram we have to read off, for half-cycle 9, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 3$. Half-cycle 9 runs from lower left to upper right, hence in the algorithm diagram we read the upper line from left to right. This gives us: u.

Half-cycle 10:

Half-cycle 10 is associated with bight-number $i = \frac{10-2}{2} = 4$. Hence from the algorithm diagram we have to read off, for half-cycle 10, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 4$. Half-cycle 10 runs from lower right to upper left, hence in the algorithm diagram we read the lower line from right to left. This gives us: $u - o$.

Half-cycle 11:

Half-cycle 11 is associated with bight-number $i = \frac{11-3}{2} = 4$. Hence from the algorithm diagram we have to read off, for half-cycle 11, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 4$. Half-cycle 11 runs from lower left to upper right, hence in the algorithm diagram we read the upper line from left to right. This gives us: $o - u$.

Half-cycle 12:

Half-cycle 12 is associated with bight-number $i = \frac{12-2}{2} = 5$. Hence from the algorithm diagram we have to read off, for half-cycle 12, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 5$. Half-cycle 12 runs from lower right to upper left, hence in the algorithm diagram we read the lower line from right to left. This gives us: $u - o$.

Half-cycle 13:

Half-cycle 13 is associated with bight-number $i = \frac{13-3}{2} = 5$. Hence from the algorithm diagram we have to read off, for half-cycle 13, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 5$. Half-cycle 13 runs from lower left to upper right, hence in the algorithm diagram we read the upper line from left to right. This gives us: $o - u$.

Half-cycle 14:

Half-cycle 14 is associated with bight-number $i = \frac{14-2}{2} = 6$. Hence from the algorithm diagram we have to read off, for half-cycle 14, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 6$. Half-cycle 14 runs from lower right to upper left, hence in the algorithm diagram we read the lower line from right to left. However, $i = 6$ is underlined, hence should be neglected for this half-cycle only (since it is a crossing of half-cycle 14 with the imaginary half-cycle of higher order). This gives us: $u - o$.

Half-cycle 15:

Half-cycle 15 is associated with bight-number $i = \frac{15-3}{2} = 6$. Hence from the algorithm diagram we have to read off, for half-cycle 15, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 6$. Half-cycle 15 runs from lower left to upper right, hence in the algorithm diagram we read the upper line from left to right. This gives us: $u - o - u$.

Half-cycle 16:

Half-cycle 16 is associated with bight-number $i = \frac{16-2}{2} = 7$. Hence from the algorithm diagram we have to read off, for half-cycle 16, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 7$. Half-cycle 16 runs from lower right to upper left, hence in the algorithm diagram we read the lower line from right to left. This gives us: $o - u - o$.

Half-cycle 17:

Half-cycle 17 is associated with bight-number $i = \frac{17-3}{2} = 7$. Hence from the algorithm diagram we have to read off, for half-cycle 17, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 7$. Half-cycle 17 runs from lower left to upper right, hence in the algorithm diagram we read the upper line from left to right. This gives us: $u - o - u$.

Half-cycle 18:

Half-cycle 18 is associated with bight-number $i = \frac{18-2}{2} = 8$. Hence from the algorithm diagram we have to read off, for half-cycle 18, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 8$. Half-cycle 18 runs from lower right to upper left, hence in the algorithm diagram we read the lower line from right to left. This gives us: $o - u - 2o$.

Half-cycle 19:

Half-cycle 19 is associated with bight-number $i = \frac{19-3}{2} = 8$. Hence from the algorithm diagram we have to read off, for half-cycle 19, the consecutive crossing-movements in accordance with the coding for the bight-numbers which are less than or equal to $i = 8$. Half-cycle 19 runs from lower left to upper right, hence in the algorithm diagram we read the upper line from left to right. This gives us: $u - o - 2u$.

And so on.

★★ *Prove that for* p_m *and* b_m **odd** *with g.c.d.* $(p_m, b_m) = 1$, *the bight-number* i *associated with the half-cycle (of the imaginary string-run) which coincides with the Standing End half-cycle of the real string-run has the value* $\frac{b_m-1}{2}$.

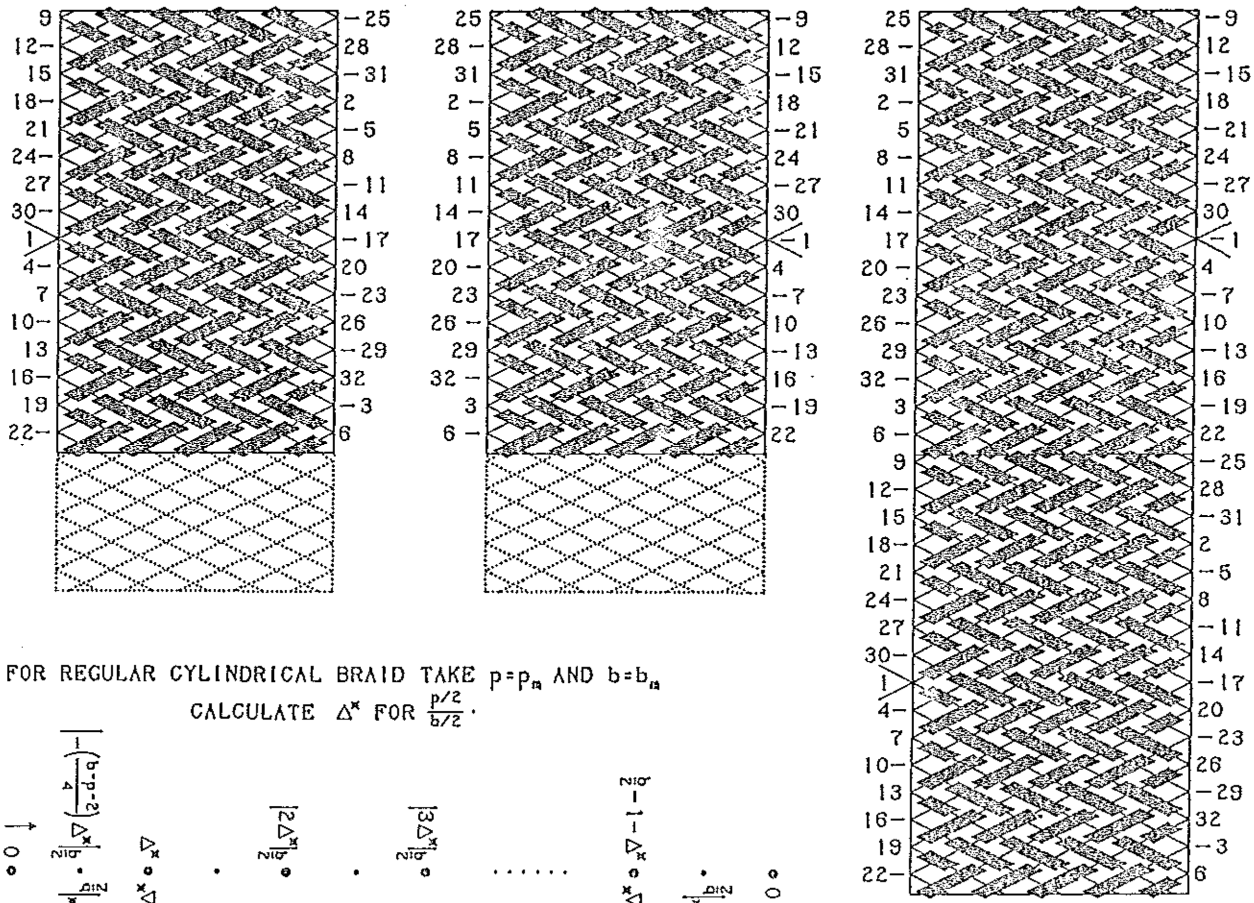
An arbitrary intersection-column in the algorithm diagram carries $i = a^*$ *for the half-cycles, associated with the imaginary string-run, which run from left to right, and carries* $i = b^*$ *for the half-cycles, associated with the imaginary string-run, which run from right to left:*

$$\begin{array}{cccccccccccccc}
\frac{b_m-1}{2} & . & . & . & . & \dots & . & a^* & . & \dots & . & . & . & \\
. & . & . & . & . & \dots & . & . & . & \dots & . & . & . & . \\
 & . & . & . & . & \dots & . & b^* & . & \dots & . & . & . & \frac{b_m-1}{2}
\end{array}$$

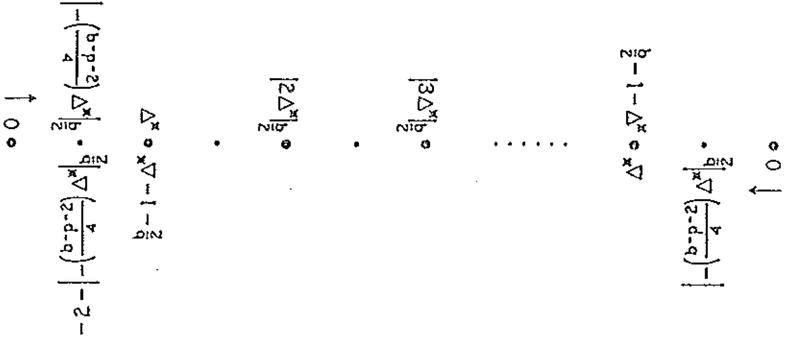
Prove that $a^* + b^* = b_m - 2$.

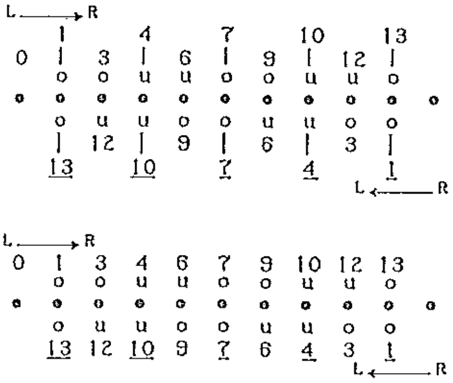
For Regular Möbius Knots with p_m and b_m **even**, and g.c.d. $(2p_m, [p_m + b_m]) = 2$, the procedure is in essence similar, but the Regular Cylindrical Braid with $p = p_m$ and $b = b_m$ is however a Semi Regular Knot which requires two strings in its construction since the g.c.d. $(p, b) = 2$.

An example of such a Regular Möbius Knot is shown in Fig. 312, depicting the 2-pass Herringbone-coded Regular Möbius Knot with $p_m/b_m = 10/32$.



FOR REGULAR CYLINDRICAL BRAID TAKE $p = p_m$ AND $b = b_m$
CALCULATE Δ^* FOR $\frac{p/2}{b/2}$.

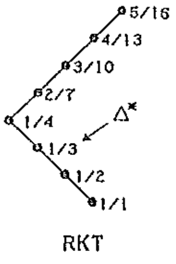




1) FREE RUN.
2) FREE RUN.
3) FREE RUN.
4) FREE RUN.
5) o.
6) o.
7) o.
8) 2o.
9) 2o.
10) 2o.
11) 2o-u.
12) 2o-u.
13) 2o-u.
14) 2o-2u.
15) 2o-2u.
16) 2o-2u.
17) 2o-2u-o.
18) 2o-2u-o.
19) 2o-2u-o.
20) 2o-2u-2o.
21) 2o-2u-2o.
22) 2o-2u-2o.
23) 2o-2u-2o-u.
24) 2o-2u-2o-u.
25) 2o-2u-2o-u.
26) 2o-2u-2o-2u.
27) 2o-2u-2o-2u.
28) 2o-2u-2o-2u.
29) 2o-2u-2o-2u-o.
30) 2o-2u-2o-2u-o.
31) 2o-2u-2o-2u-o.
32) 2o-2u-2o-2u-o.

0	16	5
3	5	16
5	1	5

PATH FORMULA:
[0; 3, 4, 1]



RKT

Fig. 312 — A 2-pass Herringbone-coded Regular Möbius Knot with $p_m/b_m = 10/32$.

The *real* string-run and the *imaginary* string-run belong each to a Regular Knot with $p^*/b^* = \frac{p_m}{2}/\frac{b_m}{2} = 5/16$.

With Euclid's algorithm we again calculate the path formula for this $p^*/b^* = \frac{p_m}{2}/\frac{b_m}{2} = 5/16$ Regular Knot, and hence its associated Δ^*-value, which in this case is equal to 3, is readily found.

The imaginary string-run does not go through the bight-point of the Standing End of the real string-run, but goes through the bight-point immediately above it. The half-cycle through this bight-point is associated with bight-number:

$$i = \left| -\left(\frac{b_m - p_m - 2}{4}\right)\Delta^* \right|_{\frac{b_m}{2}} = \left| -\left(\frac{32 - 10 - 2}{4}\right)3 \right|_{16} = 1\,.$$

With the Δ^*-value for the Regular Knot with $\frac{p_m}{2}/\frac{b_m}{2} = 5/16$ we can construct the algorithm diagram for the *real* string-run in the usual manner, and obtain the i-value sequence $0, \cdot\,, 3, \cdot\,, 6, \cdot\,, 9, \cdot\,, 12, \cdot\,.$

For the *imaginary* string-run, including the imaginary half-cycle of higher order, we obtain the i-value sequence $\cdot\,, \left| -\left(\frac{b_m - p_m - 2}{4}\right)\Delta^* \right|_{\frac{b_m}{2}} = \underline{1}, \cdot\,, \underline{4}, \cdot\,, \underline{7}, \cdot\,, \underline{10}, \cdot\,, \underline{13}\,.$

Both these i-value sequences are combined which results in the i-value sequence $0, \underline{1}, 3, \underline{4}, 6, \underline{7}, 9, \underline{10}, 12, \underline{13}\,.$

This i-value sequence is then set off in the algorithm diagram for the Möbius braid, but since a real half-cycle cannot intersect the imaginary half-cycle of higher order, we only delete the underlining of the i-values associated with the imaginary string-run for the half-cycles which run from lower left to upper right.

We like to stress again that the 2-pass Herringbone-coded Regular Möbius Knot $p_m/b_m = 10/32$ is produced by the real string-run, and that the imaginary string-run is in fact laid down by the real string-run, but that in the Semi Regular Knot $p/b = 10/32$ the imaginary string-run is laid down by a separate string.

Note that the coding along every odd numbered half-cycle of the real string-run is / / \ \ / / \ \ /, and that the coding along every even numbered half-cycle of the real string-run is \ / / \ \ / / \ \.

We can either enter these coding sequences or their associated *under* and *over* sequences for the half-cycle direction concerned. After entering these sequences in the algorithm diagram, we can read off the half-cycle algorithms for the real string-run.

In order to braid the Möbius braid in accordance with the half-cycle algorithms obtained, we put in the first revolution (the circumference of the cylinder) of the string half a twist and then follow the same apparent surface of the string. This will ensure that we automatically obtain the required half twists in the string with as end result a correctly braided Regular Möbius Knot.

⋆⋆ *Prove that for* p_m *and* b_m even *with g.c.d.* $(2p_m, [p_m + b_m]) = 2$, *the bight-number* i *associated with the half-cycle (of the imaginary string-run) which goes through the bight-point immediately above the bight-point of the Standing End half-cycle of the real string-run has the value* $\left| -(\frac{b_m - p_m - 2}{4})\Delta^* \right|_{\frac{b_m}{2}}$.

An arbitrary intersection-column in the algorithm diagram carries $i = a^*$ *for the half-cycles, associated with the imaginary string-run, which run from left to right, and carries* $i = b^*$ *for the half-cycles, associated with the imaginary string-run, which run from right to left:*

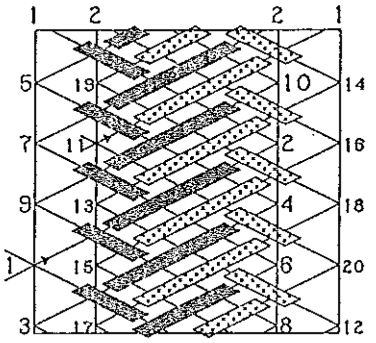
$$\begin{array}{ccccccccccccccc}
 & \left|-(\frac{b_m-p_m-2}{4})\Delta^*\right|_{\frac{b_m}{2}} & & \cdot & & \cdots & & a^* & & \cdots & & \cdot & & \cdot & \\
\cdot & \updownarrow & \cdot & \updownarrow & \cdot & \cdots & \cdot & \updownarrow & \cdot & \cdots & \cdot & \updownarrow & \cdot & \updownarrow & \cdot \\
 & \cdot & & \cdot & & \cdots & & b^* & & \cdots & & \cdot & & \left|-(\frac{b_m-p_m-2}{4})\Delta^*\right|_{\frac{b_m}{2}} &
\end{array}$$

Prove that $a^* + b^* = \frac{b_m}{2} - 2\,.$

The Braider should be well aware of the pitfalls associated with the braiding procedures for Möbius braids. Especially braiders who work with round braiding material, tend to make the error of omitting the necessary half twists in the string when braiding a Regular Möbius Braid by means of its associated virtual Regular Cylindrical Braid. We have seen that in such cases the braider does **not** obtain a Regular Möbius Braid, but will finish up with a Regular Cylindical Braid which can be transformed into a **false Möbius band** (the band retains its **separate** north, south, east and west surface lines!). This neglect of half twist is also the standard procedure found in the topological knot theory. Hence the fact that the topological knot theory is a bogus theory as far as knots and braids are concerned can readily be demonstrated with a simple Regular Möbius Braid. The topological knot theory is **only** as a purely mathematical subject of any value, and hence of no value to knots and braids. Unfortunately it are the academically brainwashed bogus mathematicians who would like us to believe differently.

A Glen Vandy Knot

The Glen Vandy Knots are the Regular Nested Knots which represent the lower limit of the Checkered Pineapple Knots. The Glen Vandy Knots depicted here are the smallest ($A = 2$) and are very beautiful little knots which are best suited for round string. All Glen Vandy Knots have interwoven components with $P_c = 4$. These knots are, for example, ideal to serve as lanyard knots.



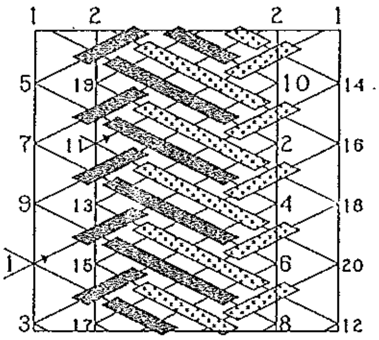
$A=2; P_{c1}=P_{c2}=4; P_{total}=8; B^*=5.$

1. L → R: FREE RUN.
2. R → L: FREE RUN.
3. L → R: FREE RUN.
4. R → L: u.
5. L → R: u.
6. R → L: 2u.
7. L → R: u-o.
8. R → L: 2u-o.
9. L → R: u-2o.
10 R → L: 2u-o.

FOUNDATION: $\begin{smallmatrix}1\\1\end{smallmatrix}\!>2$

11. L → R: u-2o.
12. R → L: o-2u.
13. L → R: u-2o.
14. R → L: 2o-2u.
15. L → R: u-3o.
16. R → L: 2o-3u.
17. L → R: u-4o.
18. R → L: 2o-4u.
19. L → R: u-4o-u.
20. R → L: 2o-4u.

INTERWEAVE: $\begin{smallmatrix}2\\2\end{smallmatrix}\!>1$



$A=2; P_{c1}=P_{c2}=4; P_{total}=8; B^*=5.$

1. L → R: FREE RUN.
2. R → L: FREE RUN.
3. L → R: FREE RUN.
4. R → L: o.
5. L → R: o.
6. R → L: 2o.
7. L → R: o-u.
8. R → L: 2o-u.
9. L → R: o-2u.
10 R → L: 2o-u.

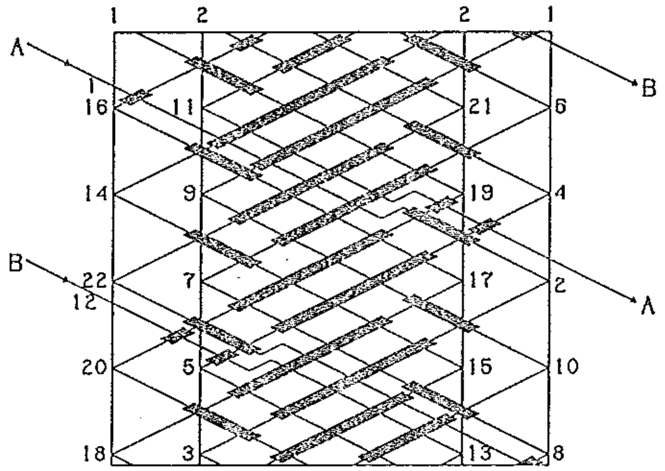
FOUNDATION: $\begin{smallmatrix}1\\1\end{smallmatrix}\!>2$

11. L → R: 2u-o.
12. R → L: 2o-u.
13. L → R: 2u-o.
14. R → L: u-2o-u.
15. L → R: 3u-o.
16. R → L: u-3o-u.
17. L → R: 4u-o.
18. R → L: u-4o-u.
19. L → R: 4u-2o.
20. R → L: u-4o-u.

INTERWEAVE: $\begin{smallmatrix}2\\2\end{smallmatrix}\!>1$

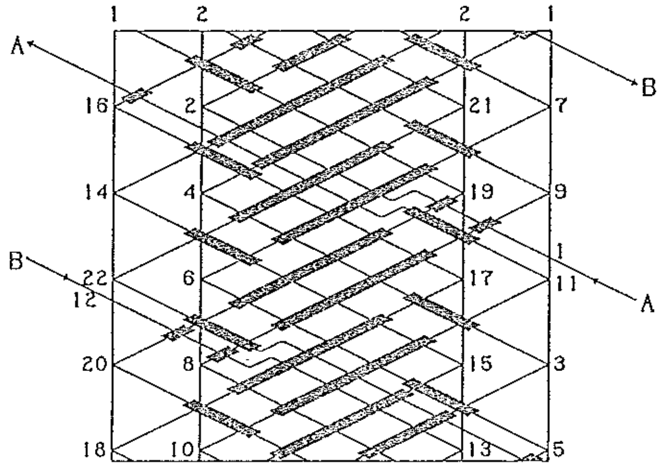
Fig. 313 — The Glen Vandy Knots with $A = 2$ and $B^* = 5$.

Fig. 314 below presents the grid-diagrams and their associated half-cycle braiding-algorithms for the uppermost Glen Vandy Knot in Fig. 313 as a lanyard knot.



A
1. L ⟶ R : FREE RUN.
2. R ⟶ L : FREE RUN.
3. L ⟶ R : FREE RUN.
4. R ⟶ L : u.
5. L ⟶ R : u.
6. R ⟶ L : u-o.
7. L ⟶ R : 2u.
8. R ⟶ L : u-2o.
9. L ⟶ R : 2u-o.
10. R ⟶ L : u-2o.
11. L ⟶ R : 3u.

B
12. L ⟶ R : 3u.
13. R ⟶ L : u-3o.
14. L ⟶ R : o-2u.
15. R ⟶ L : u-4o.
16. L ⟶ R : 2O-2u.
17. R ⟶ L : u-4o.
18. L ⟶ R : 2o-3u.
19. R ⟶ L : o-u-5o.
20. L ⟶ R : 2o-4u.
21. R ⟶ L : u-5o-u.
22. L ⟶ R : 2o-5u.



A
1. R ⟶ L : FREE RUN.
2. L ⟶ R : FREE RUN.
3. R ⟶ L : FREE RUN.
4. L ⟶ R : o.
5. R ⟶ L : o.
6. L ⟶ R : 2o.
7. R ⟶ L : o-u.
8. L ⟶ R : 3o.
9. R ⟶ L : o-2u.
10. L ⟶ R : 2o-u.
11. R ⟶ L : o-2u.

B
12. L ⟶ R : 3u.
13. R ⟶ L : u-3o.
14. L ⟶ R : o-2u.
15. R ⟶ L : u-4o.
16. L ⟶ R : 2O-2u.
17. R ⟶ L : u-4o.
18. L ⟶ R : 2o-3u.
19. R ⟶ L : o-u-5o.
20. L ⟶ R : 2o-4u.
21. R ⟶ L : u-5o-u.
22. L ⟶ R : 2o-5u.

Fig. 314 — The Glen Vandy Knot with $A = 2$; $B^* = 5$ as a lanyard knot.

THE BRAIDER

APPENDIX 1998

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A quarterly publication
for
the braiding artisan

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(Regular) Band Knots

In *The Braider*, Issue Nos 11 through 16 we discussed a very select set of the **Regular Band Knots** with a Gaucho coding (including the Spanish Ring Knot coding as a special case), a Headhunter's coding, and a Fan coding, which not only evolved from the 3-parts under–over coded Regular Knots, but in which the evolution process also proceeded in unit steps. The essential conditions fulfilled by these knots were as we have seen as follows:

For a **Spanish ring** coding or for a **Gaucho** coding, a Regular Band Knot with $[2m(\alpha+1)+1]$ parts and $[\{2m(\alpha+1)+1\}N+(\alpha+1)]$ bights evolves via Method I from a Regular Band Knot with $[2m\alpha+1]$ parts and $[\{2m\alpha+1\}N+\alpha]$ bights, while a Regular Band Knot with $[2m(\alpha+1)+1]$ parts and $[\{2m(\alpha+1)+1\}N-(\alpha+1)]$ bights evolves via Method II from a Regular Band Knot with $[2m\alpha+1]$ parts and $[\{2m\alpha+1\}N-\alpha]$ bights.[†]

For a **Headhunter's** coding, a Regular Band Knot with $[(2m+1)(\alpha+1)+1]$ parts and $[\{(2m+1)(\alpha+1)+1\}N+(\alpha+1)]$ bights evolves via Method I from a Regular Band Knot with $[(2m+1)\alpha+1]$ parts and $[\{(2m+1)\alpha+1\}N+\alpha]$ bights, while a Regular Band Knot with $[(2m+1)(\alpha+1)+1]$ parts and $[\{(2m+1)(\alpha+1)+1\}N-(\alpha+1)]$ bights evolves via Method II from a Regular Band Knot with $[(2m+1)\alpha+1]$ parts and $[\{(2m+1)\alpha+1\}N-\alpha]$ bights.[††]

For a **Fan** coding, a Regular Band Knot with $[(2m+2)(\alpha+1)-1]$ parts and $[\{(2m+2)(\alpha+1)-1\}N-(\alpha+1)]$ bights evolves via Method I from a Regular Band Knot with $[(2m+2)\alpha-1]$ parts and $[\{(2m+2)\alpha-1\}N-\alpha]$ bights, while a Regular Band Knot with $[(2m+2)(\alpha+1)-1]$ parts and $[\{(2m+2)(\alpha+1)-1\}N+(\alpha+1)]$ bights evolves via Method II from a Regular Band Knot with $[(2m+2)\alpha-1]$ parts and $[\{(2m+2)\alpha-1\}N+\alpha]$ bights.[†††]

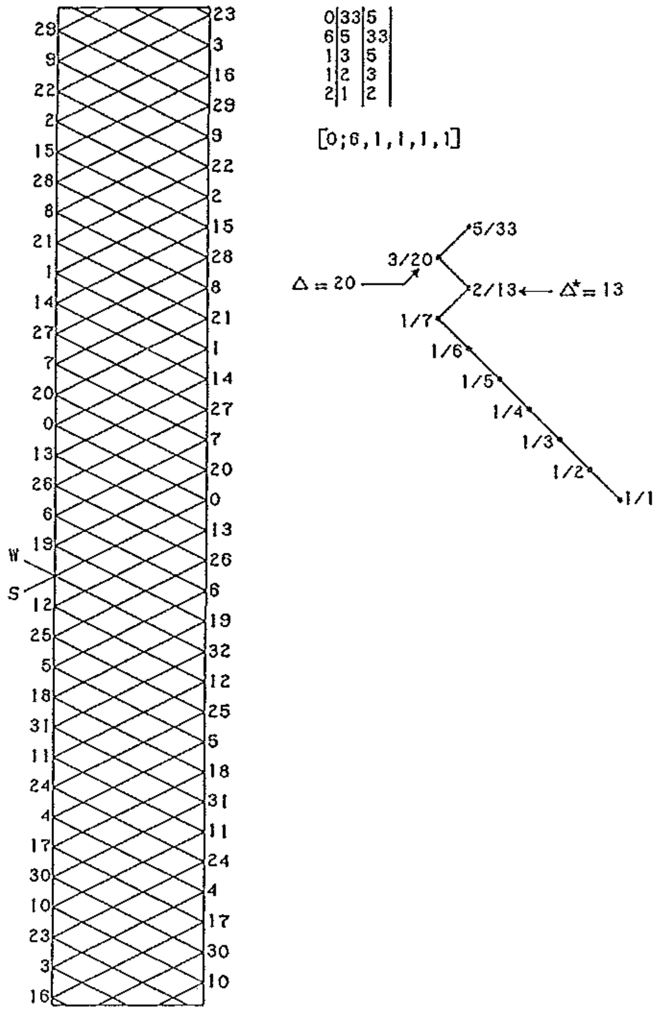
There are of course many other Regular Band Knots, with the above mentioned coding forms, which evolve from 3-parts under–over coded Regular Knots or from 2-parts under–over coded Regular Knots.[‡] These Regular Band Knots are important in applications where, for the number of bights required, no suitable one of those discussed in *The Braider*, Issue Nos 11 through 16 can be found. If for the indicated reason we would like the braiding process of the Regular Band Knot to go through the 3-parts, or the 2-parts, under–over coding stage, we must ensure, before braiding commemces, that this will be the case. This may be done by assuring that every even half-cycle of the associated 3-parts stage has an $u-o$ (or an $o-u$) coding; or that every even half-cycle of the associated 2-parts stage has an u (or an o) coding. A simple method by which this may be done for a 3-parts stage is shown by the right-hand table in Fig. 1 associated with the Regular Band Knot which is required to have a 2-pass Spanish Ring Knot coding (hence $P = 2m(\alpha+1)+1 = 2\cdot 1(1+1)+1 = 5$) with 33 bights. Above the table we set off the **algorithm diagram** of the Regular Band Knot concerned with the aid of its Δ^*-value. The table-heading displays the column-coding arrangement associated with the intersection-columns and the left-hand and right-hand bight-boundaries.

[†] see Issue No. 11, pp. 234-235.

[††] see Issue No. 12, pp. 256-257.

[†††] see Issue No. 12, pg. 257.

[‡] We prefer to braid Regular Band Knots through a 3-parts or 2-parts under–over stage since it is difficult to maintain unaided their form during the initial braiding stages. Refer to *The Braider*, Issue No. 11, pg. 232.



x	13 • 19	26 • 6	6 • 26	19 • 13	x	Δ*
x	•	•	•	•	x	
	14	27	7	20	0	
	15	28	8	21	1	
	16	29	9	22	2	
	17	30	10	23	3	
	18	31	11	24	4	
	19	32	12	25	5	
	20	0	13	26	6	
	21	1	14	27	7	
	22	2	15	28	8	
	23	3	16	29	9	Δ
	24	4	17	30	10	
	25	5	18	31	11	
	26	6	19	32	12	
	27	7	20	0	13	
	28	8	21	1	14	
	29	9	22	2	15	
	30	10	23	3	16	
	31	11	24	4	17	
	32	12	25	5	18	
	0	13	26	6	19	

Fig. 1 — The $P/B = 5/33$ 2-pass Spanish Ring Knot.

From the algorithm diagram we determine (from its bottom line, hence from the right to left half-cycles) the maximum i-value associated with its 3-parts stage. This is in general 1 less than the highest of the three lowest i-value entries; it is equal to the highest of the three lowest i-value entries when this highest entry is the leftmost entry of the three lowest i-value entries since it is then the leftmost intersection with the Standing End and hence lies on the left-hand bight-boundary of the associated 3-parts Regular Knot. Thus in the case associated with Fig. 1, the maximum i-value associated with the 3-parts stage is 19.

Below the right-hand bight-boundary indicator in the table-heading we set off the i-values of 0 to 19 inclusive. To the left of each of these i-values we set off a sequence of i-values obtained by adding (in modular fashion to B, hence in our case to 33) the Δ-value to each successive i-value. Hence the sequence of i-values associated with the i-value 0 on the right-hand bight-boundary is $|0+20|_{33} = 20$; $|20+20|_{33} = 7$; $|7+20|_{33} = 27$; $|27+20|_{33} = 14$.

The string-run diagram on the left in Fig. 1 shows the i-values associated with its half-cycles. Observe that the 2^{nd} half-cycle, hence the half-cycle associated with $i = 0$ intersects the half-cycles immediately preceding those associated with $i = 20$; $i = 7$; $i = 27$; $i = 14$, and that similarly the half-cycle associated with $i = 10$ for example intersects the half-cycles immediately preceding those associated with $i = 30$; $i = 17$; $i = 4$; $i = 24$. Since we have in our 3-parts Regular Knot only the i-values from 0 to 19 inclusive, we neglect any i-value greater than 19 and obtain the positions of the intersections on each half-cycle from right to left. These positions have been framed in the table. We see now that each right to left half-cycle in the 3-parts Regular Knot has the coding sequence $u - o$, and hence the 3-parts Regular Knot is a 3-parts **under–over coded** Regular Knot.[†]

For the examination of a 2-parts stage we follow a similar procedure.

In Fig. 2 we have examined for the 2-pass Spanish Ring Knot with $P/B = 5/34$ first the 3-parts stage (left table) and observed from this table that in the 3-parts stage not all right to left half-cycles have an $u - o$ coding (the right to left half-cycles associated with $i = 0$ to and including $i = 5$ have a $2u$ coding, and the right to left half-cycles associated with $i = 13$ to and including $i = 19$ have a $2o$ coding).

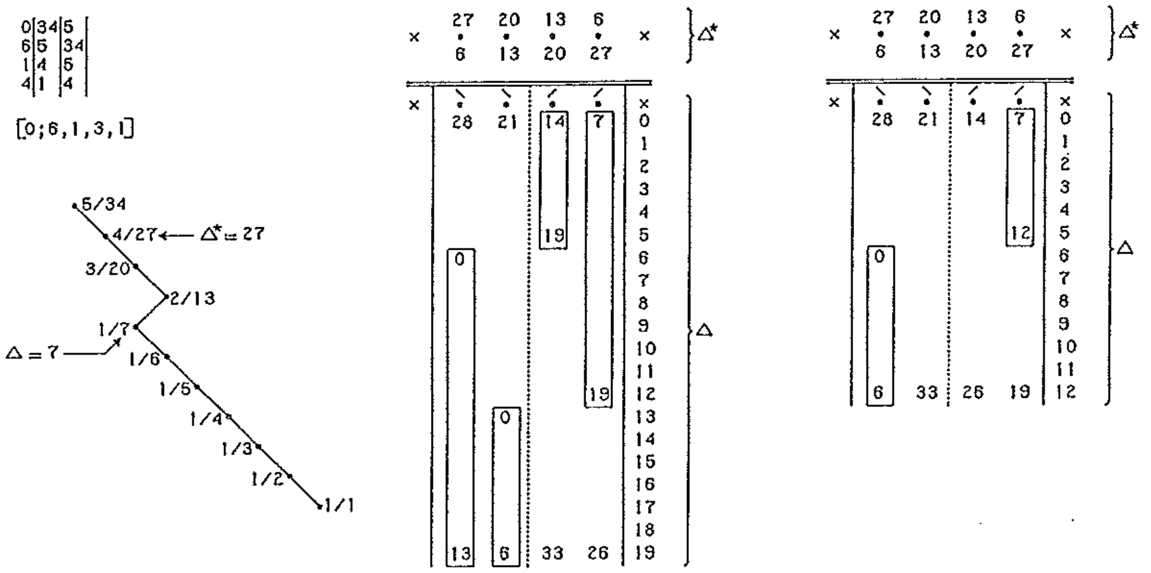


Fig. 2 — The $P/B = 5/34$ 2-pass Spanish Ring Knot.

Next we examined the 2-parts stage (right table) and observed that in this stage also not all right to left half-cycles have either an u or an o coding. Consequently this 2-pass Spanish Ring Knot is more cumbersome to braid and hence we would not employ it in an application.

In Figs. 3 and 4 we examine the 13-pass Spanish Ring Knot with $P/B = 27/181$. First we examine the associated 3-parts stage and observe that this gives us a 3-parts $u - o$ coded Regular Knot with $\frac{(2\times19+2)}{2} = 20$ bights. Then we might examine the 7/47 stage to see whether or not this gives us a 3-pass Spanish Ring Knot; which it does as we see from the lower table in Fig. 3.

[†] We could for the example in Fig. 1 also have determined from its $P/B = 5/33$ specification that this 2-pass Spanish Ring Knot goes through the 3-parts under–over coding stage from the formulae $P/B = [2m(\alpha+1)+1]/[\{2m(\alpha+1)+1\}N - (\alpha+1)]$ with $m = 1$; $\alpha + 1 = 2$; $N = 7$.

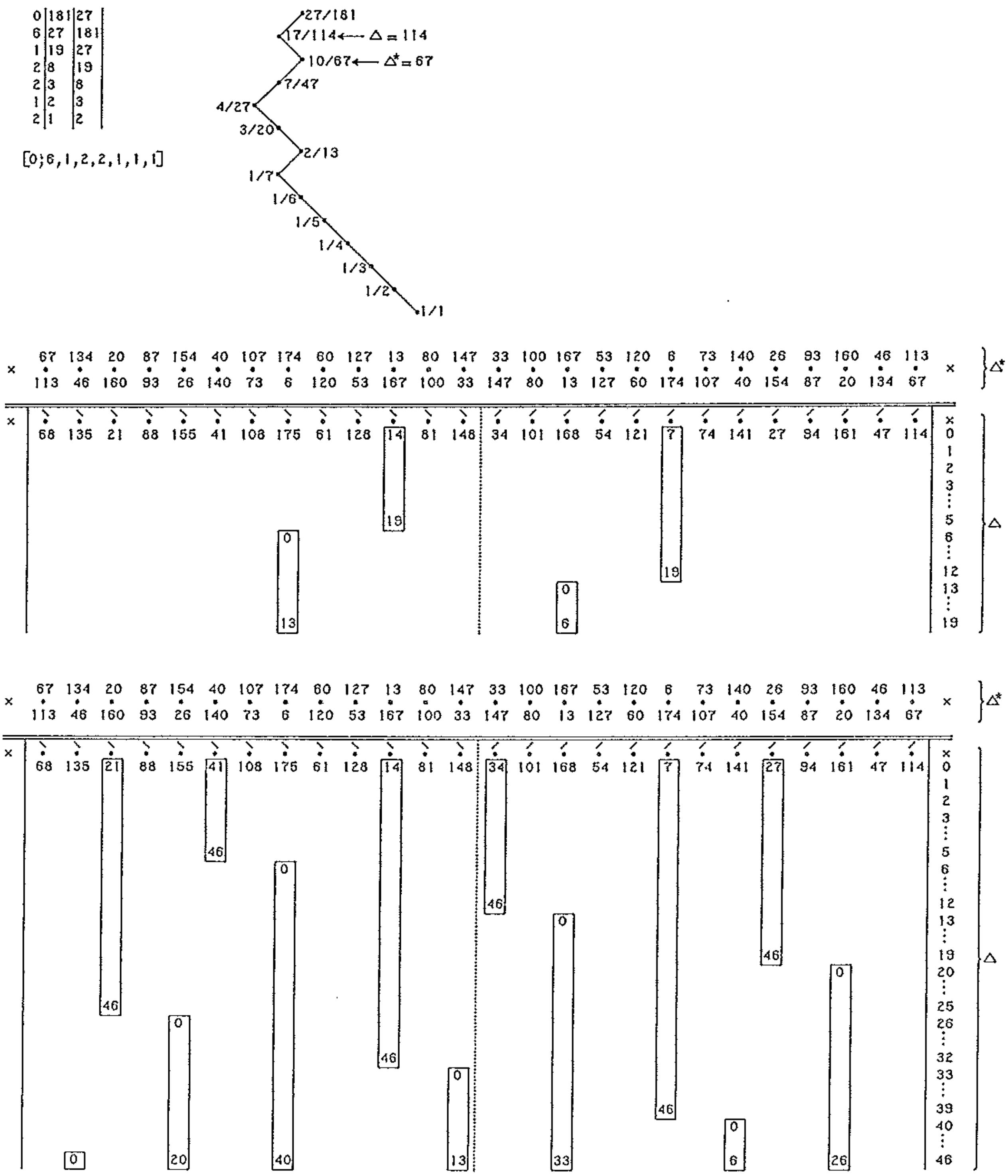


Fig. 3 — The $P/B = 27/181$ 13-pass Spanish Ring Knot.

Next we might examine the $17/114$ stage to see whether or not this gives us an 8-pass Spanish Ring Knot; which it does as we see from the table in Fig. 4.

Hence the 13-pass Spanish Ring Knot with $P/B = 27/181$ goes during its construction through the $3/20$ $u-o$ coded Regular Knot stage, the $7/47$ 3-pass Spanish Ring Knot stage and through the $17/114$ 8-pass Spanish Ring Knot stage.

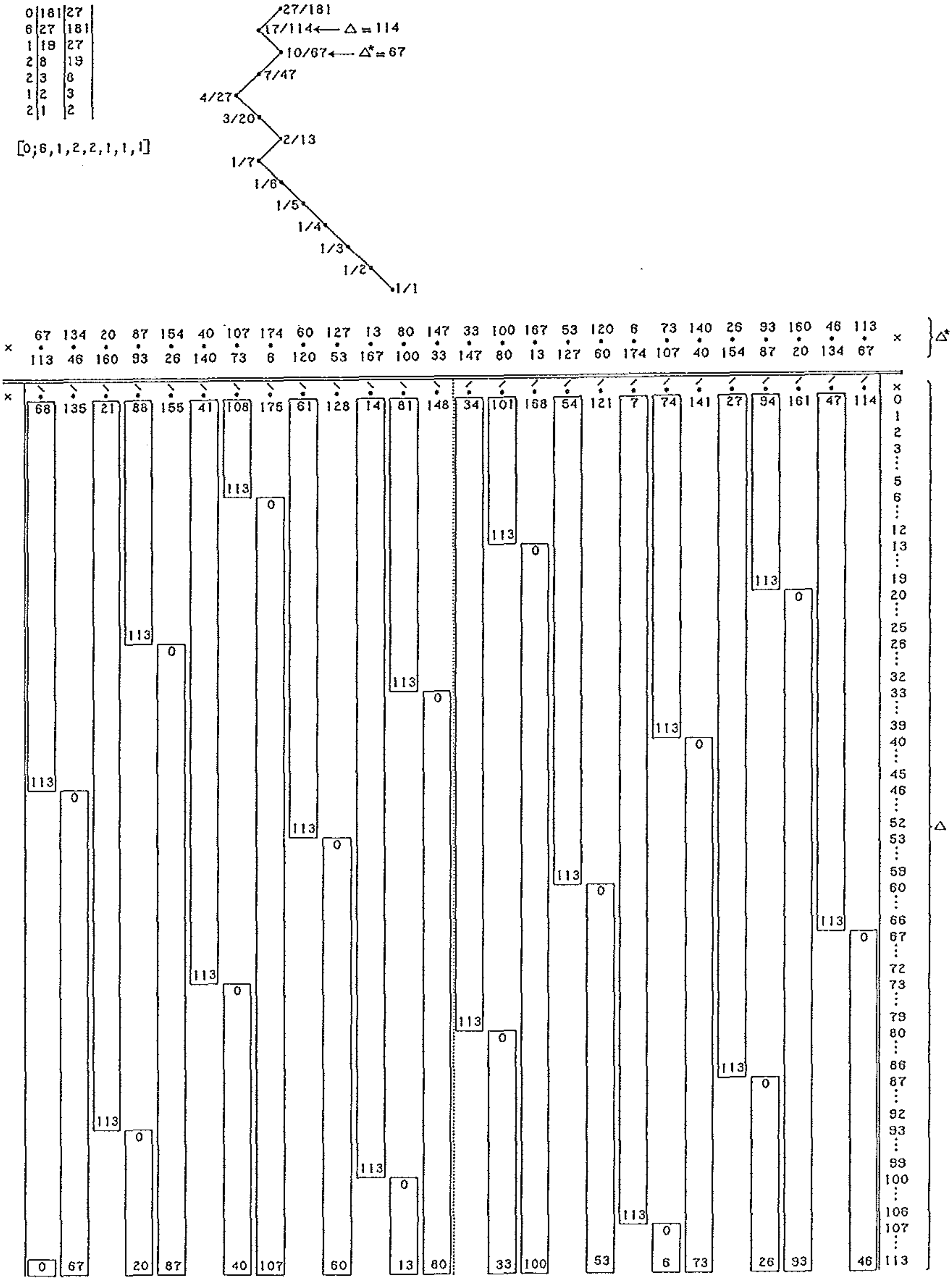


Fig. 4 — The $P/B = 27/181$ 13-pass Spanish Ring Knot.

In the upper part of Fig. 5 we examine the 4-pass Gaucho Knot with $P/B = 17/114$ and in the lower part of Fig. 5 we examine the $P/B = 7/47$ 2-pass Headhunter's Knot.

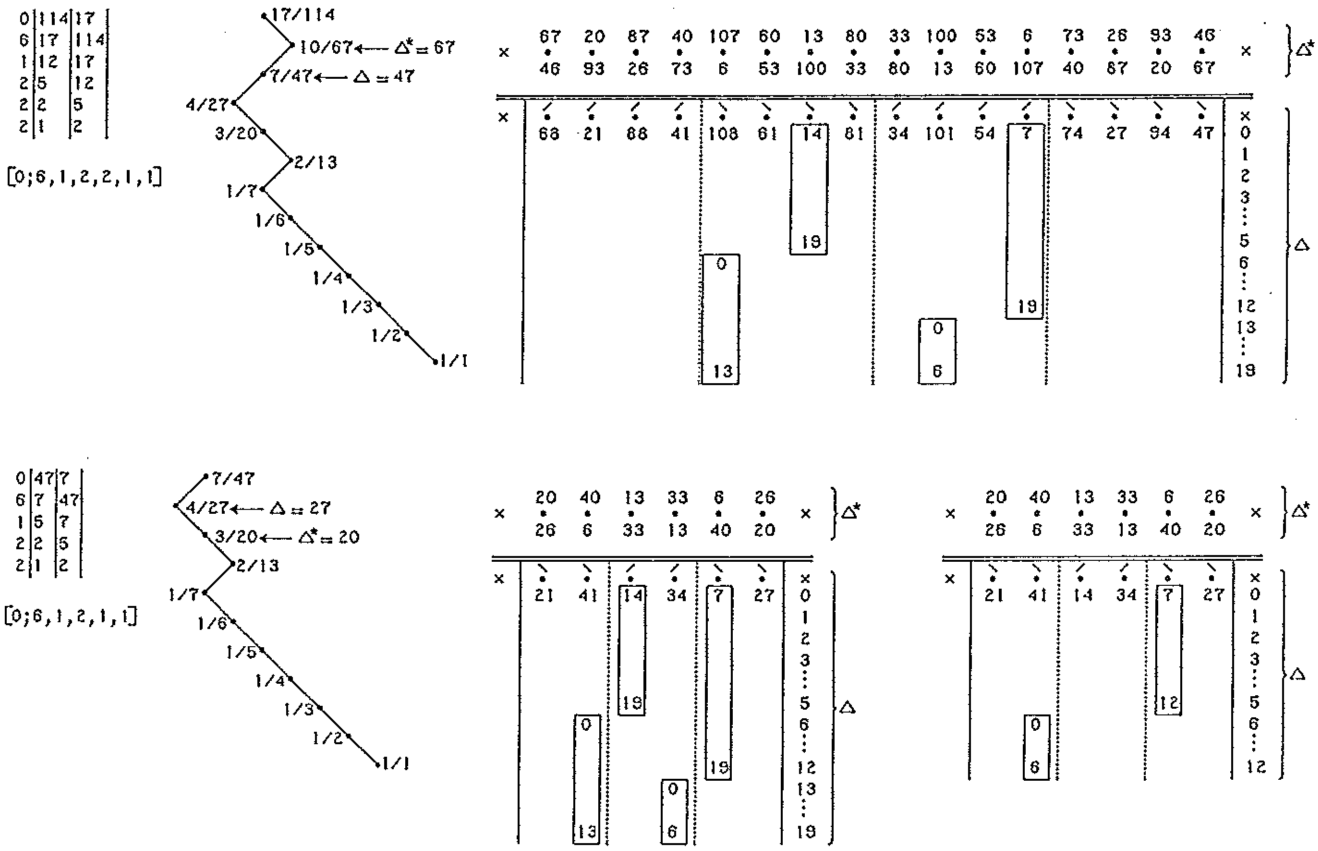


Fig. 5 — The $P/B = 17/114$ 4-pass Gaucho Knot and the $P/B = 7/47$ 2-pass Headhunter's Knot.

The upper table, associated with the $P/B = 17/114$ 4-pass Gaucho Knot shows that during its construction we pass through the $3/20$ $u - o$ coded Regular Knot.

The left lower table, associated with the $P/B = 7/47$ 2-pass Headhunter's Knot shows that the 3-parts stage does neither have an $o - u$ nor an $u - o$ coding. The lower right table shows that its 2-parts stage has an o coding. Hence this $P/B = 7/47$ 2-pass Headhunter's Knot can easily be braided via the $2/13$ o coded Regular Knot.

ERRATA—*The Braider*

The updated errata list below lists the typing errors which were noted in at least some copies of the following Issues of *The Braider*.

No. 1

pg. 3, line 9 — **it** is
pg. 3, line 10 — **unnecessarily**
pg. 3, line 17 — in order **to** present
pg. 12, line 5 under "An Introduction to Flat Braids" — Four pages are devoted **to**
pg. 16, line 9 under "Reviews" — **instructor**

No. 2

pg. 21, line 2 — delete **that**
pg. 28, line 6 — However, **a** one colour
pg. 38, line 18 under "Reviews" — **they** are first class

No. 3

pg. 42, line 2 — Naturally, **queries** not directly

No. 4

pg. 76, line 5 — **phenomenon**
pg. 81, line 16 — **phenomenon**

No. 5

pg. 87, line 3 from bottom of page — **exclude**
pg. 106, line 11 — **locality**

No. 6

pg. 129, line 15 — **2-colour**

No. 7

none

No. 8

pg. 158, line 3 — Hence **the** number of bights
pg. 161, line 12 — In Fig. 144 are depicted the **parts-raising** processes

No. 9

none

No. 10

pg. 213, line 2 — **choose**

No. 11

pg. 232, line 17 — in **general**
pg. 245, line 8 — in **general**

No. 12

pg. 257, line 6 — or $B = \{(2m+2)(\alpha+1) - 1\}N - (\alpha+1)$ bights
pg. 257, line 7 — Those with $B = \{(2m+2)(\alpha+1) - 1\}N - (\alpha+1)$ bights
pg. 257, line 9 — $b = \{(2m+2)\alpha - 1\}N - \alpha$ bights. Those with $B = \{(2m+2)(\alpha+1) - 1\}N + (\alpha+1)$ bights
pg. 257, line 10 & 11 — with $p = \{(2m+2)\alpha - 1$ parts and $b = \{(2m+2)\alpha - 1\}N + \alpha$ bights
pg. 257, line 7 from bottom — delete **thinspace**

No. 13

pg. 276, line 7 — in **general**

No. 14

pg. 308, line 3 — in **general**

No. 15

pg. 338, line 1 — **independent**

No. 16

pg. 347, line 3 — $P = 2m(\alpha + 1) + 1$
pg. 353, line 16 — **screw**
pg. 356, line 3 — **two**
pg. 359, line 5 under Fig. 310 — **boundary**
pg. 361, line 3 from bottom — **Headhunter's**-coded
pg. 366, line 30 — **braided**

No. 17

pg. 369, line 12 — **mathematical**
pg. 372, lines 8, 9, 10, 13 — **positive**
pg. 383, line 13 — Figs. **326–328**
pg. 387, line 2 below Fig. 332 — in **general**
pg. 387, line 13 below Fig. 332 — **cases**

No. 18

pg. 393, the right-hand brace } immediately before the second equal sign is missing in the three lines above the upper .... line, and the right-hand brace } immediately before the third equal sign is missing in the first line below the upper .... line.
pg. 395, line 4 — **additionally**
pg. 396, bottom line — in **general**
pg. 397, line 4 from bottom — in **general**

No. 19

pg. 419, lines 2 & 3 from bottom — in **general**
pg. 430, lines 3 & 5 — in **general**

No. 20

pg. 446, line 14 — A few **simple** examples
pg. 446, line 3 from bottom — in **general**
pg. 447, Footnote — **These** are the
pg. 450, last line — delete **thinspace**
pg. 464, line 20 — Brion Toss

THE BRAIDER

No.17 FEBRUARY 1999.

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Regular Cylindrical Bihelix Braids 378

A quarterly publication
for
the braiding artisan

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Solutions to the Questions in Issue No. 16

Question on pp. 351 – 352.

This question is a very important one since after solving the various sub-questions a multitude of questions in fields other than braiding, but *possibly* related, should present themselves. In the near future we shall present one of those questions in what therefore has to be called a "sideline" article which is associated with SE (*spongiform encephalopathy*) to which BSE (*mad cow disease*), CJD (*Creutzfeldt-Jacob disease*), Kuru, FFI (*fatal familial insomnia*) and GSS (*Gerstmann-Sträussler-Scheinker disease*) belong.

The following solutions should be carefully studied and be thoroughly comprehended since they not only clearly demonstrate the fallacious approach to knots and braids by the topological knot theorists (their theory is in relation to knots and braids a mathematical fallacy), but are of fundamental importance to knots and braids.

Although the fundamental properties of twist have been discussed in Issue No. 16 on pg. 352 we like to recall the two fundamental arrangements depicted in Fig. 315.

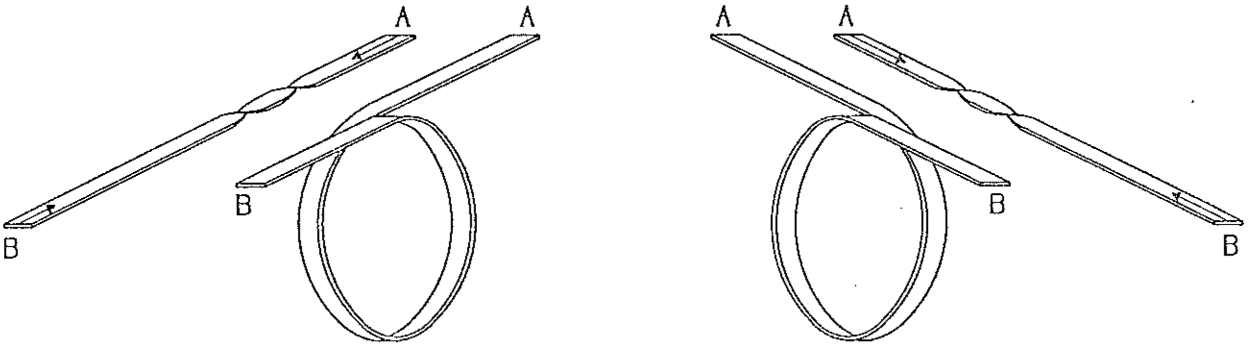


Fig. 315 — Two right helix and two left helix half twists and their equivalent loops.

The left-hand drawing clearly indicates that the loop formed creates two half twists with a **left** helix which cancel the two half twists with the right helix in the string depicted at the extreme left-hand side. Similarly the right-hand drawing indicates that the loop formed creates two half twists with a **right** helix which cancel the two half twists with the left helix in the string depicted at the extreme right-hand side. Consequently if we form in the string depicted at the extreme left a loop by bringing *A* to the **right** of *B* instead of to the left, we finish up with a loop and two additional half twists with a **right** helix, hence a loop and four half twists with a right helix. Similarly, if we form in the string depicted at the extreme right a loop by bringing *A* to the **left** of *B* instead of to the right, we finish up with a loop and two additional half twists with a **left** helix, hence a loop and four half twists with a left helix.

We can now compile the tables on pg. 371. Each table deals with its indicated number of half twists (whether with a left helix or right helix) in a piece of string. The left-hand part of these tables deals with the placing of string "ends" *A* and *B* in their relative positions which immediately decrease the number of half twists with the formation of the first loop, and the right-hand part of these tables deals with the placing of string "ends" *A* and *B* in their relative positions which first create **two additional** half twists with the formation of the first loop. In these tables *M* indicates a Regular Möbius Braid whereas *C* indicates a Regular Cylindrical Braid. The number of loops are the number of parts in the braids concerned. In the first table the –1 Half Twists indicates that its associated helix is of the opposite type to the original half twist in the string. We have an x-parts Regular Knot when with x loops the associated number of half twists is zero, and we have an x-parts Regular Möbius Knot when with x loops the associated number of half twists is x. Let's have a look at an Example in order to clarify this.

Example : A string with 8 half twists.

In the left-hand part of the table for 8 half twists we obtained the useful entry: 4 loops + 0 half twists together with the indication C. Since there are no half twists left we have a 1-string Regular Cylindrical Braid of 4-parts. The number of bights can be made any number which is coprime with 4.

In the right-hand part of this table for 8 half twists we otained two useful entries: one entry specifying 4 loops + 4 half twists together with the indication M, hence the braid is a 1-string Regular Cylindrical Braid of $p = p_m = 4$-parts and a half twist in each part. This braid can be transformed (without interfering with the string-ends and the number of loops) into a 1-string Regular Cylindrical Braid of $p = p_m = 4$-parts which contains a Matthew Walker section with $\frac{p}{2} = \frac{p_m}{2} = \frac{4}{2} = 2$-bights and a half twist in each of its $p = p_m = 4$-parts. This braid can be transformed into a 4-parts Regular Möbius Knot by deleting its Matthew Walker section (and at the same time the half twists). The other entry specifies 6 loops + 0 half twists together with the indication C. Since there are no half twists left we have a 1-string Regular Cylindrical Braid of 6-parts. For the number of bights we can take any number which is coprime with 6.

Fig. 316 shows for each of the in the tables indicated Regular Möbius Knots a grid-diagram (with Matthew Walker section containing p_m half twists) for one of the many possible b_m-values to the right of the grid-diagram of its equivalent Regular Cylindrical Braid with one bight and p_m half twists. The upper two rows and the lower two rows of grid-diagrams are associated with a string in which the half twists have a left helix, respectively a right helix. In these diagrams the Matthew Walker sections are bordered by thick lines.

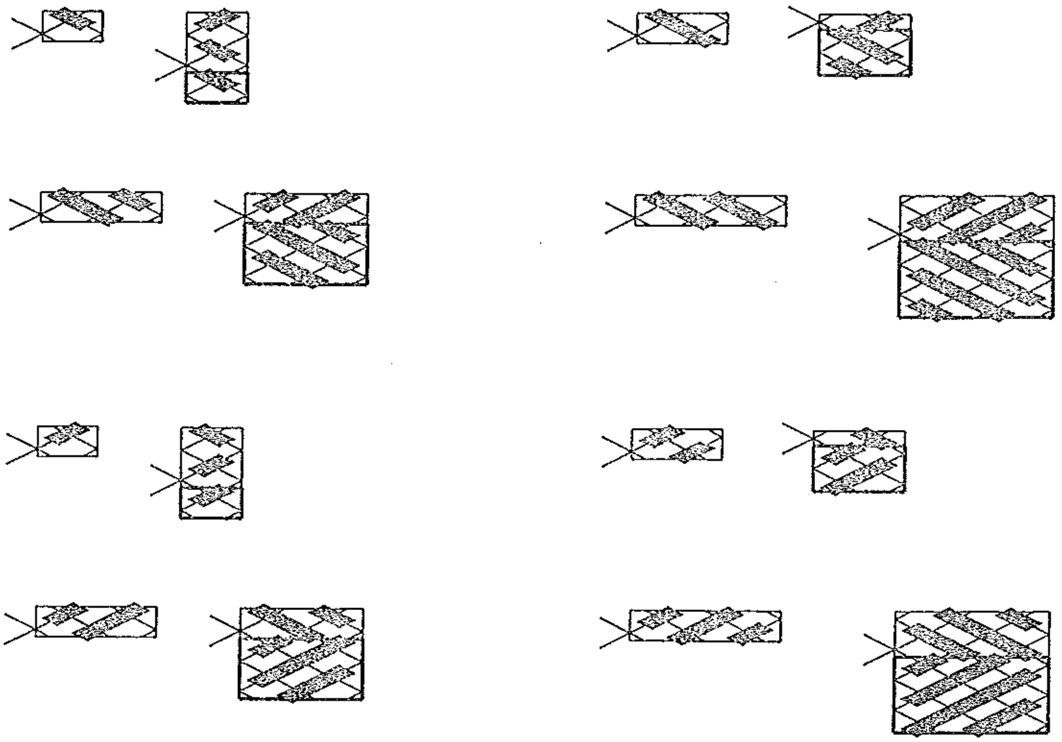


Fig. 316 — Some possible Regular Möbius Knots associated with the table entries.

1 HALF TWIST

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	−1 →M	1	3
		2	1

2 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	0 →C	1	4
		2	2 →M
		3	0 →C

3 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	1 →M	1	5
		2	3
		3	1

4 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	2	1	6
2	0 →C	2	4
		3	2
		4	0 →C

5 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	3	1	7
2	1	2	5
		3	3 →M
		4	1

6 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	4	1	8
2	2 →M	2	6
3	0 →C	3	4
		4	2
		5	0 →C

7 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	5	1	9
2	3	2	7
3	1	3	5
		4	3
		5	1

8 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	6	1	10
2	4	2	8
3	2	3	6
4	0 →C	4	4 →M
		5	2
		6	0 →C

9 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	7	1	11
2	5	2	9
3	3 →M	3	7
4	1	4	5
		5	3
		6	1

10 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	8	1	12
2	6	2	10
3	4	3	8
4	2	4	6
5	0 →C	5	4
		6	2
		7	0 →C

11 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	9	1	13
2	7	2	11
3	5	3	9
4	3	4	7
5	1	5	5 →M
		6	3
		7	1

12 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	10	1	14
2	8	2	12
3	6	3	10
4	4 →M	4	8
5	2	5	6
6	0 →C	6	4
		7	2
		8	0 →C

13 HALF TWISTS

LOOPS	+HALF TWISTS	LOOPS	+HALF TWISTS
1	11	1	15
2	9	2	13
3	7	3	11
4	5	4	9
5	3	5	7
6	1	6	5
		7	3
		8	1

From the tables we see that a string with 1 half twist is a special case and that from a string with $(6n+1)$ half twists (where n is a positive integer) neither all half twists can be eliminated, nor that the loops can each have a half twist only. The overall results for eliminating (if possible) all half twists in a string may be summarised as follows:

- A string with 1 half twist is a special case and is equivalent to a 1-part Regular Möbius Knot with the opposite helix.
- A string with n half twists (n is a positive integer greater than 1) is equivalent to:
    - (i). An $\frac{n}{2}$-parts Regular Knot when $\frac{n}{2}$ is a positive integer.
    - (ii). An $\frac{n+4}{2}$-parts Regular Knot when $\frac{n+4}{2}$ is a positive integer.
    - (iii). An $\frac{n}{3}$-parts Regular Möbius Knot with the opposite helix when $\frac{n}{3}$ is a positive integer.
    - (iv). An $\frac{n+4}{3}$-parts Regular Möbius Knot with the opposite helix when $\frac{n+4}{3}$ is a positive integer.

It will be clear that half twists in a piece of string represent **dormant** braids. It is of special significance that a set number of half twists always represents **at least** two sets of braids, which for an **even** number of half twists are genuine Regular Cylindrical Knots, and which for an **odd** number of half twists, equal to or greater than 5, are prime hybrids between Regular Cylindrical Knots and Regular Möbius Knots (they are Regular Cylindrical Knots with only one half twist in their entire string-run (the Möbius facet)). Depending on the number of half twists there may or may not be **one** set of prime Regular Möbius Knots. All the knots in a specific set have the same number of parts which is a function of the number of twists, while they may possess various numbers of bights.

We like to stress once again that the relationship between the number of half twists and the knots they represent in dormant form is one of the very fundamental properties of knots. It is really staggering that mathematicians have overlooked and hence ignored this most fundamental property. Although, are it not the obvious and most simple but essential matters of which the academic world is so often totally oblivious? Not for nothing is there the well known saying: *"some of the greatest nitwits I have met have been academics"*.

Question on pg. 364.

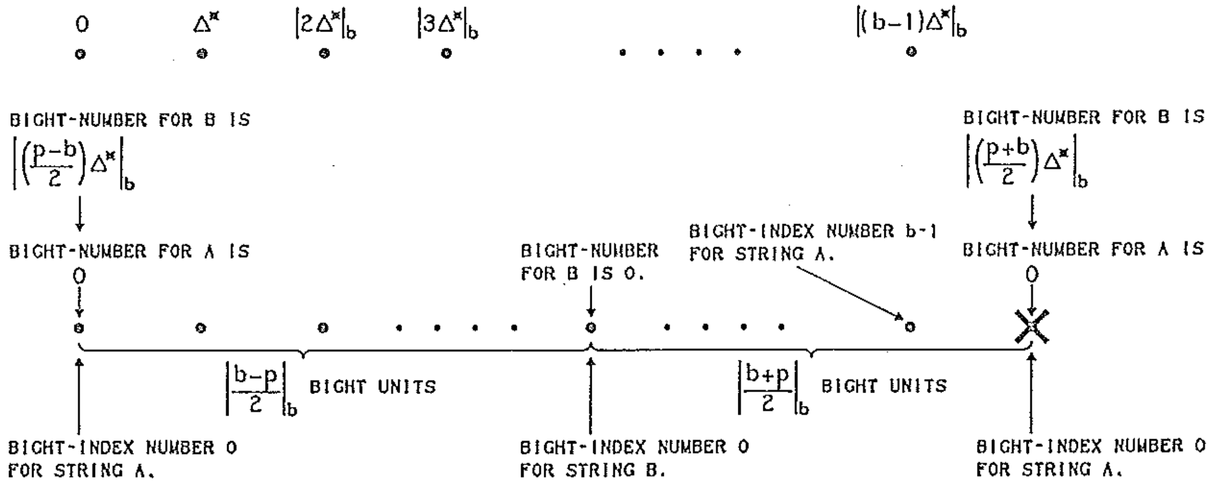

0
Δ^*
$|2\Delta^*|_b$
$|3\Delta^*|_b$
$|(b-1)\Delta^*|_b$
BIGHT-NUMBER FOR B IS
$\left|\left(\frac{p-b}{2}\right)\Delta^*\right|_b$
BIGHT-NUMBER FOR A IS
0
BIGHT-NUMBER FOR B IS 0.
BIGHT-INDEX NUMBER b-1 FOR STRING A.
BIGHT-NUMBER FOR B IS
$\left|\left(\frac{p+b}{2}\right)\Delta^*\right|_b$
BIGHT-NUMBER FOR A IS
0
$\left|\frac{b-p}{2}\right|_b$ BIGHT UNITS
$\left|\frac{b+p}{2}\right|_b$ BIGHT UNITS
BIGHT-INDEX NUMBER 0 FOR STRING A.
BIGHT-INDEX NUMBER 0 FOR STRING B.
BIGHT-INDEX NUMBER 0 FOR STRING A.

Fig. 317 — The complementary cyclic bight-number schemes for the strings A and B.

Let string-run A be the real string-run and let string-run B be the imaginary string-run. Then the uppermost diagram in Fig. 317 depicts the complementary cyclic bight-number scheme for string A.

The bight-points of string A are also the bight-points of string B, however, the strings A and B start at different bight-points. The bight-point at which the imaginary half-cycle of higher order of string B starts (bight-index number 0 for string B) receives in the complementary cyclic bight-number scheme the bight-number 0. Since $p = p_m$ and $b = b_m$, this bight-point has for string A the bight-index number $\left|\frac{b-p}{2}\right|_b$. Hence it lies $\left|\frac{b-p}{2}\right|_b$ bight-units to the right of the bight-point at which string A starts, as well as $\left|\frac{b+p}{2}\right|_b$ bight-units to the left of this starting point for string A. Let in the complementary cyclic bight-number scheme this starting bight-point of string A have the bight-number x for string B. Then:

$$\begin{aligned}
2x &= \left|\left(\frac{p-b}{2}\right)\Delta^*\right|_b + \left|\left(\frac{p+b}{2}\right)\Delta^*\right|_b \\
&= \tfrac{1}{2}|(p-b)\Delta^*|_{2b} + \tfrac{1}{2}|(p+b)\Delta^*|_{2b} \\
&= \frac{(p-b)\Delta^* + (p+b)\Delta^* + 2sb}{2} \\
&= \frac{2p\Delta^* + 2sb}{2} \qquad \text{where } s = \text{integer}. \\
x &= \frac{p\Delta^* + sb}{2}. \quad \text{Hence with } \Delta^* = \frac{b\cdot\Delta_{II,1}p - 1}{p}: \\
x &= \frac{p\cdot\frac{b\cdot\Delta_{II,1}p-1}{p} + sb}{2} \\
&= \frac{b\cdot\Delta_{II,1}p - 1 + sb}{2} \\
&= \frac{b-1}{2}, \quad \text{since } b = \mathit{odd} \text{ and } 0 \le x \le b-1 .
\end{aligned}$$

Thus we obtain the complementary cyclic bight-number scheme depicted in Fig. 318.

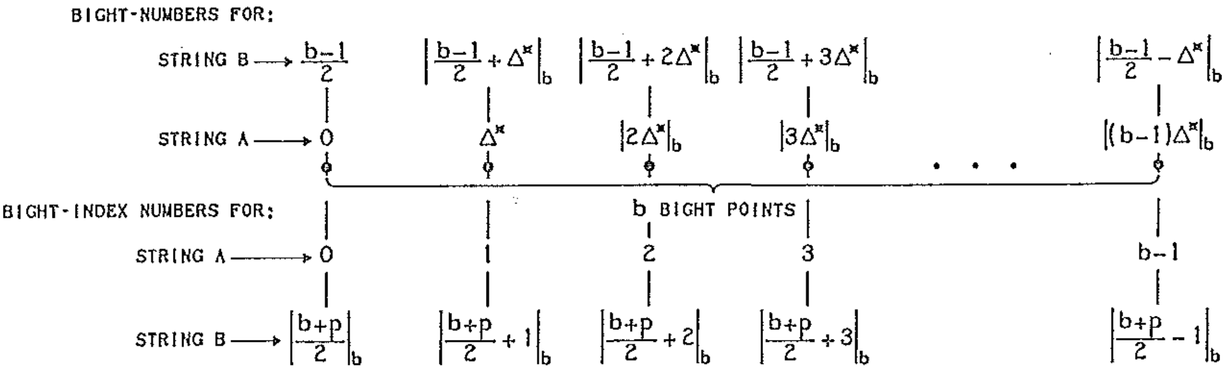

BIGHT-NUMBERS FOR:
STRING B → $\frac{b-1}{2}$
$\left|\frac{b-1}{2}+\Delta^*\right|_b$
$\left|\frac{b-1}{2}+2\Delta^*\right|_b$
$\left|\frac{b-1}{2}+3\Delta^*\right|_b$
$\left|\frac{b-1}{2}-\Delta^*\right|_b$
STRING A → 0
Δ^*
$|2\Delta^*|_b$
$|3\Delta^*|_b$
$|(b-1)\Delta^*|_b$
b BIGHT POINTS
BIGHT-INDEX NUMBERS FOR:
STRING A → 0
1
2
3
b−1
STRING B → $\left|\frac{b+p}{2}\right|_b$
$\left|\frac{b+p}{2}+1\right|_b$
$\left|\frac{b+p}{2}+2\right|_b$
$\left|\frac{b+p}{2}+3\right|_b$
$\left|\frac{b+p}{2}-1\right|_b$

Fig. 318 — The complementary cyclic bight-number scheme for $p = p_m = \mathit{odd}$ and $b = b_m = \mathit{odd}$ with g.c.d. $(p_m\,,\,b_m) = 1$.

Let for the imaginary string-run (hence string-run B) $i_y = a^*$ and $i_{p-y} = b^*$ (see Fig. 319). Then:

$$|i_y + i_{p-y}|_b = \left|\left|\frac{b-1}{2} + y\Delta^*\right|_b + \left|\frac{b-1}{2} + (p-y)\Delta^*\right|_b\right|_b$$

$$= |b - 1 + p\Delta^*|_b$$

$$= |p\Delta^* - 1|_b . \qquad \text{Hence with } \Delta^* = \frac{b \cdot \Delta_{II,1}p - 1}{p} :$$

$$|i_y + i_{p-y}|_b = \left|b \cdot \Delta_{II,1}p - 1 - 1\right|_b = |-2|_b$$

$$= b - 2 .$$

$$i_y + i_{p-y} = sb - 2 \qquad \text{where } s = 1 \text{ or } 2 \text{ since } 0 \leq i \leq b - 1 .$$

$i_y = i_{p-y}$ for $s = 2$. But $i_y \neq i_{p-y}$ for $p = odd$, hence $s = 1$. Thus:

$$a^* + b^* = i_y + i_{p-y} = b - 2 .$$

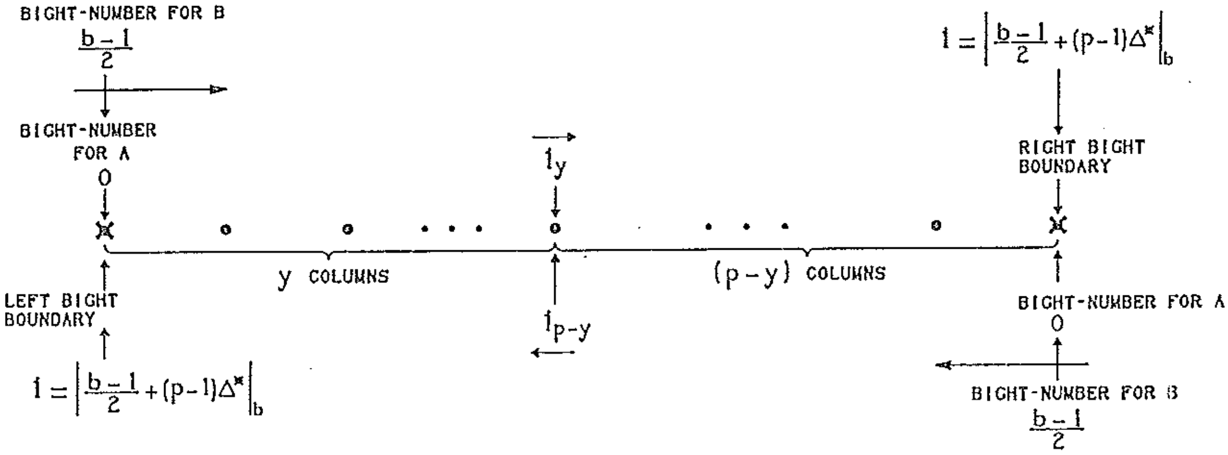


Fig. 319 — The general algorithm diagram for string B.

Question on pg. 366.

Let's first consider the case $p_m = 4n$; $b_m = 4m + 2$ where n and m are whole numbers.[†]

Let again string-run A be the real string-run and let string-run B be the imaginary string-run. Then the uppermost diagram in Fig. 320 depicts the complementary cyclic bight-number scheme for string A. Note that since the g.c.d. $(p, b) = 2$, the bight-points of string A are **not** the bight-points of string B, and we know from the g.c.d. value that the bight-points at either side of the bight-point at which string A starts are bight-points of string B. The bight-point at which the imaginary half-cycle of higher order of string B starts (bight-index number 0 for string B) receives in the complementary cyclic bight-number scheme the bight-number 0. Since $p = p_m$ and $b = b_m$, this bight-point lies $(\frac{b}{2} - \frac{p}{2}) = (2m - 2n + 1)$ bight-units to the right of the bight-point at which string A starts. Thus the bight-point immediately to the right of the bight-point at which string A starts lies $(2m - 2n)$ bight-units to the left of the bight-point at which the imaginary half-cycle of higher order of string B starts; which distance is equivalent to $(m-n)$ bight distances of string B. Hence the bight-number of the bight-point immediately to the right of the bight-point at which string A starts has the bight-number $\left|-(m-n)\Delta^*\right|_{2m+1} = \left|-\frac{b-p-2}{4}\Delta^*\right|_{\frac{b}{2}}$.

[†] **Whole numbers** are the numbers $0, 1, 2, 3, \cdots$
Natural numbers or **positive integers** are the numbers $1, 2, 3, \cdots$
Integers are the numbers $\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$

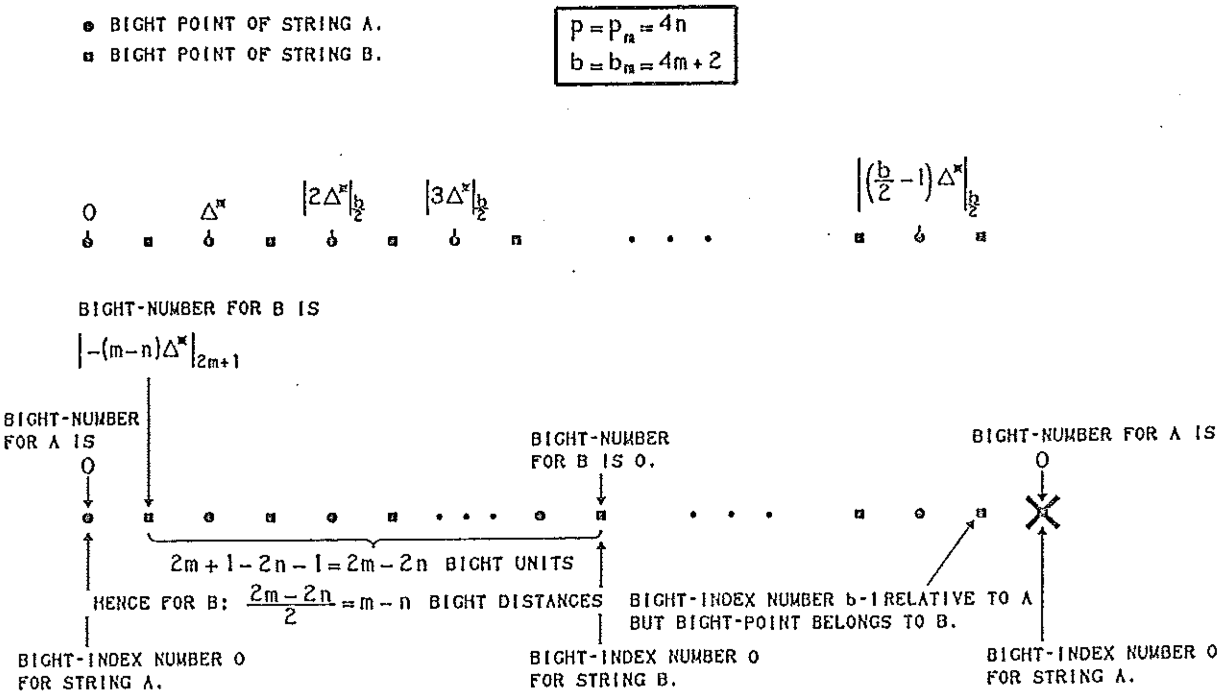

● BIGHT POINT OF STRING A.
■ BIGHT POINT OF STRING B.
$p = p_m = 4n$
$b = b_m = 4m + 2$
0
Δ^*
$|2\Delta^*|_{\frac{b}{2}}$
$|3\Delta^*|_{\frac{b}{2}}$
$|(\frac{b}{2}-1)\Delta^*|_{\frac{b}{2}}$
BIGHT-NUMBER FOR B IS
$|-(m-n)\Delta^*|_{2m+1}$
BIGHT-NUMBER FOR A IS
0
BIGHT-NUMBER FOR B IS 0.
BIGHT-NUMBER FOR A IS
0
$2m + 1 - 2n - 1 = 2m - 2n$ BIGHT UNITS
HENCE FOR B: $\frac{2m-2n}{2} = m - n$ BIGHT DISTANCES
BIGHT-INDEX NUMBER b-1 RELATIVE TO A BUT BIGHT-POINT BELONGS TO B.
BIGHT-INDEX NUMBER 0 FOR STRING A.
BIGHT-INDEX NUMBER 0 FOR STRING B.
BIGHT-INDEX NUMBER 0 FOR STRING A.

Fig. 320 — The complementary cyclic bight-number scheme for $p = p_m = 4n = even$ and $b = b_m = 4m + 2 = even$.

Let's now consider the case $p_m = 4n + 2$; $b_m = 4m$ where n and m are whole numbers.

Let, as before, string-run A be the real string-run and let string-run B be the imaginary string-run. Then the uppermost diagram in Fig. 321 depicts the complementary cyclic bight-number scheme for string A. Note again that the g.c.d.$(p, b) = 2$, and hence we know from this g.c.d. value that the bight-points at either side of the bight-point at which string A starts are bight-points of string B. The bight-point at which the imaginary half-cycle of higher order of string B starts (bight-index number 0 for string B) receives in the complementary cyclic bight-number scheme again the bight-number 0. Since $p = p_m$ and $b = b_m$, this bight-point lies $(\frac{b}{2} - \frac{p}{2}) = (2m - 2n - 1)$ bight-units to the right of the bight-point at which string A starts. Thus the bight-point immediately to the right of the bight-point at which string A starts lies $(2m - 2n - 2)$ bight-units to the left of the bight-point at which the imaginary half-cycle of higher order of string B starts; which distance is equivalent to $(m - n - 1)$ bight distances of string B. Hence the bight-number of the bight-point immediately to the right of the bight-point at which string A starts has the bight-number $\left|-(m-n-1)\Delta^*\right|_{2m} = \left|-\frac{b-p-2}{4}\Delta^*\right|_{\frac{b}{2}}$.

Hence when p_m and b_m are both even with g.c.d.$(2p_m, [p_m + b_m]) = 2$, we obtain in the complementary cyclic bight-number scheme for the bight-point immediately to the right of the bight-point at which string A starts the bight-number $i = \left|-\frac{b-p-2}{4}\Delta^*\right|_{\frac{b}{2}}$.

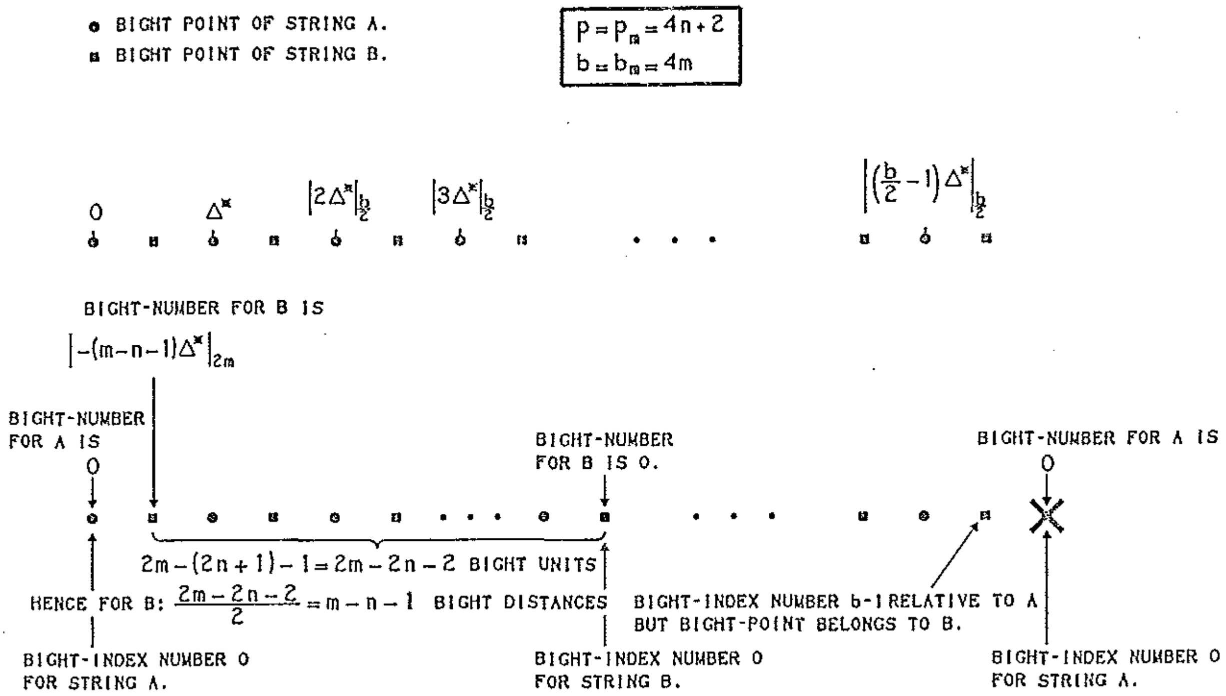

● BIGHT POINT OF STRING A.
▪ BIGHT POINT OF STRING B.
$p = p_m = 4n+2$
$b = b_m = 4m$
0
Δ^*
$|2\Delta^*|_{\frac{b}{2}}$
$|3\Delta^*|_{\frac{b}{2}}$
$\left|\left(\frac{b}{2}-1\right)\Delta^*\right|_{\frac{b}{2}}$
BIGHT-NUMBER FOR B IS
$|-(m-n-1)\Delta^*|_{2m}$
BIGHT-NUMBER FOR A IS 0
BIGHT-NUMBER FOR B IS 0.
BIGHT-NUMBER FOR A IS 0
$2m-(2n+1)-1 = 2m-2n-2$ BIGHT UNITS
HENCE FOR B: $\frac{2m-2n-2}{2} = m-n-1$ BIGHT DISTANCES
BIGHT-INDEX NUMBER b-1 RELATIVE TO A BUT BIGHT-POINT BELONGS TO B.
BIGHT-INDEX NUMBER 0 FOR STRING A.
BIGHT-INDEX NUMBER 0 FOR STRING B.
BIGHT-INDEX NUMBER 0 FOR STRING A.

Fig. 321 — The complementary cyclic bight-number scheme for $p = p_m = 4n+2 = even$ and $b = b_m = 4m = even$.

The general layout of the complementary cyclic bight-number scheme is shown in Fig. 322.

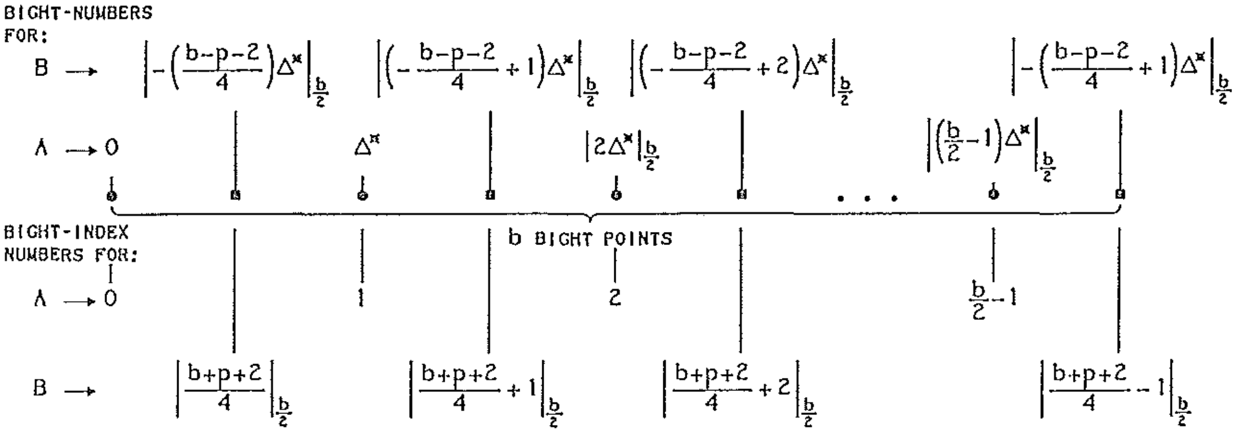

BIGHT-NUMBERS FOR:
B →
$\left|-\left(\frac{b-p-2}{4}\right)\Delta^*\right|_{\frac{b}{2}}$
$\left|\left(-\frac{b-p-2}{4}+1\right)\Delta^*\right|_{\frac{b}{2}}$
$\left|\left(-\frac{b-p-2}{4}+2\right)\Delta^*\right|_{\frac{b}{2}}$
$\left|-\left(\frac{b-p-2}{4}+1\right)\Delta^*\right|_{\frac{b}{2}}$
A → 0
Δ^*
$|2\Delta^*|_{\frac{b}{2}}$
$\left|\left(\frac{b}{2}-1\right)\Delta^*\right|_{\frac{b}{2}}$
b BIGHT POINTS
BIGHT-INDEX NUMBERS FOR:
A → 0
1
2
$\frac{b}{2}-1$
B →
$\left|\frac{b+p+2}{4}\right|_{\frac{b}{2}}$
$\left|\frac{b+p+2}{4}+1\right|_{\frac{b}{2}}$
$\left|\frac{b+p+2}{4}+2\right|_{\frac{b}{2}}$
$\left|\frac{b+p+2}{4}-1\right|_{\frac{b}{2}}$

Fig. 322 — The complementary cyclic bight-number scheme for $p = p_m = even$ and $b = b_m = even$ with g.c.d. $(2p_m\,,\ [p_m + b_m]) = 2$.

Let for the imaginary string-run (hence string-run B) $i_y = a^*$ and $i_{\frac{p}{2}-y-1} = b^*$. Then for the case $p_m = 4n$; $b_m = 4m+2$, where n and m are whole numbers, we obtain (see Fig. 323):

$$\left|i_y + i_{\frac{p}{2}-y-1}\right|_{\frac{b}{2}} = |i_y + i_{2n-y-1}|_{\frac{b}{2}}$$

$$= \Big||[-(m-n)+y]\Delta^*|_{2m+1} + |[-(m-n)+2n-1-y]\Delta^*|_{2m+1}\Big|_{2m+1}$$

$$
\begin{aligned}
\left|i_y + i_{\frac{p}{2}-y-1}\right|_{\frac{b}{2}} &= |[-2m+2n+(2n-1)]\Delta^*|_{2m+1} \\
&= |-(2m+1)\Delta^* + 4n\Delta^*|_{2m+1} \\
&= |4n\Delta^*|_{2m+1}\,;\ \text{with}\ \Delta^* = \frac{b'\cdot\Delta_{II,1}p'-1}{p'}\,,\text{where}\ p'=\tfrac{p}{2}\,,\ b'=\tfrac{b}{2}\ : \\
&= |-2|_{2m+1} = 2m-1 = \frac{b}{2}-2 = b'-2\,.
\end{aligned}
$$

$$i_y + i_{\frac{p}{2}-y-1} = sb'-2 \qquad \text{where } s=1 \text{ or } 2 \text{ since } 0 \le i \le b'-1\,.$$

$$i_y = i_{\frac{p}{2}-y-1} \ \text{ for } \ s=2\,.$$

But $i_y \neq i_{\frac{p}{2}-y-1}$ since $2y = even \neq 2n-1 = odd$, hence $s=1$. Thus:

$$a^* + b^* = i_y + i_{\frac{p}{2}-y-1} = b'-2 = \frac{b}{2}-2\,.$$

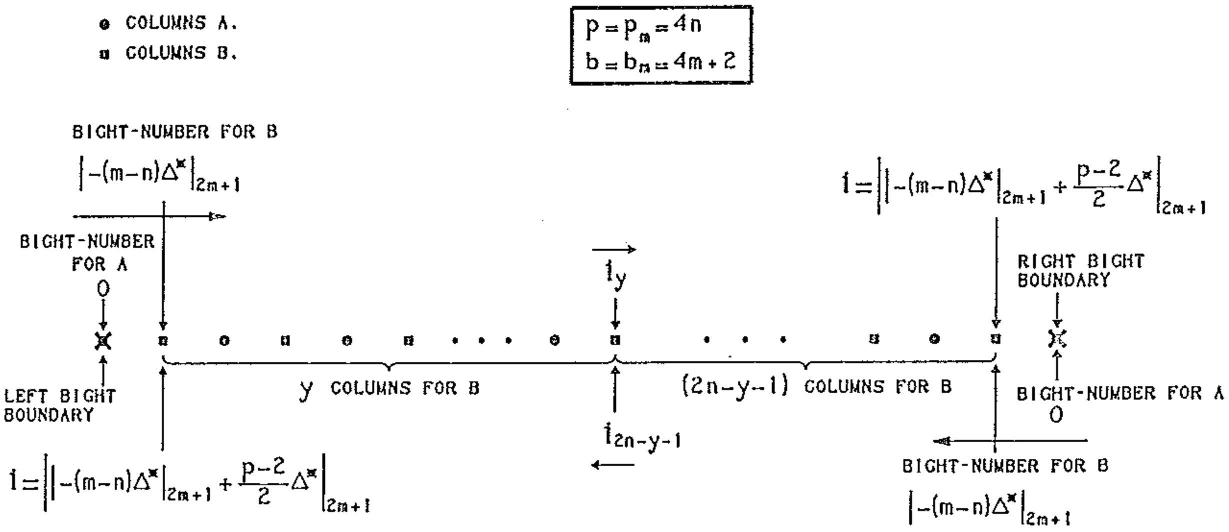


Fig. 323 — The general algorithm diagram for string B.

For the case $p_m = 4n+2$; $b_m = 4m$, where n and m are whole numbers, we obtain (see Fig. 324):

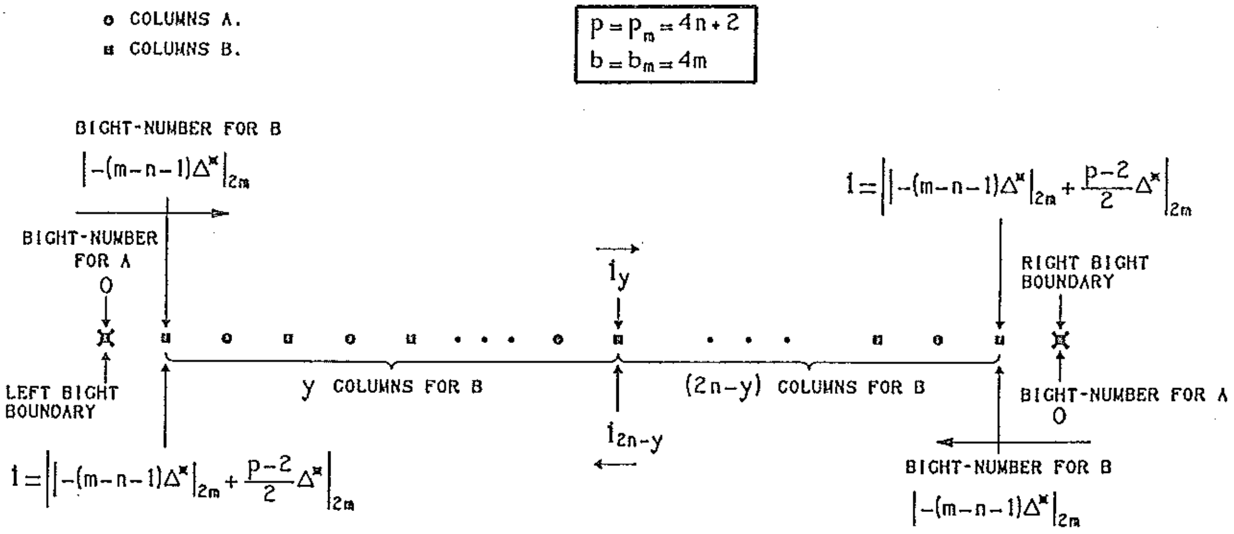


Fig. 324 — The general algorithm diagram for string B.

$$\begin{aligned}
\left|i_y + i_{\frac{p}{2}-y-1}\right|_{\frac{b}{2}} &= |i_y + i_{2n-y}|_{\frac{b}{2}} \\
&= ||[-(m-n-1)+y]\Delta^*|_{2m} + |[-(m-n-1)+2n-y]\Delta^*|_{2m}|_{2m} \\
&= |[-2m+2n+2+2n]\Delta^*|_{2m} \\
&= |(4n+2)\Delta^*|_{2m}\ ;\ \text{with}\ \Delta^* = \frac{b'\cdot\Delta_{II,1}p'-1}{p'}\ \text{, where}\ p'=\tfrac{p}{2}\ ,\ b'=\tfrac{b}{2}\ : \\
&= |-2|_{2m} = 2m-2 = \frac{b}{2}-2 = b'-2\,.
\end{aligned}$$

$$i_y + i_{\frac{p}{2}-y-1} = sb'-2 \qquad \text{where}\ s=1\ \text{or}\ 2\ \text{since}\ 0\le i\le b'-1\,.$$

$$i_y = i_{\frac{p}{2}-y-1}\ \text{for}\ s=2\,.\ \text{Hence}\ 2y=2n\,,\ \text{consequently}\ y=n\,.$$

It follows from the path in the RKT with its associated parities that Δ^* is **odd** (see Fig. 325).

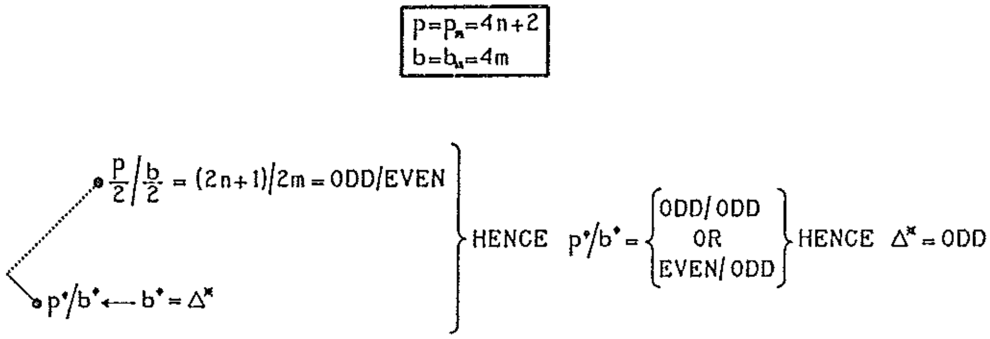

p=pm=4n+2
b=bm=4m
p/2 / b/2 = (2n+1)/2m = ODD/EVEN
p*/b* ← b* = Δ*
HENCE p*/b* = ODD/ODD OR EVEN/ODD
HENCE Δ* = ODD

Fig. 325 — The parity of Δ^* in association with $p=p_m=4n+2$ and $b=b_m=4m$.

Say $\Delta^* = 2t+1$, then for $y=n$ we obtain:

$$\begin{aligned}
i_y = i_n &= |[-(m-n-1)+n]\Delta^*|_{2m} \\
&= |(-m+2n+1)\Delta^*|_{2m}\ ;\ \text{with}\ \Delta^* = \frac{b'\cdot\Delta_{II,1}p'-1}{p'} = \frac{2m\cdot\Delta_{II,1}p'-1}{2n+1}\ : \\
&= |-m\Delta^*-1|_{2m}\ ;\ \text{with}\ \Delta^*=2t+1\ : \\
&= |-m-1|_{2m} = m-1\,.\ \text{But}\ m-1<2m-1\,,\ \text{hence}\ s\ne 2\,.\ \text{Thus :}
\end{aligned}$$

$$a^*+b^* = i_y + i_{\frac{p}{2}-y-1} = b'-2 = \frac{b}{2}-2\,.$$

Hence for $p=p_m=\mathit{even}$ and $b=b_m=\mathit{even}$ with g.c.d.$(2p_m\,,\,[p_m+b_m])=2$ we obtain for string B that $a^*+b^* = \frac{b_m}{2}-2$.

Regular Cylindrical Bihelix Braids

In contrast with the Regular Cylindrical Braids, which have for the left to right half-cycles the same helix angle as for the right to left half-cycles, the Regular Cylindrical Bihelix Braids have for those respective half-cycles a different angle.

Just as a Cylindrical Knot may be regarded as a finite length of Round Braid with a minimum number of free string-ends, so may a Cylindrical Bihelix Knot be regarded as a finite length of Round Bihelix Braid with a minimum number of free string-ends.[†] Due to the different helix angles for the respective left to right and the right to left half-cycles, the weaving pattern will always have a spiral effect. This spiral effect in the Cylindrical Bihelix Knots will of course influence their string-run diagrams by making them somewhat more complicated. Hence let's first see how their string-run diagrams are constructed.

The two uppermost diagrams in Fig. 326 show the essential construction process for the string-run of a [3 , 7] Round Bihelix Braid. That is a Round Bihelix Braid which has 3 leads with a left-hand helix and 7 leads with a right-hand helix. In the uppermost left-hand diagram we did set off 3 unit-lengths horizontally followed by 7 unit-lengths vertically. Then we connected the starting-point with the end-point, thus creating the right-angled triangle. In this triangle the basic grid is produced by the vertical and horizontal lines through the points set off. Next we draw lines perpendicular to the hypotenuse through its end-points. Finally, about an axis through the lower vertex of this right-angled triangle we rotate in the clockwise direction the obtained diagram through an angle of *arctan* $3/7 = 23.1986°$.[‡] This gives us the uppermost right-hand diagram. By extending the grid in this uppermost right-hand diagram between the two parallel horizontal lines (which represent the cylinder generating line of section) we obtain the string-run of the [3 , 7] Round Bihelix Braid. Any two vertically aligned points on these two parallel horizontal lines represent the same single point on the cylinder generating line of section. A line between the two horizontal lines, connecting crossing-points from lower right to upper left, represents a bight-boundary line when these crossing-points on such a line are made into bight-points. Such a bight-boundary line will always have a slant in a Cylindrical Bihelix Braid.[††] Here we find the very significant difference between the bight-boundaries of a Regular Cylindrical Bihelix Braid and a Regular Cylindrical Braid that in the former the end-points of a bight-boundary line don't coincide (the end-points are not vertically aligned), whereas in the latter they do coincide. In a Round Bihelix Braid a bight-boundary line is a segment of a column-helix, and in the [3 , 7] Round Bihelix Braid we got $\sigma = 4$ such column-helixes since $|3 - 7| = 4 = \sigma$ (see Fig. 327). A Cylindrical Bihelix Braid is a Regular Cylindrical Bihelix Braid when in a finite lenghth of Round Bihelix Braid the set of left-hand bights and the set of right-hand bights each lie in a not stacked fashion on one or more of σ **adjacent** bight-boundary lines. In Figs. 326 & 328 the set of left-hand bights and the set of right-hand bights of the Regular Cylindrical Bihelix Braid lie each on one bight-boundary line. Note that in this braid the left-hand bight-boundary line and the right-hand bight-boundary line do not both belong to the same column-helix. Of the four adjacent bight-boundary lines on the left-hand side in Fig. 327, the bight-boundary lines σ_1 , σ_2 , σ_4 each carry one bight, and of the four adjacent bight-boundary lines on the right-hand side the bight-boundary lines σ_1 , σ_2 , σ_3 each carry one bight.

[†] Recall that we can also regard a Cylindrical Knot as a length of Flat Braid wrapped around a cylinder with strings of opposite ends suitably joined. Although an in essence similar procedure may be used for a Cylindrical Bihelix Knot, it is not very suitable.

[‡] *arctan* x (also written as tan^{-1}) is the *arc* of which the *tangent* is equal to x.

[††] Only when both helix angles are the same is a bight-boundary line vertical and do we have a Regular Cylindrical Braid.

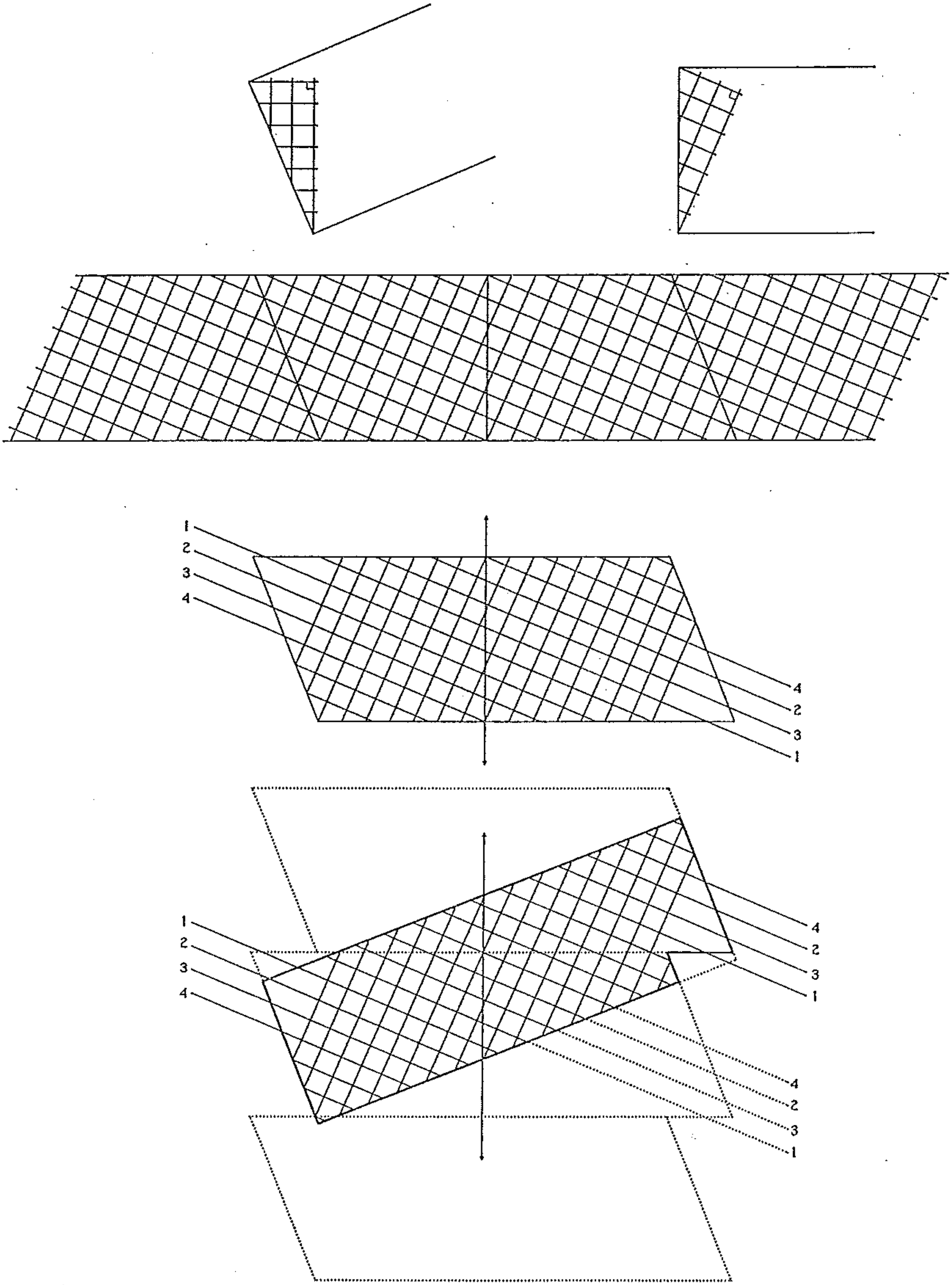


Fig. 326 — The string-run diagram of a [3, 7] Bihelix Braid.

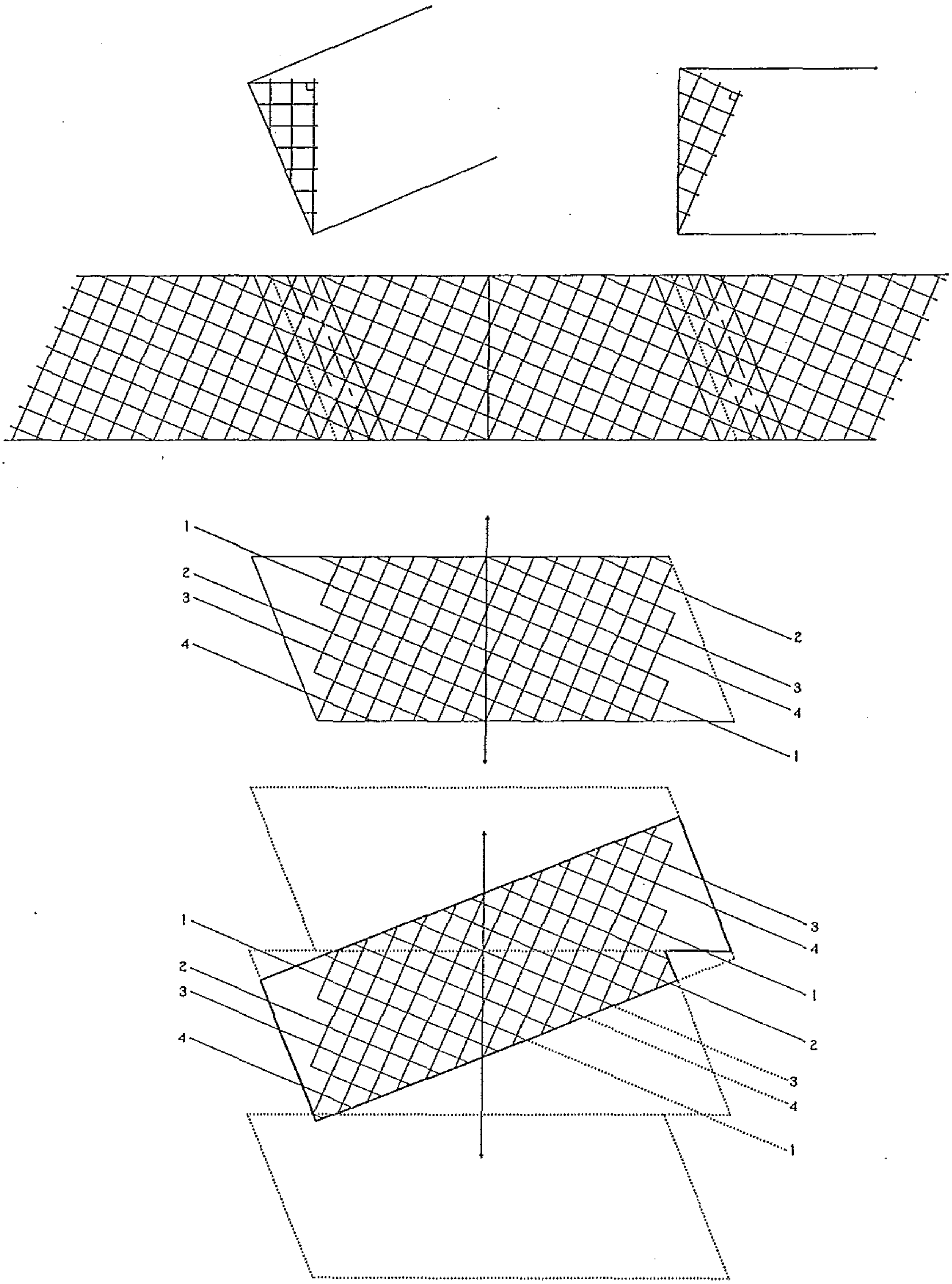


Fig. 327 — The string-run diagram of a [3 , 7] Bihelix Braid.

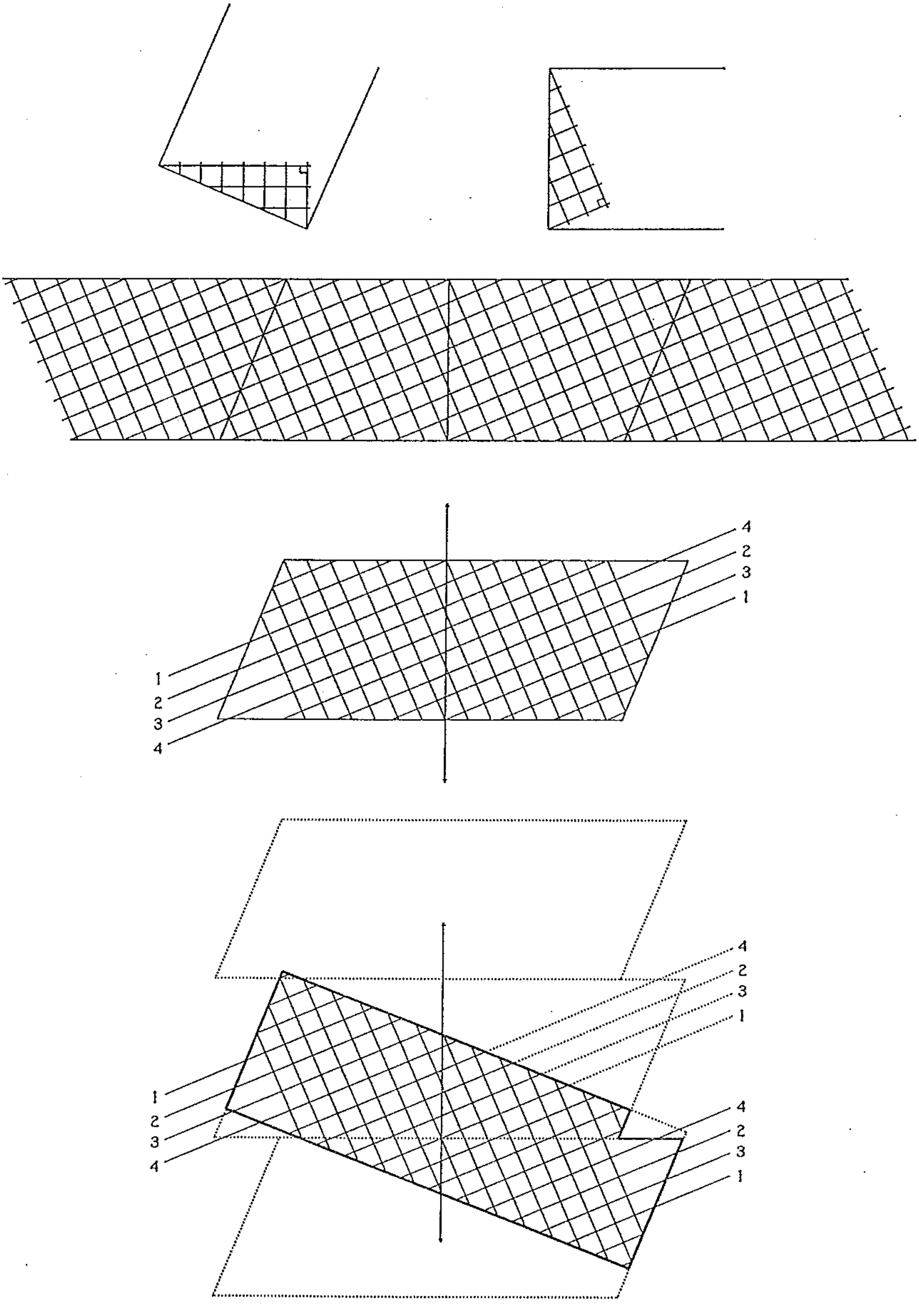


Fig. 328 — The string-run diagram of a [7, 3] Bihelix Braid.

The essential construction process for the string-run of a [7 , 3] Round Bihelix Braid is shown in Fig. 328. That is a Round Bihelix Braid which has 7 leads with a left-hand helix and 3 leads with a right-hand helix. In the uppermost left-hand diagram we did set off 7 unit-lengths horizontally followed by 3 unit-lengths vertically. Then we connected the starting-point with the end-point, thus creating the right-angled triangle. In this triangle the basic grid is produced by the vertical and horizontal lines through the points set off. Next we draw lines perpendicular to the hypotenuse through its end-points. Finally, about an axis through the lower vertex of this right-angled triangle we rotate in the clockwise direction the obtained diagram through an angle of $arctan\ 7/3 = 66.8014°$ and extend the grid between the two horizontal lines.

The two horizontal lines (the cylinder generating line of section) of the Regular Cylindrical Bihelix Braids interfere with some string-run intersection-points and hence the lowermost modified string-run diagrams depicted in the Figs. 326 - 328 are often preferred.

In this Issue of *The Braider* we shall limit our discussion to the Round Bihelix Braids for which $\sigma = 1$, and as an Example of such a braid we shall take the [11 , 10] Round Bihelix Braid.

The construction of the string-run of this Round Bihelix Braid is shown in the upper part of Fig. 329. Since $\sigma = 1$, there is only one column-helix, and consequently there is only the σ_1 bight-boundary line.

Let's take the Regular Bihelix Knot depicted by the third row of diagrams in Fig. 329. The transfer of points between the top line and the bottom line in these diagrams is indicated by the vertical line with arrows at each of its ends. The half-cycles are numbered in the usual convential way. The bottom row of diagrams in Fig. 329 depict exactly the same knot: only the cylinder generating line of section is in a different position!

Although in this Bihelix Knot both left and right bight-boundary lines start and end on the same cylinder generating line of section, the reader will no doubt realize that this is not a necessity. Hence for this reason alone (there is an other reason as well as we shall see a little later) we need to specify for a Bihelix Knot the number of its crossing-points. For the knot in Fig. 329 the number of crossing-points is 168.

After braiding this knot, the skewness indicated by the slanting bight-boundary lines will more or less disappear due to some automatic realignment. From the bottom row of diagrams it will be evident that the best possible realignment will be obtained when both the left and right bight-boundary lines start and end on the same cylinder generating line of section. **Hence only those Regular Cylindrical Bihelix Braids whose left and right bight-boundary lines start and end on the same cylinder generating line of section will be called Prime Regular Cylindrical Bihelix Braids; the one string variety are then the Prime Regular Bihelix Knots.**

In Fig. 330 are depicted the first 25 consecutive Prime Regular Cylindrical Bihelix Braids associated with the [11 , 10] Round Bihelix Braid. In the first 21 diagrams the number of column-lines increase in a regular manner by one in each consecutive diagram. Diagram #22, however, although having the same number of column-lines as diagram #21, has one more crossing-point (this is the other reason, referred to earlier, for specifying the number of crossing-points in order for unambiguously specifying a Prime Regular Cylindrical Bihelix Braid). This phenomenon will occur at regular intervals.

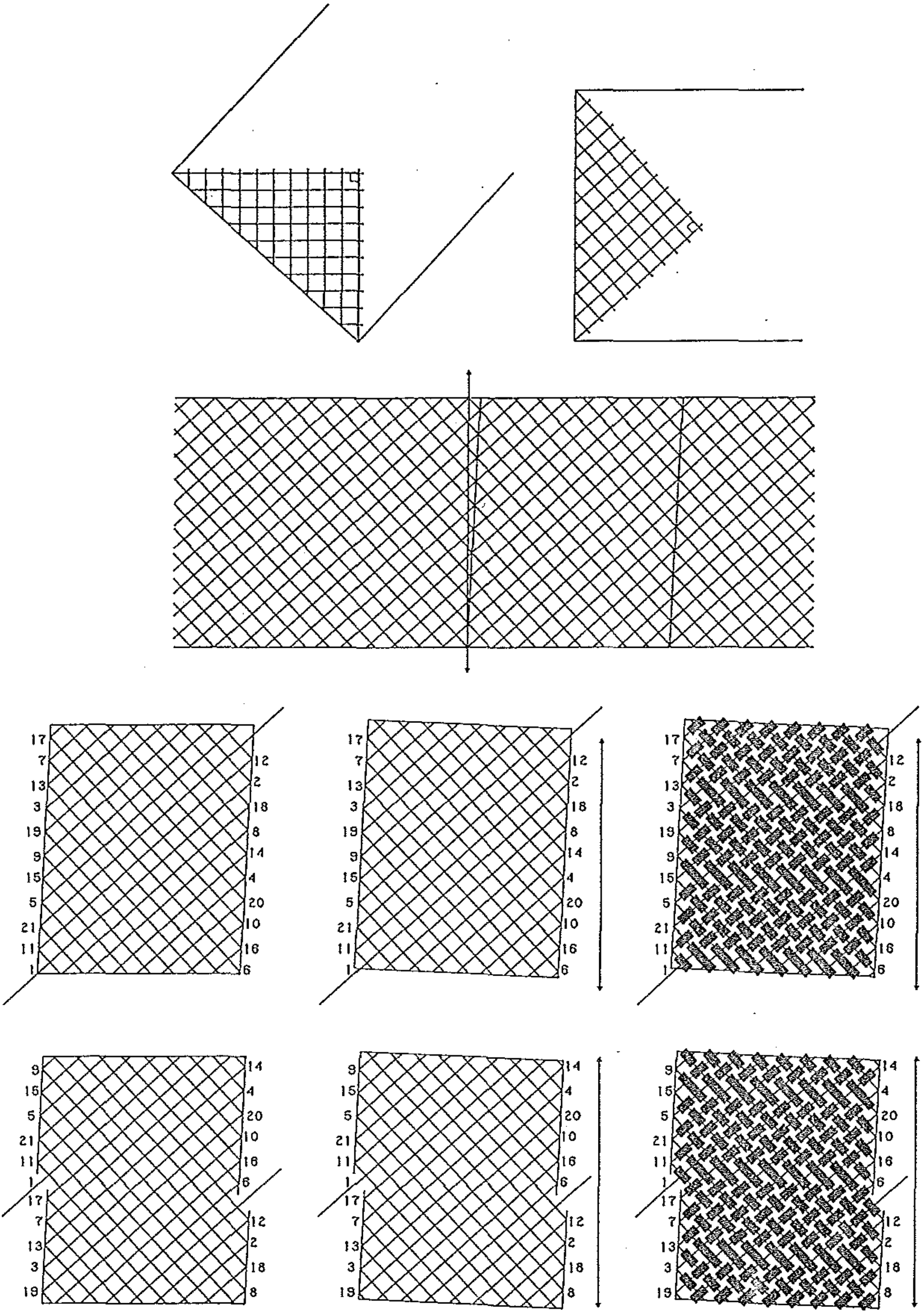


Fig. 329 — The string-run diagram of a [11, 10] Bihelix Braid.

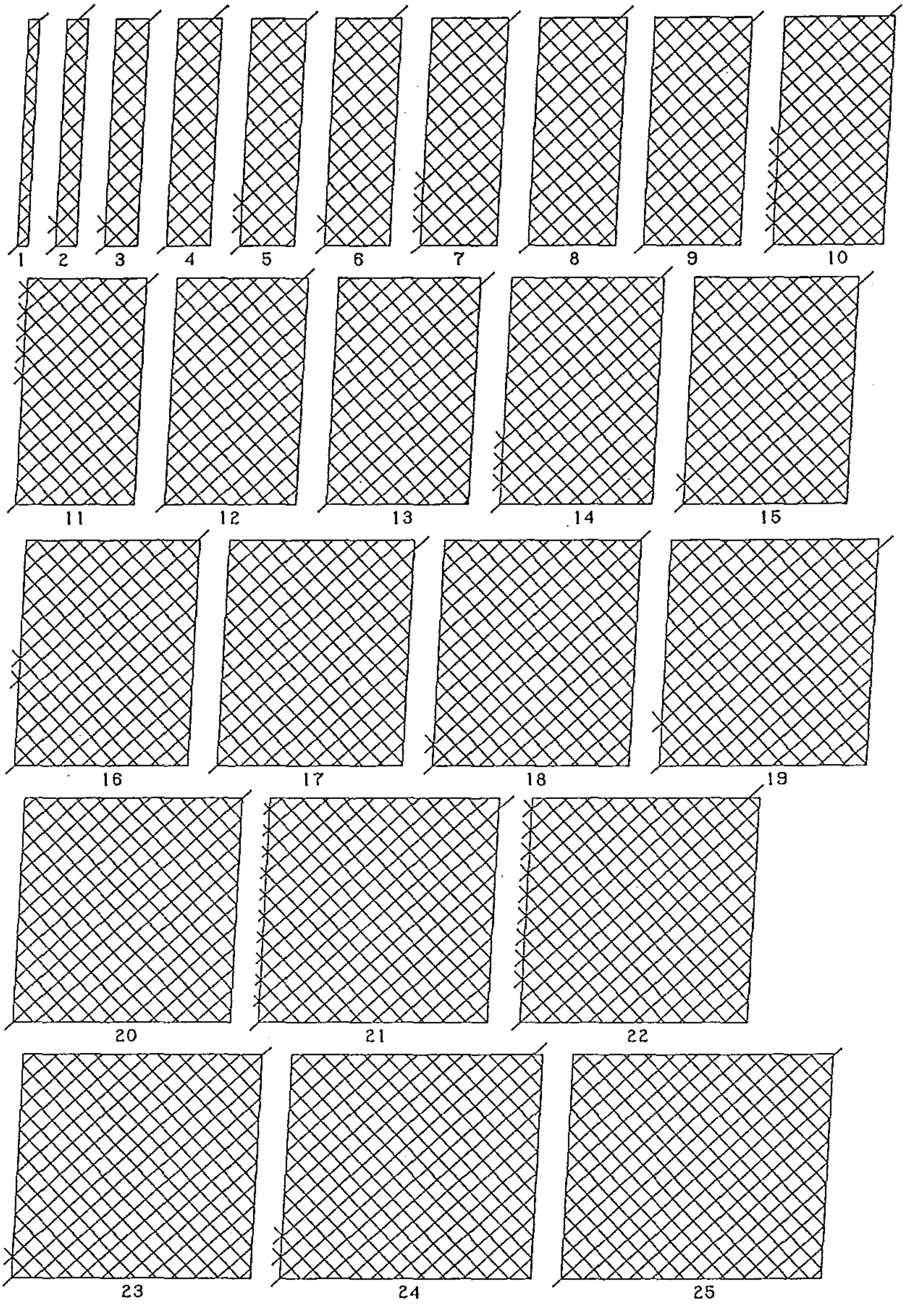


Fig. 330 — The first 25 consecutive Prime Regular Cylindrical [11, 10] Bihelix Braids.

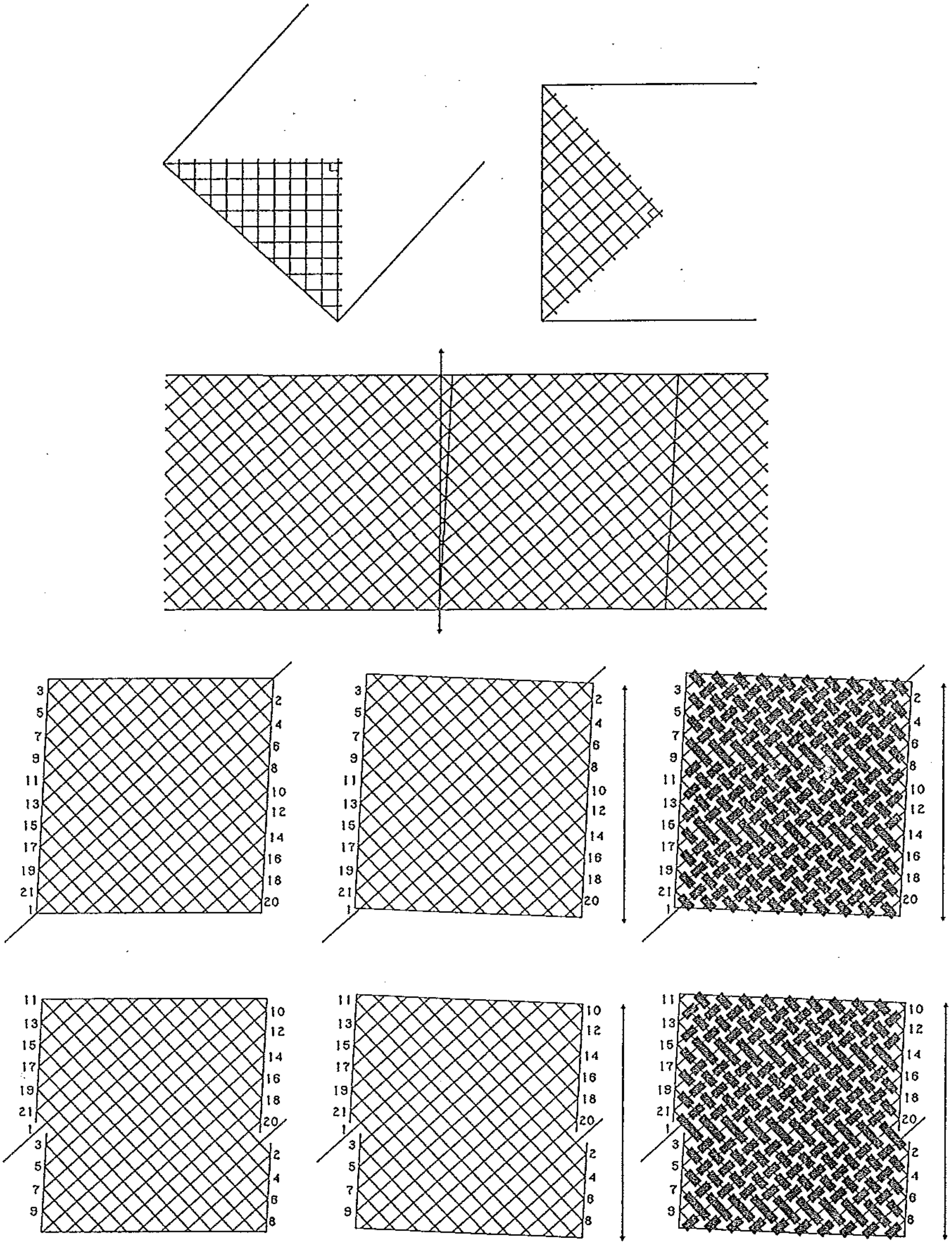


Fig. 331 — The Prime Regular [11 , 10] Bihelix Knot with 200 crossing-points.

In our Example it occurs for the first time between the Prime Regular Cylindrical Bihelix Braids with 210 crossing-points and 211 crossing-points, next it will occur between the Prime Regular Cylindrical Bihelix Braids with 431 and 432 crossing-points, next between the Prime Regular Cylindrical Bihelix Braids with 652 and 653 crossing-points, and so on. In due course we will meet these Prime Regular Cylindrical Bihelix Braids with 211, 431 and 653 crossing-points again in connection with another braid-form, and we shall then discuss the special properties they possess.

The Prime Regular Bihelix Knot in Fig. 331 has 200 crossing-points. Its string-run diagram is #20 in Fig. 330. This is another special knot. It is identical to the Regular Knot in Fig. 332 in which the last half-cycle has not been laid down. Regular Cylindrical Braids in which the last half-cycle of one of the string has not been laid down are called **Premature Regular Cylindrical Braids.**

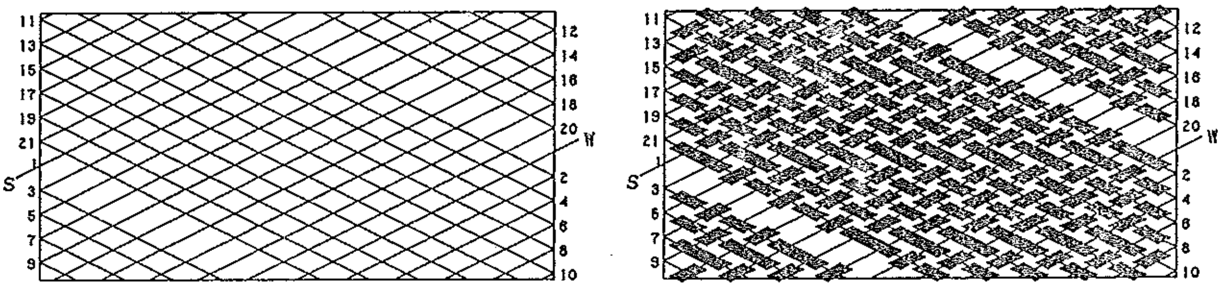


Fig. 332 — A $p/b = 21/11$ Premature Regular Knot.

Only a very few Premature Regular Cylindrical Braids are **Prime** Regular Cylindrical Bihelix Braids; in general they are only some of the Regular Cylindrical Bihelix Braids, an Example is shown in Fig. 333. Here the Premature Regular Cylindrical Braid is a Premature Regular Knot with 17-parts and 11-bights. It is identical to the Regular Bihelix Knot depicted by the second and third row diagrams in Fig. 333. It will readily be seen that a good realignment of the string-run is not possible. Consequently such knots should not be used in high class work.

In the string-run diagrams of Fig. 334 the dotted line depicts the missing last half-cycle of the Regular Cylindrical Braids having 11-bights. The bight-boundary line A is the bottom part of the right-hand bight-boundary line of the knot in the second row of diagrams of Fig. 333, and the bight-boundary line B is the right-hand bight-boundary line of the knot in Fig. 331. Note that between every two adjacent column-lines there is only one Regular Cylindrical Bihelix Braid (in most cases **not** Prime) which is equivalent to a Premature Regular Cylindrical Braid.

It will be evident that the string-run of a Prime Regular Cylindrical Bihelix Braid can be represented by an (in most cases) extended string-run diagram of a Regular Cylindrical Braid.

⋆⋆ *From the sequence of diagrams in Fig. 335 explain the procedure used for constructing the diagrams. The same numbered diagrams in Figs. 335 and 330 depict the same braid.*

We shall now have a brief look into the way we can calculate the number of string-run crossings of a Prime Regular Cylindrical $[(n+1)\,,\, n]$ Bihelix Braid.

Let N be the approximate number of crossing-points required, and let the **T-line** (see the diagrams of Fig. 335) be $\left|2m + \left\lfloor \frac{m}{n} \right\rfloor\right|_{2(n+1)}$ rows (in cyclic fashion) above the **S-line** (the dotted line through the bight-point of the Standing End S).[†]

[†] $\lfloor x \rfloor$ denotes the greatest whole number equal to or smaller than x.

Then: $m = \left\lfloor \dfrac{N+n}{2n^2+2n+1} \right\rfloor$.

Let $y = N + n - m(2n^2+2n+1) = zn + r$, where $0 \le r \le n$.

Then we have the following three cases:[†]

$r < \left\lceil \frac{z-1}{2} \right\rceil$	$r > \left\lceil \frac{z-1}{2} \right\rceil$	$r = \left\lceil \frac{z-1}{2} \right\rceil$
$y_L^* = (z-1)n + \left\lceil \frac{z-2}{2} \right\rceil$	$y_L^* = zn + \left\lceil \frac{z-1}{2} \right\rceil$	$y^* = y = zn + \left\lceil \frac{z-1}{2} \right\rceil$
$y_H^* = zn + \left\lceil \frac{z-1}{2} \right\rceil$	$y_H^* = (z+1)n + \left\lceil \frac{z}{2} \right\rceil$	

Then :

$$\left.\begin{aligned} N_L &= m(2n^2+2n+1) - n + y_L^* \\ N_H &= m(2n^2+2n+1) - n + y_H^* \end{aligned}\right\} \text{ let } N^* \text{ be the } N \text{ selected, then :}$$

$$\begin{aligned} N^* &= m(2n^2+2n+1) - n + z^*n + \left\lceil \frac{z^*-1}{2} \right\rceil \\ &= n\left(2mn + 2m + z^* - 1 + \left\lfloor \frac{m + \left\lceil \frac{z^*-1}{2} \right\rceil}{n} \right\rfloor \right) + \left| m + \left\lceil \frac{z^*-1}{2} \right\rceil \right|_n \\ &= n(p-1) + \left\{ \left| m + \left\lceil \frac{z^*-1}{2} \right\rceil \right|_n \right\} . \\ p &= 2mn + 2m + z^* + \left\lfloor \frac{m + \left\lceil \frac{z^*-1}{2} \right\rceil}{n} \right\rfloor . \end{aligned}$$

Here p is the number of parts of the unextended Premature Regular Cylindrical Braid, which with its extension of $\left\{ \left| m + \left\lceil \frac{z^*-1}{2} \right\rceil \right|_n \right\}$ bight-points (this are the extreme right-hand bights between the M-line and T-line) is equivalent to the Prime Regular Cylindrical $[(n+1), n]$ Bihelix Braid with N^* crossings in its string-run.

Example 1 :

Say $N = 873$ and $n = 10$. Then: $2n^2 + 2n + 1 = 200 + 20 + 1 = 221$.

$$m = \left\lfloor \frac{N+n}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{873+10}{221} \right\rfloor = \lfloor 3.996 \rfloor = 3 .$$

$$y = N + n - m(2n^2 + 2n - 1) = 220 = 22n + 0 = 21n + 10 = zn + r .$$

$$r = 10 = \left\lceil \frac{21-1}{2} \right\rceil = \left\lceil \frac{z-1}{2} \right\rceil .$$

$$y^* = y = zn + \left\lceil \frac{z-1}{2} \right\rceil = 21n + \left\lceil \frac{21-1}{2} \right\rceil = 21n + 10 = 220 .$$

$$N = m(2n^2+2n+1) - n + y^* = 873 \quad \to p = 88 ; \left\{ \left| m + \left\lceil \frac{z^*-1}{2} \right\rceil \right|_n \right\} = 3 .$$

[†] $\lceil x \rceil$ denotes the smallest whole number equal to or greater than x.

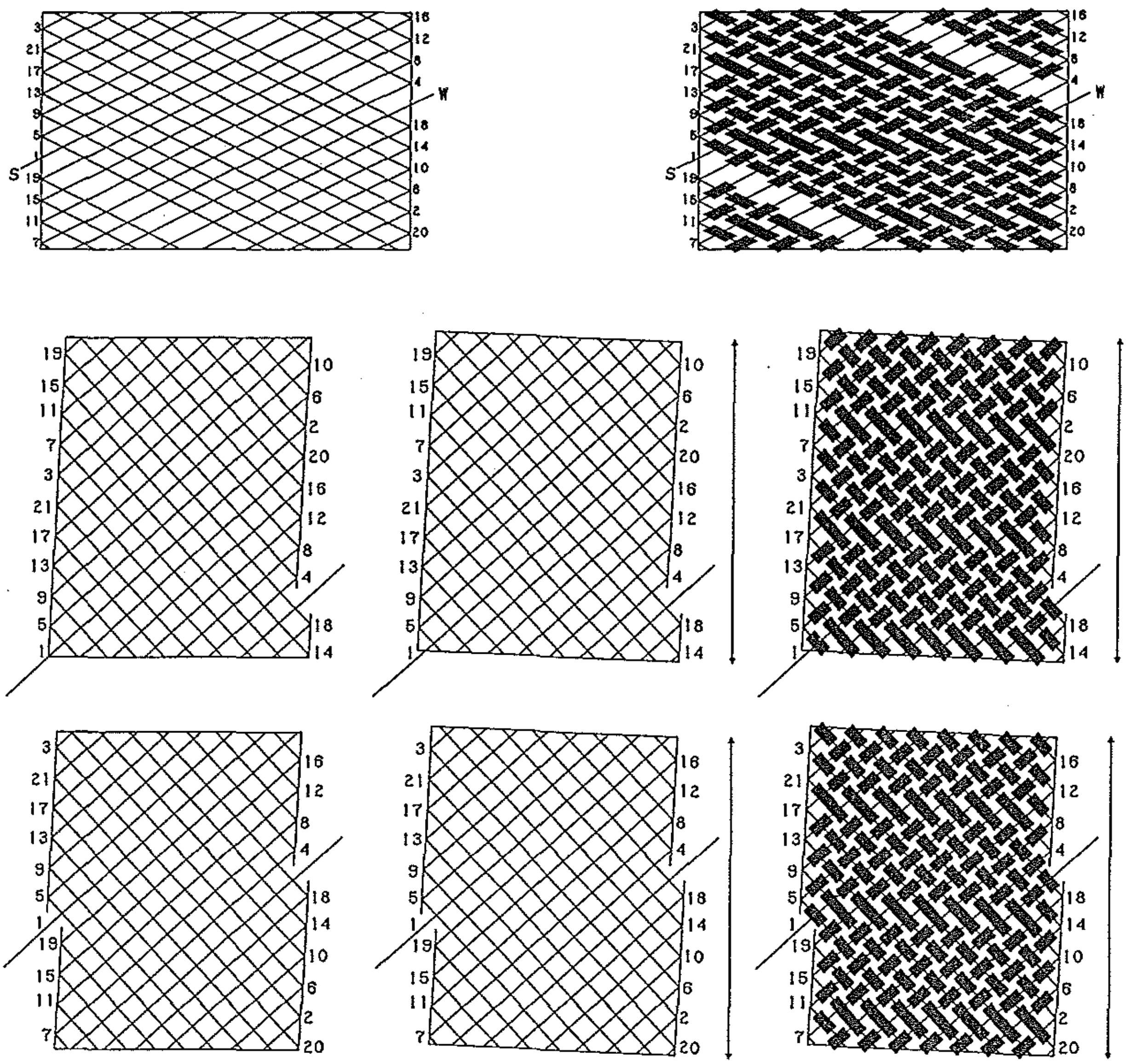


Fig. 333 – A $p/b = 17/11$ Premature Regular Knot.

Example 2 :

Say $N = 432$ and $n = 10$. Then: $2n^2 + 2n + 1 = 200 + 20 + 1 = 221$.

$$m = \left\lfloor \frac{N+n}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{432+10}{221} \right\rfloor = \lfloor 2 \rfloor = 2 .$$

$$y = N + n - m(2n^2 + 2n - 1) = 432 + 10 - 2 \cdot 221 = 0 = 0n + 0 = zn + r .$$

$$r = 0 = \left\lceil \frac{0-1}{2} \right\rceil = \left\lceil \frac{z-1}{2} \right\rceil .$$

$$y^* = y = zn + \left\lceil \frac{z-1}{2} \right\rceil = 0n + \left\lceil \frac{0-1}{2} \right\rceil = 0n + 0 = 0 .$$

$$N = m(2n^2 + 2n + 1) - n + y^* = 432 \quad \to p = 44 \, ; \; \left\{ \left| m + \left\lceil \frac{z^*-1}{2} \right\rceil \right|_n \right\} = 2 .$$

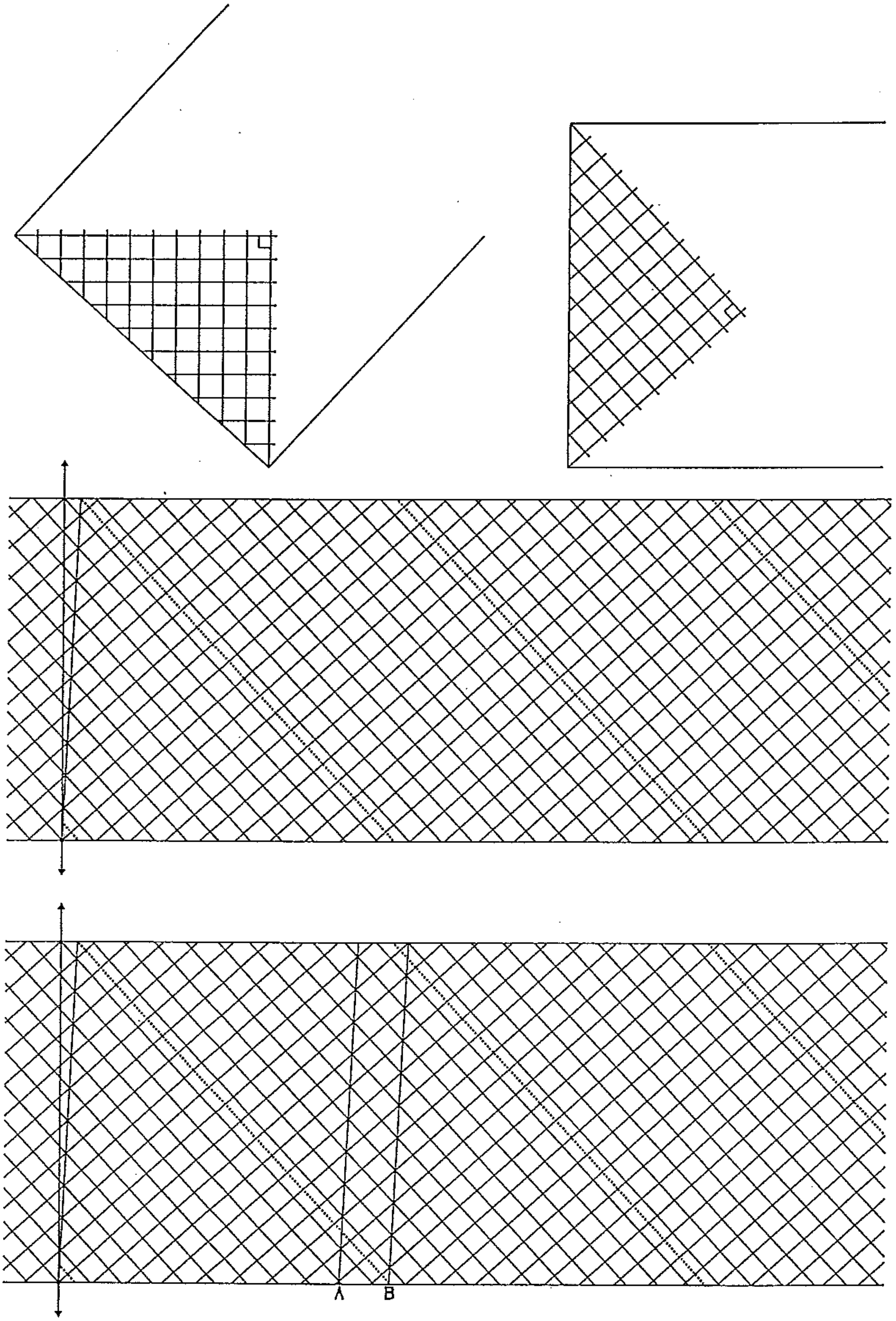
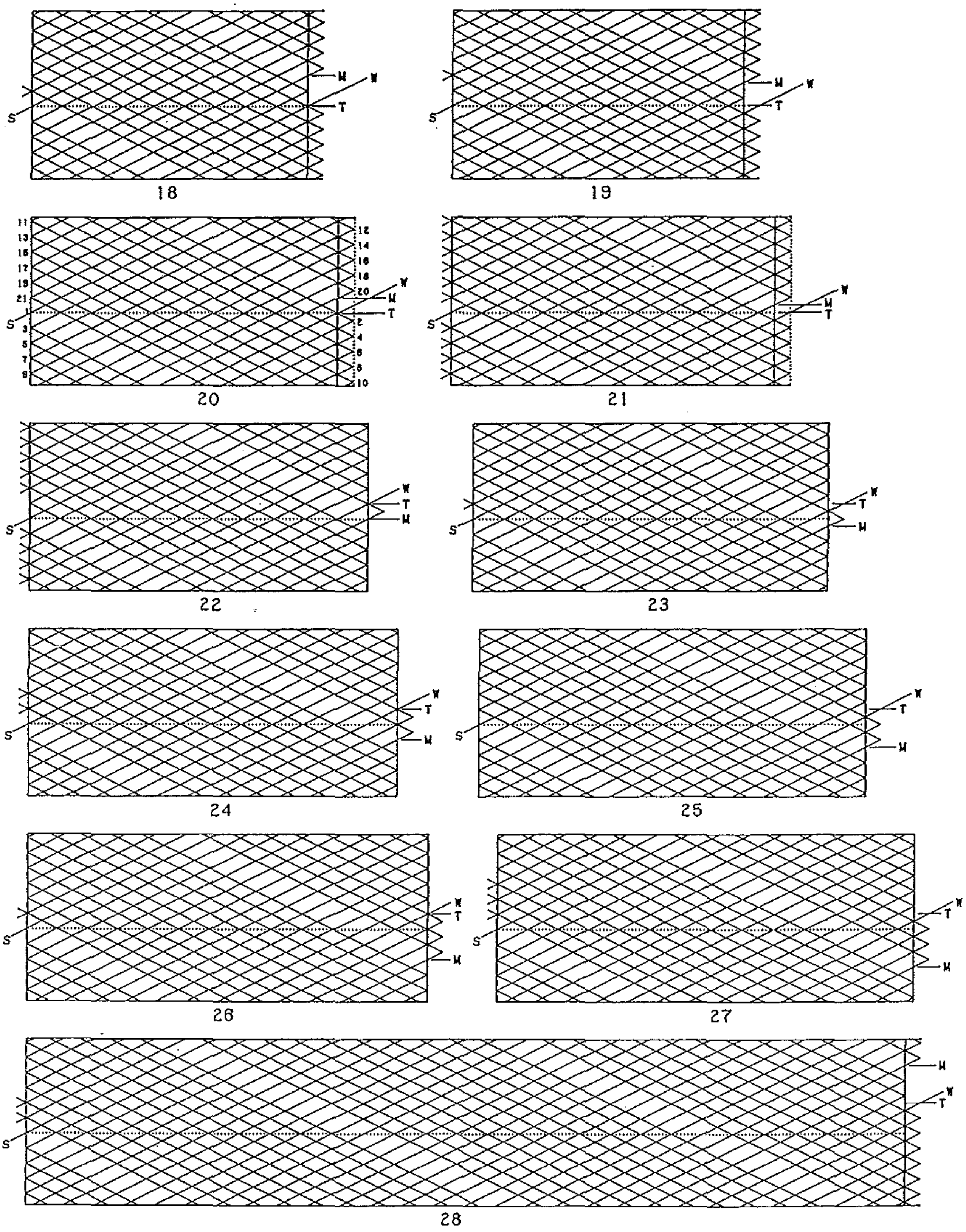


Fig. 334 – Premature Regular Cylindrical Braids in the [11, 10] Bihelix Round Braid.



18 19 20 21 22 23 24 25 26 27 28

Fig. 335 — Prime Regular Cylindrical [11, 10] Bihelix Braids as extended Premature Regular Cylindrical Braids.

Example 3:

Say $N = 564$ and $n = 10$. Then: $2n^2 + 2n + 1 = 200 + 20 + 1 = 221$.

$$m = \left\lfloor \frac{N+n}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{564+10}{221} \right\rfloor = \lfloor 2.597 \rfloor = 2\,.$$

$$y = N + n - m(2n^2 + 2n - 1) = 564 + 10 - 2 \cdot 221 = 132 = 13n + 2 = zn + r\,.$$

$$r = 2 < \left\lceil \frac{13-1}{2} \right\rceil = \left\lceil \frac{z-1}{2} \right\rceil\,.$$

$$y_L^* = (z-1)n + \left\lceil \frac{z-2}{2} \right\rceil = (13-1)n + \left\lceil \frac{13-2}{2} \right\rceil = 12n + 6 = 126\,.$$

$$y_H^* = zn + \left\lceil \frac{z-1}{2} \right\rceil = 13n + \left\lceil \frac{13-1}{2} \right\rceil = 13n + 6 = 136\,.$$

$$N_L = m(2n^2 + 2n + 1) - n + y_L^* = 558 \quad \rightarrow p = 56\,; \left\{ \left| m + \left\lceil \frac{z^*-1}{2} \right\rceil \right|_n \right\} = 8\,.$$

$$N_H = m(2n^2 + 2n + 1) - n + y_H^* = 568 \quad \rightarrow p = 57\,; \left\{ \left| m + \left\lceil \frac{z^*-1}{2} \right\rceil \right|_n \right\} = 8\,.$$

Example 4:

Say $N = 6842$ and $n = 10$. Then: $2n^2 + 2n + 1 = 200 + 20 + 1 = 221$.

$$m = \left\lfloor \frac{N+n}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{6842+10}{221} \right\rfloor = \lfloor 31.005 \rfloor = 31\,.$$

$$y = N + n - m(2n^2 + 2n - 1) = 6842 + 10 - 31 \cdot 221 = 1 = 0n + 1 = zn + r\,.$$

$$r = 1 > \left\lceil \frac{0-1}{2} \right\rceil = \left\lceil \frac{z-1}{2} \right\rceil = 0\,.$$

$$y_L^* = zn + \left\lceil \frac{z-1}{2} \right\rceil = 0n + \left\lceil \frac{0-1}{2} \right\rceil = 0n + 0 = 0\,.$$

$$y_H^* = (z+1)n + \left\lceil \frac{z}{2} \right\rceil = 1n + \left\lceil \frac{0}{2} \right\rceil = 1n + 0 = 10\,.$$

$$N_L = m(2n^2 + 2n + 1) - n + y_L^* = 6841 \quad \rightarrow p = 685\,; \left\{ \left| m + \left\lceil \frac{z^*-1}{2} \right\rceil \right|_n \right\} = 1\,.$$

$$N_H = m(2n^2 + 2n + 1) - n + y_H^* = 6851 \quad \rightarrow p = 686\,; \left\{ \left| m + \left\lceil \frac{z^*-1}{2} \right\rceil \right|_n \right\} = 1\,.$$

For the $[n, (n+1)]$ Prime Regular Cylindrical Bihelix Braids we obtain of course exactly the same general formulae since their string-runs are the mirror image of the $[(n+1), n]$ Prime Regular Cylindrical Bihelix Braids.

We can also obtain for the Prime Regular Cylindrical Bihelix Braids their equivalent extended Premature Regular Cylindrical Braids by means of the position of the M-line in relation to the the position of the T-line. This is the more graphical method associated with the question on pg. 387.

Although the grid-diagrams of the extended Premature Regular Cylindrical Braids do not give us a good picture of the actual braid (only the proper bihelix diagrams are able to do this), they are nevertheless of great value not only for the actual construction process employed but also as an aid in the derivation of the half-cycle algorithms.

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No.18 **MAY 1999.**

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for
the braiding artisan

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Solution to the Question in Issue No. 17

Question on pg. 387.

The attentive reader will have realised that the position of the **T-line** in relation to the **S-line** (specified at the bottom of pg. 387) was not really required for the formulae on pg. 388, and hence was obviously intended to give the reader a start in solving the question posed. On pg. 388 we could simply have begun with saying: let $m = \left\lfloor \dfrac{N+n}{2n^2+2n+1} \right\rfloor$. To explain the question, we need to understand the background to the formulae on pg. 388, and for this reason we have tabulated in the list below the sequential number of crossings in $[(n+1)\,,\,n]$ Prime Regular Cylindrical Bihelix Braids.

$$1n+1=-n+\{2n+1\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2n+1=-n+\{3n+1\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$3n+2=-n+\{4n+2\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$4n+2=-n+\{5n+2\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$5n+3=-n+\{6n+3\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$6n+3=-n+\{7n+3\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$\vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots$$

$$2n^2-n-1-n=-n+\{(2n-2)n+(n-1)\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2n^2-n-1=-n+\{(2n-1)n+(n-1)\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2n^2+n-n=2n^2=-n+\{(2n)n+n\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$2n^2+2n+1-n-1=2n^2+n=-n+\{(2n+1)n+n\}=-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2n^2+2n+1-n=2n^2+n+1=\{2n^2+2n+1\}-n+\{0n+0\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$2n^2+2n+1=\{2n^2+2n+1\}-n+\{n+0\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

* *

$$2n^2+2n+1+n+1=\{2n^2+2n+1\}-n+\{2n+1\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2n^2+2n+1+2n+1=\{2n^2+2n+1\}-n+\{3n+1\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2n^2+2n+1+3n+2=\{2n^2+2n+1\}-n+\{4n+2\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2n^2+2n+1+4n+2=\{2n^2+2n+1\}-n+\{5n+2\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$\vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots$$

$$\{2n^2+2n+1\}+2n^2-2n-1=\{2n^2+2n+1\}-n+\{(2n-2)n+(n-1)\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$\{2n^2+2n+1\}+2n^2-n-1=\{2n^2+2n+1\}-n+\{(2n-1)n+(n-1)\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$\{2n^2+2n+1\}+2n^2=\{2n^2+2n+1\}-n+\{(2n)n+n\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$2\{2n^2+2n+1\}-n-1=\{2n^2+2n+1\}-n+\{(2n+1)n+n\}=\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2\{2n^2+2n+1\}-n=2\{2n^2+2n+1\}-n+\{0n+0\}=2\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$2\{2n^2+2n+1\}=2\{2n^2+2n+1\}-n+\{n+0\}=2\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

* *

$$2\{2n^2+2n+1\}+n+1=2\{2n^2+2n+1\}-n+\{2n+1\}=2\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2\{2n^2+2n+1\}+2n+1=2\{2n^2+2n+1\}-n+\{3n+1\}=2\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$\vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots$$

$$2\{2n^2+2n+1\}+2n^2-2n-1=2\{2n^2+2n+1\}-n+\{(2n-2)n+(n-1)\}=2\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2\{2n^2+2n+1\}+2n^2-n-1=2\{2n^2+2n+1\}-n+\{(2n-1)n+(n-1)\}=2\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$2\{2n^2+2n+1\}+2n^2=2\{2n^2+2n+1\}-n+\{(2n)n+n\}=2\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$3\{2n^2+2n+1\}-n-1=2\{2n^2+2n+1\}-n+\{(2n+1)n+n\}=2\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$3\{2n^2+2n+1\}-n=3\{2n^2+2n+1\}-n+\{0n+0\}=3\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$3\{2n^2+2n+1\}=3\{2n^2+2n+1\}-n+\{n+0\}=3\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

* *

$$3\{2n^2+2n+1\}+n+1=3\{2n^2+2n+1\}-n+\{2n+1\}=3\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$3\{2n^2+2n+1\}+2n+1=3\{2n^2+2n+1\}-n+\{3n+1\}=3\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$3\{2n^2+2n+1\}+2n^2-2n-1=3\{2n^2+2n+1\}-n+\{(2n-2)n+(n-1)\}=3\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$3\{2n^2+2n+1\}+2n^2-n-1=3\{2n^2+2n+1\}-n+\{(2n-1)n+(n-1)\}=3\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$3\{2n^2+2n+1\}+2n^2=3\{2n^2+2n+1\}-n+\{(2n)n+n\}=3\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$4\{2n^2+2n+1\}-n-1=3\{2n^2+2n+1\}-n+\{(2n+1)n+n\}=3\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$4\{2n^2+2n+1\}-n=4\{2n^2+2n+1\}-n+\{0n+0\}=4\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$4\{2n^2+2n+1\}=4\{2n^2+2n+1\}-n+\{n+0\}=4\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

* *

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

* *

$$m\{2n^2+2n+1\}+n+1=m\{2n^2+2n+1\}-n+\{2n+1\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$m\{2n^2+2n+1\}+2n+1=m\{2n^2+2n+1\}-n+\{3n+1\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$m\{2n^2+2n+1\}+3n+2=m\{2n^2+2n+1\}-n+\{4n+2\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$m\{2n^2+2n+1\}+4n+2=m\{2n^2+2n+1\}-n+\{5n+2\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$m\{2n^2+2n+1\}+2n^2-4n-2=m\{2n^2+2n+1\}-n+\{(2n-4)n+(n-2)\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$m\{2n^2+2n+1\}+2n^2-3n-2=m\{2n^2+2n+1\}-n+\{(2n-3)n+(n-2)\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$m\{2n^2+2n+1\}+2n^2-2n-1=m\{2n^2+2n+1\}-n+\{(2n-2)n+(n-1)\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$m\{2n^2+2n+1\}+2n^2-n-1=m\{2n^2+2n+1\}-n+\{(2n-1)n+(n-1)\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$m\{2n^2+2n+1\}+2n^2=m\{2n^2+2n+1\}-n+\{(2n)n+n\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$(m+1)\{2n^2+2n+1\}-n-1=m\{2n^2+2n+1\}-n+\{(2n+1)n+n\}=m\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$(m+1)\{2n^2+2n+1\}-n=(m+1)\{2n^2+2n+1\}-n+\{0n+0\}=(m+1)\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

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$$(m+1)\{2n^2+2n+1\}=(m+1)\{2n^2+2n+1\}-n+\{n+0\}=(m+1)\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

* *

$$(m+1)\{2n^2+2n+1\}+n+1=(m+1)\{2n^2+2n+1\}-n+\{2n+1\}=(m+1)\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$(m+1)\{2n^2+2n+1\}+2n+1=(m+1)\{2n^2+2n+1\}-n+\{3n+1\}=(m+1)\{2n^2+2n+1\}-n+\left\{zn+\left\lceil\tfrac{z-1}{2}\right\rceil\right\}.$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

From this tabulation we can readily derive the formulae on pg. 388.

For the construction of the **Extended Premature Regular Cylindrical String-run Diagrams** for the $[(n+1), n]$ **Prime Regular Cylindrical Bihelix Braids** we need to develop a method which enables us to add the additionally required intersection-points to a **Premature Regular Cylindrical String-run Diagram.** Let the to be extended string-run diagram of the Premature Regular Cylindrical Braid have p columns. Then the number of intersection-points in this Premature Regular Cylindrical Braid is equal to $N = (p-1)n$. Let N^* (where $N^* \geq N$) be the number of intersection-points in the Extended Premature Regular Cylindrical Braid (the Prime Regular Cylindrical Bihelix Braid). Then:

$$m=\left\lfloor \frac{N+n}{2n^2+2n+1} \right\rfloor=\left\lfloor \frac{np}{2n^2+2n+1} \right\rfloor .$$

$$N=m(2n^2+2n+1)-n+zn+r .$$

$$N^*=m(2n^2+2n+1)-n+z^*n+\left\lceil \frac{z^*-1}{2} \right\rceil .$$

$$\text{For } z^*=z \quad \longrightarrow \quad N^*-N=\left\lceil \tfrac{z-1}{2} \right\rceil-r=2\left\lceil \tfrac{z-1}{2} \right\rceil-2r \ \text{ rows} .$$

$$\text{For } z^*=z+1 \quad \longrightarrow \quad N^*-N=\left\lceil \tfrac{z}{2} \right\rceil-r+n=2\left\lceil \tfrac{z}{2} \right\rceil-2r+2n \ \text{ rows} .$$

$$\text{M-line lies } |-p|_{2n+2} \ \text{ rows above S-line} .$$

$$\text{T}^*\text{-line lies } T^* \text{ rows above S-line} .$$

$$\text{Distance from M-line to T}^*\text{-line}=\left|T^*+|p|_{2n+2}\right|_{2n+2}=|T^*+p|_{2n+2} \ \text{ rows} .$$

$$\text{For } z^*=z \quad \longrightarrow \quad |T^*+p|_{2n+2}=2\left\{\left\lceil \tfrac{z-1}{2} \right\rceil-r\right\} .$$

$$T^*=\left|2\left\lceil \tfrac{z-1}{2} \right\rceil-2r-p\right|_{2n+2} .$$

$$\text{For } z^*=z+1 \quad \longrightarrow \quad |T^*+p|_{2n+2}=2\left\{\left\lceil \tfrac{z}{2} \right\rceil-r+n\right\} .$$

$$T^*=\left|2\left\lceil \tfrac{z}{2} \right\rceil-2r-p-2\right|_{2n+2} .$$

For z is even:

$$z-1=\text{odd} \longrightarrow \left\lceil \tfrac{z-1}{2} \right\rceil=\tfrac{z}{2} \ ; \ z=\text{even} \longrightarrow \left\lceil \tfrac{z}{2} \right\rceil=\tfrac{z}{2} .$$

$$\text{For } z^*=z: \qquad T^*=\left|2\left\lceil \tfrac{z-1}{2} \right\rceil-2r-p\right|_{2n+2}=|z-2r-p|_{2n+2} .$$

$$np=m(2n^2+2n+1)+zn+r \longrightarrow 2r=2n(p-z)-2m(2n^2+2n+1) .$$

$$z-2r-p=(2n+2)(z-p)+(p-z)+2mn(2n+2)+2m .$$

$$T^*=|z-2r-p|_{2n+2}=|2m+(p-z)|_{2n+2} .$$

$$np=m(2n^2+2n+1)+zn+r \longrightarrow p-z=m(2n+2)+\tfrac{m+r}{n} .$$

$$T^*=|2m+(p-z)|_{2n+2}=\left|2m+\tfrac{m+r}{n}\right|_{2n+2} .$$

$$\text{For } z^*=z+1: \qquad T^*=\left|2m+\tfrac{m+r}{n}-2\right|_{2n+2} .$$

For z is odd:

$$z-1=\text{even} \longrightarrow \left\lceil \tfrac{z-1}{2} \right\rceil=\tfrac{z-1}{2} \ ; \ z=\text{odd} \longrightarrow \left\lceil \tfrac{z}{2} \right\rceil=\tfrac{z+1}{2} .$$

$$\text{For } z^*=z: \qquad T^*=\left|2m+\tfrac{m+r}{n}-1\right|_{2n+2} .$$

$$\text{For } z^*=z+1: \qquad T^*=\left|2m+\tfrac{m+r}{n}-1\right|_{2n+2} .$$

Since the distance from the M-line to the T*-line must be an **even** number of rows, it follows that:

p=odd ⟶ M-line=odd number of rows above S-line ⟶ T*-line=odd number of rows above S-line.

p=even ⟶ M-line=even number of rows above S-line ⟶ T*-line=even number of rows above S-line.

Furthermore:

$np = m(2n^2+2n+1)+zn+r$.

When $m =$ a multiple of n, then: $0 \leq z \leq 2n+1$ with $r=0$, or
$z=2n+1$ with $r=n$.

When $m \neq$ a multiple of n, say $m=tn+c$, then: $0 \leq z \leq 2n+1$ with $r=n-c$.

$np = m(2n^2+2n+1)+z^*n+r^*$, $\quad 0 \leq z^* \leq 2n+1$ with $r^* = \left\lceil \frac{z^*-1}{2} \right\rceil$.

$0 \leq r \leq n$, $\quad$ hence $0 \leq \frac{r}{n} \leq 1$.

For $\frac{m}{n} \neq$ whole number $\longrightarrow$ $\left\lfloor \frac{m}{n} \right\rfloor = \left\lceil \frac{m}{n} \right\rceil - 1 = \frac{m+r}{n} - 1$.

For $\frac{m}{n} =$ whole number $\longrightarrow$ $\left\lfloor \frac{m}{n} \right\rfloor = \left\lceil \frac{m}{n} \right\rceil = \frac{m}{n}$.

In the procedure envisaged we select a p-value. Then we **add** an additional δ crossing-points in order to obtain an Extended Premature Regular Cylindrical Braid which is a Prime Regular Cylindrical Bihelix Braid.

Note that when there are two Prime Regular Cylindrical Bihelix Braids for column $p = a$, the number of additional crossing-points are respectively δ and $(\delta+1)$, and with our above described procedure we will obtain only the first of these two Prime Regular Cylindrical Bihelix Braids for that column. When for the first of these two Prime Regular Cylindrical Bihelix Braids $\delta = n-1$, then the second will be obtained as the Prime Regular Cylindrical Bihelix Braid with $\delta = 0$ for column $p = a+1$. The occurrence of two Prime Cylindrical Bihelix Braids for column $p = a$ takes place when, for the first of the two, $z^* = 2n+1$.

We found for the T*-line three formulae:

A. $T^* = \left|2m + \frac{m+r}{n}\right|_{2n+2}$, when $z =$ **even**, and $r \leq \left\lceil \frac{z-1}{2} \right\rceil$.

B. $T^* = \left|2m + \frac{m+r}{n} - 2\right|_{2n+2}$, when $z =$ **even**, and $r > \left\lceil \frac{z-1}{2} \right\rceil$.

C. $T^* = \left|2m + \frac{m+r}{n} - 1\right|_{2n+2}$, when $z =$ **odd**.

Furthermore we have seen that $\frac{m+r}{n} - 1 = \left\lfloor \frac{m}{n} \right\rfloor$ when $\frac{m}{n} \neq$ whole number. Hence a much more convenient method is to use a single T-line, which is defined by:

$$T = \left|2m + \left\lfloor \frac{m}{n} \right\rfloor\right|_{2n+2}.$$

When $\frac{m}{n} \neq$ whole number:

- For **A**, the T-line lies **one row below** the T*-line.
- For **B**, the T-line lies **one row above** the T*-line.
- For **C**, the T-line coincides with the T*-line.

When $\frac{m}{n} =$ whole number:

- For **A**, the T-line coincides with the T*-line.
- **B** does **not** apply.
- For **C**, the T-line coincides with the T*-line when $z = 2n+1$ with $r = n$, and lies **one row above** the T*-line when $0 \leq z \leq 2n+1$ with $r = 0$.

Hence the process of adding additional crossing-points requires only the positioning of the T-line relative to the S-line, and all we require for that is to calculate the T-value in accordance with the formula given above. Then it becomes a graphical process in which we draw bights adjacent to the right-hand bight-boundary from the M-line to the T-line with in general no bight-point on the T-line. The only exception to this rule, when there is a bight-point on the T-line, may occur when the T-line lies one row below the T*-line. As we shall show below, there is however a simple rule for accommodating this event.

$$\text{Let:} \quad pn=(tn+c)(2n^2+2n+1)+zn+(n-c)\,; \quad r=n-c\,.$$

$$\text{Then:} \quad p=(tn+c)(2n+2)+t+z+1\,.$$

$$|-p|_{2n+2}=|-(t+z+1)|_{2n+2}\,.$$

$$r\leq\left\lceil\tfrac{z-1}{2}\right\rceil \text{ for } 2n-2c\leq z\leq 2n+1\,.$$

We only have to consider the case A (in which z=even). Hence for the z-values: $2n-2c\,, 2n-2c+2\,, 2n-2c+4\,, \cdots, 2n-4\,, 2n-2\,, 2n$. Thus for $z=2n-2c+2s$, where $0\leq s\leq c$.

Note that the number of z-values involved is equal to $(c+1)$.

$$z=2n-2c+2s \longrightarrow \quad p=(tn+c)(2n+2)+t+2n-2c+1+2s\,, \quad m=\left\lfloor\tfrac{pn}{2n^2+2n+1}\right\rfloor=tn+c\,.$$

$$\text{M-line} \rightarrow \quad M=|-p|_{2n+2}=|-t-2n+2c-2s-1|_{2n+2}=|-t+2c-2s+1|_{2n+2}\,.$$

$$\text{T}^*\text{-line} \rightarrow \quad T^*=|2tn+2c+t+1|_{2n+2}=|-t+2c+1|_{2n+2}\,.$$

$$\text{T-line} \rightarrow \quad T=|2tn+2c+t|_{2n+2}=|-t+2c|_{2n+2}=|T^*-1|_{2n+2}\,.$$

$$2\delta=|T^*-M|_{2n+2}=|2s|_{2n+2}\,.$$

$$|T-M|_{2n+2}=|2s-1|_{2n+2}\,.$$

Note that for $s=0$ the value of $|T^*-M|_{2n+2}=0$, hence there is no additional intersection, and consequently there is no bight-point on the T-line to the right of the right-hand bight-boundary (note that for $s=0$ the M-line coincides with the T*-line and consequently it is impossible to have immediately to the right of the right-hand bight-boundary a bight-point on the T-line which is one row below the M-line). This leaves us with c z-values for which there is a bight-point on the T-line immediately to the right of the right-hand bight-boundary. Since the number of additional intersections is equal to the number of bight-points adjacent to the right of the right-hand bight-boundary, this number of bight-points are for the respective $z=2n-2c+2s$ equal to s. The maximum value of s is equal to c, and the value for c can be obtained from m by means of $c=|m|_n$. Although we are supposed to add the additional intersection-points from the M-line to the T*-line (0 intersection-points when these lines coincide), the method of adding the additional intersection-points from the M-line to the T-line (n intersection-points when the T-line lies one row below the M-line) gives also a valid solution as readily will be seen from the formulae.

It will now be evident that the rule for adding the additionally required intersection-points to the string-run of a Premature Regular Cylindrical Braid with p columns (the associated Regular Cylindrical Braid has $b=n+1$ bights), in order to make it the string-run of an $[n+1\,,\,n]$ Prime Regular Cylindrical Bihelix Braid, is as follows:

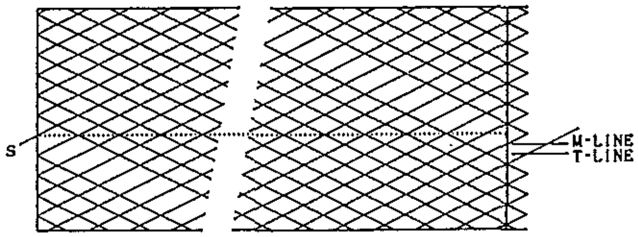
- Draw in the string-run diagram of the Premature Regular Cylindrical Braid the M-line. This line, which is parallel to the S-line, goes trough the last right-hand bight-point on the string-run of the associated Regular Cylindrical Braid.
- Calculate the T-value by means of the formula $T=\left|2m+\left\lfloor\frac{m}{n}\right\rfloor\right|_{2n+2}$, where $m=\left\lfloor\frac{pn}{2n^2+2n+1}\right\rfloor$. Draw in the string-run diagram of the Premature Regular Cylindrical Braid the T-line. This line, which is parallel to the S-line and M-line, lies T rows (or $\frac{T}{2}$ bights) above (in cyclic fashion) the S-line.
- Continue with the Working End and lay down from the M-line to the T-line (in cyclic fashion) a zigzag column of bights immediately adjacent to the right of the right-hand bight-boundary of the Premature Regular Cylindrical Braid. In general the last bight-point in the zigzag column of added bights is **below** the T-line, and only then is the last bight-point **on** the T-line (if such a position is possible) when the number of bight-points in the added zigzag column is equal to or less than c.

In order to illustrate the graphical method clearly, we have presented below six Examples for which large p-values have been selected. Since for such large p-values the associated string-run diagrams would be much too long for the width of the paper, we had to remove the major mid-section of their lengths. For this reason we have calculated and listed for each Example the M-value in order to enable us to draw the M-line in its correct position relative to the S-line. For the p-value range which would be used in practical applications, hence for string-run diagrams which would normally fit within the width of the paper, we don't of course require the M-value in order to draw the M-line in its proper position.

Example 1. $[n+1, n] = [11, 10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 8559$. $M = |-p|_{2n+2} = |-8559|_{22} = 21$.

$$m = \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{85590}{221} \right\rfloor = 387.$$

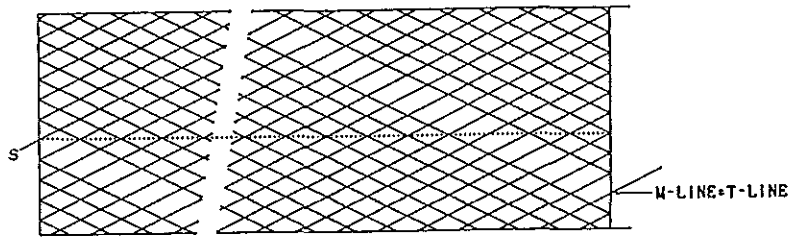
$$T = \left|2m + \left\lfloor \frac{m}{n} \right\rfloor\right|_{2n+2} = |774 + 38|_{22} = |812|_{22} = 20.$$



Example 2. $[n+1, n] = [11, 10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 1326$. $M = |-p|_{2n+2} = |-1326|_{22} = 16$.

$$m = \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{13260}{221} \right\rfloor = 60.$$

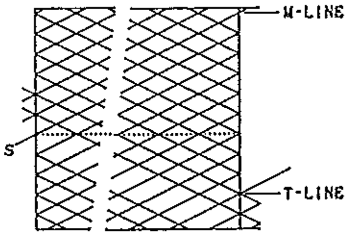
$$T = \left|2m + \left\lfloor \frac{m}{n} \right\rfloor\right|_{2n+2} = |120 + 6|_{22} = |126|_{22} = 16.$$



Example 3. $[n+1, n] = [11, 10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 1330$. $M = |-p|_{2n+2} = |-1330|_{22} = 12$.

$$m = \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{13300}{221} \right\rfloor = 60.$$

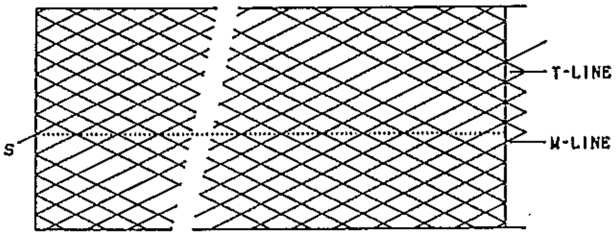
$$T = \left|2m + \left\lfloor \frac{m}{n} \right\rfloor\right|_{2n+2} = |120 + 6|_{22} = |126|_{22} = 16.$$



Example 4. $[n+1\,,\,n] = [11\,,\,10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 1475$. $M = |-p|_{2n+2} = |-1475|_{22} = 21$.

$$m = \left\lfloor \tfrac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \tfrac{14750}{221} \right\rfloor = 66\,.$$

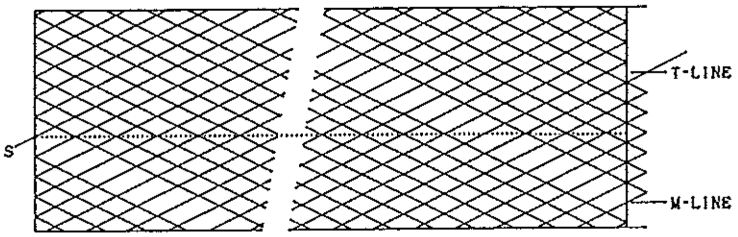
$$T = \left|2m + \left\lfloor \tfrac{m}{n} \right\rfloor\right|_{2n+2} = |132+6|_{22} = |138|_{22} = 6\,.$$



Example 5. $[n+1\,,\,n] = [11\,,\,10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 1459$. $M = |-p|_{2n+2} = |-1459|_{22} = 15$.

$$m = \left\lfloor \tfrac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \tfrac{14590}{221} \right\rfloor = 66\,.$$

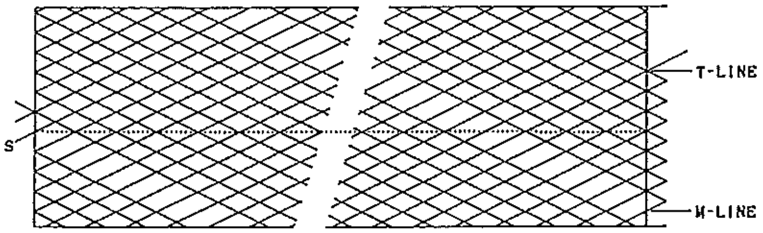
$$T = \left|2m + \left\lfloor \tfrac{m}{n} \right\rfloor\right|_{2n+2} = |132+6|_{22} = |138|_{22} = 6\,.$$



Example 6. $[n+1\,,\,n] = [11\,,\,10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 1460$. $M = |-p|_{2n+2} = |-1460|_{22} = 14$.

$$m = \left\lfloor \tfrac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \tfrac{14600}{221} \right\rfloor = 66\,.$$

$$T = \left|2m + \left\lfloor \tfrac{m}{n} \right\rfloor\right|_{2n+2} = |132+6|_{22} = |138|_{22} = 6\,.$$



We mentioned on pg. 396 that there are sometimes two Prime Regular Cylindrical Bihelix Braids for a column $p = a$, and that this occurs when, for the first of the two, $z^* = 2n+1$. Our graphical method is not capable of finding this second Prime Regular Cylindrical Bihelix Braid when for the first one $0 \le \delta < n-1$. However, since the second one has a number of intersections which is equal to $N = (m+1)(2n^2+2n+1)-n$ we can, with a simple calculation, readily establish the position of the second one:

$$p = (m+1)(2n+2) + \left\lfloor \frac{m+1}{n} \right\rfloor \quad \text{with } \delta = |m+1|_n\,.$$

The reader will undoubtedly have noticed that the graphical procedure also leads to a simple calculation procedure:

- Select a value for p.
- Calculate $m = \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor$.
- Calculate $z = p - m(2n+2) - \left\lceil \frac{m}{n} \right\rceil$. When $z = 2n+2$, take $z = 2n+1$.
- Calculate $r = |-m|_n$. When initially $z = 2n+2$, take $r = n$
- When $r \leq \left\lceil \frac{z-1}{2} \right\rceil$, take $z^* = z$ and $r^* = \left\lceil \frac{z-1}{2} \right\rceil$. Then $\delta = r^* - r$.
- When $r > \left\lceil \frac{z-1}{2} \right\rceil$, take $z^* = z+1$ and $r^* = \left\lceil \frac{z}{2} \right\rceil$. Then $\delta = n - (r - r^*)$.

Example 7. $[n+1, n] = [11, 10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 1458$.

$m = \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{14580}{221} \right\rfloor = 65$.

$z = p - m(2n+2) - \left\lceil \frac{m}{n} \right\rceil = 1458 - 65 \cdot 22 - \left\lceil \frac{65}{10} \right\rceil = 21$.

$r = |-m|_n = |-65|_{10} = 5$.

Hence $z^* = z = 21$, and $r^* = \left\lceil \frac{z-1}{2} \right\rceil = \left\lceil \frac{21-1}{2} \right\rceil = 10$.

$\delta = r^* - r = 10 - 5 = 5$.

Note that $p = (m+1)(2n+2) + \left\lfloor \frac{m+1}{n} \right\rfloor = 66 \cdot 22 + \left\lfloor \frac{66}{10} \right\rfloor = 1452 + 6 = 1458$ with $\delta = |m+1|_n = |66|_{10} = 6$ gives also a Prime Regular Cylindrical Bihelix Braid. Furthermore, since $z^* = 2n+1$ for the Prime Regular Cylindrical Bihelix Braid we found, it follows that there is a second Prime Regular Cylindrical Bihelix Braid for column $p = 1458$ with one more crossing-point.

Example 8. $[n+1, n] = [11, 10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 2587$.

$m = \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{25870}{221} \right\rfloor = 117$.

$z = p - m(2n+2) - \left\lceil \frac{m}{n} \right\rceil = 2587 - 117 \cdot 22 - \left\lceil \frac{117}{10} \right\rceil = 1$.

$r = |-m|_n = |-117|_{10} = 3$.

Hence $z^* = z + 1 = 2$, and $r^* = \left\lceil \frac{z}{2} \right\rceil = \left\lceil \frac{1}{2} \right\rceil = 1$.

$\delta = n - (r - r^*) = 10 - (3 - 1) = 8$.

Example 9. $[n+1, n] = [11, 10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 314$.

$m = \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{3140}{221} \right\rfloor = 14$.

$z = p - m(2n+2) - \left\lceil \frac{m}{n} \right\rceil = 314 - 14 \cdot 22 - \left\lceil \frac{14}{10} \right\rceil = 4$.

$r = |-m|_n = |-14|_{10} = 6$.

Hence $z^* = z + 1 = 5$, and $r^* = \left\lceil \frac{z}{2} \right\rceil = \left\lceil \frac{4}{2} \right\rceil = 2$.

$\delta = n - (r - r^*) = 10 - (6 - 2) = 6$.

Example 10. $[n+1, n] = [11, 10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 984$.

$m = \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{9840}{221} \right\rfloor = 44$.

$z = p - m(2n+2) - \lceil \frac{m}{n} \rceil = 984 - 44 \cdot 22 - \lceil \frac{44}{10} \rceil = 11$.
$r = |-m|_n = |-44|_{10} = 6$.
Hence $z^* = z + 1 = 12$, and $r^* = \lceil \frac{z}{2} \rceil = \lceil \frac{11}{2} \rceil = 6$.
$\delta = n - (r - r^*) = 10 - (6 - 6) = 10$.

Example 11. $[n+1, n] = [11, 10]$ Prime Regular Cylindrical Bihelix Braid. The Premature Regular Cylindrical Braid has $p = 833$.
$m = \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{8330}{221} \right\rfloor = 37$.
$z = p - m(2n+2) - \lceil \frac{m}{n} \rceil = 833 - 37 \cdot 22 - \lceil \frac{37}{10} \rceil = 15$.
$r = |-m|_n = |-37|_{10} = 3$.
Hence $z^* = z = 15$, and $r^* = \lceil \frac{z-1}{2} \rceil = \lceil \frac{15-1}{2} \rceil = 7$.
$\delta = r^* - r = 7 - 3 = 4$.

At least some readers may wonder where the expression $2n^2 + 2n + 1$ comes from. It is in fact a special case of a general expression which belongs to another braidform, namely the **torus braids**. In due course we shall discuss that braidform in some detail, and see in which way the Prime Regular Cylindrical Bihelix Braids are linked to them.

Braids and Diseases

In this article we will look at a property of braids which we discussed more recently and a discovery about some (at the moment) incurable diseases. We have to stress that we do not know whether or not the two phenomena which play the principal role in our discussion are linked, but their similarity is striking nevertheless.

We all know how on the 20th of March 1996 the British Ministry of Health triggered an international panic when it announced that the till then obscure *Spongiform Encephalopathy Advisory Committee* had reported ten unusual cases of *Creutzfeldt-Jakob disease (CJD)*. The unusual aspect of these ten cases was their mean age of onset of 29 years as compared to the usual mean age of onset of 65 years in the 185 cases of *CJD* till then diagnosed in the UK from May 1990. Furthermore, these ten unusual cases did differ somewhat in symptoms and brain pathology from normal *CJD*. The Committee concluded that the most likely explanation for this unusual event was that the affected people had been exposed to *Bovine Spongiform Encephalopathy (BSE)*.

CJD, a rare degenerative disease which is invariably fatal, is one of a group of human and animal diseases which are thought to have similar causes. Others which belong to this group besides *BSE* are *Kuru*, *Fatal Familial Insomnia (FFI)*, *Gerstmann-Sträussler-Scheinker disease (GSS)* and *Scrapie*, the last of which seems to affect certain animals, such as sheep and goats, but not humans.

Scrapie is the most studied of the group, the affected animals show a lack of coordination in their movements (ataxia), wasting and severe itching of the skin which causes the animal to rub (hence the name *Scrapie*).

BSE is thought to have arisen during the late 1970s when *Scrapie* infected sheep meal was incorporated into cattle feed. Diseases similar to *BSE* have been found in domestic cats, captive wild cats (puma, cheetah, ocelot), captive wild ruminants (gemsbok, eland, Arabian oryx, nyala, greater kudu, scimitar-horned oryx), farmed minks. Experimental injection of brain extracts from *BSE* infected cattle has transferred the disease to mice, cattle, sheep, pigs and marmoset monkeys. The brain of an infected animal shows degenerative changes in the form of clumps of fibrous material (plaques) and small holes which give the brain a spongelike appearance under a microscope (hence the name *spongiform encephalopathies*).

CJD relates to two very similar diseases which were described by respectively Hans Creutzfeldt and Alfons Jacob around the early 1920s. It are diseaes of middle to late adult human life characterised by abnormalities of behaviour and intellect, weakness, ataxia, spasticity, tremors and hesitant speech. *CJD* is invariably fatal, death occurring within four to twelve months of the onset of the symptoms.

Kuru appears to be related to *CJD*, but its circumstances are very different. The disease was described by two American medical anthropologists, Gajdusek and Zigas in 1957. The disease occurred at high frequency among the remote Fore people of New Guinea; all age groups were susceptible. *Kuru* means shivering in the Fore language; it is characterised by ataxia and tremors which progress to total physical incapacity and death within a year. The disease is spread by ritual cannibalism in the eating of lightly cooked brains along with other flesh during the rite of mourning and respect for dead kinsmen. It has been suggested that *Kuru* arose from a sporadic case of *CJD* that became an epidemic. With the cessation of cannibalism, the disease has largely disappeared.

FFI is a more recent human disease akin to *CJD* and *Kuru*. It appears to be inherited, and it impairs the autonomic and motor systems. In advanced cases complex hallucinations occur followed by stupor, coma and death. The mean age of onset is 49, and death takes place within about a year.

GSS is like *FFI*. It appears to be inherited. A loss of coordination precedes dementia. Death can take up to six years.

What causes this group of diseases is not known. They are vastly different to the great majority of infectious diseases which are caused by microbes — protozoans, bacteria, viruses and fungi, all very minute but containing the essentials of life, namely nucleic acids and proteins. Identifying the infectious agent which causes *Scrapie* presented a great problem since it was resistant to ultraviolet radiation, radioactivity, heat, formaldehyde, and enzymes that destroy most proteins and nucleic acids; it proved to be resistant to agents which kill all life forms known. In highly infectious brain extracts no virus particles could be detected with electron microscopy. There was no evidence that the host's immune system was even the slightest bit concerned by its owners slow demise; an almost unheard of situation with infectious diseases. Biologists regarded for a long time the elusive agent to be an unconventional slow acting virus, and some biologists still do.

But a few heretics suggested that since no nucleic acid could be detected, the infectious agent simply lacked both DNA and RNA. This suggestion is as fundamentally radical as the idea of a bird without wings that can fly, a horse without legs that can gallop. Stanley Prusiner has been the main proponent of the idea. He has called the infectious agents which underlie all these diseases **prions**. He has obtained strong evidence that the infectious agent is an *altered form* of a single protein. This protein he has called **PrP** (Prion Protein). Normal PrP is a regular protein which is coded by an everyday gene made of DNA. It is common on the surface of braincells. Genetically engineered mice which lack PrP entirely develop, reproduce and learn normally, but show disturbance of sleep and activity patterns similar to FFI.

The characteristic form of normal PrP is like two parallel coil springs resting on top of two more springs which lean to the left. In patients with spongiform encephalopathies the two top coils of the PrP protein have changed into a flat "sheet-like" structure, and it is the changed structure of this protein which seems to be responsible for the formation of plaques and holes in the brain that eventually leads to dementia and death.

In the previous two issues of *The Braider* we have seen the influence of half twists in a string, and what they represent. Of special interest is that a set number of half twists always represent at least two sets of braids (see pg. 372). The question is: do we with the changed PrP protein (with its "sheet-like" structure) as compared to the normal PrP protein encounter a phenomenon that is similar to that of half-twists in a string???

What's in a Name?

In various publications associated with knots and braids several people have voiced their opinion about the necessity of a "universal" knot and braid classification in order to identify unambiguously a knot or braid. As is generally the case, a desire is the easy part, getting it is the hard part. We agree of course that in order to avoid confusion, a "universal" classification system would be desirable, but unfortunately most, if not all, of its advocates don't realise the complexity of the problems involved. In their, often pragmatic, mind they think that there are only a couple of hundred, or at most a few thousand knots and braids. In reality however, there are at least as many different knots and braids as there are stars in the universe. Furthermore, with repect to a classification system, there are more often than not no distinct boundaries between various knots and braids. Even before a real successful **restricted** classification can be achieved, we need to know a great deal more than we do at the moment about relationships between seemingly different knots and braids.

Whether there is a classification system or not, we will still have various names for the same knot or braid, since these different names will often relate better to a particular application or historical aspect. Unfortunately, historical aspects are sometimes grossly distorted, and it is in this connection that we will have a look at a well-known braidform.

The braidform in question is known under several names, some of which are: Loop Stitch, Loop Lacing, Overlay Stitch, Buttonhole Stitch, Spanish Edge Lacing, Mexican Edge Lacing, Cordovan Stitch, Cordover Stitch, etc. This 'stitch' is normally encountered in one of the following three forms: single, double, or triple. It is described in most leathercraft and braiding books (Pamphlet No. 2 gives improved construction procedures for the double Cordovan stitch).

The name *Cordovan* refers to the Spanish town by the name of *Córdoba* in the southern province *Andalusia.* Hence instead of *Cordovan* we should write **Córdoban**. Why then is the name misspelled? The reason is that *Cordova* spells the name *Córdoba* as it is pronounced in Spanish (in Spanish the *v* and the *b* are pronounced in the same way; at the beginning of a phrase or after *m* or *n* they are pronounced as *b* in *boy*, in any other position the sound is softened and the lips do not meet).

As far as we know the name *Cordover* has only been used by Ron Edwards in his booklet "Leather Lacing Manual", ISBN 0 909901 36 8, published by *The Rams Skull*

Press. In that booklet Ron states: "This is a popular and ancient stitch which may have originated in the Spanish province now known as Cordober. Cordober was once famous for its leatherwork, and the stitch is also known as Spanish Edging.". The fact is that there is no Spanish province by the name of Cordober, but that the town Córdoba in the province Andalusia was famous for its leatherwork. It is unlikely however that the stitch originated there, but rather was introduced there by the Moors. The name *Córdoba* can be found back in Argentina: a town in a province by the same name, in Mexico: a town in the province *Durango*, as well as a town in the province *Vera Cruz*, and in Columbia: the province *Córdoba.* Not only is the name *Córdoba* corrupted into *Cordova* for the stitch, but we do also find towns in the United States of America (Alabama, North Carolina, New Mexico and Alaska), and even a bay and a mountain peak in Alaska, with the name *Cordova.*

Attaching a Round Braid to a Ring

A simple attachment of a 4-string Round Braid to a ring is shown in Fig. 336.

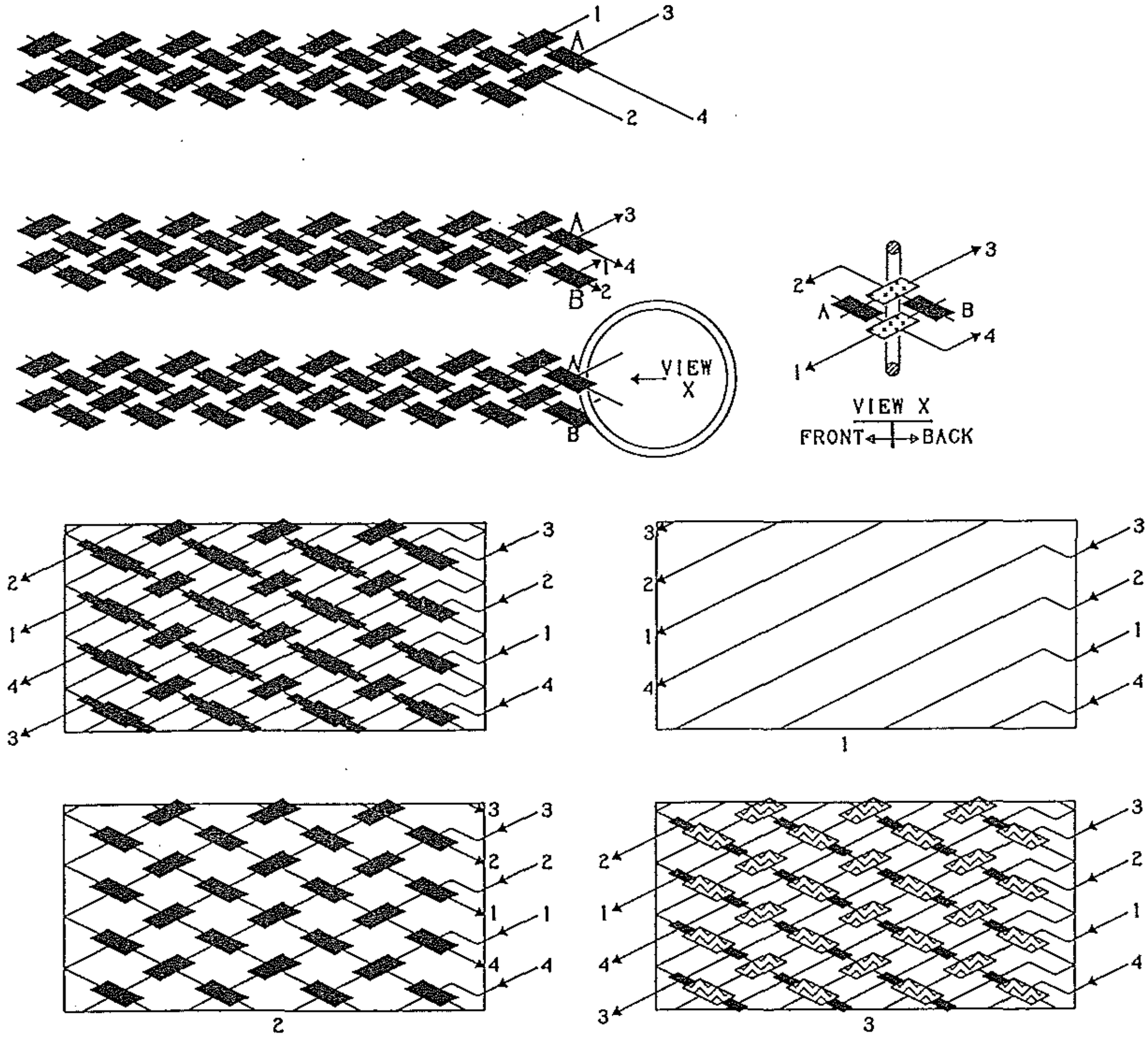


Fig. 336 — Attaching a 4-string Round Braid to a Ring.

Fig. 337 depicts a variation to the previous method; the diameter of the covering knot is a little bigger due to the extra four strings under it.

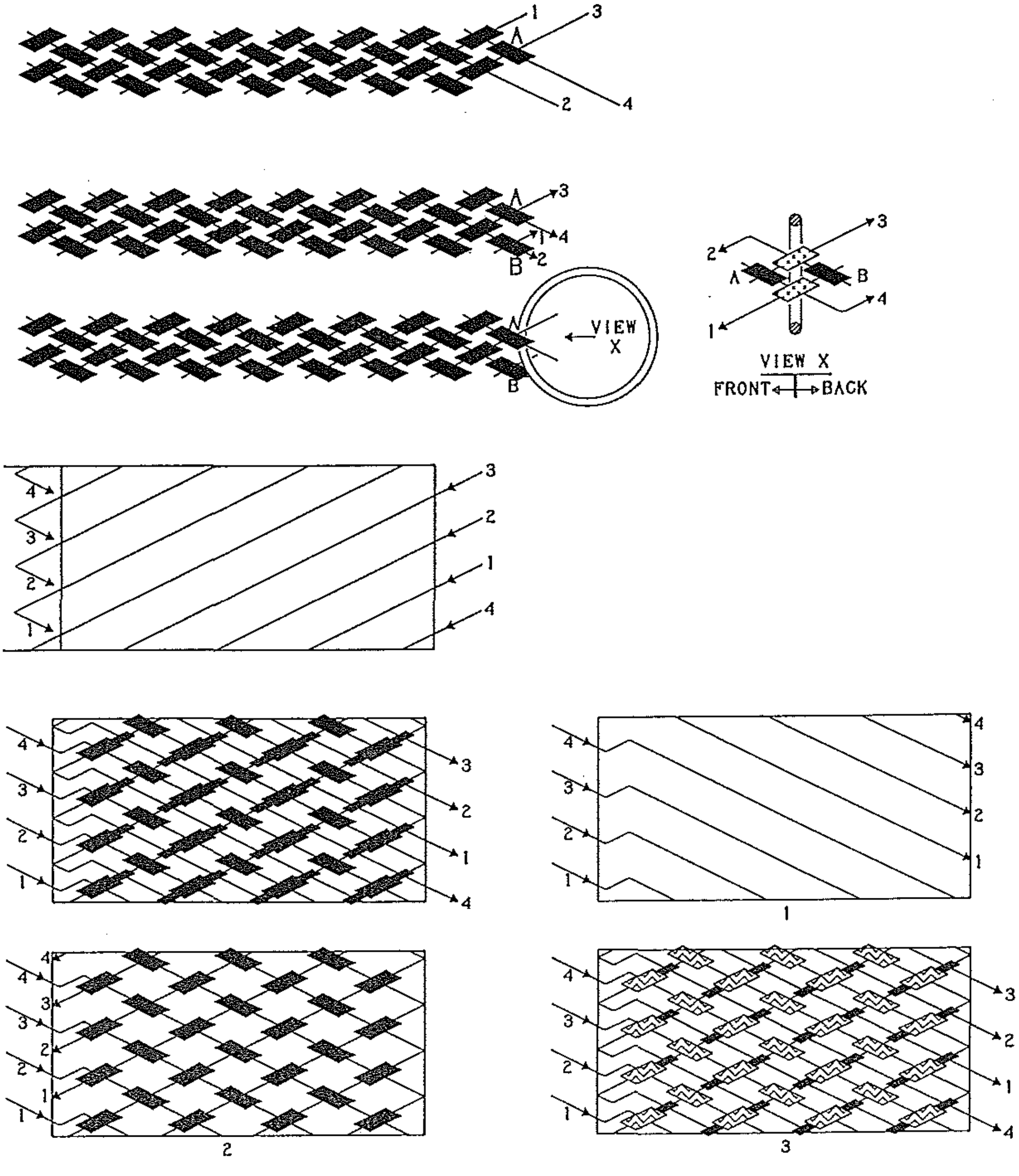


Fig. 337 — Attaching a 4-string Round Braid to a Ring.

When strings 1 & 3 are of the same colour and strings 2 & 4 are of the same but a different colour, the covering knot will have a spiral colour pattern when it has a Casa-coding, a lengthwise V-pattern when it has a 2-pass Gaucho or Headhunters coding, and a spiral V-pattern when it has a 3-pass Gaucho or Headhunters coding.

Figs. 338, 339 and 340 show the attachment of a 6-string Round Braid to a ring. For thin ring material we use the construction procedure depicted in Fig. 338, for thick ring material the construction procedure in Fig. 340 is used, and for ring material in between the method in Fig. 339 is the most appropriate.

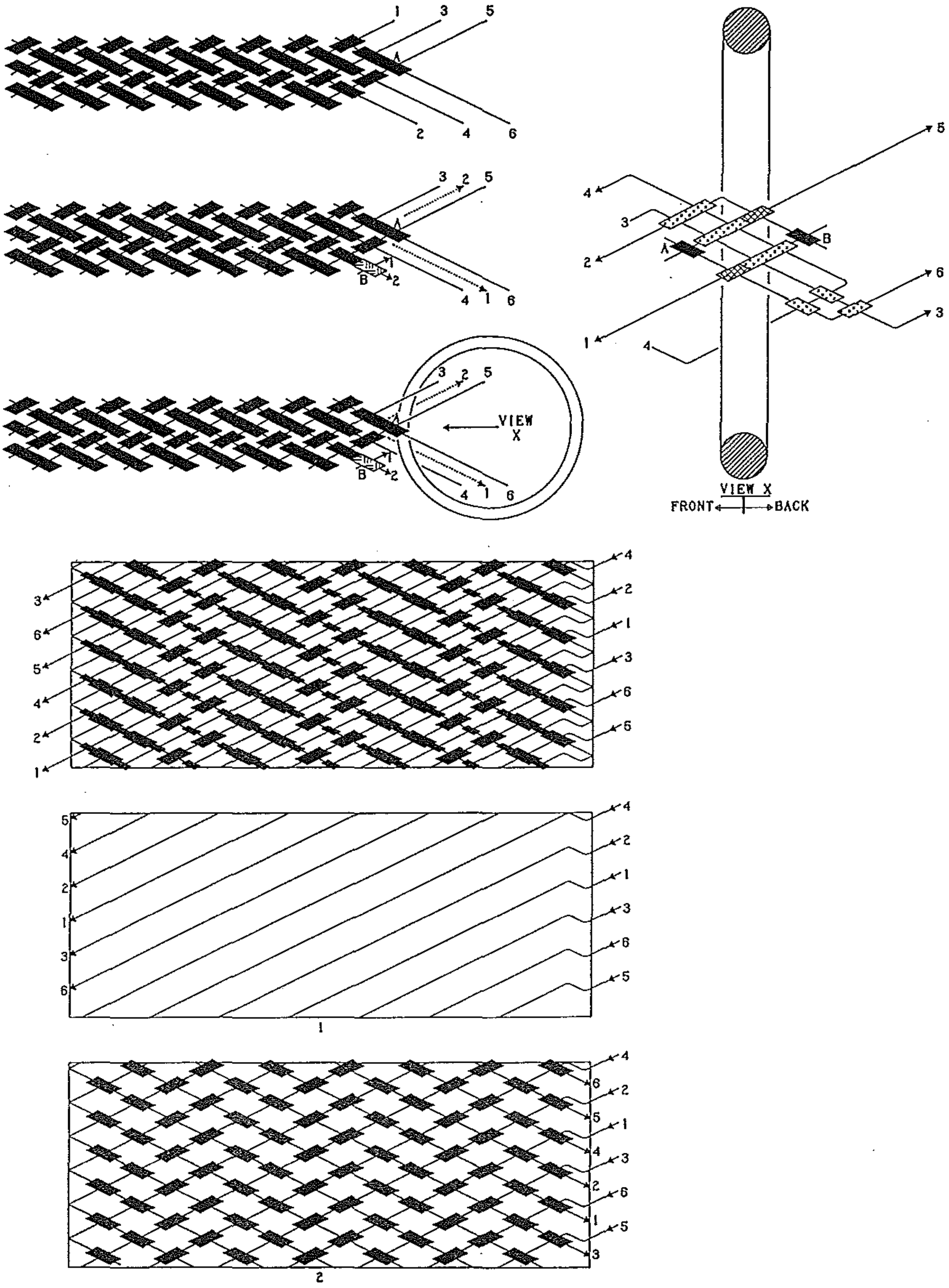


Fig. 338 — Attaching a 6-string Round Braid to a Ring of thin material.

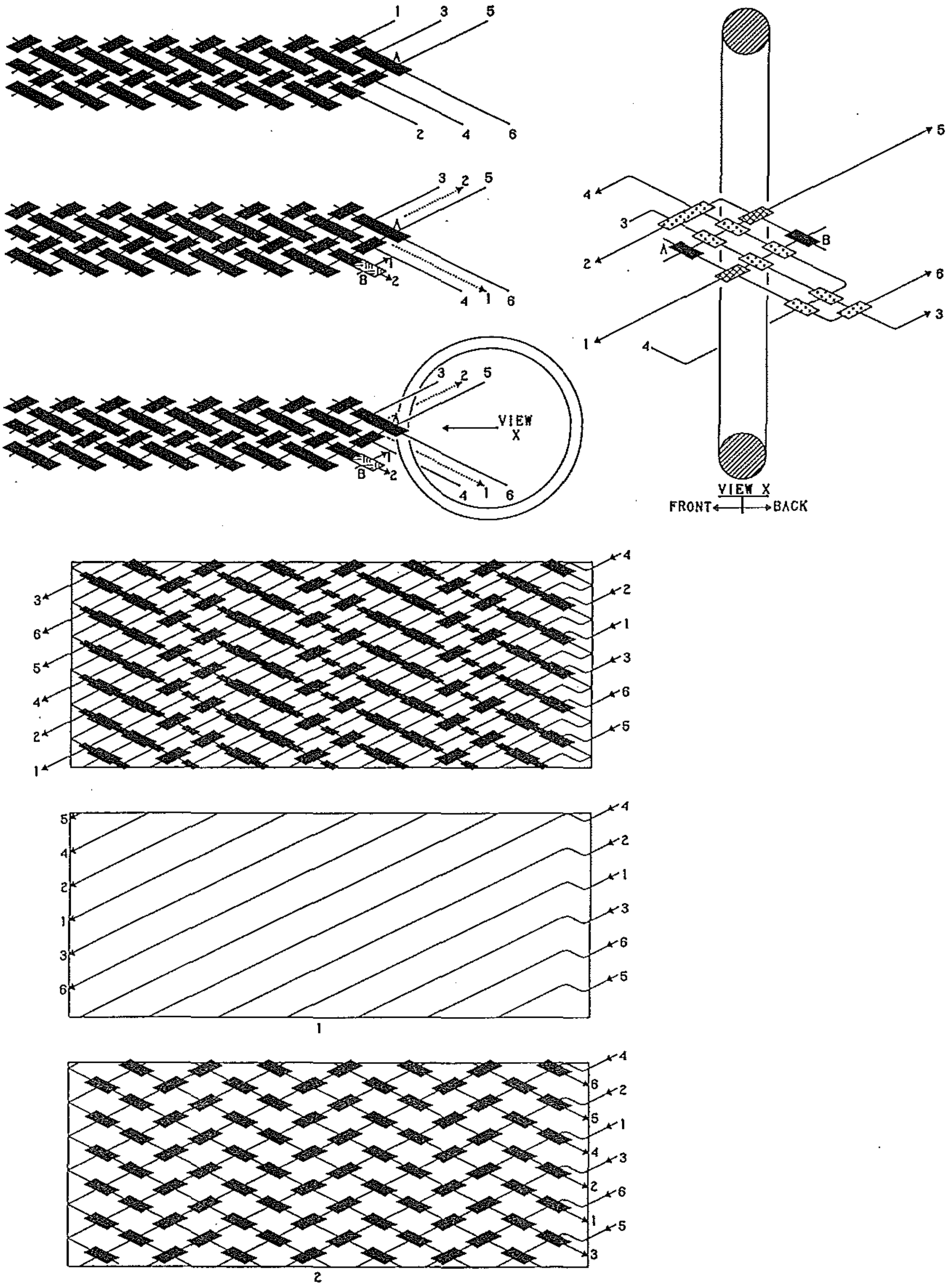


Fig. 339 — Attaching a 6-string Round Braid to a Ring of average thickness.

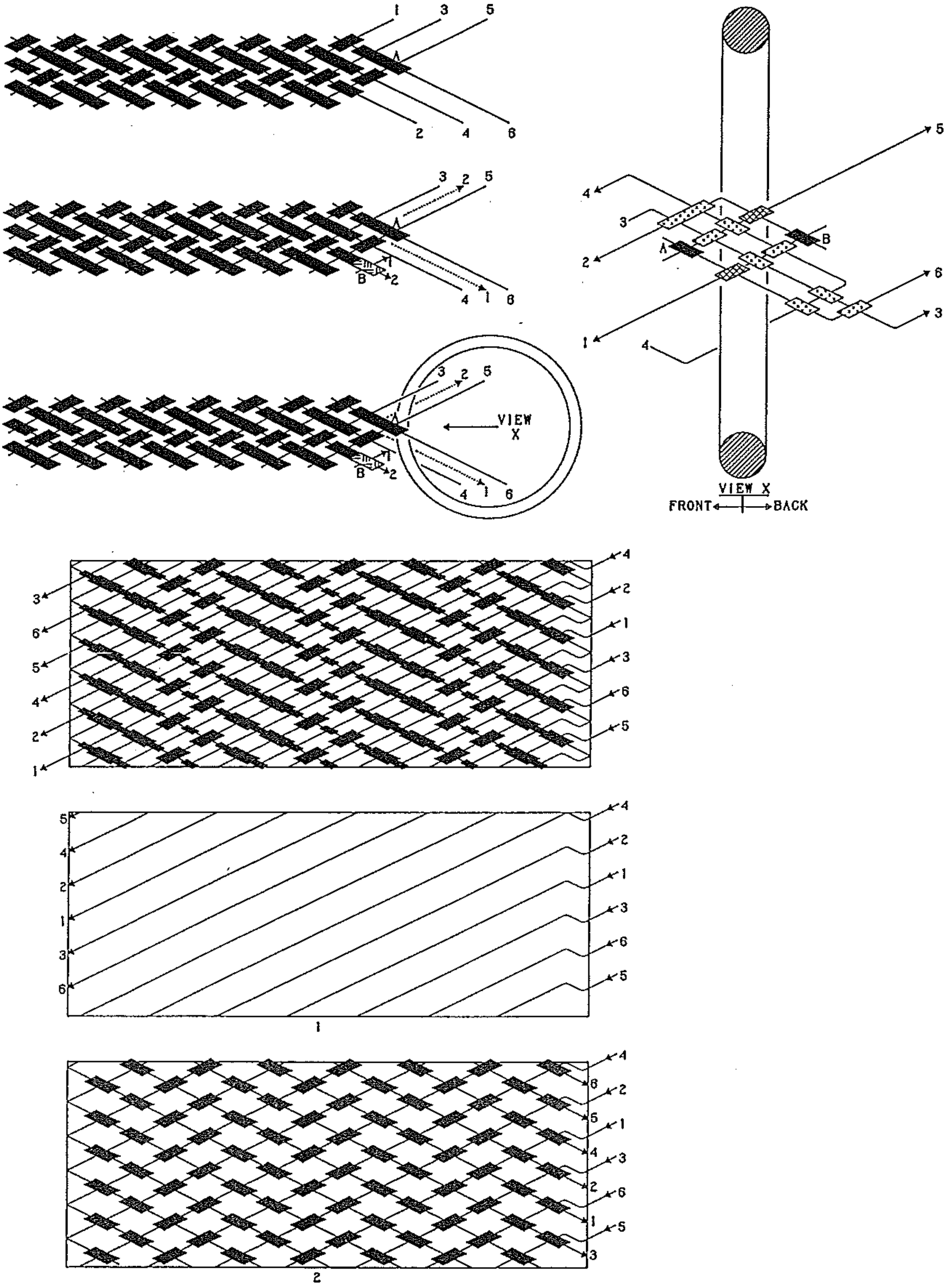
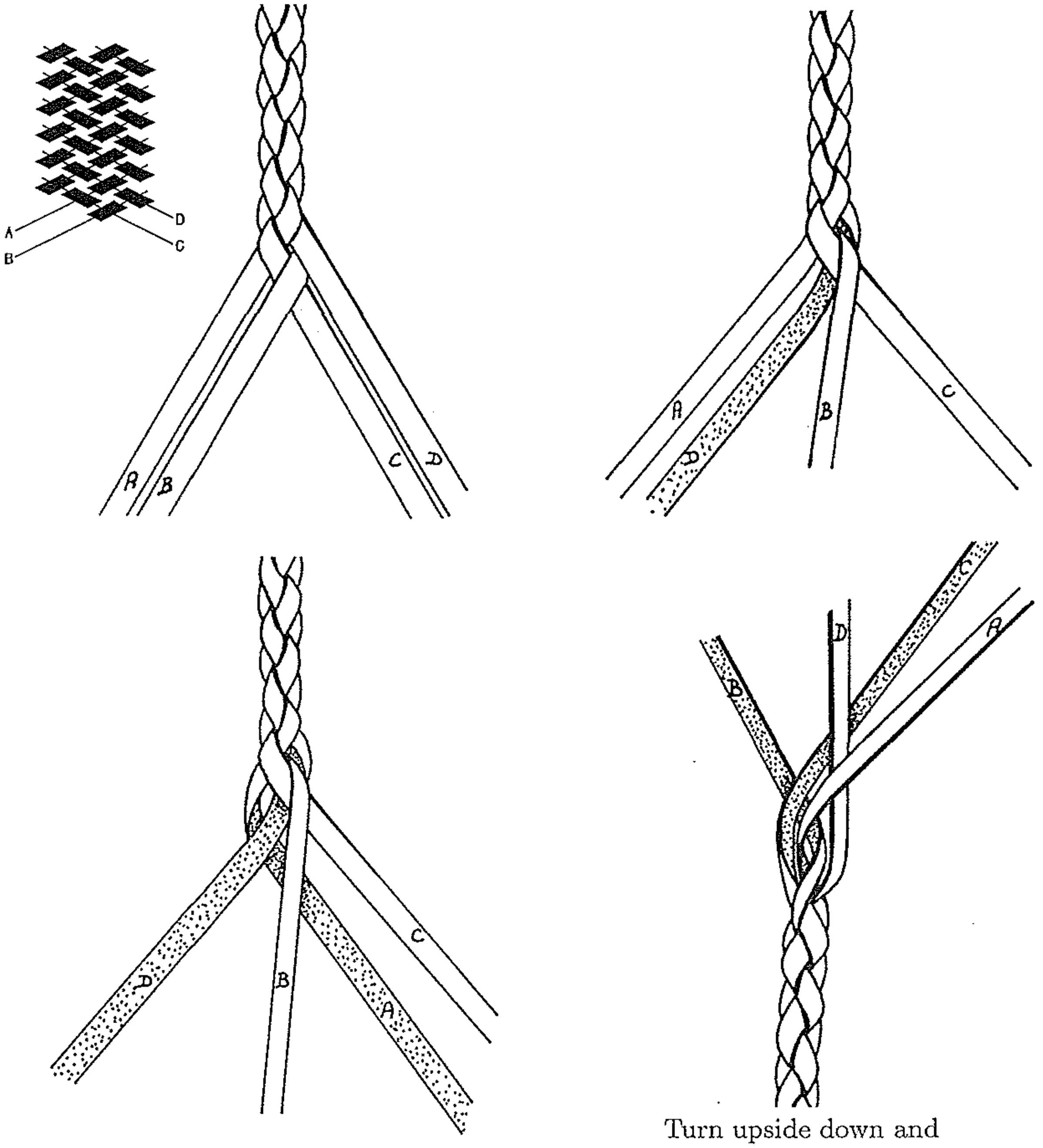


Fig. 340 — Attaching a 6-string Round Braid to a Ring of thick material.

Integrated Braids

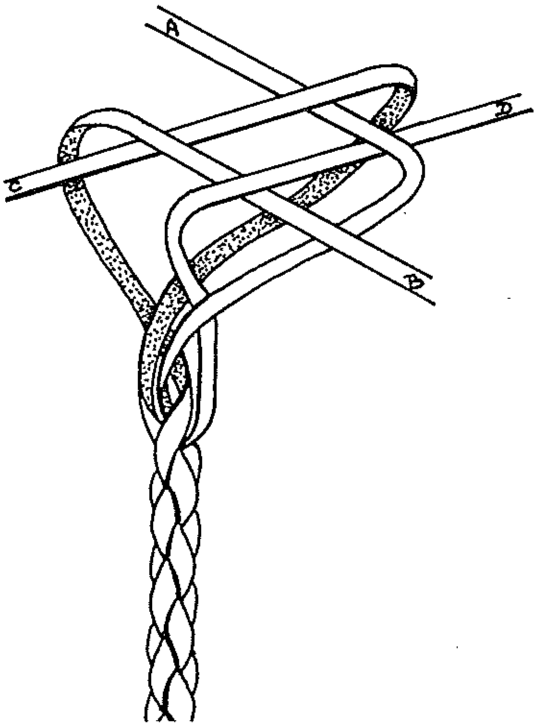
An **Integrated Braid** is a braid which forms an integral part with another braidform, in other words, both braidforms are made with the same strings. Hence the covering knots in the previous described attachments were in fact *Integrated Braids* since they were a continuation of the Round Braids.

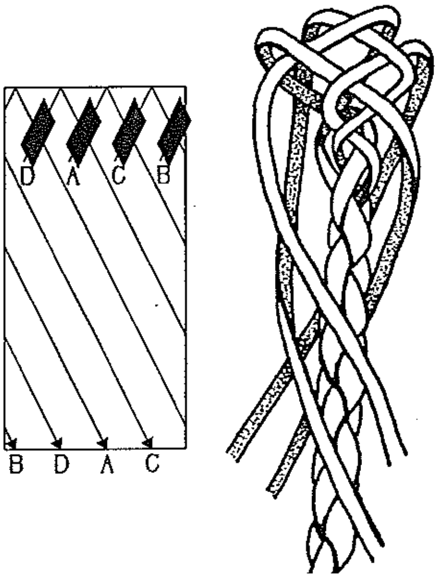
We shall start with a very elementary Integrated Braid used in finishing a 4-string Round Braid. It should be pointed out here that knots which are used to finish a braid with, should always be of the **integrated** type, hence **not** the so often encountered knots whose string(s) are not a part of the braid they are covering the end of.



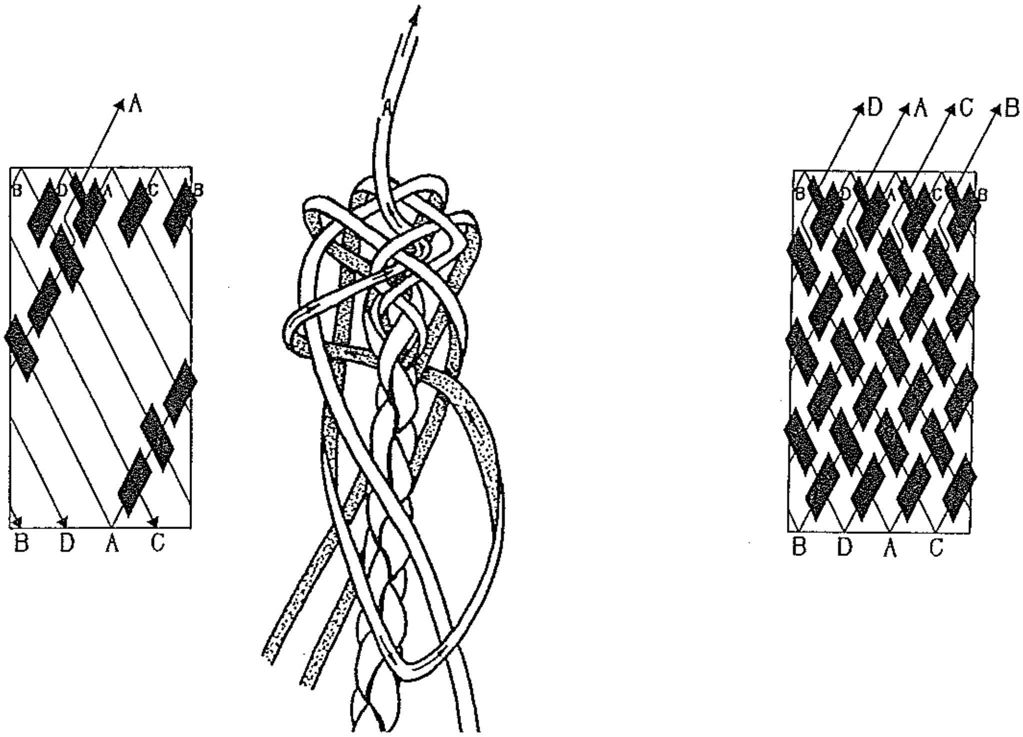
Turn upside down and
back to front.

Fig. 341 — Preparation of 4-string Round Braid for $p/b = 8/4$ Terminal Casa Knot.





Turn each string down to
the right.

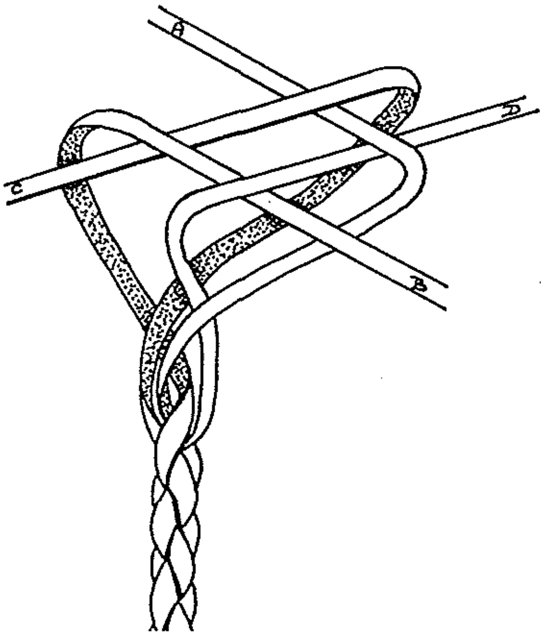


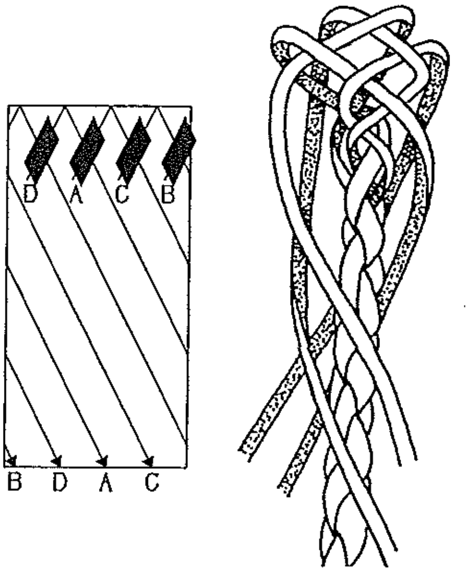
Turn each string up to the right, then
$o - u - o - u - o - u$, along the left of the Standing End
as indicated for string 'A'. Repeat for strings 'B', 'C', and 'D'.

Fig. 342 — The integrated $p/b = 8/4$ Terminal Casa Knot.

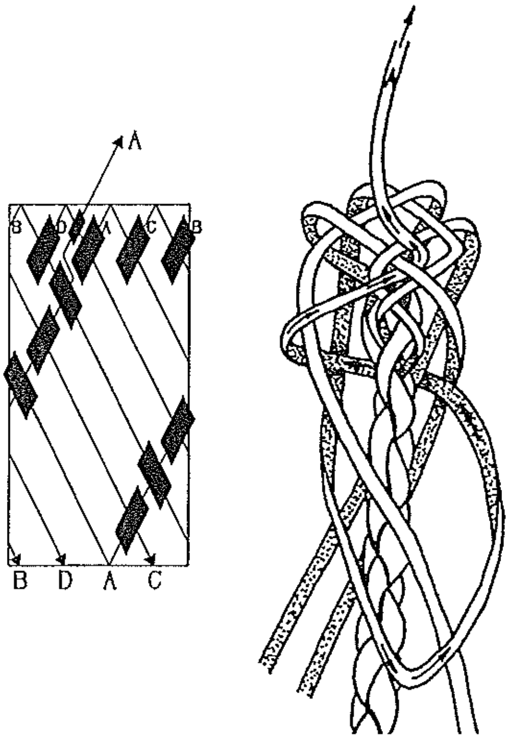
The end of the strings may be cut in order to form a tassel. The length of the integrated knot can be varied to suit.

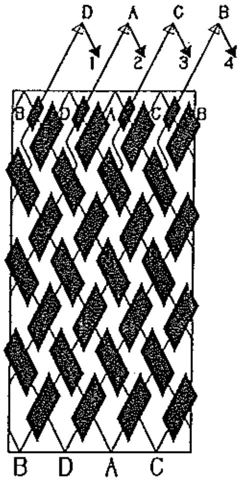
For many applications a tasselled end would not be appropriate, and in such cases we often want an end-knot of greater diameter. This may readily be done by braiding a second integrated knot over the first one. The string-ends of the first or foundation knot are then not fed through the eye of its crown (see Fig. 343).





Turn each string down to the right.





Turn each string up to the right, then $o - u - o - u - o - u$, along the left of the Standing End as indicated for string 'A'. Repeat for strings 'B', 'C', and 'D'.

Fig. 343 — The integrated $p/b = 8/4$ Foundation Casa Knot.

The second integrated knot, which will cover the foundation knot shown in Fig. 343, is depicted in Fig. 344. Any excess string should be cut off after the second knot has been properly tightened so that no string-ends are visible. Note that the 4-string Round Braid has been turned horizontally with the foundation knot to the right.

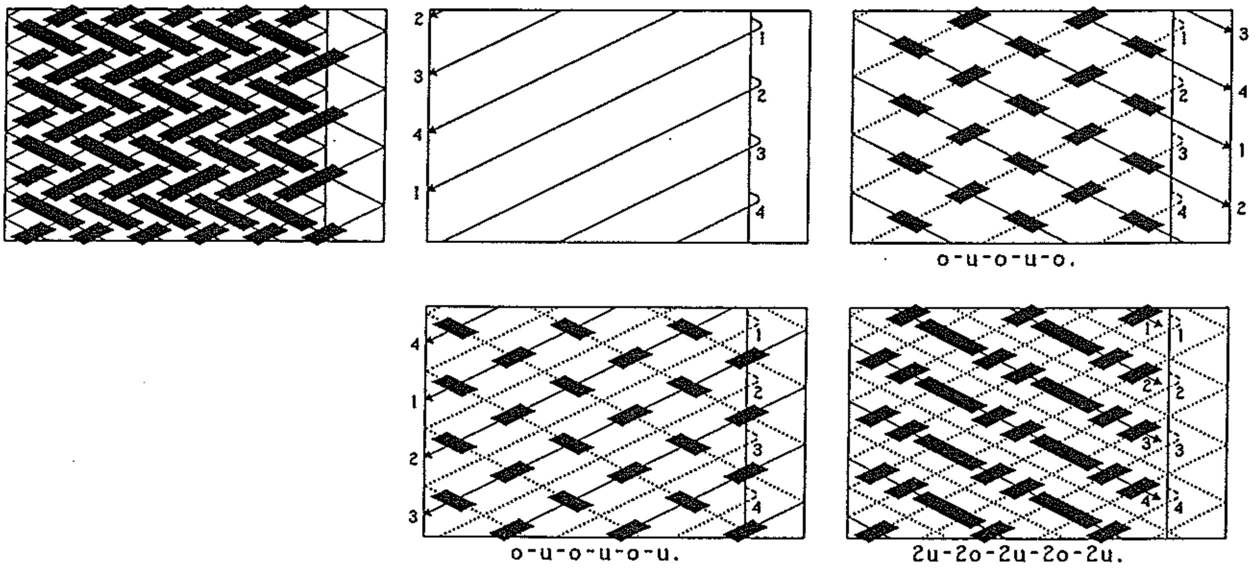


Fig. 344 — The second integrated knot covering the $p/b = 8/4$ Foundation Casa Knot.

In Fig. 345 we have depicted another example of an integrated terminal knot consisting of a foundation knot covered by a second knot.

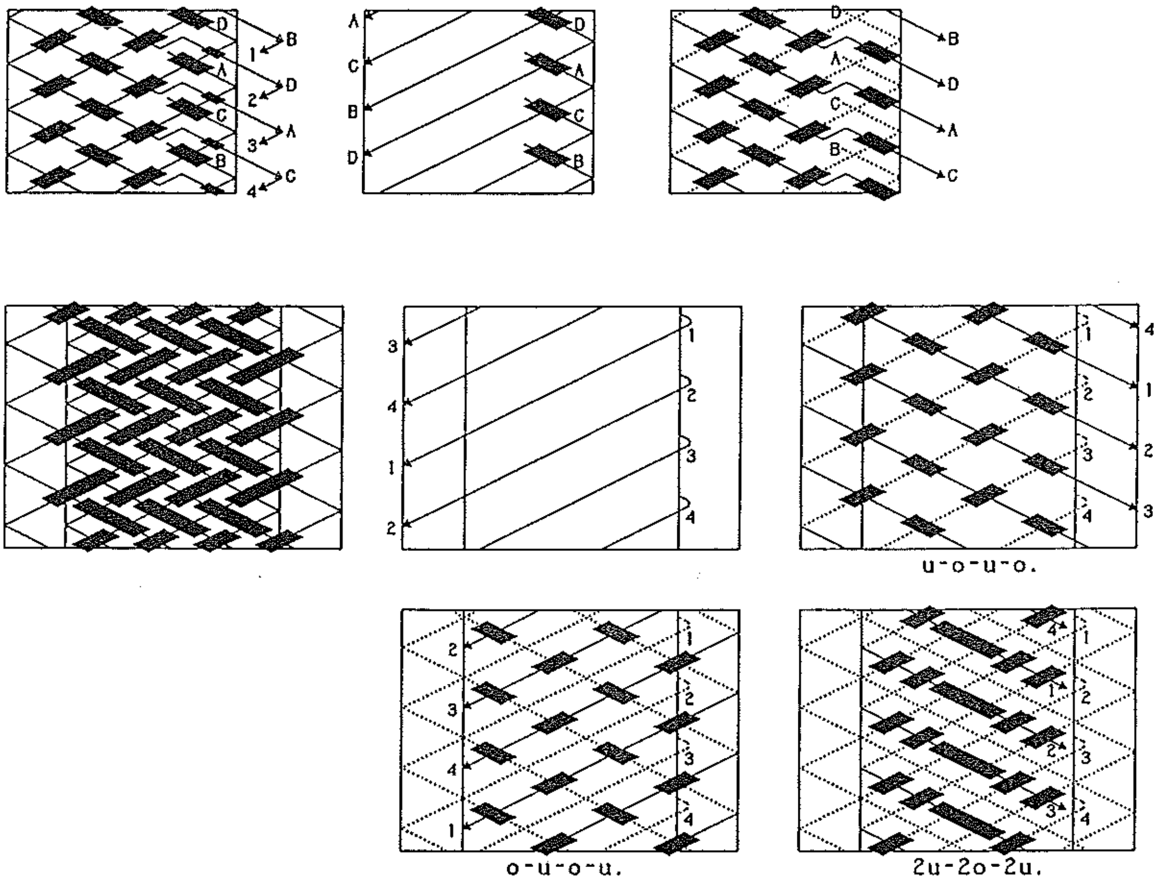


Fig. 345 — An integrated first and second terminal knot for a 4-string Round Braid.

It will be self evident that the sizes of the integrated knots have to be compatible with the string-size. In order to enable the reader to braid the discussed integrated knots as inexpensive exercise pieces, the shown examples are suitable for the fabric-reinforced plastic "Dura Flex" lace.

In the following example the terminal integrated knot is perfectly spherical. This is achieved by using three integrated knots on top of each other.

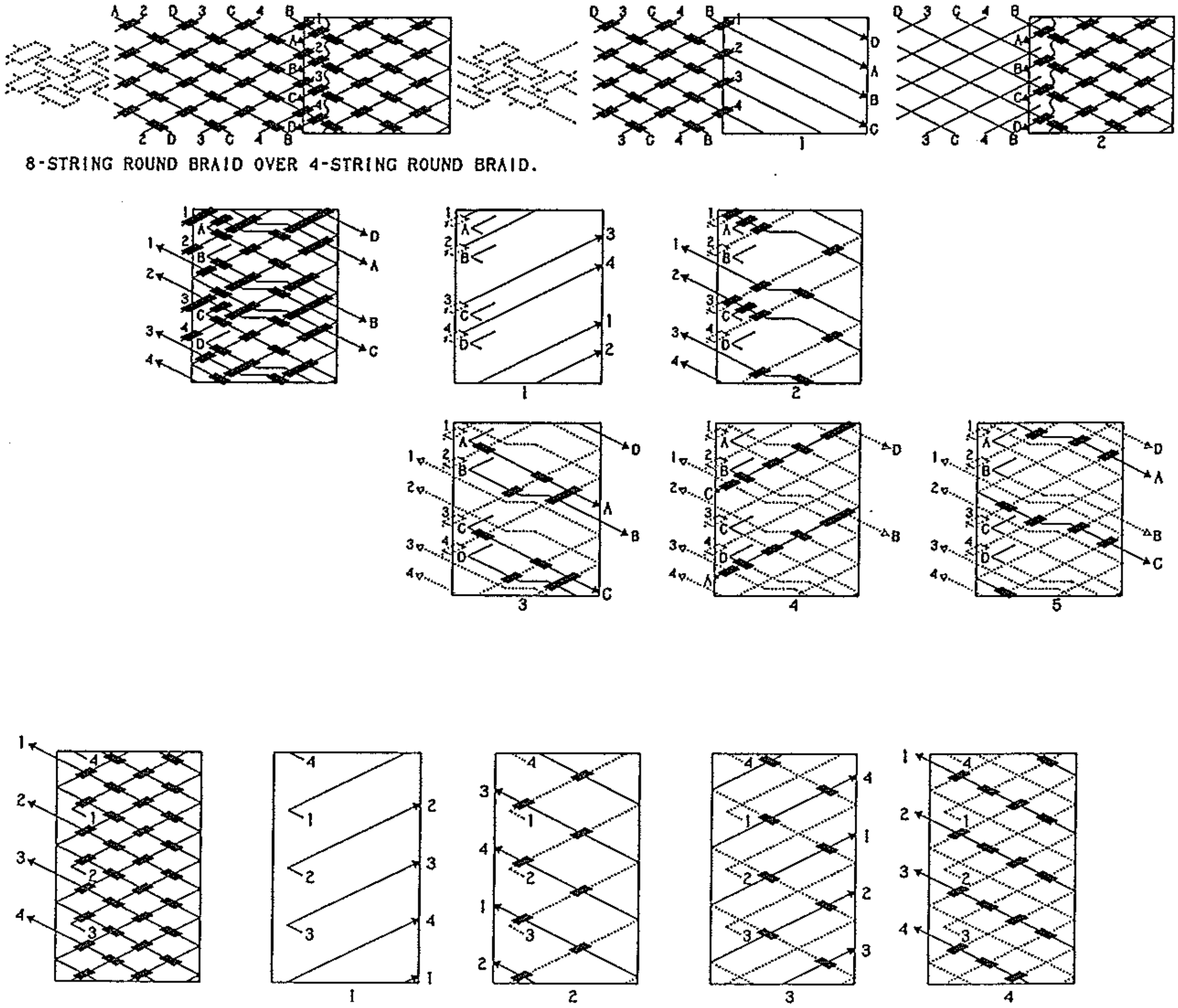

8-STRING ROUND BRAID OVER 4-STRING ROUND BRAID.

Fig. 346 — The first and second integrated foundation knot.

The 8-string Casa-coded Round Braid over the 4-string Round Braid core is finished off as indicated in the upper row of diagrams (see Fig. 346).

The next two rows of diagrams in Fig. 346 depict the braiding of the first integrated foundation knot, while the last row of diagrams depict the braiding of the second integrated foundation knot.

After the integrated foundation knots have been completed, the 4-string Round Braid core is unbraided up to the integrated foundation knots, and the final integrated knot, depicted in Fig. 347, is then braided over the foundation knots.

Note that all the strings of the 8-string Round Braid take part in the first foundation knot. The strings of the 4-string Round Braid core, which are marked I, II, II, IV, do not partake in the construction of the foundation knots, but only partake in the construction of the final finishing knot, together with all the strings of the 8-string Round Braid.

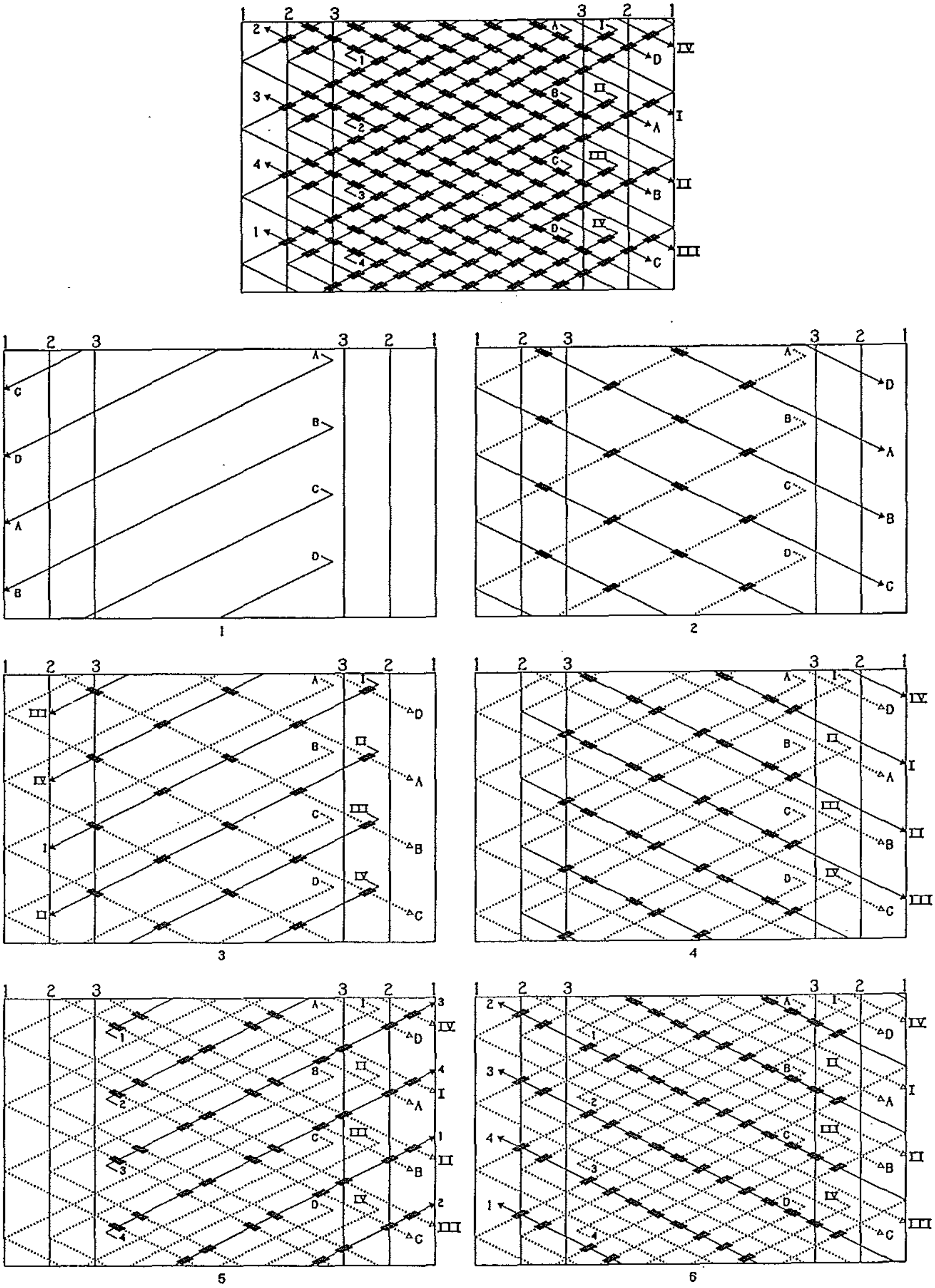


Fig. 347 — The final integrated knot over the integrated foundation knots.

THE BRAIDER

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A quarterly publication for the braiding artisan

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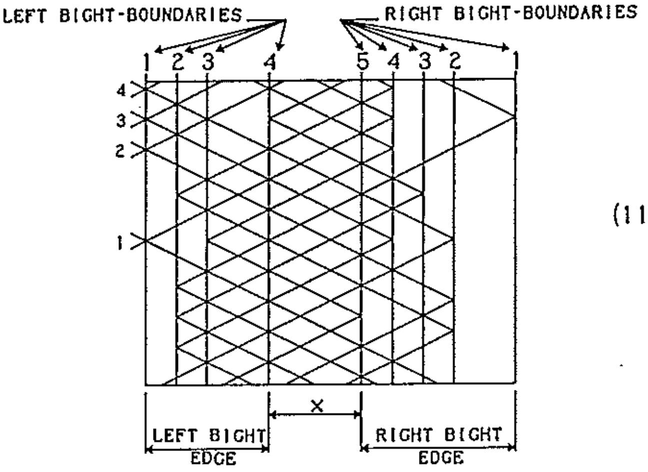
Nested Cylindrical Braids

Nested Cylindrical Braids are cylindrial braids in which at least on one of its two **bight-edges** not all the bights are on one **bight-boundary** (see Fig. 348).



(112/3/1112){1241112223/4422524314}

Fig. 348 — A Nested Cylindrical Braid.

The string-run of the Nested Cylindrical Braid in Fig. 348 is specified by:
(112/3/1112){1241112223/4422524314}10

In this specification (112/3/1112) gives the position of the bight-boundaries relative to each other in column-distances from left to right.

The two sets of numerals in {1241112223/4422524314} specify the positions of the bight-points in respectively a **period** at the left bight-edge and in a **period** at the right bight-edge. The final number (10) indicates the total number of bights (B_{total}) at each of the two bight-edges.

Note that in this braid there is only one period on each bight-edge since the number of numerals in 1241112223 and in 4422524314 is equal to B_{total}.

The first numeral in 1241112223 (hence 1) and the first numeral in 4422524314 (hence 4) indicate that there is a lower-left to upper-right half-cycle which runs from the left bight-boundary 1 to the right bight-boundary 4. This half-cycle is the **reference half-cycle** for the **right-hand bight-points** (the right reference half-cycle RRHC).

The left set of numerals (hence 1241112223) specify in a left period the left bight-boundaries of the left bight-points of the RRHC and the sequential adjacent parallel half-cycles in cyclic fashion above it. Similarly, the right set of numerals (hence 4422524314) specify in a right period the right bight-boundaries of the right bight-points of the RRHC and the sequential adjacent parallel half-cycles in cyclic fashion above it.

With reference to the RRHC, each numeral in the right sequence set 4422524314 is given a **ranking-number** j as subscript, starting with 1 for the RRHC. Hence we obtain for the right period the sequence $4_1 4_2 2_3 2_4 5_5 2_6 4_7 3_8 1_9 4_{10}$.

Each numeral in the left sequence set 1241112223 also receives a **ranking-number** j as subscript. These ranking-numbers j are calculated in the following way.

Say the n^{th} numeral in the left sequence set is k. Then the ranking-number j associated with this numeral is equal to:

$|n + \text{the sum of the first } (k-1) \text{ numerals of the left bight-boundary position specification}|_{A_l}$,
where A_l is the number of bights in a left period.

Consequently for the numerals of the left sequence set 1241112223 we obtain with the left bight-boundary position specification 112 and $A_l = 10$ the following ranking-numbers j:

$n = 1$ for numeral $k = 1$, hence its ranking-number $j = |1 + 0|_{10} = 1$.
$n = 2$ for numeral $k = 2$, hence its ranking-number $j = |2 + 1|_{10} = 3$.
$n = 3$ for numeral $k = 4$, hence its ranking-number $j = |3 + (1 + 1 + 2)|_{10} = 7$.
$n = 4$ for numeral $k = 1$, hence its ranking-number $j = |4 + 0|_{10} = 4$.
$n = 5$ for numeral $k = 1$, hence its ranking-number $j = |5 + 0|_{10} = 5$.
$n = 6$ for numeral $k = 1$, hence its ranking-number $j = |6 + 0|_{10} = 6$.
$n = 7$ for numeral $k = 2$, hence its ranking-number $j = |7 + 1|_{10} = 8$.
$n = 8$ for numeral $k = 2$, hence its ranking-number $j = |8 + 1|_{10} = 9$.
$n = 9$ for numeral $k = 2$, hence its ranking-number $j = |9 + 1|_{10} = 0 = 10$.
$n = 10$ for numeral $k = 3$, hence its ranking-number $j = |10 + (1 + 1)|_{10} = 2$.

Hence we obtain for the left period the sequence $1_1 2_3 4_7 1_4 1_5 1_6 2_8 2_9 2_{10} 3_2$.

The half-cycle from lower-right to upper-left whose left-hand bight-point has the ranking-number 1 is the **reference half-cycle** for the **left-hand bight-points** (the left reference half-cycle LRHC).

The **left nesting-number** A_l is the number of bights in a period at the left bight-edge, and the **right nesting-number** A_r is the number of bights in a period at the right bight-edge.

A **first return string-run** is the shortest consecutive half-cycle sequence to a half-cycle which is identical to the first half-cycle in the sequence. A first return string-run can readily be calculated after its first half-cycle has been chosen.

Let in a first return string-run the **first** half-cycle run from lower-left to upper-right and let its left bight-point position be specified by $l_{1_{j_l}}$, where l_1 is its left bight-boundary number, and j_l its ranking-number; similarly let its right bight-point position be specified by $r_{1_{j_r}}$, where r_1 is its right bight-boundary number, and j_r its ranking-number. Furthermore, let a general cycle with its associated bight-points, starting from the left bight-edge, be indicated as shown at the left in Fig. 349, and let a general cycle with its associated bight-points, starting from the right bight-edge, be indicated as shown at the right in Fig. 349.

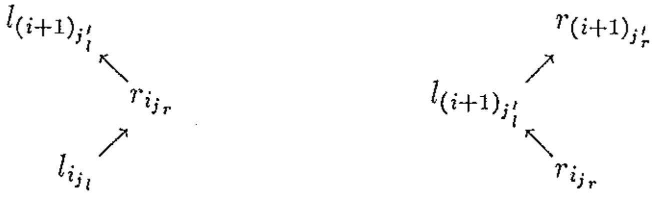


Fig. 349 — A general left and right cycle.

The bight-boundary number l_{i+1} is the left bight-boundary number which is associated with the left ranking-number j'_l. Similarly the bight-boundary number r_{i+1} is the right bight-boundary number which is associated with the right ranking-number j'_r.

Let $\mathcal{K}_l$ be the total number of left bight-boundaries, and let $\mathcal{K}_r$ be the total number of right bight-boundaries.

$$j_l' = |j_l + \Delta_{l_i} + x + \Delta_{r_i}|_{A_l}\,.$$

$$j_r' = \left|j_r + \Delta_{r_i} + x + \Delta_{l_{i+1}}\right|_{A_r}\,.$$

Δ_{l_i} = sum of the last $(\mathcal{K}_l - l_i)$ numerals in left bight-boundary position specification sequence.

$\Delta_{l_{i+1}}$ = sum of the last $(\mathcal{K}_l - l_{i+1})$ numerals in left bight-boundary position specification sequence.

Δ_{r_i} = sum of the first $(\mathcal{K}_r - r_i)$ numerals in right bight-boundary position specification sequence.

$$B_l^* = \text{number of periods at left bight-edge}\,.$$

$$B_r^* = \text{number of periods at right bight-edge}\,.$$

$$B_{total} = A_l B_l^* = A_r B_r^* = A^{**} B^{**}\,.$$

$$d = \text{g.c.d.}(A_l\,,\, A_r)\,.$$

$$A^{**} = \frac{B_{total}}{B^{**}} = \frac{A_l \cdot A_r}{d}\,.$$

$$B^{**} = \frac{B_{total} \cdot d}{A_l \cdot A_r} = \frac{B_l^* \cdot d}{A_r} = \frac{B_r^* \cdot d}{A_l}\,.$$

$$\alpha = \text{number of bights in first-return string-run}\,.$$

$$P_{component} = \frac{\alpha \cdot x + \sum (\Delta_{l_i} + \Delta_{r_i})}{A^{**}}\,.$$

$$P_{total} = \sum P_{component}\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs}\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{sub-components}\\ \text{in a component}\end{array}\right\} = \text{g.c.d.}(P_{component}\,, B^{**})\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components}\,.$$

For the string-run in Fig. 348 we obtain:

$$A_l = 10\,.$$

$$A_r = 10\,.$$

$$d = \text{g.c.d.}\,(A_l\,,\, A_r) = \text{g.c.d.}\,(10\,,\, 10) = 10\,.$$

$$A^{**} = \frac{A_l \cdot A_r}{d} = \frac{10 \cdot 10}{10} = 10\,.$$

$$B_l^* = \text{number of periods at left bight-edge} = 1\,.$$

$$B_r^* = \text{number of periods at right bight-edge} = 1\,.$$

$$B_{total} = A_l B_l^* = A_r B_r^* = A^{**} B^{**} = 10 \cdot 1 = 10\,.$$

$$B^{**} = \frac{B_{total}}{A^{**}} = 1\,.$$

$$\mathcal{K}_l = 4\,.$$

$$\mathcal{K}_r = 5\,.$$

Hence:

$$\begin{array}{llcll}
\Delta_{l_i} = 4 & \text{for } l_i = 1\,. & & \Delta_{r_i} = 5 & \text{for } r_i = 1\,.\\
\Delta_{l_i} = 3 & \text{for } l_i = 2\,. & & \Delta_{r_i} = 3 & \text{for } r_i = 2\,.\\
\Delta_{l_i} = 2 & \text{for } l_i = 3\,. & & \Delta_{r_i} = 2 & \text{for } r_i = 3\,.\\
\Delta_{l_i} = 0 & \text{for } l_i = 4\,. & & \Delta_{r_i} = 1 & \text{for } r_i = 4\,.\\
 & & & \Delta_{r_i} = 0 & \text{for } r_i = 5\,.
\end{array}$$

Each lower-left to upper-right half-cycle type in a Nested Cylindrical Braid occurs only once in all the first-return string-runs of such a braid. The lower-left to upper-right half-cycle types can be read from the Nested Cylindrical Braid specification $(112/3/1112)\{1_1 2_3 4_7 1_4 1_5 1_6 2_8 2_9 2_{10} 3_2 / 4_1 4_2 2_3 2_4 5_5 2_6 4_7 3_8 1_9 4_{10}\}10$:

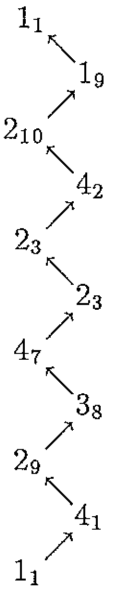
$$\begin{array}{lcl}
1_1 \longrightarrow 4_1 & \qquad & 1_6 \longrightarrow 2_6\\
2_3 \longrightarrow 4_2 & & 2_8 \longrightarrow 4_7\\
4_7 \longrightarrow 2_3 & & 2_9 \longrightarrow 3_8\\
1_4 \longrightarrow 2_4 & & 2_{10} \longrightarrow 1_9\\
1_5 \longrightarrow 5_5 & & 3_2 \longrightarrow 4_{10}
\end{array}$$

Anyone of these listed types may be taken as the first lower-left to upper-right half-cycle in the 1^{st} first-return string-run, but normally we take the first listed one. Every lower-left to upper-right half-cycle encountered in this 1^{st} first-return string-run gets deleted from the type-list.

Anyone of the remaining types in the type-list may be taken as the first lower-left to upper-right half-cycle in the 2^{nd} first-return string-run, and again every lower-left to upper-right half-cycle encountered in this 2^{nd} first-return string-run gets deleted from the remaining type-list.

This process is carried on till all the lower-left to upper-right half-cycle types have been deleted from the type-list.

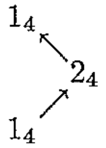
For the first-return string-runs we thus obtain:



$$\begin{array}{lclcl}
l_{1_{j_l}} = 1_1 \longrightarrow 4_1 = r_{1_{j_r}} & \to & j'_l = |1+4+3+1|_{10} = 9 & \to & l_{2_{j'_l}} = 2_9\,.\\
l_{2_{j'_l}} = 2_9 \longleftarrow 4_1 = r_{1_{j_r}} & \to & j'_r = |1+1+3+3|_{10} = 8 & \to & r_{2_{j'_r}} = 3_8\,.\\
l_{2_{j_l}} = 2_9 \longrightarrow 3_8 = r_{2_{j_r}} & \to & j'_l = |9+3+3+2|_{10} = 7 & \to & l_{3_{j'_l}} = 4_7\,.\\
l_{3_{j'_l}} = 4_7 \longleftarrow 3_8 = r_{2_{j_r}} & \to & j'_r = |8+2+3+0|_{10} = 3 & \to & r_{3_{j'_r}} = 2_3\,.\\
l_{3_{j_l}} = 4_7 \longrightarrow 2_3 = r_{3_{j_r}} & \to & j'_l = |7+0+3+3|_{10} = 3 & \to & l_{4_{j'_l}} = 2_3\,.\\
l_{4_{j'_l}} = 2_3 \longleftarrow 2_3 = r_{3_{j_r}} & \to & j'_r = |3+3+3+3|_{10} = 2 & \to & r_{4_{j'_r}} = 4_2\,.\\
l_{4_{j_l}} = 2_3 \longrightarrow 4_2 = r_{4_{j_r}} & \to & j'_l = |3+3+3+1|_{10} = 10 & \to & l_{5_{j'_l}} = 2_{10}\,.\\
l_{5_{j'_l}} = 2_{10} \longleftarrow 4_2 = r_{4_{j_r}} & \to & j'_r = |2+1+3+3|_{10} = 9 & \to & r_{5_{j'_r}} = 1_9\,.\\
l_{5_{j_l}} = 2_{10} \longrightarrow 1_9 = r_{5_{j_r}} & \to & j'_l = |10+3+3+5|_{10} = 1 & \to & l_{6_{j'_l}} = 1_1\,.\\
l_{6_{j'_l}} = 1_1 \longleftarrow 1_9 = r_{5_{j_r}} & \to & j'_r = |9+5+3+4|_{10} = 1 & \to & r_{6_{j'_r}} = 4_1\,.
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{5 \cdot 3 + (4+3+0+3+3) + (1+2+3+1+5)}{10} = 4\,.$$

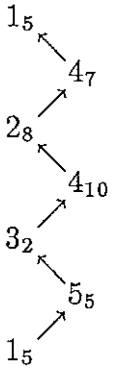
g.c.d.$(P_c\,,\,B^{**}) =$ g.c.d.$(4\,,\,1) = 1\,.$



$$\begin{aligned}
&l_{1_{j_l}} = 1_4 \longrightarrow 2_4 = r_{1_{j_r}} &&\rightarrow\ j'_l = |4+4+3+3|_{10} = 4 &&\rightarrow\ l_{2_{j'_l}} = 1_4\,.\\
&l_{2_{j'_l}} = 1_4 \longleftarrow 2_4 = r_{1_{j_r}} &&\rightarrow\ j'_r = |4+3+3+4|_{10} = 4 &&\rightarrow\ r_{2_{j'_r}} = 2_4\,.
\end{aligned}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 3 + (4) + (3)}{10} = 1\,.$$

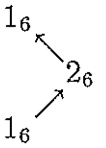
g.c.d. $(P_c\,,\, B^{**}) =$ g.c.d. $(1\,,\,1) = 1\,.$



$$\begin{aligned}
&l_{1_{j_l}} = 1_5 \longrightarrow 5_5 = r_{1_{j_r}} &&\rightarrow\ j'_l = |5+4+3+0|_{10} = 2 &&\rightarrow\ l_{2_{j'_l}} = 3_2\,.\\
&l_{2_{j'_l}} = 3_2 \longleftarrow 5_5 = r_{1_{j_r}} &&\rightarrow\ j'_r = |5+0+3+2|_{10} = 10 &&\rightarrow\ r_{2_{j'_r}} = 4_{10}\,.\\
&l_{2_{j_l}} = 3_2 \longrightarrow 4_{10} = r_{2_{j_r}} &&\rightarrow\ j'_l = |2+2+3+1|_{10} = 8 &&\rightarrow\ l_{3_{j'_l}} = 2_8\,.\\
&l_{3_{j'_l}} = 2_8 \longleftarrow 4_{10} = r_{2_{j_r}} &&\rightarrow\ j'_r = |10+1+3+3|_{10} = 7 &&\rightarrow\ r_{3_{j'_r}} = 4_7\,.\\
&l_{3_{j_l}} = 2_8 \longrightarrow 4_7 = r_{3_{j_r}} &&\rightarrow\ j'_l = |8+3+3+1|_{10} = 5 &&\rightarrow\ l_{4_{j'_l}} = 1_5\,.\\
&l_{4_{j'_l}} = 1_5 \longleftarrow 4_7 = r_{3_{j_r}} &&\rightarrow\ j'_r = |7+1+3+4|_{10} = 5 &&\rightarrow\ r_{4_{j'_r}} = 5_5\,.
\end{aligned}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{3 \cdot 3 + (4+2+3) + (0+1+1)}{10} = 2\,.$$

g.c.d. $(P_c\,,\, B^{**}) =$ g.c.d. $(2\,,\,1) = 1\,.$



$$\begin{aligned}
&l_{1_{j_l}} = 1_6 \longrightarrow 2_6 = r_{1_{j_r}} &&\rightarrow\ j'_l = |6+4+3+3|_{10} = 6 &&\rightarrow\ l_{2_{j'_l}} = 1_6\,.\\
&l_{2_{j'_l}} = 1_6 \longleftarrow 2_6 = r_{1_{j_r}} &&\rightarrow\ j'_r = |6+3+3+4|_{10} = 6 &&\rightarrow\ r_{2_{j'_r}} = 2_6\,.
\end{aligned}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 3 + (4) + (3)}{10} = 1\,.$$

g.c.d. $(P_c\,,\, B^{**}) =$ g.c.d. $(1\,,\,1) = 1\,.$

Hence:

$$P_{total} = \sum P_{component} = 4+1+1+2 = 8\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 4\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1+1+1+1 = 4\,.$$

In general a Nested Cylindrical Braid will have different nesting-numbers A_l and A_r, and in general $B^{**} \neq 1$. A string-run example of such a Nested Cylindrical Braid for which $A_l = 4$ and $A_r = 6$ with $B_{total} = 24$ is depicted in Fig. 350.

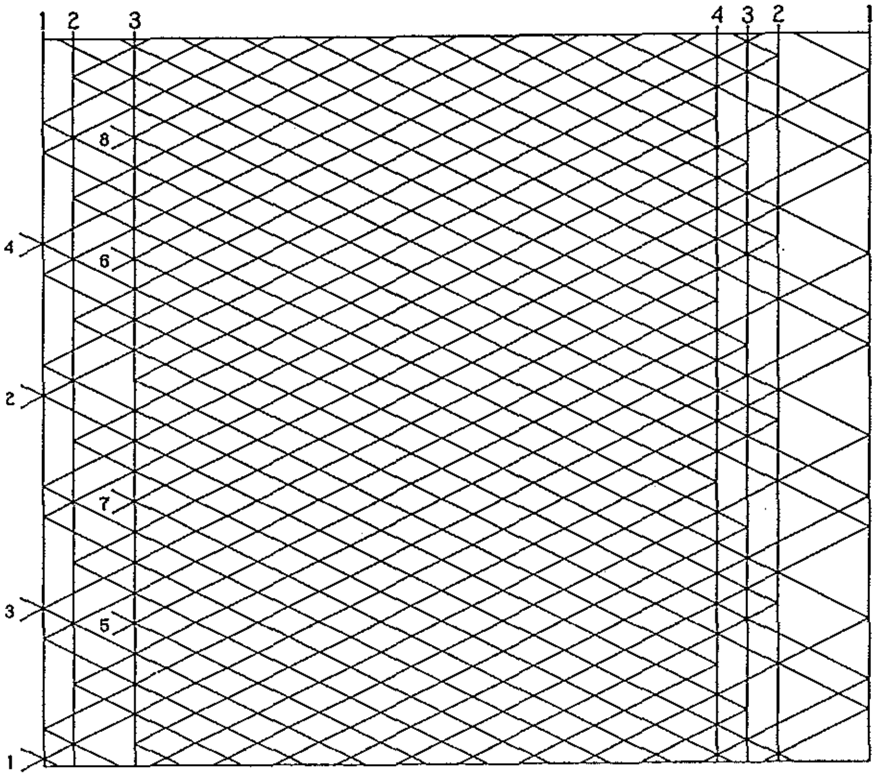


Fig. 350 — A Nested Cylindrical Braid with $A_l = 4$, $A_r = 6$, $x = 19$ and $B_{total} = 24$.

The specification of this braid is (12/19/113){1123/113142}24, or with the ranking-numbers as subscripts attached to the bight-bondary numbers:

$$12/19/113)\{1_1 1_2 2_4 3_3 / 1_1 1_2 3_3 1_4 4_5 2_6\}24\,.$$

For the string-run we obtain:

$$A_l = 4\,.$$
$$A_r = 6\,.$$
$$d = \text{g.c.d.}\,(A_l\,,\,A_r) = \text{g.c.d.}\,(4\,,\,6) = 2\,.$$
$$A^{**} = \frac{A_l \cdot A_r}{d} = \frac{4 \cdot 6}{2} = 12\,.$$
$$B_l^* = \text{number of periods at left bight-edge} = 6\,.$$
$$B_r^* = \text{number of periods at right bight-edge} = 4\,.$$
$$B_{total} = A_l B_l^* = A_r B_r^* = A^{**} B^{**} = 12 \cdot 2 = 24\,.$$
$$B^{**} = \frac{B_{total}}{A^{**}} = 2\,.$$
$$\mathcal{K}_l = 3\,.$$
$$\mathcal{K}_r = 4\,.$$

Hence:

$\Delta_{l_i} = 3$ for $l_i = 1$.
$\Delta_{l_i} = 2$ for $l_i = 2$.
$\Delta_{l_i} = 0$ for $l_i = 3$.

$\Delta_{r_i} = 5$ for $r_i = 1$.
$\Delta_{r_i} = 2$ for $r_i = 2$.
$\Delta_{r_i} = 1$ for $r_i = 3$.
$\Delta_{r_i} = 0$ for $r_i = 4$.

The lower-left to upper-right half-cycle types can be read from the Nested Cylindrical Braid specification $(12/19/113)\{1_1 1_2 2_4 3_3 / 1_1 1_2 3_3 1_4 4_5 2_6\}24$:

$$\begin{array}{ccc}
1_1 \longrightarrow 1_1 & 1_1 \longrightarrow 4_5 & 1_1 \longrightarrow 3_3 \\
1_2 \longrightarrow 1_2 & 1_2 \longrightarrow 2_6 & 1_2 \longrightarrow 1_4 \\
2_4 \longrightarrow 3_3 & 2_4 \longrightarrow 1_1 & 2_4 \longrightarrow 4_5 \\
3_3 \longrightarrow 1_4 & 3_3 \longrightarrow 1_2 & 3_3 \longrightarrow 2_6
\end{array}$$

For the first-return string-runs we thus obtain:

$$\begin{tikzcd}
1_1 & \\
& 1_4 \arrow[ul] \\
1_2 \arrow[ur] & \\
& 1_1 \arrow[ul] \\
2_4 \arrow[ur] & \\
& 3_3 \arrow[ul] \\
1_1 \arrow[ur] & \\
& 4_5 \arrow[ul] \\
2_4 \arrow[ur] & \\
& 2_6 \arrow[ul] \\
3_3 \arrow[ur] & \\
& 4_5 \arrow[ul] \\
1_1 \arrow[ur] & \\
& 1_2 \arrow[ul] \\
1_2 \arrow[ur] & \\
& 3_3 \arrow[ul] \\
2_4 \arrow[ur] & \\
& 1_1 \arrow[ul] \\
1_1 \arrow[ur] &
\end{tikzcd}$$

$$\begin{array}{lclcl}
l_{1_{j_l}} = 1_1 \longrightarrow 1_1 = r_{1_{j_r}} & \to & j'_l = |1+3+19+5|_4 = 4 & \to & l_{2_{j'_l}} = 2_4 . \\
l_{2_{j'_l}} = 2_4 \longleftarrow 1_1 = r_{1_{j_r}} & \to & j'_r = |1+5+19+2|_6 = 3 & \to & r_{2_{j'_r}} = 3_3 . \\
l_{2_{j_l}} = 2_4 \longrightarrow 3_3 = r_{2_{j_r}} & \to & j'_l = |4+2+19+1|_4 = 2 & \to & l_{3_{j'_l}} = 1_2 . \\
l_{3_{j'_l}} = 1_2 \longleftarrow 3_3 = r_{2_{j_r}} & \to & j'_r = |3+1+19+3|_6 = 2 & \to & r_{3_{j'_r}} = 1_2 . \\
l_{3_{j_l}} = 1_2 \longrightarrow 1_2 = r_{3_{j_r}} & \to & j'_l = |2+3+19+5|_4 = 1 & \to & l_{4_{j'_l}} = 1_1 . \\
l_{4_{j'_l}} = 1_1 \longleftarrow 1_2 = r_{3_{j_r}} & \to & j'_r = |2+5+19+3|_6 = 5 & \to & r_{4_{j'_r}} = 4_5 . \\
l_{4_{j_l}} = 1_1 \longrightarrow 4_5 = r_{4_{j_r}} & \to & j'_l = |1+3+19+0|_4 = 3 & \to & l_{5_{j'_l}} = 3_3 . \\
l_{5_{j'_l}} = 3_3 \longleftarrow 4_5 = r_{4_{j_r}} & \to & j'_r = |5+0+19+0|_6 = 6 & \to & r_{5_{j'_r}} = 2_6 . \\
l_{5_{j_l}} = 3_3 \longrightarrow 2_6 = r_{5_{j_r}} & \to & j'_l = |3+0+19+2|_4 = 4 & \to & l_{6_{j'_l}} = 2_4 . \\
l_{6_{j'_l}} = 2_4 \longleftarrow 2_6 = r_{5_{j_r}} & \to & j'_r = |6+2+19+2|_6 = 5 & \to & r_{6_{j'_r}} = 4_5 . \\
l_{6_{j_l}} = 2_4 \longrightarrow 4_5 = r_{6_{j_r}} & \to & j'_l = |4+2+19+0|_4 = 1 & \to & l_{7_{j'_l}} = 1_1 . \\
l_{7_{j'_l}} = 1_1 \longleftarrow 4_5 = r_{6_{j_r}} & \to & j'_r = |5+0+19+3|_6 = 3 & \to & r_{7_{j'_r}} = 3_3 . \\
l_{7_{j_l}} = 1_1 \longrightarrow 3_3 = r_{7_{j_r}} & \to & j'_l = |1+3+19+1|_4 = 4 & \to & l_{8_{j'_l}} = 2_4 . \\
l_{8_{j'_l}} = 2_4 \longleftarrow 3_3 = r_{7_{j_r}} & \to & j'_r = |3+1+19+2|_6 = 1 & \to & r_{8_{j'_r}} = 1_1 . \\
l_{8_{j_l}} = 2_4 \longrightarrow 1_1 = r_{8_{j_r}} & \to & j'_l = |4+2+19+5|_4 = 2 & \to & l_{9_{j'_l}} = 1_2 . \\
l_{9_{j'_l}} = 1_2 \longleftarrow 1_1 = r_{8_{j_r}} & \to & j'_r = |1+5+19+3|_6 = 4 & \to & r_{9_{j'_r}} = 1_4 . \\
l_{9_{j_l}} = 1_2 \longrightarrow 1_4 = r_{9_{j_r}} & \to & j'_l = |2+3+19+5|_4 = 1 & \to & l_{10_{j'_l}} = 1_1 . \\
l_{10_{j'_l}} = 1_1 \longleftarrow 1_4 = r_{9_{j_r}} & \to & j'_r = |4+5+19+3|_6 = 1 & \to & r_{10_{j'_r}} = 1_1 .
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{9 \cdot 19 + (3+2+3+3+0+2+3+2+3) + (5+1+5+0+2+0+1+5+5)}{12} = 18 .$$

g.c.d.$(P_c\,, B^{**}) =$ g.c.d.$(18\,, 2) = 2$.

$$\begin{tikzcd}
1_2 & \\
& 2_6 \arrow[ul] \\
1_2 \arrow[ur] &
\end{tikzcd}$$

$$\begin{array}{lclcl}
l_{1_{j_l}} = 1_2 \longrightarrow 2_6 = r_{1_{j_r}} & \to & j'_l = |2+3+19+2|_4 = 2 & \to & l_{2_{j'_l}} = 1_2 . \\
l_{2_{j'_l}} = 1_2 \longleftarrow 2_6 = r_{1_{j_r}} & \to & j'_r = |6+2+19+3|_6 = 6 & \to & r_{2_{j'_r}} = 2_6 .
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 19 + (3) + (2)}{12} = 2 .$$

g.c.d.$(P_c\,, B^{**}) =$ g.c.d.$(2\,, 2) = 2$.

$$\begin{tikzcd}
3_3 & \\
 & 1_4 \arrow[ul] \\
3_3 \arrow[ur] &
\end{tikzcd}$$

$$\begin{array}{lclcl}
l_{1_{j_l}} = 3_3 \longrightarrow 1_4 = r_{1_{j_r}} & \rightarrow & j'_l = |3+0+19+5|_4 = 3 & \rightarrow & l_{2_{j'_l}} = 3_3\,. \\
l_{2_{j'_l}} = 3_3 \longleftarrow 1_4 = r_{1_{j_r}} & \rightarrow & j'_r = |4+5+19+0|_6 = 4 & \rightarrow & r_{2_{j'_r}} = 1_4\,.
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 19 + (0) + (5)}{12} = 2\,.$$

g.c.d. $(P_c\,,\, B^{**}) =$ g.c.d. $(2\,,\,2) = 2$.

$$\begin{tikzcd}
3_3 & \\
 & 1_2 \arrow[ul] \\
3_3 \arrow[ur] &
\end{tikzcd}$$

$$\begin{array}{lclcl}
l_{1_{j_l}} = 3_3 \longrightarrow 1_2 = r_{1_{j_r}} & \rightarrow & j'_l = |3+0+19+5|_4 = 3 & \rightarrow & l_{2_{j'_l}} = 3_3\,. \\
l_{2_{j'_l}} = 3_3 \longleftarrow 1_2 = r_{1_{j_r}} & \rightarrow & j'_r = |2+5+19+0|_6 = 2 & \rightarrow & r_{2_{j'_r}} = 1_2\,.
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 19 + (0) + (5)}{12} = 2\,.$$

g.c.d. $(P_c\,,\, B^{**}) =$ g.c.d. $(2\,,\,2) = 2$.

Hence:

$$P_{total} = \sum P_{component} = 18 + 2 + 2 + 2 = 24\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 4\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 2 + 2 + 2 + 2 = 8\,.$$

⋆⋆ *Calculate the first-return string-runs, the P_c-values and the number of essential strings required for the Nested Cylindrical Braids with the following string-run specifications:*

(1). (22/5/22){123/312}12.
(2). (2/7/22){12/312}12.
(3). (3/6/22){112/312}12.
Give some different specifications for the above braids.

Regular Cylindrical Bihelix Braids

The first thing we want to know about an $[n+1\,,\,n]$ Prime Regular Cylindrical Bihelix Braid is the number of essential strings required. In the two previous Issues of *The Braider* we have seen how an $[n+1\,,\,n]$ Prime Regular Cylindrical Bihelix Braid can be represented by an Extended Premature Regular Cylindrical Braid which is a Premature Regular Cylindrical Braid to which the necessary additional intersection-points have been added on its right-hand bight-boundary. Furthermore we have seen that these additional intersection-points are associated with the number of bights which were placed to the right and adjacent to the right-hand bight-boundary of the Premature Regular Cylindrical Braid. We can thus calculate the number of essential strings required after we know the p and b values ($b = n+1$) of the Regular Cylindrical Braid concerned and the number of bights adjacent to the right-hand bight-boundary of its premature form. In the previous Issue of *The Braider* on pg. 396 we denoted this number of bights adjacent to its right-hand bight-boundary by δ. The value of δ may be found from the string-run diagram which can be constructed in accordance with the procedures set out in the previous Issue of *The Braider* on pg. 397, or the value of δ may be calculated in accordance with the procedures set out on pg. 400.

The left-hand 'bight'-point of the Premature Regular Cylindrical Braid which has the index-number $I_{0_l} = 0$, contains the Standing-end of the string to which the Working-end of this string does **not** return. The Standing-end half-cycle associated with this left-hand 'bight'-point (hence the half-cycle, from lower-left to upper-right, which leaves the left index-number $I_{0_l} = 0$) runs to the right-hand bight-boundary of the Premature Regular Cylindrical Braid where it enters index-number $I_{0_{ri}} = 0$. The succeeding half-cycle (which runs from lower-right to upper-left) leaves or passes through index-number $I_{0_{ro}}$, and enters index-number I_{1_l} on the left-hand bight-boundary. Note that any two successive half-cycles enter and leave the same bight-point on the left-hand bight-boundary (hence the incoming index-number and the outgoing index-number have the same value), while on the right-hand bight-boundary of the Premature Regular Cylindrical Braid any two successive half-cycles do not necessarily enter and leave the same point. Consequently on the right-hand bight-boundary of the Premature Regular Cylindrical Braid, index-number $I_{x_{ri}}$ associated with an incoming half-cycle and index-number $I_{x_{ro}}$ associated with its succeeding outgoing half-cycle do not necessarily have the same value (see Fig. 351).

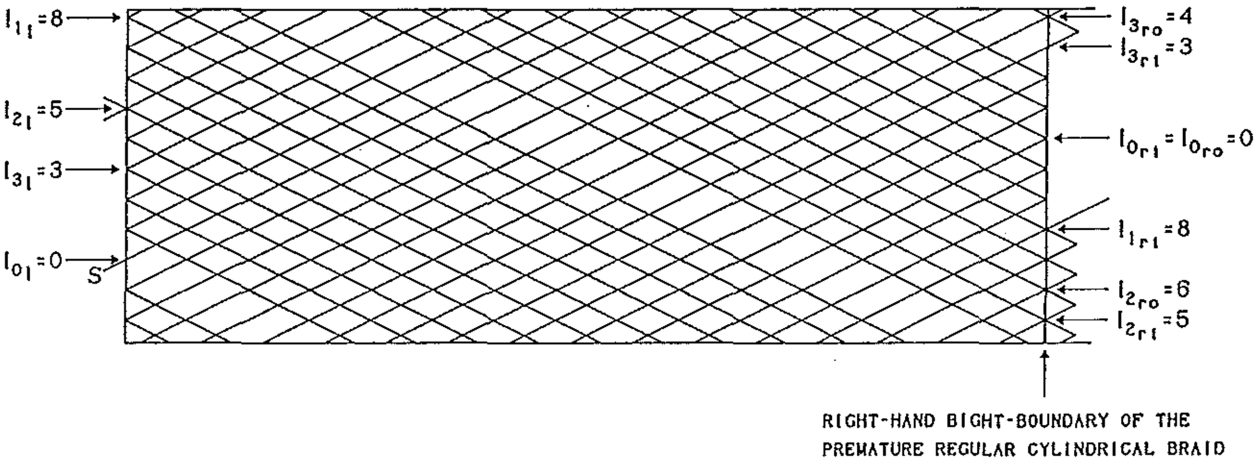


Fig. 351 — The string-run of an Extended Premature Regular Cylindrical Braid.

For the Example in Fig. 351 we obtain the following:

$$
\begin{aligned}
n+1 &= 11\,,\\
n &= 10\,,\\
p &= 30\,,\\
m &= \left\lfloor \frac{pn}{2n^2+2n+1} \right\rfloor = \left\lfloor \frac{300}{221} \right\rfloor = 1\,,\\
c &= |m|_n = |1|_{10} = 1\,,
\end{aligned}
$$

$$
\begin{aligned}
M &= |-p|_{2n+2} = |-30|_{22} = 14\,,\\
T &= \left|2m + \left\lfloor\frac{m}{n}\right\rfloor\right|_{2n+2} = \left|2\times 1 + \left\lfloor\frac{1}{10}\right\rfloor\right|_{22} = |2+0|_{22} = 2\,,\\
z &= p - m(2n+2) - \left\lceil\frac{m}{n}\right\rceil = 30 - 1(22) - \left\lceil\frac{1}{10}\right\rceil = 30 - 22 - 1 = 7\,,\\
r &= |-m|_n = |-1|_{10} = 9\,.
\end{aligned}
$$

Since $r > \left\lceil\frac{z-1}{2}\right\rceil$ we obtain :

$$
\begin{aligned}
z^* &= z + 1 = 7 + 1 = 8\,,\\
r^* &= \left\lceil\frac{z}{2}\right\rceil = \left\lceil\frac{7}{2}\right\rceil = 4\,,\\
\delta &= n - (r - r^*) = 10 - (9-4) = 5\,.
\end{aligned}
$$

The missing half-cycle in the Premature Regular Cylindrical Braid is the lower-right to upper-left half-cycle which would begin at the right-hand bight-boundary in index-number $|-p|_{n+1} = |-30|_{11} = 3$. Hence the lower-left to upper-right half-cycle which passes at the right-hand bight-boundary of the Premature Regular Cylindrical Braid through index-number $I_{j_{r_i}} = 3$ is followed by the lower-right to upper-left half-cycle which passes at that same boundary through index-number $I_{j_{r_o}} = 4$. Similarly for the adjacent $\delta - 1 = 5 - 1 = 4$ left to right and right to left half-cycles:
left to right (in) through $I_{j_{r_i}} = 4$ followed by right to left (out) through $I_{j_{r_o}} = 5$,
left to right (in) through $I_{j_{r_i}} = 5$ followed by right to left (out) through $I_{j_{r_o}} = 6$,
left to right (in) through $I_{j_{r_i}} = 6$ followed by right to left (out) through $I_{j_{r_o}} = 7$,
left to right (in) through $I_{j_{r_i}} = 7$ followed by right to left (out) through $I_{j_{r_o}} = 8$.

The lower-left to upper-right half-cycle which passes through $I_{j_{r_i}} = 8$ exits the braid to the right.

Thus we obtain:

$$
\begin{array}{rll}
1^{st}\text{ half-cycle} & : & I_{0_{l_o}} = 0 \longrightarrow I_{0_{r_i}} = 0\\
2^{nd}\text{ half-cycle} & : & I_{1_{l_i}} = \left|I_{0_{r_o}} + |p|_{n+1}\right|_{n+1} = |0+8|_{11} = 8 \longleftarrow I_{0_{r_o}} = 0\\
3^{rd}\text{ half-cycle} & : & I_{1_{l_o}} = 8 \longrightarrow I_{1_{r_i}} = 8
\end{array}
$$

$$
\begin{array}{rll}
4^{th}\text{ half-cycle} & : & I_{2_{l_o}} = 5 \longrightarrow I_{2_{r_i}} = 5\\
5^{th}\text{ half-cycle} & : & I_{3_{l_i}} = \left|I_{2_{r_o}} + |p|_{n+1}\right|_{n+1} = |6+8|_{11} = 3 \longleftarrow I_{2_{r_o}} = 6\\
6^{th}\text{ half-cycle} & : & I_{3_{l_o}} = 3 \longrightarrow I_{3_{r_i}} = 3\\
7^{th}\text{ half-cycle} & : & I_{4_{l_i}} = \left|I_{3_{r_o}} + |p|_{n+1}\right|_{n+1} = |4+8|_{11} = 1 \longleftarrow I_{3_{r_o}} = 4\\
8^{th}\text{ half-cycle} & : & I_{4_{l_o}} = 1 \longrightarrow I_{4_{r_i}} = 1\\
9^{th}\text{ half-cycle} & : & I_{5_{l_i}} = \left|I_{4_{r_o}} + |p|_{n+1}\right|_{n+1} = |1+8|_{11} = 9 \longleftarrow I_{4_{r_o}} = 1\\
10^{th}\text{ half-cycle} & : & I_{5_{l_o}} = 9 \longrightarrow I_{5_{r_i}} = 9\\
11^{th}\text{ half-cycle} & : & I_{6_{l_i}} = \left|I_{5_{r_o}} + |p|_{n+1}\right|_{n+1} = |9+8|_{11} = 6 \longleftarrow I_{5_{r_o}} = 9\\
12^{th}\text{ half-cycle} & : & I_{6_{l_o}} = 6 \longrightarrow I_{6_{r_i}} = 6\\
13^{th}\text{ half-cycle} & : & I_{7_{l_i}} = \left|I_{6_{r_o}} + |p|_{n+1}\right|_{n+1} = |7+8|_{11} = 4 \longleftarrow I_{6_{r_o}} = 7\\
14^{th}\text{ half-cycle} & : & I_{7_{l_o}} = 4 \longrightarrow I_{7_{r_i}} = 4\\
15^{th}\text{ half-cycle} & : & I_{8_{l_i}} = \left|I_{7_{r_o}} + |p|_{n+1}\right|_{n+1} = |5+8|_{11} = 2 \longleftarrow I_{7_{r_o}} = 5
\end{array}
$$

16th half-cycle : $I_{8_{lo}} = 2 \longrightarrow I_{8_{ri}} = 2$
17th half-cycle : $I_{9_{li}} = \left| I_{8_{ro}} + |p|_{n+1} \right|_{n+1} = |2+8|_{11} = 10 \longleftarrow I_{8_{ro}} = 2$
18th half-cycle : $I_{9_{lo}} = 10 \longrightarrow I_{9_{ri}} = 10$
19th half-cycle : $I_{10_{li}} = \left| I_{9_{ro}} + |p|_{n+1} \right|_{n+1} = |10+8|_{11} = 7 \longleftarrow I_{9_{ro}} = 10$
20th half-cycle : $I_{10_{lo}} = 7 \longrightarrow I_{10_{ri}} = 7$
21st half-cycle : $I_{11_{li}} = \left| I_{10_{ro}} + |p|_{n+1} \right|_{n+1} = |8+8|_{11} = 5 \longleftarrow I_{10_{ro}} = 8$

The Standing-end of the second string may of course be placed in any left-hand bight-point which is not occupied by the first string.

Not only do we now know that we require two strings for this braid, but we also know the number of half-cycles in each of the two interbraided braids, furthermore we know how these two strings are interbraided.

The above displayed calculation procedure can of course be set out in a much more compact and hence simpler manner as shown below:

$\delta = 5$; $|-30|_{11} = 3$; $I_{j_{ri}} = 3,4,5,6,7$; $|30|_{11} = 8$.

$$\begin{array}{c} 0,0 \\ 8 \\ \hline \text{3 half-cycles} \end{array} \qquad \begin{array}{l} 5,6 \\ \quad 3,4 \\ \qquad 1,1 \\ \qquad\quad 9,9 \\ \qquad\qquad 6,7 \\ \qquad\qquad\quad 4,5 \\ \qquad\qquad\qquad 2,2 \\ \qquad\qquad\qquad\quad 10,10 \\ \qquad\qquad\qquad\qquad\quad 7,8 \\ \hline \qquad\qquad \text{18 half-cycles} \end{array}$$

The total number of half-cycles is equal to $2n+1 = 21$.

The coding which is superimposed on the string-run is generally a row-coding (see for example *The Braider*, No. 17, Figs. 329 & 331). Although in the grid-diagram of the equivalent Extended Premature Regular Cylindrical Braid such a row-coding is not a true row-coding, it can however be treated as such by using the appropriate adjustments in the calculation procedures for the determination of the half-cycle algorithms.

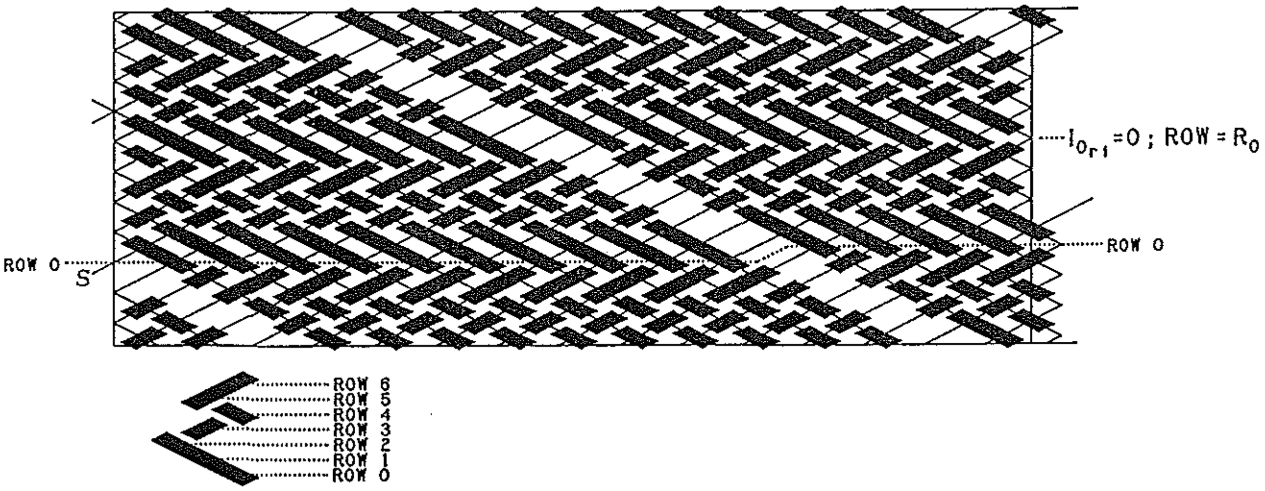


Fig. 352 — The string-run of Fig. 351 with a superimposed 'row'-coding.

Since the number of 'rows' is equal to $2n+1$, we can obtain with a suitable selection for n a 'row'-coding with a periodic pattern. In our Example $2n+1 = 21 = 3 \times 7$, consequently for a repeating coding-pattern we can have a seven 'row' periodic coding-pattern which repeats three times or we can have a three 'row' periodic coding-pattern which repeats seven times. In Fig. 352 we have selected the first option, hence the 'row'-coding period $\pi = 7$ which repeats three times; the 'row'-coding period is shown under the grid-diagram.

The coding of row 0 is the coding on the dotted line which passes on the left-hand bight-boundary through the Standing-end point S (hence index-number $I_{0_{lo}} = 0$), and which passes on the right-hand bight-boundary of the Premature Regular Cylindrical Braid through the point which lies $\left(|p|_{2n+2} - \left\lfloor \frac{p}{2n+2} \right\rfloor\right)$ rows below the intersection-point of this boundary with the Standing-end half-cycle, hence $\left(|p|_{2n+2} - \left\lfloor \frac{p}{2n+2} \right\rfloor\right)$ rows below index-number $I_{0_{ri}} = 0$.

The intersection-rows associated with the half-cycle to be laid down may be calculated as follows:

For a lower-left to upper-right half-cycle:

$$R = \left| 2I_{j_{lo}} + C_l - \left\lfloor \frac{I_{j_{lo}} + C_l}{n+1} \right\rfloor \right|_{\pi},$$

where C_l is the intersection-column number from the left-hand bight-boundary of the Premature Regular Cylindrical Braid.

For a lower-right to upper-left half-cycle:

$$R = \left| p - \left\lfloor \frac{p}{n+1} \right\rfloor + 2I_{j_{ro}} + C_r - \lambda \right|_{\pi},$$

where C_r is the intersection-column number from the right-hand bight-boundary of the Premature Regular Cylindrical Braid,
$\lambda = 0$ for $|-p|_{n+1} = 0$,
$\lambda = 0$ for $I_{j_{ro}} < |-p|_{n+1}$,
$\lambda = 1$ for $I_{j_{ro}} > |-p|_{n+1}$.

The C_l and C_r values are calculated with the following formulae:

$C_l = \left| I_{j_{li_{\text{previous}}}} - I_{j_{lo_{\text{new}}}} \right|_{n+1} + t(n+1)$ where $t = 0, 1, 2, 3, \cdots$ and $0 < C_l \leq p$ when $C_l = p$ is a valid intersection, or $0 < C_l < p$ when $C_l = p$ is not a valid intersection.

$C_r = \left| I_{j_{ri_{\text{previous}}}} - I_{j_{ro_{\text{new}}}} \right|_{n+1} + t(n+1)$ where $t = 0, 1, 2, 3, \cdots$ and $0 \leq C_r < p$ when $C_r = 0$ is a valid intersection, or $0 < C_r < p$ when $C_r = 0$ is not a valid intersection.

A calculation example in full for a left to right and a right to left half-cycle is shown below.

For the 12th half-cycle in our Example we thus obtain:

$I_{j_{lo}} = I_{6_{lo}} = 6$; $C_l = \left| I_{j_{li_{\text{previous}}}} - 6 \right|_{11} + t(11)$ with $I_{j_{li_{\text{previous}}}} = 8, 3, 1, 9, 6$.
$C_l = |8 - 6|_{11} + 11t = 2 + 11t = 2, 13, 24$.
$C_l = |3 - 6|_{11} + 11t = 8 + 11t = 8, 19, 30$.
$C_l = |1 - 6|_{11} + 11t = 6 + 11t = 6, 17, 28$.
$C_l = |9 - 6|_{11} + 11t = 3 + 11t = 3, 14, 25$.
$C_l = |6 - 6|_{11} + 11t = 0 + 11t = 11, 22$.

Hence $C_l = 2,3,6,8,11\,;\ 13,14,17,19,22\,;\ 24,25,28,30$.
$\lfloor \frac{6+C_l}{11} \rfloor = 1,2,3$ for $C_l = 5,16,27$; $R = \left|12 + C_l - \lfloor \frac{6+C_l}{11} \rfloor\right|_7 = \left|5 + C_l - \lfloor \frac{6+C_l}{11} \rfloor\right|_7$.
$C_l = 2,3 \mid 6,8,11,13,14 \mid 17,19,22,24,25 \mid 28,30$.
$R = 0,1\,;\ 3,5,1,3,4\,;\ 6,1,4,6,0\,;\ 2,4$. Hence the sequence is:
Left to **Right** : $2u - 2o - u - o - u - o - 2u - o - 3u$.

For the 15^{th} half-cycle in our Example we thus obtain:
$I_{jr_o} = I_{7r_o} = 5$; $C_r = \left|I_{jri_{\text{previous}}} - 5\right|_{11} + t(11)$ with $I_{jri_{\text{previous}}} = 0,8,5,3,1,9,6,4$.
$C_r = |0-5|_{11} + 11t = 6 + 11t = 6,17,28$.
$C_r = |8-5|_{11} + 11t = 3 + 11t = 3,14,25$.
$C_r = |5-5|_{11} + 11t = 0 + 11t = 0,11,22$.
$C_r = |3-5|_{11} + 11t = 9 + 11t = 9,20$.
$C_r = |1-5|_{11} + 11t = 7 + 11t = 7,18,29$.
$C_r = |9-5|_{11} + 11t = 4 + 11t = 4,15,26$.
$C_r = |6-5|_{11} + 11t = 1 + 11t = 1,12,23$.
$C_r = |4-5|_{11} + 11t = 10 + 11t = 10,21$.
Hence $C_r = 0,1,3,4,6,7,9,10\,;\ 11,12,14,15,17,18,20,21\,;\ 22,23,25,26,28,29$.
$\lambda = 1$; $R = |28 + 10 + C_r - \lambda|_7 = |2 + C_r|_7$.
$C_r = 0,1,3,4,6,7,9,10\,;\ 11,12,14,15,17,18,20,21\,;\ 22,23,25,26,28,29$.
$R = 2,3,5,6,1,2,4,5,6,0,2,3,5,6,1,2,3,4,6,0,2,3$. Hence the sequence is:
Right to **Left** : $o - 3u - 3o - 2u - 2o - 3u - 2o - u - o - u - 2o - u$.

The calculation results for the half-cycle algorithms of this Prime Regular Cylindrical Bihelix Braid (Fig. 352) are as follows:

Half-cycle 1 :
Left → Right : Free run.

Half-cycle 2 : $I_{jr_o} = 0$; $I_{jri_{\text{previous}}} = 0$; $\lambda = 0$; $R = |C_r|_7$.
$C_r = 11; 22$.
$R = 4, 1$.
Right → Left : $2o$.

Half-cycle 3 : $I_{jl_o} = 8$; $I_{jli_{\text{previous}}} = 8$; $\left\lfloor \frac{I_{jl_o}+C_l}{11} \right\rfloor = 1,2,3$ for $C_l = 3,14,25$;
$R = \left|2 + C_l - \lfloor \frac{8+C_l}{11} \rfloor\right|_7$.
$C_l = 11 \mid 22$.
$R = 5; 1$.
Left → Right : $o - u$. String exits braid.

Half-cycle 4 : $I_{jl_o} = 5$; $I_{jli_{\text{previous}}} = 8$; $\left\lfloor \frac{I_{jl_o}+C_l}{11} \right\rfloor = 1,2,3$ for $C_l = 6,17,28$;
$R = \left|3 + C_l - \lfloor \frac{5+C_l}{11} \rfloor\right|_7$. New string.
$C_l = 3 \mid 14 \mid 25$.
$R = 6; 2; 5$.
Left → Right : $o - u - o$.

Half-cycle 5 : $I_{jr_o} = 6$; $I_{jri_{\text{previous}}} = 0,8,5$; $\lambda = 1$; $R = |4 + C_r|_7$.
$C_r = 2,5,10; 13,16,21; 24,27$.
$R = 6,2,0,3,6,4,0,3$.
Right → Left : $u - 2o - 2u - 2o - u$.

Half-cycle 6: $I_{j_{l_o}} = 3$; $I_{j_{l i_{\text{previous}}}} = 8, 3$; $\left\lfloor \frac{I_{j_{l_o}}+C_l}{11} \right\rfloor = 1, 2, 3$ for $C_l = 8, 19, 30$; $R = \left|6 + C_l - \left\lfloor \frac{3+C_l}{11} \right\rfloor\right|_7$.
$C_l = 5 \,|\, 11, 16 \,|\, 22, 27$.
$R = 4; 2, 0; 5, 3$.
Left → Right: $3u - 2o$.

Half-cycle 7: $I_{j_{r_o}} = 4$; $I_{j_{r i_{\text{previous}}}} = 0, 8, 5, 3$; $\lambda = 1$; $R = |C_r|_7$.
$C_r = 1, 4, 7, 10; 12, 15, 18, 21; 23, 26, 29$.
$R = 1, 4, 0, 3, 5, 1, 4, 0, 2, 5, 1$.
Right → Left: $3o - 2u - 4o - u - o$.

Half-cycle 8: $I_{j_{l_o}} = 1$; $I_{j_{l i_{\text{previous}}}} = 8, 3, 1$; $\left\lfloor \frac{I_{j_{l_o}}+C_l}{11} \right\rfloor = 1, 2$ for $C_l = 10, 21$; $R = \left|2 + C_l - \left\lfloor \frac{1+C_l}{11} \right\rfloor\right|_7$.
$C_l = 2, 7 \,|\, 11, 13, 18 \,|\, 22, 24, 29$.
$R = 4, 2; 5, 0, 5; 1, 3, 1$.
Left → Right: $2u - o - u - o - u - o - u$.

Half-cycle 9: $I_{j_{r_o}} = 1$; $I_{j_{r i_{\text{previous}}}} = 0, 8, 5, 3, 1$; $\lambda = 0$; $R = |2 + C_r|_7$.
$C_r = 2, 4, 7, 10, 11; 13, 15, 18, 21, 22; 24, 26, 29$.
$R = 4, 6, 2, 5, 6, 1, 3, 6, 2, 3, 5, 0, 3$.
Right → Left: $o - u - o - 2u - o - 2u - o - 2u - o - u$.

Half-cycle 10: $I_{j_{l_o}} = 9$; $I_{j_{l i_{\text{previous}}}} = 8, 3, 1, 9$; $\left\lfloor \frac{I_{j_{l_o}}+C_l}{11} \right\rfloor = 1, 2, 3$ for $C_l = 2, 13, 24$; $R = \left|4 + C_l - \left\lfloor \frac{9+C_l}{11} \right\rfloor\right|_7$.
$C_l = 3, 5, 10, 11 \,|\, 14, 16, 21, 22 \,|\, 25, 27$.
$R = 6, 1, 6, 0; 2, 4, 2, 3; 5, 0$.
Left → Right: $o - u - o - 4u - 2o - u$.

Half-cycle 11: $I_{j_{r_o}} = 9$; $I_{j_{r i_{\text{previous}}}} = 0, 8, 5, 3, 1, 9$; $\lambda = 1$; $R = |3 + C_r|_7$.
$C_r = 2, 3, 5, 7, 10, 11; 13, 14, 16, 18, 21, 22; 24, 25, 27, 29$.
$R = 5, 6, 1, 3, 6, 0, 2, 3, 5, 0, 3, 4, 6, 0, 2, 4$.
Right → Left: $2u - o - 2u - 2o - 2u - o - u - o - u - 3o$.

Half-cycle 12: $I_{j_{l_o}} = 6$; $I_{j_{l i_{\text{previous}}}} = 8, 3, 1, 9, 6$; $\left\lfloor \frac{I_{j_{l_o}}+C_l}{11} \right\rfloor = 1, 2, 3$ for $C_l = 5, 16, 27$; $R = \left|5 + C_l - \left\lfloor \frac{6+C_l}{11} \right\rfloor\right|_7$.
$C_l = 2, 3 \,|\, 6, 8, 11, 13, 14 \,|\, 17, 19, 22, 24, 25 \,|\, 28, 30$.
$R = 0, 1; 3, 5, 1, 3, 4; 6, 1, 4, 6, 0; 2, 4$.
Left → Right: $2u - 2o - u - o - u - o - 2u - o - 3u$.

Half-cycle 13: $I_{j_{r_o}} = 7$; $I_{j_{r i_{\text{previous}}}} = 0, 8, 5, 3, 1, 9, 6$; $\lambda = 1$; $R = |6 + C_r|_7$.
$C_r = 1, 2, 4, 5, 7, 9, 10; 12, 13, 15, 16, 18, 20, 21; 23, 24, 26, 27, 29$.
$R = 0, 1, 3, 4, 6, 1, 2, 4, 5, 0, 1, 3, 5, 6, 1, 2, 4, 5, 0$.
Right → Left: $2o - u - o - u - 3o - u - 2o - 3u - 3o - u - o$.

Half-cycle 14: $I_{j_{l_o}} = 4$; $I_{j_{l i_{\text{previous}}}} = 8, 3, 1, 9, 6, 4$; $\left\lfloor \frac{I_{j_{l_o}}+C_l}{11} \right\rfloor = 1, 2, 3$ for $C_l = 7, 18, 29$; $R = \left|1 + C_l - \left\lfloor \frac{4+C_l}{11} \right\rfloor\right|_7$.
$C_l = 2, 4, 5 \,|\, 8, 10, 11, 13, 15, 16 \,|\, 19, 21, 22, 24, 26, 27 \,|\, 30$.
$R = 3, 5, 6; 1, 3, 4, 6, 1, 2; 4, 6, 0, 2, 4, 5; 0$.
Left → Right: $3o - u - o - u - o - 3u - o - 3u - o - u$.

Half-cycle 15 : $I_{j_{r_o}} = 5$; $I_{j_{ri_{\text{previous}}}} = 0,8,5,3,1,9,6,4$; $\lambda = 1$; $R = |2 + C_r|_7$.
$C_r = 0,1,3,4,6,7,9,10; 11,12,14,15,17,18,20,21; 22,23,25,26,28,29$.
$R = 2,3,5,6,1,2,4,5,6,0,2,3,5,6,1,2,3,4,6,0,2,3$.
Right → Left : $o - 3u - 3o - 2u - 2o - 3u - 2o - u - o - u - 2o - u$.

Half-cycle 16 : $I_{j_{l_o}} = 2$; $I_{j_{li_{\text{previous}}}} = 8,3,1,9,6,4,2$; $\left\lfloor \frac{I_{j_{l_o}}+C_l}{11} \right\rfloor = 1,2$ for $C_l = 9,20$;
$R = \left|4 + C_l - \left\lfloor \frac{2+C_l}{11} \right\rfloor\right|_7$.
$C_l = 1,2,4,6,7 \,|\, 10,11,12,13,15,17,18 \,|\, 21,22,23,24,26,28,29$.
$R = 5,6,1,3,4; 6,0,1,2,4,6,0; 2,3,4,5,0,2,3$.
Left → Right : $2o - u - o - u - o - 4u - o - 2u - o - u - o - 2u - o$.

Half-cycle 17 : $I_{j_{r_o}} = 2$; $I_{j_{ri_{\text{previous}}}} = 0,8,5,3,1,9,6,4,2$; $\lambda = 0$; $R = |4 + C_r|_7$.
$C_r = 1,2,3,4,6,7,9,10,11; 12,13,14,15,17,18,20,21,22; 23,24,25,26,28,29$.
$R = 5,6,0,1,3,4,6,0,1,2,3,4,5,0,1,3,4,5,6,0,1,2,4,5$.
Right → Left : $2u - 2o - u - o - u - 3o - u - o - u - 2o - u - o - 2u - 4o - u$.

Half-cycle 18 : $I_{j_{l_o}} = 10$; $I_{j_{li_{\text{previous}}}} = 8,3,1,9,6,4,2,10$; $\left\lfloor \frac{I_{j_{l_o}}+C_l}{11} \right\rfloor = 1,2,3$ for
$C_l = 1,12,23$; $R = \left|6 + C_l - \left\lfloor \frac{10+C_l}{11} \right\rfloor\right|_7$.
$C_l = |\, 2,3,4,5,7,9,10,11 \,|\, 13,14,15,16,18,20,21,22 \,|\, 24,25,26,27,29$.
$R = 0,1,2,3,5,0,1,2; 3,4,5,6,1,3,4,5; 6,0,1,2,4$.
Left → Right : $3u - 2o - 3u - o - u - 2o - u - o - u - 2o - 4u$.

Half-cycle 19 : $I_{j_{r_o}} = 10$; $I_{j_{ri_{\text{previous}}}} = 0,8,5,3,1,9,6,4,2,10$; $\lambda = 1$; $R = |5 + C_r|_7$.
$C_r = 1,2,3,4,5,6,7,9,10,11; 12,13,14,15,16,17,18,20,21,22;$
$23,24,25,26,27,28,29$.
$R = 6,0,1,2,3,4,5,0,1,2,3,4,5,6,0,1,2,4,5,6,0,1,2,3,4,5,6$.
Right → Left : $u - 3o - u - o - u - 3o - u - o - 2u - 4o - 2u - 3o - u - o - 2u$.

Half-cycle 20 : $I_{j_{l_o}} = 7$; $I_{j_{li_{\text{previous}}}} = 8,3,1,9,6,4,2,10,7$; $\left\lfloor \frac{I_{j_{l_o}}+C_l}{11} \right\rfloor = 1,2,3$ for
$C_l = 4,15,26$; $R = \left|C_l - \left\lfloor \frac{7+C_l}{11} \right\rfloor\right|_7$.
$C_l = 1,2,3 \,|\, 5,6,7,8,10,11,12,13,14 \,|\, 16,17,18,19,21,22,23,24,25 \,|\, 27,28,29,30$.
$R = 1,2,3; 4,5,6,0,2,3,4,5,6; 0,1,2,3,5,6,0,1,2; 3,4,5,6$.
Left → Right : $2u - o - u - 2o - 2u - o - u - 2o - 3u - 3o - 3u - o - u - 2o$.

Half-cycle 21 : $I_{j_{r_o}} = 8$; $I_{j_{ri_{\text{previous}}}} = 0,8,5,3,1,9,6,4,2,10,7$; $\lambda = 1$; $R = |1 + C_r|_7$.
$C_r = 0,1,2,3,4,5,6,7,8,9,10; 11,12,13,14,15,16,17,18,19,20,21;$
$22,23,24,25,26,27,28,29$.
$R = 1,2,3,4,5,6,0,1,2,3,4,5,6,0,1,2,3,4,5,6,0,1,2,3,4,5,6,0,1,2$.
Right → Left : $2o - u - o - 2u - 3o - u - o - 2u - 3o - u - o - 2u - 3o - u - o - 2u - 3o$.

Although the calculations are simple, there are many to be performed with the consequence that errors may go undetected. It is therefore much more convenient to let a computer do this work, and especially for colour-patterns the computer can then readily draw the **actual** grid-diagram of the Prime Regular Cylindrical Bihelix Braid.

Integrated Braids

In this issue we shall discuss the Gaucho Button Knots as integrated braids. Although the Gaucho Knot Button described by Bruce Grant in the *Encyclopedia of*

Rawhide and Leather Braiding (Fig. 353 depicts its grid-diagram) can be regarded as an integrated braid with respect to the loop which goes over into the four strings of the braid, the resulting button however can in general not be regarded as being a satisfactory one. The reason being that due to the diameter of the Gaucho Knot it will in general not be possible to obtain a sufficiently firm button.

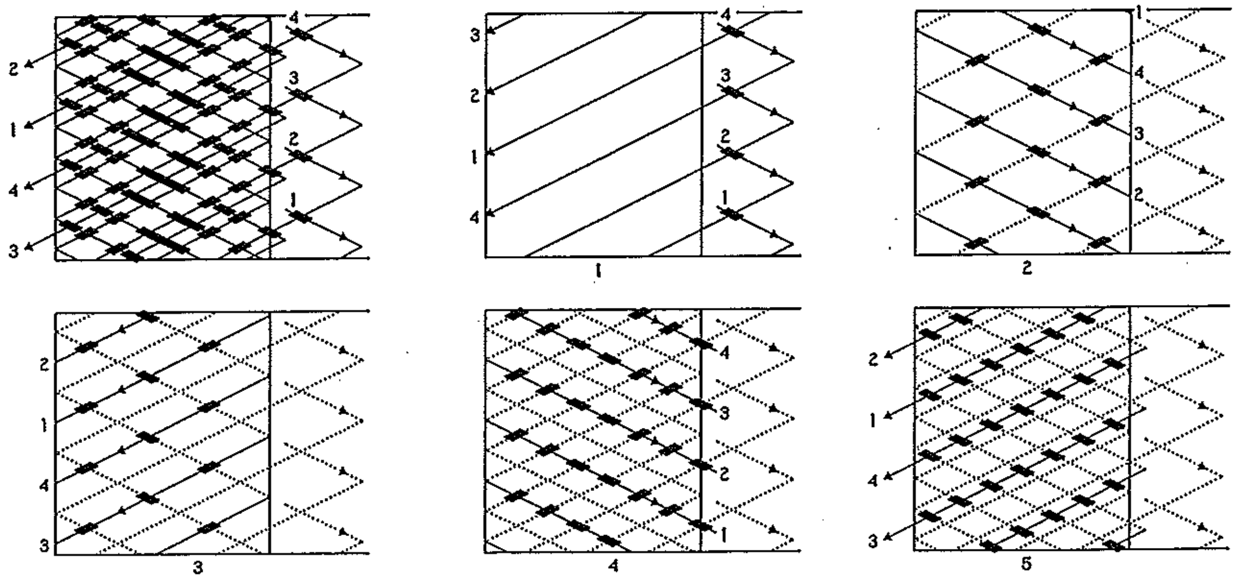


Fig. 353 — The Gaucho Knot Button as described by Bruce Grant in the *Encyclopedia of Rawhide and Leather Braiding.*

In order to improve this button sufficiently, we must place a foundation knot under it. The construction of such a Gaucho Button Knot as an integrated braid at the end of a four string round braid has been depicted in Fig. 354.

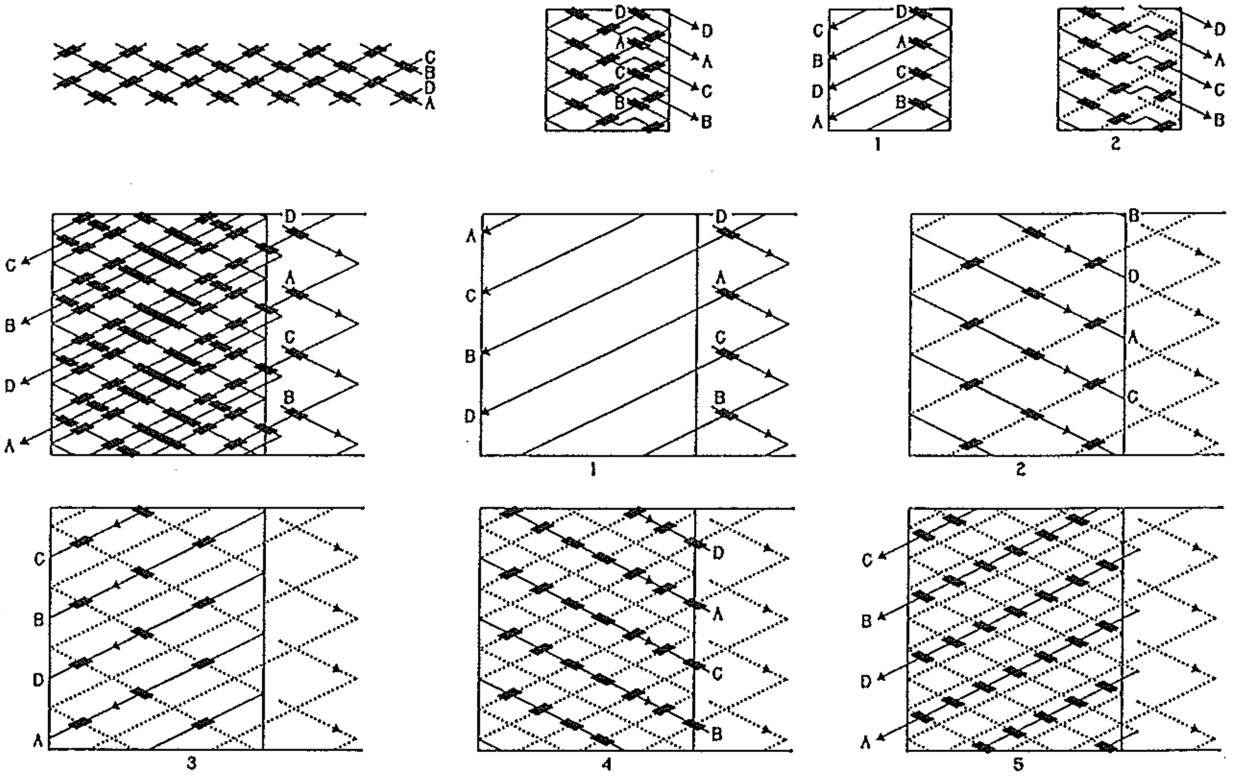


Fig. 354 — A simple integrated Gaucho Button Knot.

In most applications a bigger or bulkier button is often required. This can readily be accomplished by omitting the last braiding step 5 in Fig. 354 and continuing the braiding process after step 4 by braiding a further integrated Gaucho-coded knot over the braid so far obtained. Examples of such further integrated Gaucho-coded knots are shown in Figs. 357 – 361.

The shape of the final button depends on the lenghts of the successive integrated knots. With the integrated overall foundation up to and including step 4 of Fig. 354 the shape of the button will be as follows: with the final integrated knots of Fig. 357 & 358, the shape of the button will be as depicted by the left-most drawing in Fig. 355; with the final integrated knot of Fig. 359, the shape of the button will be as depicted by the second drawing from the left in Fig. 355; and with the final integrated knots of Figs. 360 & 361, the shape of the button will be as depicted by the third drawing from the left in Fig. 355.

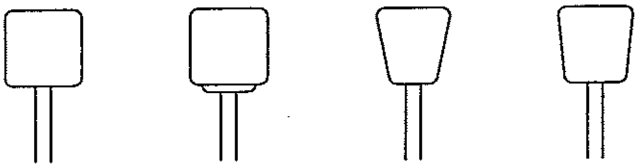


Fig. 355 — The various shapes of the integrated Gaucho-coded Button Knots.

With a longer integrated overall foundation as depicted in Fig. 356 and a final integrated Gaucho-coded knot as in Figs. 360 & 361, the shape of the button will be as depicted by the right-most drawing in Fig. 355.

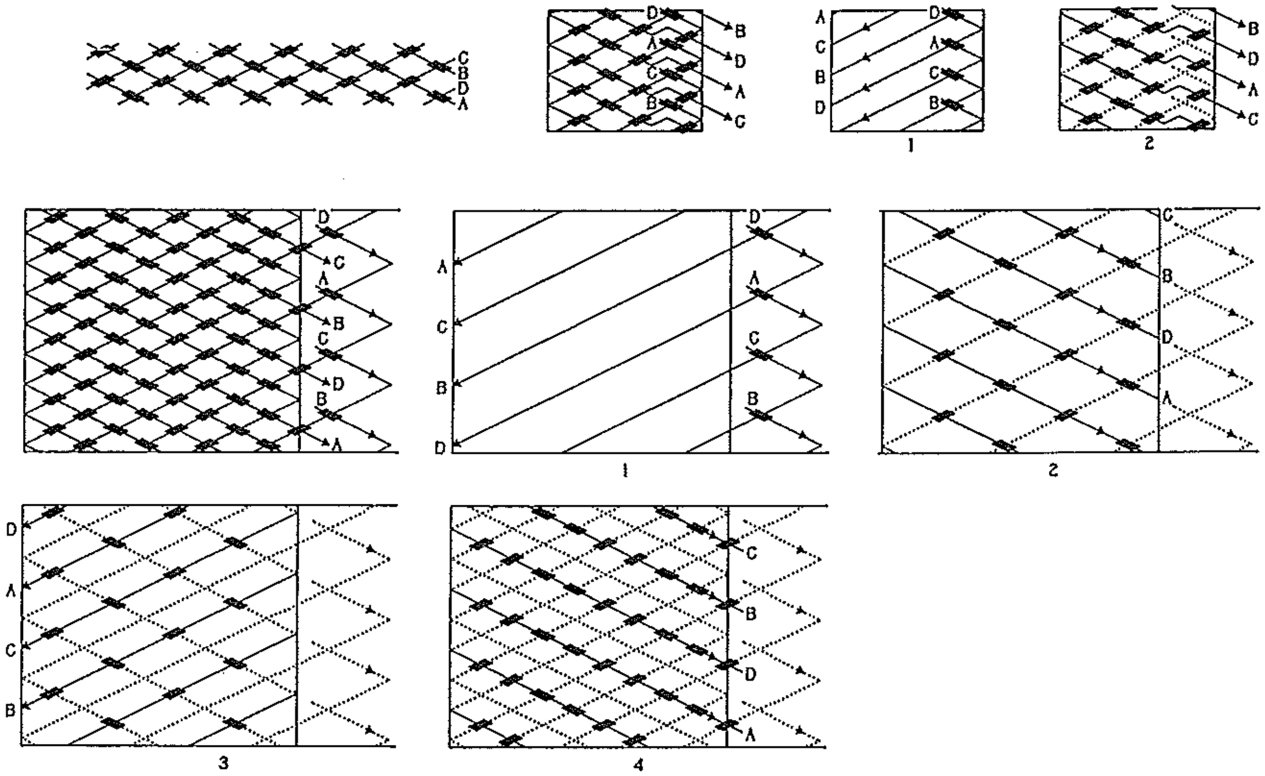


Fig. 356 — A longer integrated foundation.

We like to remind the reader that our depicted examples are intended for the string-dimensions of the inexpensive fabric reinforced "Dura Flex" lace. For other string-dimensions the sizes of the various integrated knots have to be adjusted accordingly.

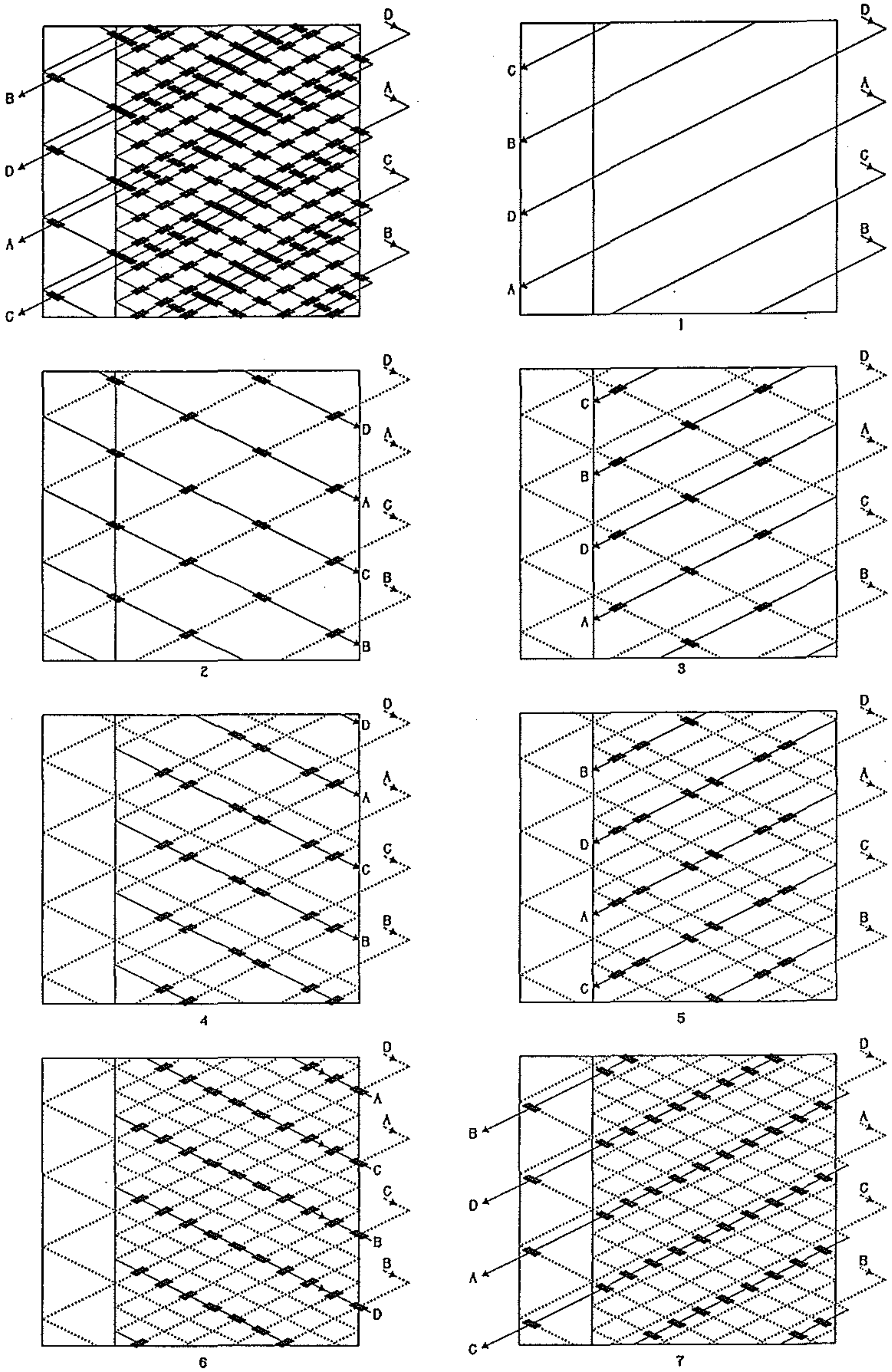


Fig. 357 — A final Gaucho-coded integrated knot for the Gaucho Button Knot.

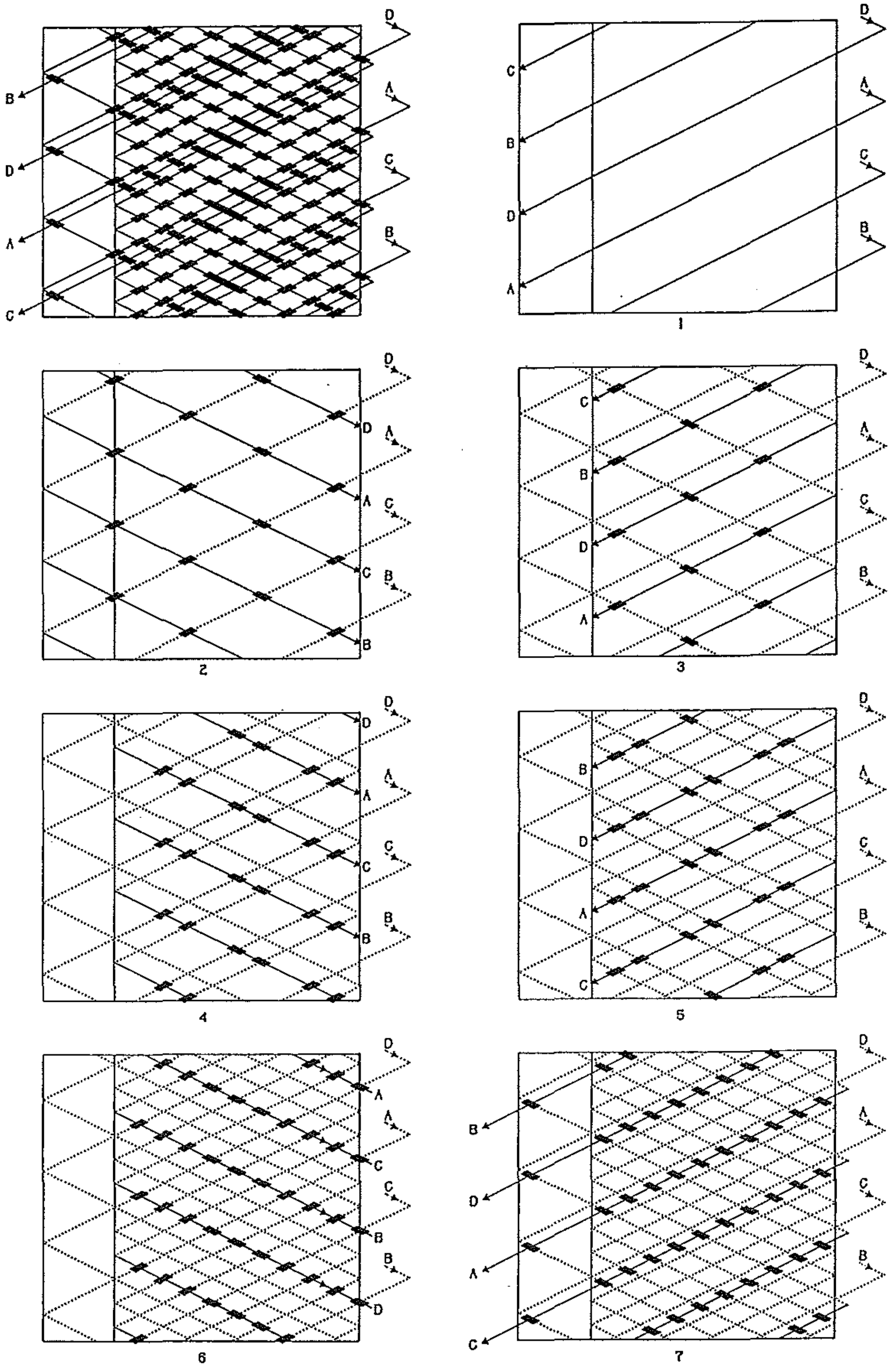


Fig. 358 — A final Gaucho-coded integrated knot for the Gaucho Button Knot.

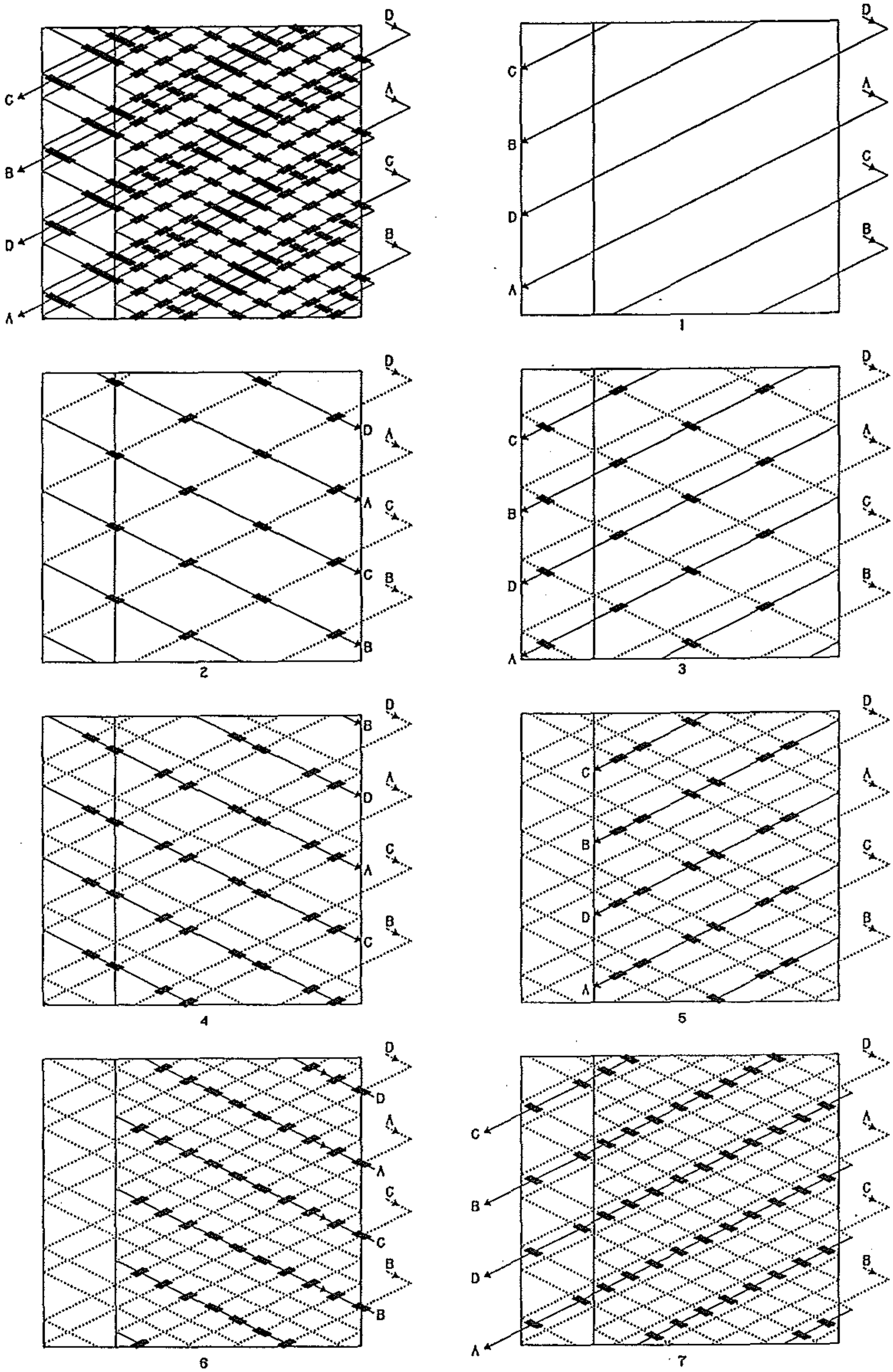


Fig. 359 — A final Gaucho-coded integrated knot for the Gaucho Button Knot.

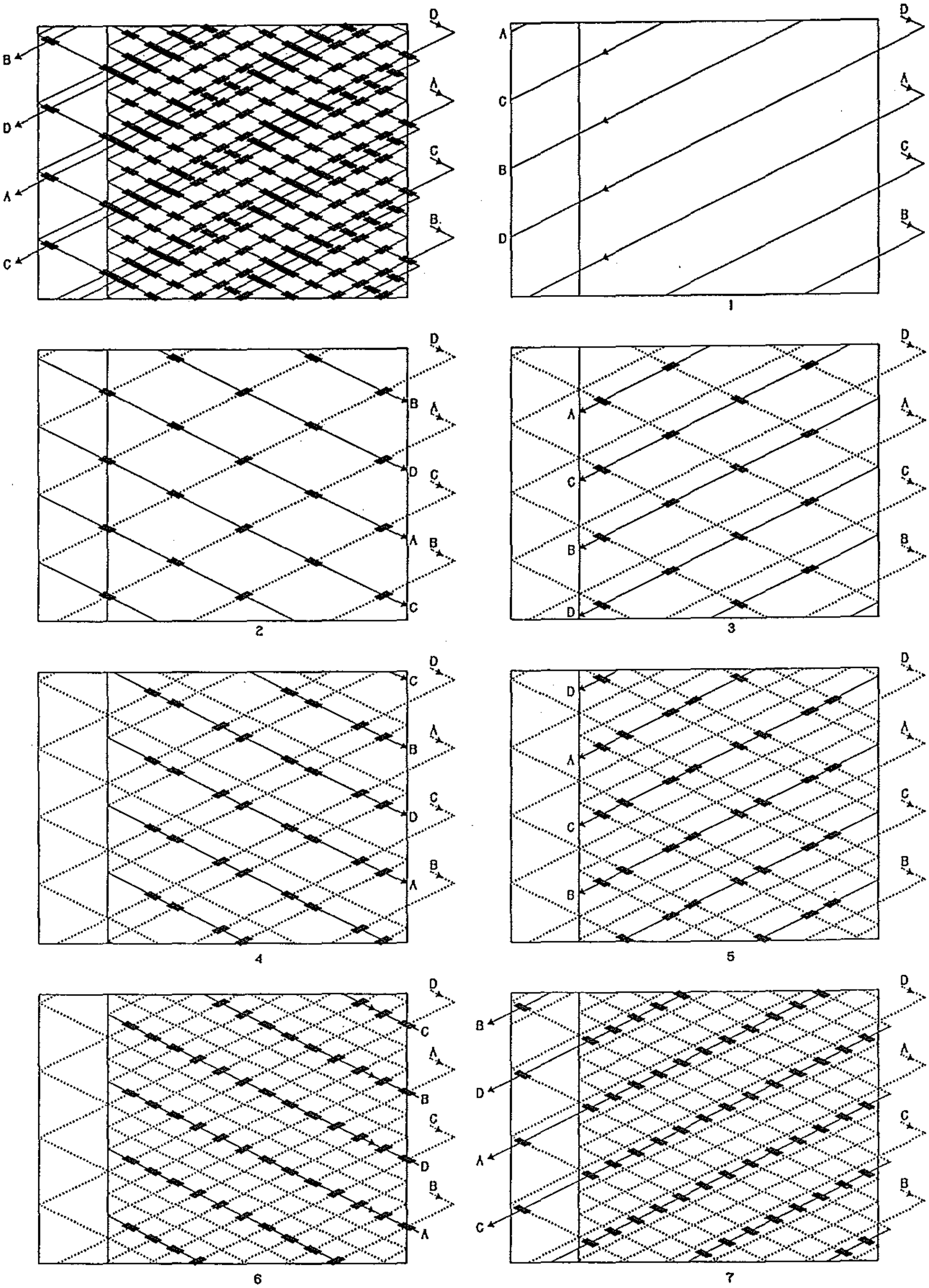


Fig. 360 — A final Gaucho-coded integrated knot for the Gaucho Button Knot.

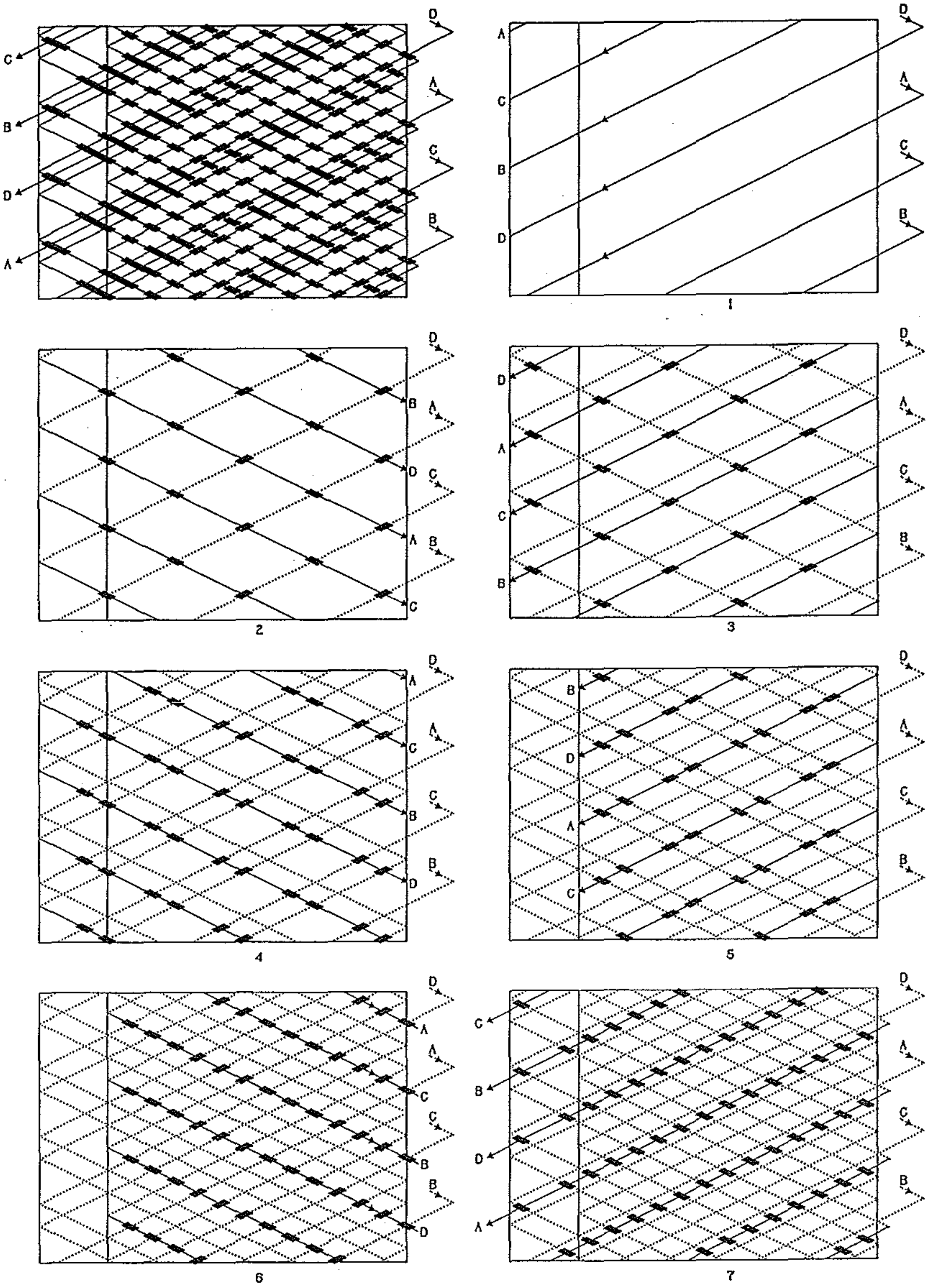


Fig. 361 — A final Gaucho-coded integrated knot for the Gaucho Button Knot.

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Solution to the Question in Issue No. 19

Question on pg. 422.

The string-run specification (22/5/22){123/312}12 under (1) is obviously impossible. This becomes immediately apparent when the numerals in the left periodic sequence 123 are provided with their ranking-numbers:

$n = 1$ for numeral $k = 1$, hence its ranking-number $j = |1 + 0|_3 = 1$.
$n = 2$ for numeral $k = 2$, hence its ranking-number $j = |2 + 2|_3 = 1$.
$n = 3$ for numeral $k = 3$, hence its ranking-number $j = |3 + (2 + 2)|_3 = 1$.

Hence we obtain for the left period the sequence $1_1 2_1 3_1$. But:

In a valid periodic sequence each ranking-number must occur once only.

In the previous issue of *The Braider* we have seen that the ranking-numbers j in the left periodic sequence range from $j = 1$ to $j = A_l$, and are calculated with the formula:

$$j = |n + \text{the sum of the first } (k-1) \text{ numerals of the left bight-boundary position specification}|_{A_l},$$

where A_l is the number of bights in a left period, and k is the n^{th} numeral in the left periodic sequence.

Furthermore we have seen that in the right periodic sequence the consecutive ranking-numbers j are $1, 2, 3, \cdots, A_r$.

Each numeral (bight-boundary number) in the left periodic sequence and in the right periodic sequence has furthermore a **complementary ranking-number** j_c.

In a valid periodic sequence each complementary ranking-number must occur once only.

In the left periodic sequence the consecutive complementary ranking-numbers j_c are $1, 2, 3, \cdots, A_l$, and in the right periodic sequence the complementary ranking-numbers j_c range from $j_c = 1$ to $j_c = A_r$, and are calculated with the formula:

$$j_c = |n - \text{the sum of the last } (k-1) \text{ numerals of the right bight-boundary position specification}|_{A_r},$$

where A_r is the number of bights in a right period, and k is the n^{th} numeral in the right periodic sequence.

From the conditions imposed on the ranking-numbers and complementary ranking-numbers it follows that the only valid left sequences for (1) are 132, 321 and 213, which with their ranking-numbers are respectively $1_1 3_3 2_2$, $3_2 2_1 1_3$ and $2_3 1_2 3_1$, while the only valid right sequences for (1) are 312, 123 and 231, which with their complementary ranking-numbers are respectively $3_3 1_2 2_1$, $1_1 2_3 3_2$ and $2_2 3_1 1_3$.

In a **set** of valid left sequences the consecutive A_l numerals in each of these sequences are always in an identical cyclic order; similarly in a set of valid right sequences the consecutive A_r numerals in each of these sequences are always in an identical cyclic order. In our example there is only one **set** of valid left sequences and there is only one **set** of valid right sequences.

Since any one left sequence of a **set** of **left** sequences can be associated with any one right sequence of a **set** of **right** sequences, we obtain a total of $A_l \cdot A_r$ left–right sequence **pairs** in each left–right **set** of sequence **pairs**. Thus if we have η_l left **sets** of valid left sequences and η_r right **sets** of valid right sequences, we have a total of $\eta_l \cdot \eta_r$ **sets** of left–right sequence **pairs**, and consequently we have a total of $\eta_l \cdot \eta_r \cdot A_l \cdot A_r$ left–right sequence **pairs**. In our example we have only one left sequence **set** ($\eta_l = 1$) and only one right sequence **set** ($\eta_r = 1$), and hence we have a total of $1 \cdot 1 = 1$ left–right sequence **set** of left–right sequence **pairs**. Consequently we have a total of $1\cdot 1\cdot 3\cdot 3 = 9$ left–right sequence **pairs**.

The stack of adjacent A^{**} different lower-left to upper-right half-cycles are read from their associated left–right sequence **pairs** as we have shown in the previous issue of *The Braider*.

Two left–right sequence pairs are identical when their respective stacks of adjacent A^{**} different lower-left to upper-right half-cycles are in cyclic order identical as far as the bight-boundary numbers are concerned. Note that the ranking-numbers play no role!!!

The number of different stacks (hence different cyclic adjacent sequences of lower-left to upper-right half-cycles and consequently different 'braid-types') associated with a **set** of left–right sequence **pairs** is equal to $d = \text{g.c.d.}(A_l, A_r)$. Hence the total number of different stacks (hence different cyclic adjacent sequences of lower-left to upper-right half-cycles and consequently different 'braid-types') associated with the total number of $\eta_l\cdot\eta_r$ **sets** of left–right sequence **pairs** is equal to $\eta_l\cdot\eta_r\cdot d$.

Thus in our example with $d = 3$, the number of different stacks of adjacent A^{**} lower-left to upper-right half-cycles for the single **set** $(\eta_l\cdot\eta_r = 1\cdot 1 = 1)$ of 9 $(A_l\cdot A_r = 3\cdot 3 = 9)$ left–right sequence **pairs** is equal to 3.

It will thus be obvious that in order to obtain the d different stacks of adjacent A^{**} lower-left to upper-right half-cycles for a **set** of left–right sequence pairs, we only have to obtain the stacks for the left–right sequence pairs in which the left sequence remains the same while the right sequence is respectively one of the first d consecutive cyclic ordered right sequences. Hence in our example for instance, we can obtain the 3 different stacks of the adjacent $A^{**} = 3$ lower-left to upper-right half-cycles from the 3 left–right sequence **pairs** for which the left sequence is 132 and the right sequence is successively 312, 123, 231. These left–right sequence pairs give us the following 3 different stacks of adjacent lower-left to upper-right half-cycles:

$$\begin{array}{ccc}
1 \longrightarrow 3 & \qquad 1 \longrightarrow 1 & \qquad 1 \longrightarrow 2 \\
3 \longrightarrow 1 & \qquad 3 \longrightarrow 2 & \qquad 3 \longrightarrow 3 \\
2 \longrightarrow 2 & \qquad 2 \longrightarrow 3 & \qquad 2 \longrightarrow 1
\end{array}$$

Although the ranking-numbers do not play a role here, we normally leave them, at least initially, attached to the bight-boundary numbers. The reason for this is readily demonstrated with the sequence **set** 1123/113142 of the $A_l\cdot A_r = 4\cdot 6 = 24$ left–right sequence pairs of the example in the previous issue of *The Braider* on pg. 420. The 2 different stacks of adjacent lower-left to upper-right half-cycles are:

$$\begin{array}{cc}
1_1 \longrightarrow 1_1 & \qquad 1_1 \longrightarrow 1_1 \\
1_2 \longrightarrow 1_2 & \qquad 1_2 \longrightarrow 3_2 \\
2_4 \longrightarrow 3_3 & \qquad 2_4 \longrightarrow 1_3 \\
3_3 \longrightarrow 1_4 & \qquad 3_3 \longrightarrow 4_4 \\
1_1 \longrightarrow 4_5 & \qquad 1_1 \longrightarrow 2_5 \\
1_2 \longrightarrow 2_6 & \qquad 1_2 \longrightarrow 1_6 \\
2_4 \longrightarrow 1_1 & \qquad 2_4 \longrightarrow 1_1 \\
3_3 \longrightarrow 1_2 & \qquad 3_3 \longrightarrow 3_2 \\
1_1 \longrightarrow 3_3 & \qquad 1_1 \longrightarrow 1_3 \\
1_2 \longrightarrow 1_4 & \qquad 1_2 \longrightarrow 4_4 \\
2_4 \longrightarrow 4_5 & \qquad 2_4 \longrightarrow 2_5 \\
3_3 \longrightarrow 2_6 & \qquad 3_3 \longrightarrow 1_6
\end{array}$$

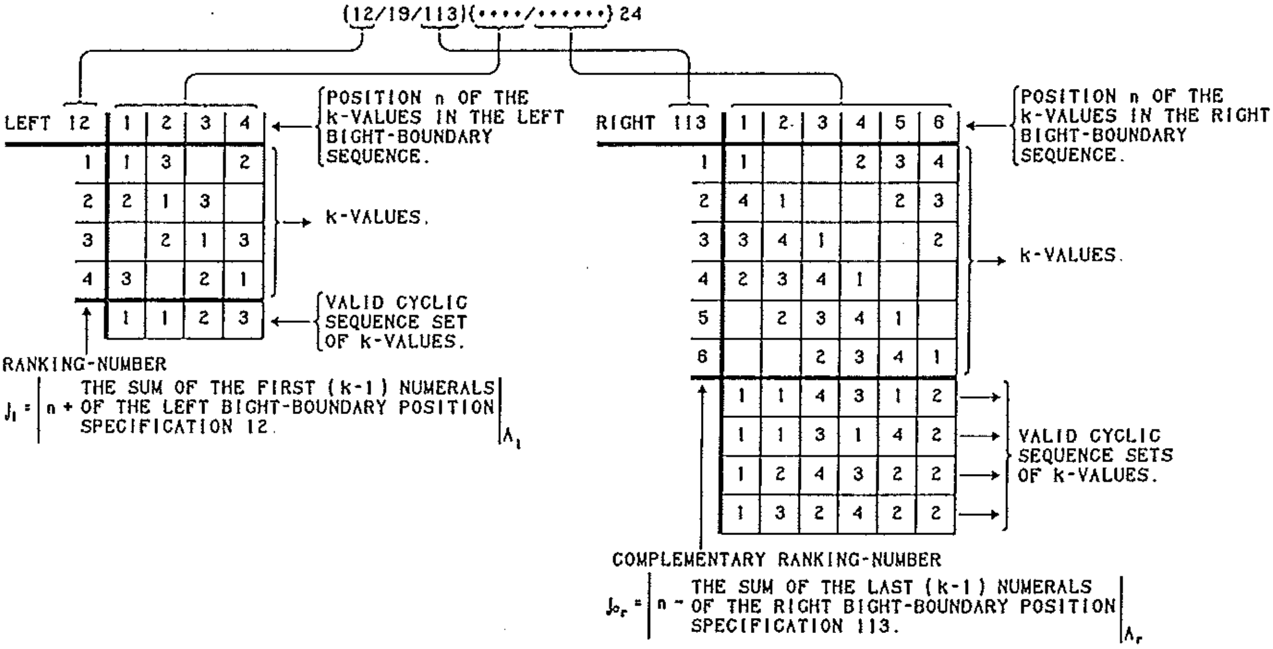
Since $A_l = 4$ and $A_r = 6$, we obtain $d = \text{g.c.d.}(A_l, A_r) = 2$. Thus for this **set** of left–right sequence pairs we obtain 2 different stacks of $A^{**} = 12$ adjacent lower-left to upper-right half-cycles. These $d = 2$ different stacks may be obtained from the 2 left–right sequence pairs in which the left sequence is $1_1 1_2 2_4 3_3$ and the right sequence is respectively one of the first $d = 2$ consecutive cyclic ordered right sequences, hence respectively $1_1 1_2 3_3 1_4 4_5 2_6$ and $1_1 3_2 1_3 4_4 2_5 1_6$. Note their respective ranking-numbers!!!

After leaving the ranking-numbers off, we obtain:

$$\begin{array}{ccc}
1 \longrightarrow 1 & \qquad & 1 \longrightarrow 1 \\
1 \longrightarrow 1 & & 1 \longrightarrow 3 \\
2 \longrightarrow 3 & & 2 \longrightarrow 1 \\
3 \longrightarrow 1 & & 3 \longrightarrow 4 \\
1 \longrightarrow 4 & & 1 \longrightarrow 2 \\
1 \longrightarrow 2 & & 1 \longrightarrow 1 \\
2 \longrightarrow 1 & & 2 \longrightarrow 1 \\
3 \longrightarrow 1 & & 3 \longrightarrow 3 \\
1 \longrightarrow 3 & & 1 \longrightarrow 1 \\
1 \longrightarrow 1 & & 1 \longrightarrow 4 \\
2 \longrightarrow 4 & & 2 \longrightarrow 2 \\
3 \longrightarrow 2 & & 3 \longrightarrow 1
\end{array}$$

The valid **left** sequence **sets** (the valid left cyclic sequences of k-values) are determined by the **left** bight-boundary position specification, and similarly the valid **right** sequence **sets** (the valid right cyclic sequences of k-values) are determined by the **right** bight-boundary position specification. These valid sequence **sets** of k-values can readily be obtained by means of the calculation procedure set out in Fig. 362 for $(12/\ast\ast/113)\{\ast\ast\ast\ast\,/\,\ast\ast\ast\ast\ast\ast\}$ as an Example.

To a valid sequence **set**, each row **must** contribute only one column-cell containing a k-value and in a valid sequence **set**, **each** k-value **must** at least be present once.


(12/19/113)(••••/••••••)24
POSITION n OF THE k-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.
k-VALUES.
VALID CYCLIC SEQUENCE SET OF k-VALUES.
RANKING-NUMBER
POSITION n OF THE k-VALUES IN THE RIGHT BIGHT-BOUNDARY SEQUENCE.
k-VALUES.
VALID CYCLIC SEQUENCE SETS OF k-VALUES.
COMPLEMENTARY RANKING-NUMBER

LEFT 12	1	2	3	4
1	1	3		2
2	2	1	3	
3		2	1	3
4	3		2	1
	1	1	2	3

$$J_l = \left| n + \text{THE SUM OF THE FIRST } (k-1) \text{ NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 12.} \right|_{A_l}$$

RIGHT 113	1	2	3	4	5	6
1	1			2	3	4
2	4	1			2	3
3	3	4	1			2
4	2	3	4	1		
5		2	3	4	1	
6			2	3	4	1
	1	1	4	3	1	2
	1	1	3	1	4	2
	1	2	4	3	2	2
	1	3	2	4	2	2

$$J_{\circ_r} = \left| n - \text{THE SUM OF THE LAST } (k-1) \text{ NUMERALS OF THE RIGHT BIGHT-BOUNDARY POSITION SPECIFICATION 113.} \right|_{A_r}$$

Fig. 362 — An Example calculation procedure for the valid sequence **sets**.

When there is on **left** bight-boundary 1 a Standing End (and hence its associated Working End), we take by convention for a **left** sequence always one which begins with bight-boundary 1. Although we could obviously begin with another bight-boundary, this convention ensures that the RRHC[†] always starts at left bight-boundary 1, and that the LRHC[†] has the ranking-number 1 for its left-hand bight-point.

In the Example of Fig. 362, the **left** bight-boundary position specification 12 determines that there is only one valid **left** sequence **set** with $A_l = 4$ k-values ($1 \leq k \leq \mathcal{K}_l$, hence $1 \leq k \leq 3$), while the **right** bight-boundary position specification 113 determines that there are four valid **right** sequence **sets** with $A_r = 6$ k-values ($1 \leq k \leq \mathcal{K}_r$, hence $1 \leq k \leq 4$). Thus, these left and right bight-boundary position specifications determine that there are with $A_l = 4$ and $A_r = 6$ a total of $1 \times 4 \times d = 1 \times 4 \times 2 = 8$ different stacks of adjacent lower-left to upper-right half-cycles possible, which means that there are a total of 8 different 'braid-types' possible.

For the bight-boundary position specifications of sub-questions (1), (2) and (3), the calculations for the valid left and right sequence **sets** of k-values are shown in Fig. 363.

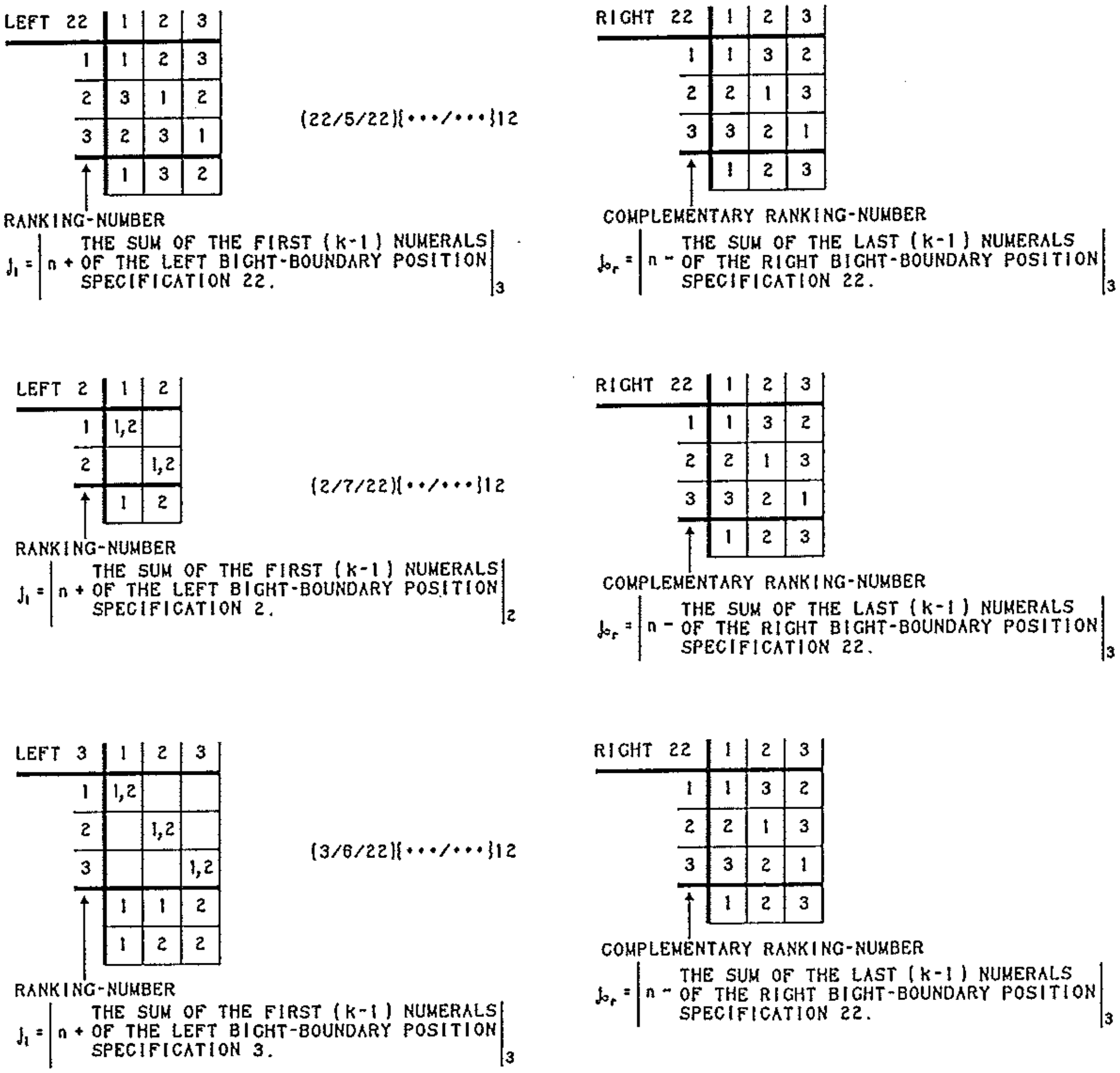

LEFT 22 | 1 | 2 | 3
1 | 1 | 2 | 3
2 | 3 | 1 | 2
3 | 2 | 3 | 1
1 | 3 | 2
(22/5/22)(•••/•••)12
RANKING-NUMBER
THE SUM OF THE FIRST (k-1) NUMERALS
j_l = n + OF THE LEFT BIGHT-BOUNDARY POSITION
SPECIFICATION 22.
3
RIGHT 22 | 1 | 2 | 3
1 | 1 | 3 | 2
2 | 2 | 1 | 3
3 | 3 | 2 | 1
1 | 2 | 3
COMPLEMENTARY RANKING-NUMBER
THE SUM OF THE LAST (k-1) NUMERALS
j_{c_r} = n − OF THE RIGHT BIGHT-BOUNDARY POSITION
SPECIFICATION 22.
3
LEFT 2 | 1 | 2
1 | 1,2 |
2 | | 1,2
1 | 2
(2/7/22)(••/•••)12
RANKING-NUMBER
THE SUM OF THE FIRST (k-1) NUMERALS
j_l = n + OF THE LEFT BIGHT-BOUNDARY POSITION
SPECIFICATION 2.
2
RIGHT 22 | 1 | 2 | 3
1 | 1 | 3 | 2
2 | 2 | 1 | 3
3 | 3 | 2 | 1
1 | 2 | 3
COMPLEMENTARY RANKING-NUMBER
THE SUM OF THE LAST (k-1) NUMERALS
j_{c_r} = n − OF THE RIGHT BIGHT-BOUNDARY POSITION
SPECIFICATION 22.
3
LEFT 3 | 1 | 2 | 3
1 | 1,2 | |
2 | | 1,2 |
3 | | | 1,2
1 | 1 | 2
1 | 2 | 2
(3/6/22)(•••/•••)12
RANKING-NUMBER
THE SUM OF THE FIRST (k-1) NUMERALS
j_l = n + OF THE LEFT BIGHT-BOUNDARY POSITION
SPECIFICATION 3.
3
RIGHT 22 | 1 | 2 | 3
1 | 1 | 3 | 2
2 | 2 | 1 | 3
3 | 3 | 2 | 1
1 | 2 | 3
COMPLEMENTARY RANKING-NUMBER
THE SUM OF THE LAST (k-1) NUMERALS
j_{c_r} = n − OF THE RIGHT BIGHT-BOUNDARY POSITION
SPECIFICATION 22.
3

Fig. 363 — The calculations for the valid sequence **sets** associated with the sub-questions (1), (2), (3).

[†] See previous Issue of *The Braider*, No. 19, pp. 415 & 416.

For the bight-boundary position specifications in sub-question (1) we obtain the one valid left sequence **set** ($\eta_l = 1$) and the one valid right sequence **set** ($\eta_r = 1$) which give the sequences discussed on pg. 437. Since $d = 3$, we thus obtain three different stacks of adjacent lower-left to upper-right half-cycles ($\eta_l \cdot \eta_r \cdot d = 1 \cdot 1 \cdot 3 = 3$). Each such stack consists of three adjacent lower-left to upper-right half-cycles ($A^{**} = 3$). Refer to pg. 438.

For the bight-boundary position specifications in sub-question (2) we obtain the one valid left sequence **set** ($\eta_l = 1$) and the one valid right sequence **set** ($\eta_r = 1$). Since $d = 1$, we thus obtain only one stack of adjacent lower-left to upper-right half-cycles ($\eta_l \cdot \eta_r \cdot d = 1 \cdot 1 \cdot 1 = 1$). The stack consists of six adjacent lower-left to upper-right half-cycles ($A^{**} = 6$).

For the bight-boundary position specifications in sub-question (3) we obtain the two valid left sequence **sets** ($\eta_l = 2$) and the one valid right sequence **set** ($\eta_r = 1$). Since $d = 3$, we thus obtain six different stacks of adjacent lower-left to upper-right half-cycles ($\eta_l \cdot \eta_r \cdot d = 2 \cdot 1 \cdot 3 = 6$). Each such stack consists of three adjacent lower-left to upper-right half-cycles ($A^{**} = 3$).

Thus for the bight-boundary position specification (22/5/22) in sub-question (1), we have the following three different string-run specifications:

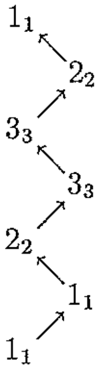
(1a). (22/5/22){132/123}12.
(1b). (22/5/22){132/231}12.
(1c). (22/5/22){132/312}12.

(1a):

The stack of adjacent lower-left to upper-right half-cycles for the string-run specification $(22/5/22)\{1_1 3_3 2_2 / 1_1 2_2 3_3\}12$ is:

$$\begin{aligned} 1_1 &\longrightarrow 1_1 \\ 3_3 &\longrightarrow 2_2 \\ 2_2 &\longrightarrow 3_3 \end{aligned}$$

For the first-return string-runs we thus obtain:



$$\begin{array}{llll}
l_{1_{j_l}} = 1_1 \longrightarrow 1_1 = r_{1_{j_r}} & \rightarrow \; j'_l = |1+4+5+4|_3 = 2 & \rightarrow & l_{2_{j'_l}} = 2_2 \,. \\
l_{2_{j'_l}} = 2_2 \longleftarrow 1_1 = r_{1_{j_r}} & \rightarrow \; j'_r = |1+4+5+2|_3 = 3 & \rightarrow & r_{2_{j'_r}} = 3_3 \,. \\
l_{2_{j_l}} = 2_2 \longrightarrow 3_3 = r_{2_{j_r}} & \rightarrow \; j'_l = |2+2+5+0|_3 = 3 & \rightarrow & l_{3_{j'_l}} = 3_3 \,. \\
l_{3_{j'_l}} = 3_3 \longleftarrow 3_3 = r_{2_{j_r}} & \rightarrow \; j'_r = |3+0+5+0|_3 = 2 & \rightarrow & r_{3_{j'_r}} = 2_2 \,. \\
l_{3_{j_l}} = 3_3 \longrightarrow 2_2 = r_{3_{j_r}} & \rightarrow \; j'_l = |3+0+5+2|_3 = 1 & \rightarrow & l_{4_{j'_l}} = 1_1 \,. \\
l_{4_{j'_l}} = 1_1 \longleftarrow 2_2 = r_{3_{j_r}} & \rightarrow \; j'_r = |2+2+5+4|_3 = 1 & \rightarrow & r_{4_{j'_r}} = 1_1 \,.
\end{array}$$

$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{3 \cdot 5 + (4+2+0) + (4+0+2)}{3} = 9\,.$
g.c.d.$(P_c\,, B^{**}) =$ g.c.d.$(9\,, 4) = 1\,.$

Hence:

$$P_{total} = \sum P_{component} = 9\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1\,.$$

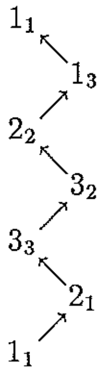
$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1\,.$$

(1b):

The stack of adjacent lower-left to upper-right half-cycles for the string-run specification $(22/5/22)\{1_1 3_3 2_2/2_1 3_2 1_3\}12$ is:

$$\begin{array}{c} 1_1 \longrightarrow 2_1\\ 3_3 \longrightarrow 3_2\\ 2_2 \longrightarrow 1_3 \end{array}$$

For the first-return string-runs we thus obtain:



$$\begin{array}{lllll}
l_{1_{j_l}} = 1_1 \longrightarrow 2_1 = r_{1_{j_r}} & \rightarrow & j'_l = |1+4+5+2|_3 = 3 & \rightarrow & l_{2_{j'_l}} = 3_3\,.\\
l_{2_{j'_l}} = 3_3 \longleftarrow 2_1 = r_{1_{j_r}} & \rightarrow & j'_r = |1+2+5+0|_3 = 2 & \rightarrow & r_{2_{j'_r}} = 3_2\,.\\
l_{2_{j_l}} = 3_3 \longrightarrow 3_2 = r_{2_{j_r}} & \rightarrow & j'_l = |3+0+5+0|_3 = 2 & \rightarrow & l_{3_{j'_l}} = 2_2\,.\\
l_{3_{j'_l}} = 2_2 \longleftarrow 3_2 = r_{2_{j_r}} & \rightarrow & j'_r = |2+0+5+2|_3 = 3 & \rightarrow & r_{3_{j'_r}} = 1_3\,.\\
l_{3_{j_l}} = 2_2 \longrightarrow 1_3 = r_{3_{j_r}} & \rightarrow & j'_l = |2+2+5+4|_3 = 1 & \rightarrow & l_{4_{j'_l}} = 1_1\,.\\
l_{4_{j'_l}} = 1_1 \longleftarrow 1_3 = r_{3_{j_r}} & \rightarrow & j'_r = |3+4+5+4|_3 = 1 & \rightarrow & r_{4_{j'_r}} = 2_1\,.
\end{array}$$

$P_c = \frac{\alpha\cdot x + \sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \frac{3\cdot 5+(4+0+2)+(2+0+4)}{3} = 9\,.$
g.c.d. $(P_c\,,\,B^{**}) =$ g.c.d. $(9\,,\,4) = 1\,.$

Hence:

$$P_{total} = \sum P_{component} = 9\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1\,.$$

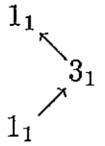
$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1\,.$$

(1c):

The stack of adjacent lower-left to upper-right half-cycles for the string-run specification $(22/5/22)\{1_1 3_3 2_2/3_1 1_2 2_3\}12$ is:

$$\begin{array}{c} 1_1 \longrightarrow 3_1\\ 3_3 \longrightarrow 1_2\\ 2_2 \longrightarrow 2_3 \end{array}$$

For the first-return string-runs we thus obtain:



$$\begin{array}{lllll}
l_{1_{j_l}} = 1_1 \longrightarrow 3_1 = r_{1_{j_r}} & \rightarrow & j'_l = |1+4+5+0|_3 = 1 & \rightarrow & l_{2_{j'_l}} = 1_1\,.\\
l_{2_{j'_l}} = 1_1 \longleftarrow 3_1 = r_{1_{j_r}} & \rightarrow & j'_r = |1+0+5+4|_3 = 1 & \rightarrow & r_{2_{j'_r}} = 3_1\,.
\end{array}$$

$$P_{c_1} = \tfrac{\alpha\cdot x+\sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \tfrac{1\cdot5+(4)+(0)}{3} = 3\,.$$
g.c.d. $(P_{c_1}\,,\,B^{**}) =$ g.c.d. $(3\,,\,4) = 1\,.$

$$\begin{tikzcd} 3_3 & \\ & 1_2 \arrow[lu] \\ 3_3 \arrow[ru] & \end{tikzcd}$$

$$\begin{aligned} l_{1_{j_l}} &= 3_3 \longrightarrow 1_2 = r_{1_{j_r}} \rightarrow j_l' = |3+0+5+4|_3 = 3 \rightarrow l_{2_{j_l'}} = 3_3\,.\\ l_{2_{j_l'}} &= 3_3 \longleftarrow 1_2 = r_{1_{j_r}} \rightarrow j_r' = |2+4+5+0|_3 = 2 \rightarrow r_{2_{j_r'}} = 1_2\,. \end{aligned}$$

$$P_{c_2} = \tfrac{\alpha\cdot x+\sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \tfrac{1\cdot5+(0)+(4)}{3} = 3\,.$$
g.c.d. $(P_{c_2}\,,\,B^{**}) =$ g.c.d. $(3\,,\,4) = 1\,.$

$$\begin{tikzcd} 2_2 & \\ & 2_3 \arrow[lu] \\ 2_2 \arrow[ru] & \end{tikzcd}$$

$$\begin{aligned} l_{1_{j_l}} &= 2_2 \longrightarrow 2_3 = r_{1_{j_r}} \rightarrow j_l' = |2+2+5+2|_3 = 2 \rightarrow l_{2_{j_l'}} = 2_2\,.\\ l_{2_{j_l'}} &= 2_2 \longleftarrow 2_3 = r_{1_{j_r}} \rightarrow j_r' = |3+2+5+2|_3 = 3 \rightarrow r_{2_{j_r'}} = 2_3\,. \end{aligned}$$

$$P_{c_3} = \tfrac{\alpha\cdot x+\sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \tfrac{1\cdot5+(2)+(2)}{3} = 3\,.$$
g.c.d. $(P_{c_3}\,,\,B^{**}) =$ g.c.d. $(3\,,\,4) = 1\,.$

Hence:

$$P_{total} = \sum P_{component} = 3+3+3 = 9\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1+1+1 = 3\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1+1+1 = 3\,.$$

The string-run diagram of (22/5/22){132/312}12 is shown at the left in Fig. 364.

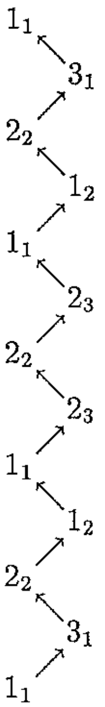
With the convention discussed on pp. 439 and 440, there are no alternative string-run specifications to the ones given above for (1a), (1b) and (1c).

Sub-question (2):

The stack of adjacent lower-left to upper-right half-cycles for the string-run specification $(2/7/22)\{1_1 2_2/3_1 1_2 2_3\}12$ is:

$$\begin{array}{c} 1_1 \longrightarrow 3_1\\ 2_2 \longrightarrow 1_2\\ 1_1 \longrightarrow 2_3\\ 2_2 \longrightarrow 3_1\\ 1_1 \longrightarrow 1_2\\ 2_2 \longrightarrow 2_3 \end{array}$$

For the first-return string-runs we thus obtain:



$$\begin{array}{llll}
l_{1_{j_l}} = 1_1 \longrightarrow 3_1 = r_{1_{j_r}} & \to\ j'_l = |1+2+7+0|_2 = 2 & \to & l_{2_{j'_l}} = 2_2 . \\
l_{2_{j'_l}} = 2_2 \longleftarrow 3_1 = r_{1_{j_r}} & \to\ j'_r = |1+0+7+0|_3 = 2 & \to & r_{2_{j'_r}} = 1_2 . \\
l_{2_{j_l}} = 2_2 \longrightarrow 1_2 = r_{2_{j_r}} & \to\ j'_l = |2+0+7+4|_2 = 1 & \to & l_{3_{j'_l}} = 1_1 . \\
l_{3_{j'_l}} = 1_1 \longleftarrow 1_2 = r_{2_{j_r}} & \to\ j'_r = |2+4+7+2|_3 = 3 & \to & r_{3_{j'_r}} = 2_3 . \\
l_{3_{j_l}} = 1_1 \longrightarrow 2_3 = r_{3_{j_r}} & \to\ j'_l = |1+2+7+2|_2 = 2 & \to & l_{4_{j'_l}} = 2_2 . \\
l_{4_{j'_l}} = 2_2 \longleftarrow 2_3 = r_{3_{j_r}} & \to\ j'_r = |3+2+7+0|_3 = 3 & \to & r_{4_{j'_r}} = 2_3 . \\
l_{4_{j_l}} = 2_2 \longrightarrow 2_3 = r_{4_{j_r}} & \to\ j'_l = |2+0+7+2|_2 = 1 & \to & l_{5_{j'_l}} = 1_1 . \\
l_{5_{j'_l}} = 1_1 \longleftarrow 2_3 = r_{4_{j_r}} & \to\ j'_r = |3+2+7+2|_3 = 2 & \to & r_{5_{j'_r}} = 1_2 . \\
l_{5_{j_l}} = 1_1 \longrightarrow 1_2 = r_{5_{j_r}} & \to\ j'_l = |1+2+7+4|_2 = 2 & \to & l_{6_{j'_l}} = 2_2 . \\
l_{6_{j'_l}} = 2_2 \longleftarrow 1_2 = r_{5_{j_r}} & \to\ j'_r = |2+4+7+0|_3 = 1 & \to & r_{6_{j'_r}} = 3_1 . \\
l_{6_{j_l}} = 2_2 \longrightarrow 3_1 = r_{6_{j_r}} & \to\ j'_l = |2+0+7+0|_2 = 1 & \to & l_{7_{j'_l}} = 1_1 . \\
l_{7_{j'_l}} = 1_1 \longleftarrow 3_1 = r_{6_{j_r}} & \to\ j'_r = |1+0+7+2|_3 = 1 & \to & r_{7_{j'_r}} = 3_1 .
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{6 \cdot 7 + (2+0+2+0+2+0) + (0+4+2+2+4+0)}{6} = 10 .$$

g.c.d. $(P_c\,,\, B^{**}) =$ g.c.d. $(10\,,\,2) = 2$.

Hence:

$$P_{total} = \sum P_{component} = 10 .$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1 .$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 2 .$$

The string-run diagram of (2/7/22){12/312}12 is shown at the centre in Fig. 364.

With the convention discussed on pp. 439 and 440, there are two alternative string-run specifications for the string-run specification (2/7/22){12/312}12:

(2/7/22){12/123}12.
(2/7/22){12/231}12.

Sub-question (3):

The stack of adjacent lower-left to upper-right half-cycles for the string-run specification $(3/6/22)\{1_1 1_2 2_3/3_1 1_2 2_3\}12$ is:

$$\begin{array}{c}
1_1 \longrightarrow 3_1 \\
1_2 \longrightarrow 1_2 \\
2_3 \longrightarrow 2_3
\end{array}$$

For the first-return string-runs we thus obtain:

$$\begin{tikzcd} 1_1 & \\ & 3_1 \arrow[ul] \\ 1_1 \arrow[ur] & \end{tikzcd}$$

$$\begin{aligned}
l_{1_{j_l}} &= 1_1 \longrightarrow 3_1 = r_{1_{j_r}} &&\to j'_l = |1+3+6+0|_3 = 1 &&\to l_{2_{j'_l}} = 1_1 . \\
l_{2_{j'_l}} &= 1_1 \longleftarrow 3_1 = r_{1_{j_r}} &&\to j'_r = |1+0+6+3|_3 = 1 &&\to r_{2_{j'_r}} = 3_1 .
\end{aligned}$$

$P_{c_1} = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 6 + (3) + (0)}{3} = 3 .$

g.c.d. $(P_{c_1}, B^{**}) =$ g.c.d. $(3, 4) = 1 .$

$$\begin{tikzcd} 1_2 & \\ & 2_3 \arrow[ul] \\ 2_3 \arrow[ur] & \\ & 1_2 \arrow[ul] \\ 1_2 \arrow[ur] & \end{tikzcd}$$

$$\begin{aligned}
l_{1_{j_l}} &= 1_2 \longrightarrow 1_2 = r_{1_{j_r}} &&\to j'_l = |2+3+6+4|_3 = 3 &&\to l_{2_{j'_l}} = 2_3 . \\
l_{2_{j'_l}} &= 2_3 \longleftarrow 1_2 = r_{1_{j_r}} &&\to j'_r = |2+4+6+0|_3 = 3 &&\to r_{2_{j'_r}} = 2_3 . \\
l_{1_{j_l}} &= 2_3 \longrightarrow 2_3 = r_{1_{j_r}} &&\to j'_l = |3+0+6+2|_3 = 2 &&\to l_{2_{j'_l}} = 1_2 . \\
l_{2_{j'_l}} &= 1_2 \longleftarrow 2_3 = r_{1_{j_r}} &&\to j'_r = |3+2+6+3|_3 = 2 &&\to r_{2_{j'_r}} = 1_2 .
\end{aligned}$$

$P_{c_2} = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{2 \cdot 6 + (3+0) + (4+2)}{3} = 7 .$

g.c.d. $(P_{c_2}, B^{**}) =$ g.c.d. $(7, 4) = 1 .$

Hence:

$$P_{total} = \sum P_{component} = 3 + 7 = 10 .$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1 + 1 = 2 .$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1 + 1 = 2 .$$

The string-run diagram of (3/6/22){112/312}12 is shown at the right in Fig. 364.

With the convention discussed on pp. 439 and 440, there is one alternative string-run specifications for the string-run specification (3/6/22){112/312}12:

(3/6/22){121/123}12.

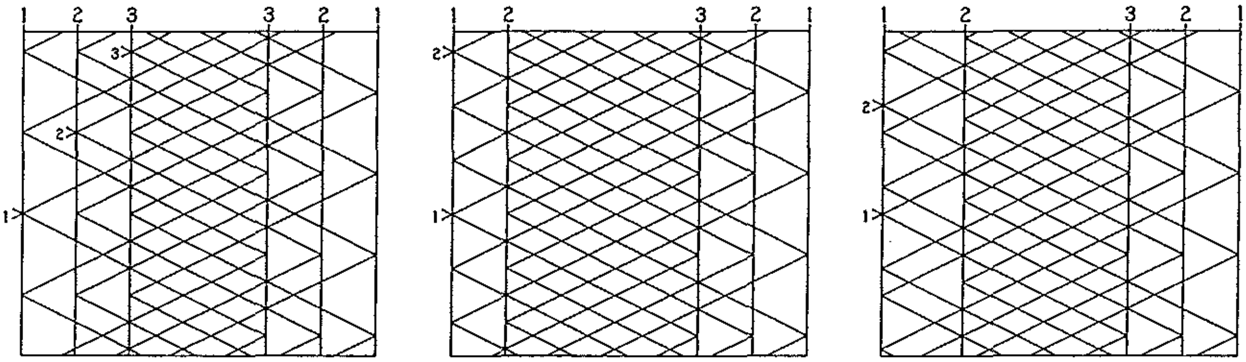


Fig. 364 — The string-run diagrams of (1c), (2), (3).

Scenes from the Knot Universe

In Issue No. 18 of *The Braider*, pg. 403, we mentioned that any attempt in creating a knot-classification system constitutes a futile exercise, an exercise that can only be contemplated by those who haven't got the faintest idea of what knots are really about. In this article we shall show that knots with even a very simple string-run layout are not easy to classify completely. We shall therefore not make any attempt to even partly classify the knots discussed, but rather show some relationships instead.

Relationships between knots are solely governed by their grid-diagrams and not by tying procedures, superficial appearances or behavioural aspects.

A grid-diagram consists of two important aspects: (1) the string-run and (2) the coding which is superimposed upon the string-run. The most basic aspect is the string-run. The coding may enable the string-run to be rearranged, which will then result in further relationships. It is therefore important to stress that such rearrangements do never furnish equivalent knots or braids.[†] A few simple examples are depicted in Fig. 365. The right-hand grid-diagram in the uppermost example is not a rearrangement of the left-hand grid-diagram since the left-hand grid-diagram cannot be rearranged into the right-hand grid-diagram without causing a full turn in the string with a left helix. In the next example, the two interbraided knots can be slid apart and hence the two right-hand grid-diagrams are a rearrangement of the left-hand grid-diagram. In the lowermost example the two right-hand grid-diagrams are not a rearrangement of the left-hand grid-diagram since such a rearrangement is impossible to accomplish.

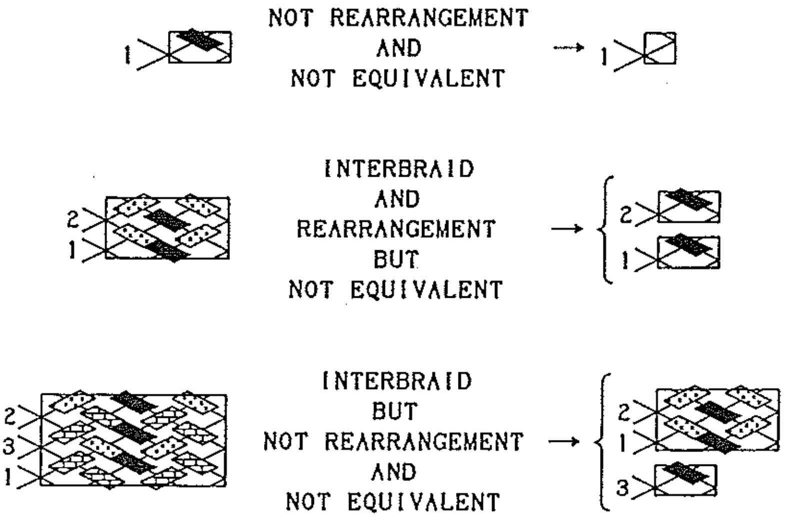


Fig. 365 — A few simple Examples.

The string-run layout we shall concern ourselves with is depicted at the left in Fig. 366; it is an interbraid of two components with equivalent string-runs. This string-run layout can be depicted for example by each of the two string-run diagrams to its right. The central string-run diagram is that of a flat braid, whereas the rightmost string-run diagram is that of its cylindrical form. The **everted** string-run layout and the two everted string-run diagrams associated with the string-runs in Fig. 366 are depicted in Fig. 367.

It should be stressed here that by everting a knot we obtain in general a different knot (refer for more details to Pamphlet No. 8: *Knots — Facts and Fallacies*).

[†] The topological knot theory goes also here totally astray.

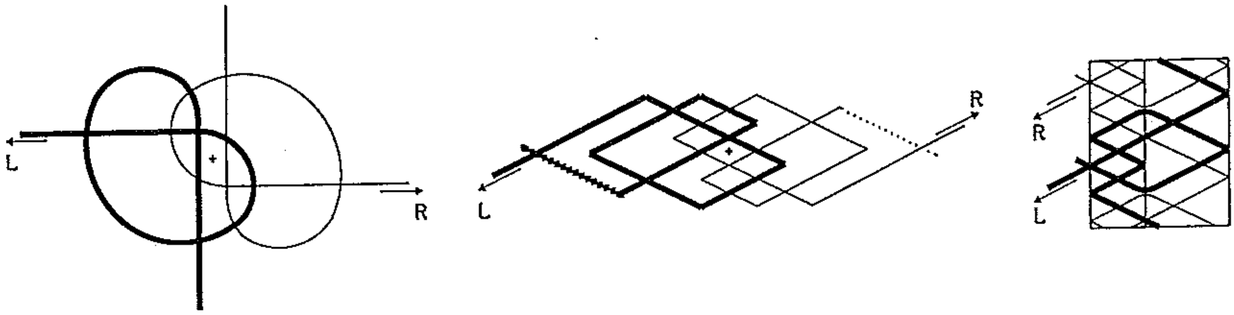


Fig. 366 — The string-run layout to be discussed.

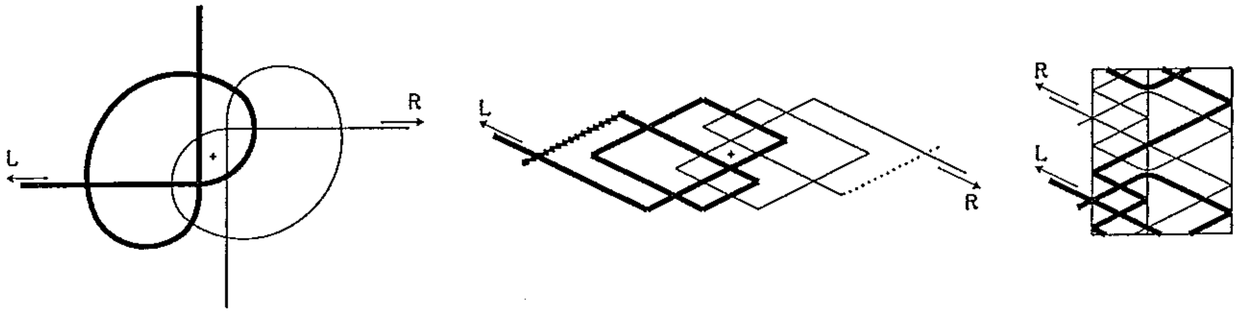


Fig. 367 — The everted string-run layout and everted string-run diagrams of Fig. 366.

For the superimposed coding in the upper-left diagram of Fig. 368, the upper-right diagram of its cylindrical form can be rearranged into the diagram of the cylindrical braid immediately below it. Furthermore, since we can lift in the upper-left diagram the two dotted string-parts up and slide the left-hand string-part to the right while sliding the right-hand string-part to the left, we can rearrange the upper-left string-run diagram with its superimposed coding into the diagram immediately below it. The cylindrical form of this diagram is depicted to its right. In turn this diagram can again be rearranged into the string-run diagram of the cylindrical braid immediately below it.

Similarly, for the superimposed coding in the leftmost third diagram from the top in Fig. 368, the diagram of its cylindrical form (depicted on its right) can be rearranged into the string-run diagram of the cylindrical braid immediately below it. In this case we drop the two dotted string-parts and slide the left-hand string-part to the right while sliding the right-hand string-part to the left.

The codings indicated in Fig. 369 enable us to slide the two central SW-NE running string-parts past each other in the SE-NW direction.This results in the lowermost rearranged string-run diagrams.

We shall limit our discussion not only to string material that is symmetric with respect to shape, colour and pattern/texture, but also to the string-run diagrams depicted in Figs. 368 & 369, except in one case where a further string-run layout, depicted by the two lowermost string-run diagrams in Fig. 370, will be considered (see pg. 457). This string-run layout can be obtained from the lowermost string-run diagrams in Fig. 369 by the codings indicated in Fig. 370. We shall furthermore limit our discussion to coding arrangements which are **balanced**[†] and in which each of the two interbraided components are 'overhand' knots.

[†] These are the coding arrangements in which the crossing-sequences from the ends L and R are either identical or opposites for each of the two interbraided strings.

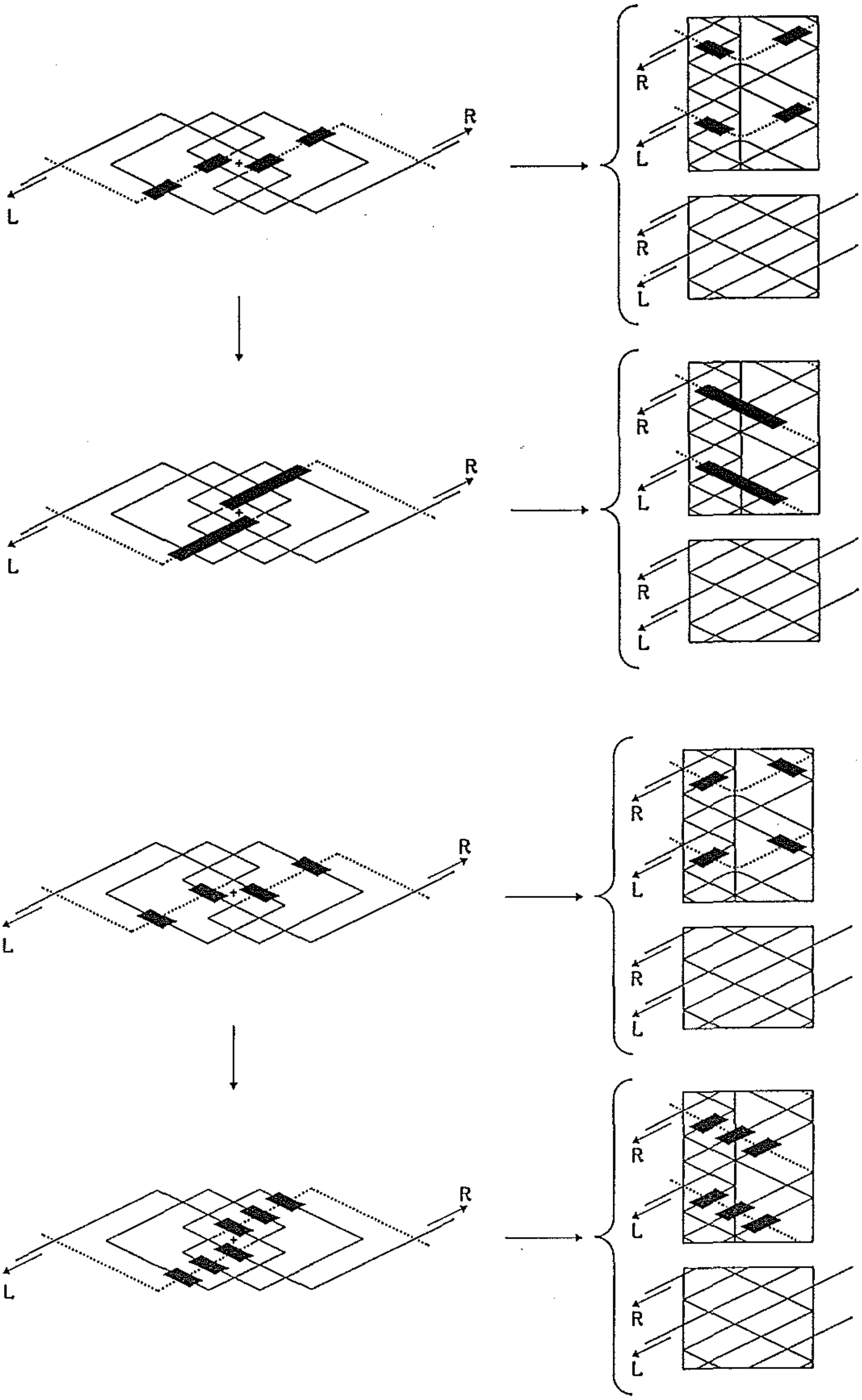


Fig. 368 — Some rearrangements facilitated by the indicated superimposed coding.

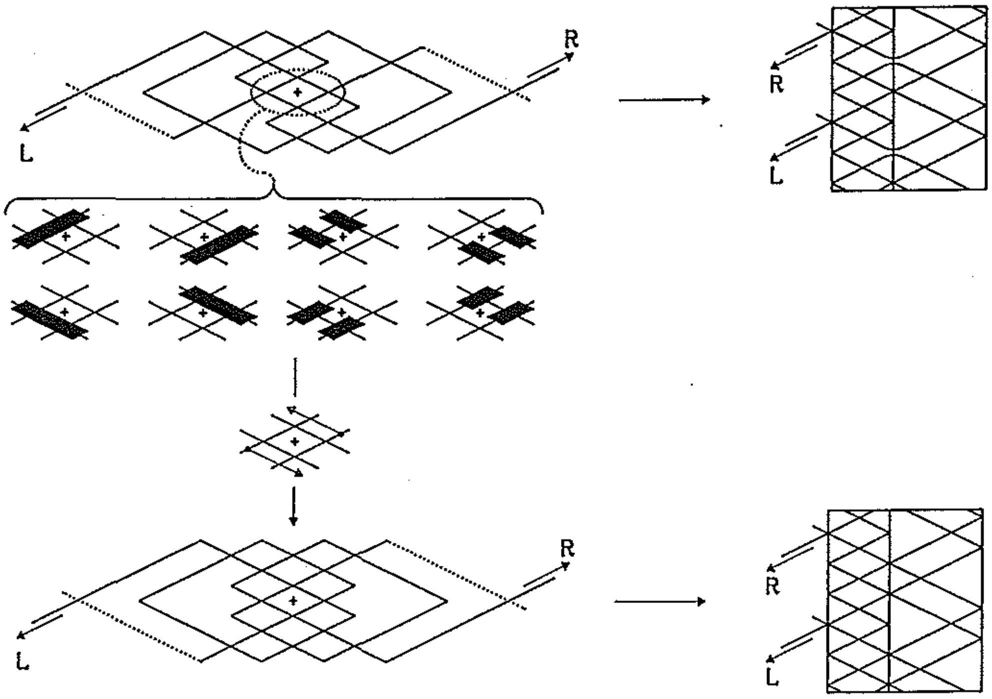


Fig. 369 — Further codings which facilitate a rearrangement of the string-run.

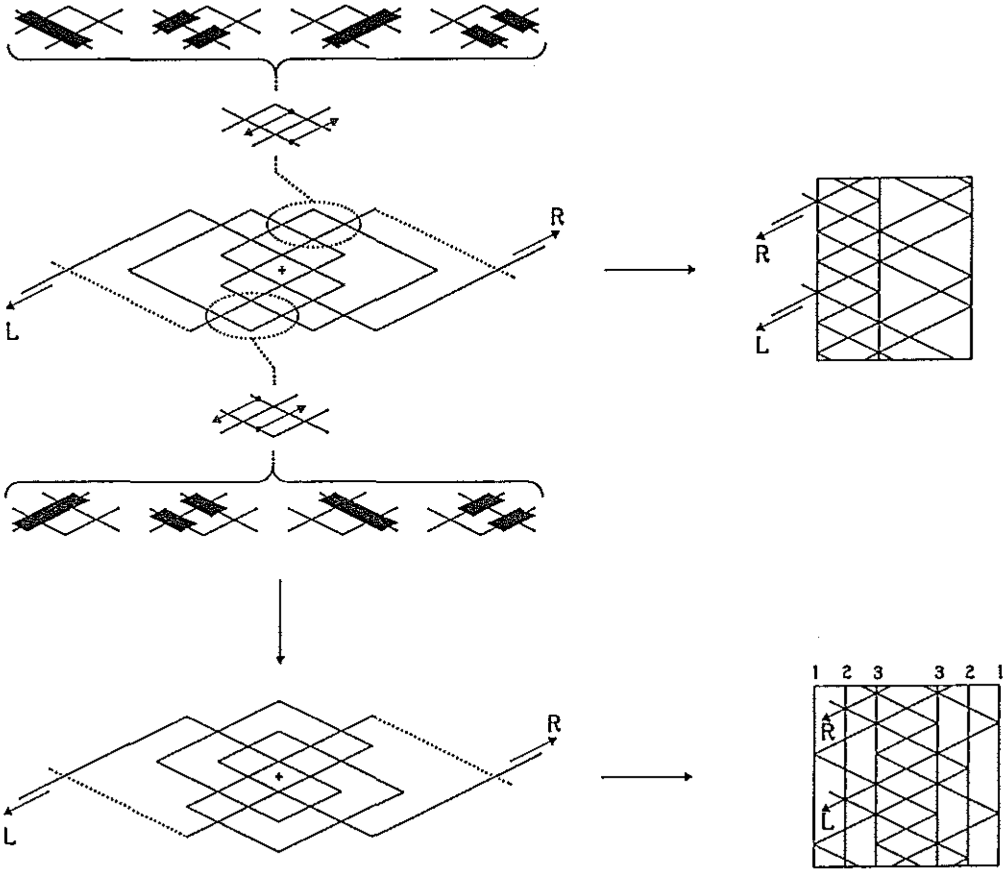


Fig. 370 — Codings which facilitate a further rearrangement of the string-run.

The necessary superimposed codings for the 'overhand' knots condition are shown in Fig. 371.

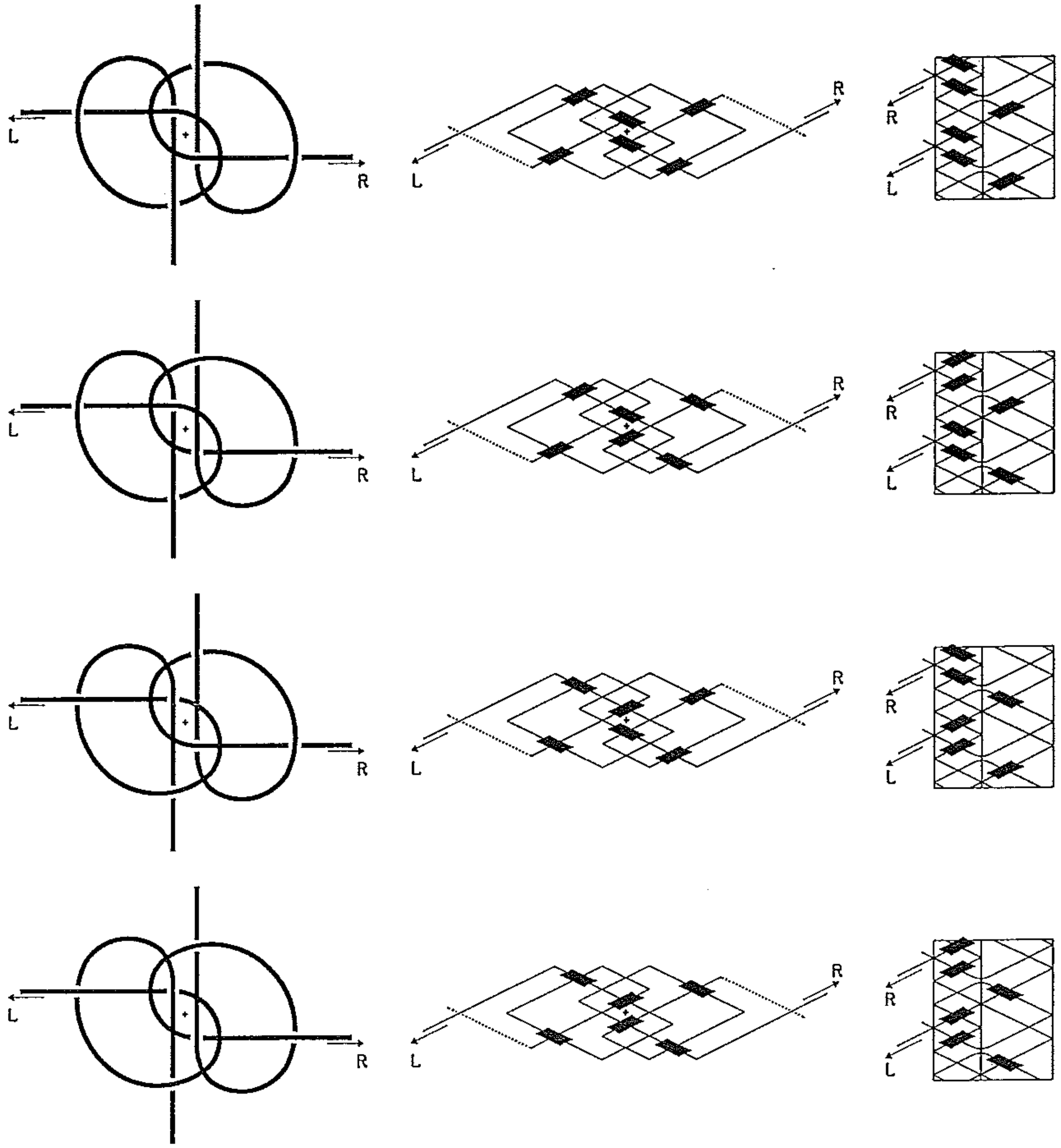


Fig. 371 — Superimposed codings for the 'overhand' knots condition.

Thus with respect to our limitations, the necessary superimposed codings for the string-runs in Fig. 368 are those depicted in Fig. 372.

In Fig. 372 the lower set of diagrams are the **everted** forms of the central set of diagrams. By comparing this lower set of diagrams with the upper set of diagrams it will be evident that they are each others **complement**.

Figs. 373&374 together show the four grid-diagrams associated with the upper layout in Fig. 372.

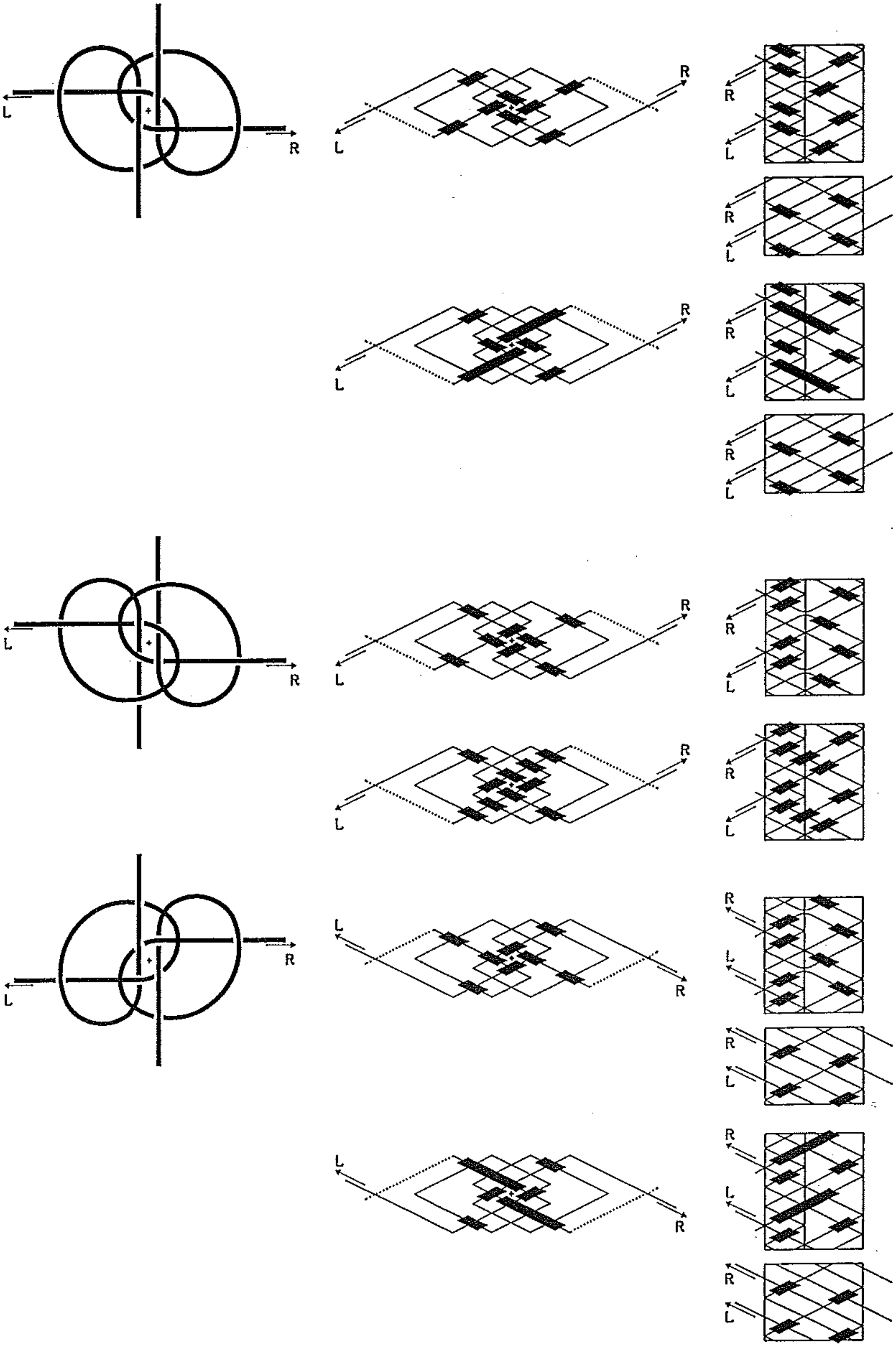


Fig.372 — The necessary superimposed codings for the string-runs in Fig. 368.

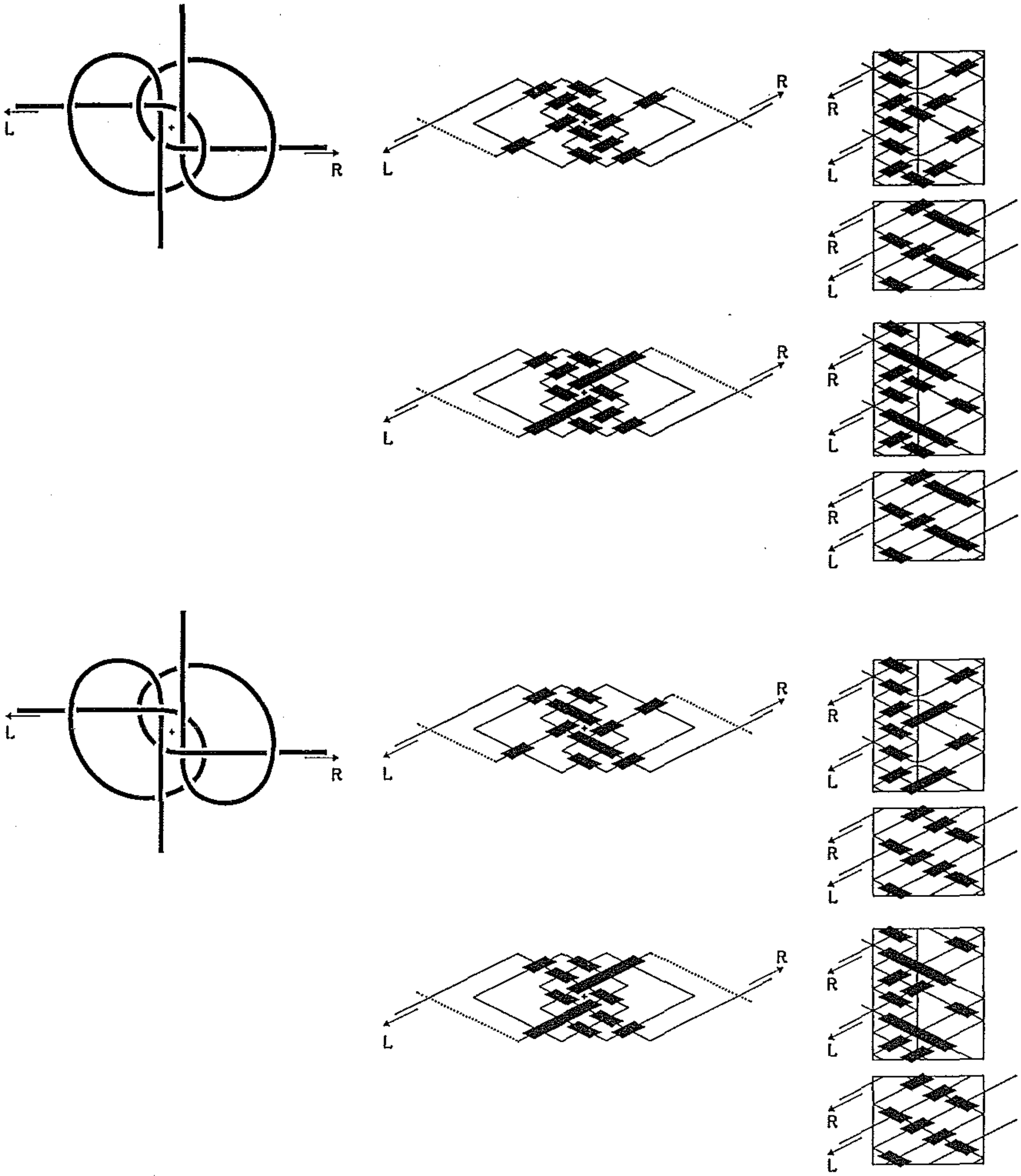


Fig. 373 — The grid-diagrams associated with the upper layout in Fig. 372.

The active ends L and R of the upper layout in Fig. 373 are the inactive ends of the lower layout in Fig. 374. The lower layout in Fig. 374 is the complement to **Ashley's Bend** (#1452 in Ashley) and the complement to **The Footrope Knot** (#783 in Ashley). By 'rolling over' the crown of the upper layout in Fig. 373 we obtain the lower layout in Fig. 374. Refer also to *The Braider*, Issue No. 5, pg. 100.

The lower layout in Fig. 373 can be rearranged in **The True-Lover's Knot** (#798 in Ashley).

The upper layout in Fig. 374 is the complement to Ashley #776 & 795 (a **Matthew Walker** open ender), which in turn is the everted form of the third layout from the top in Fig. 375.

The lower layout in Fig. 374 results in the complement to **Ashley's Bend** (#1452 in Ashley) and the complement to **The Footrope Knot** (#783 in Ashley).

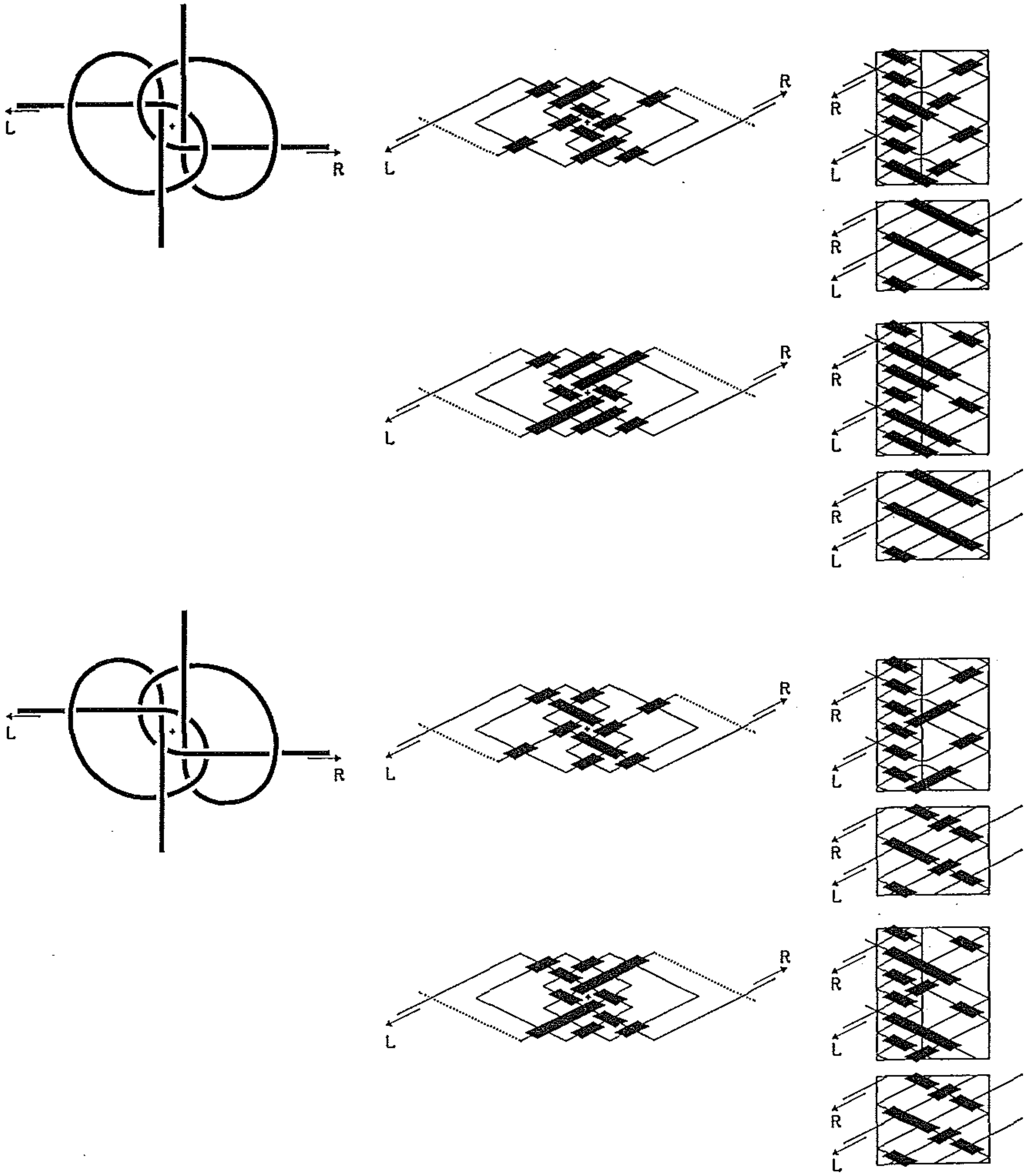


Fig. 374 — The grid-diagrams associated with the upper layout in Fig. 372.

Fig. 375 shows the four grid-diagrams associated with the central layout in Fig. 372. Figs. 376 & 377 show their everted layouts (associated with the lower layout in Fig. 372).

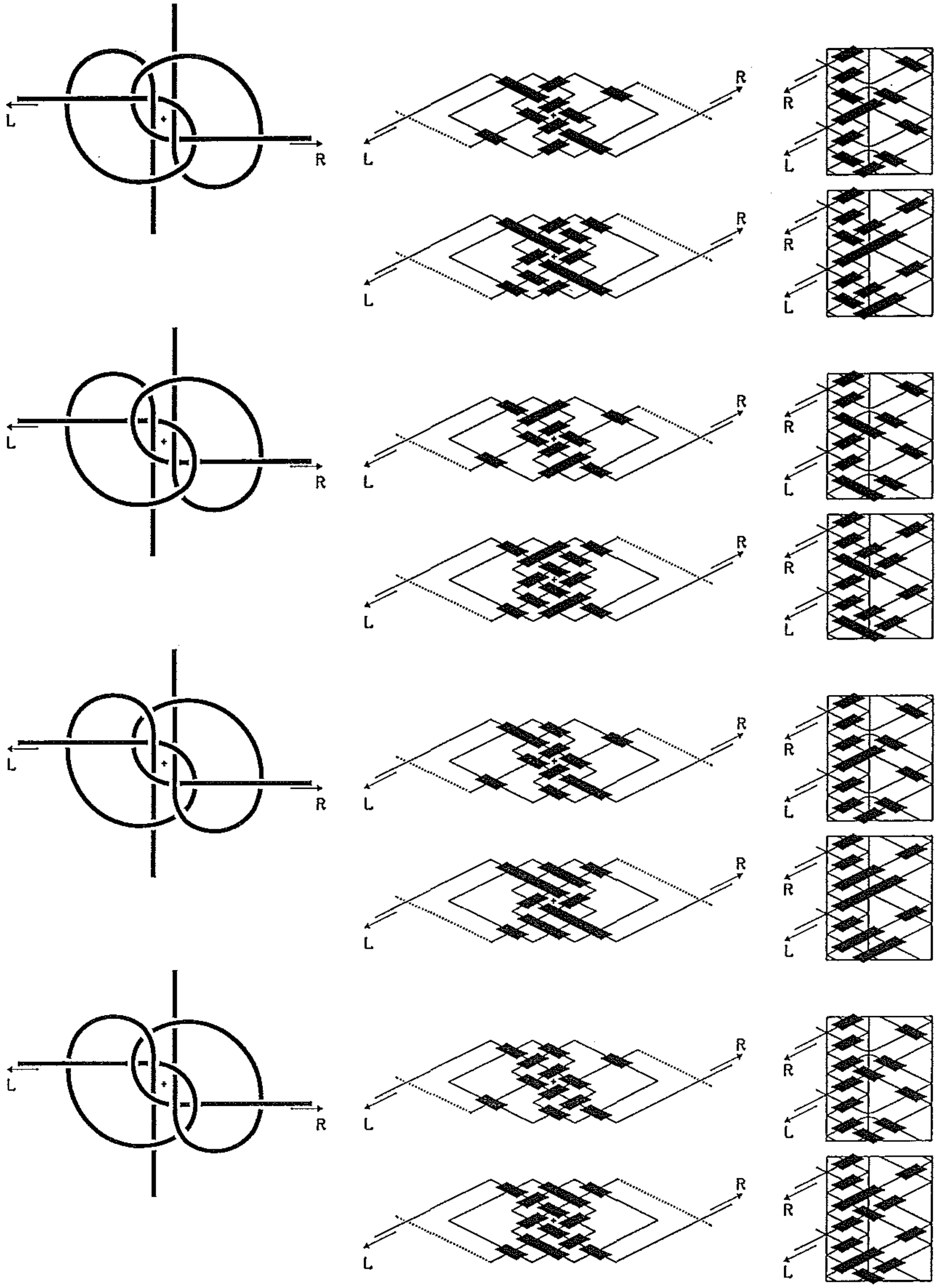


Fig. 375 — The grid-diagrams associated with the central layout in Fig. 372.

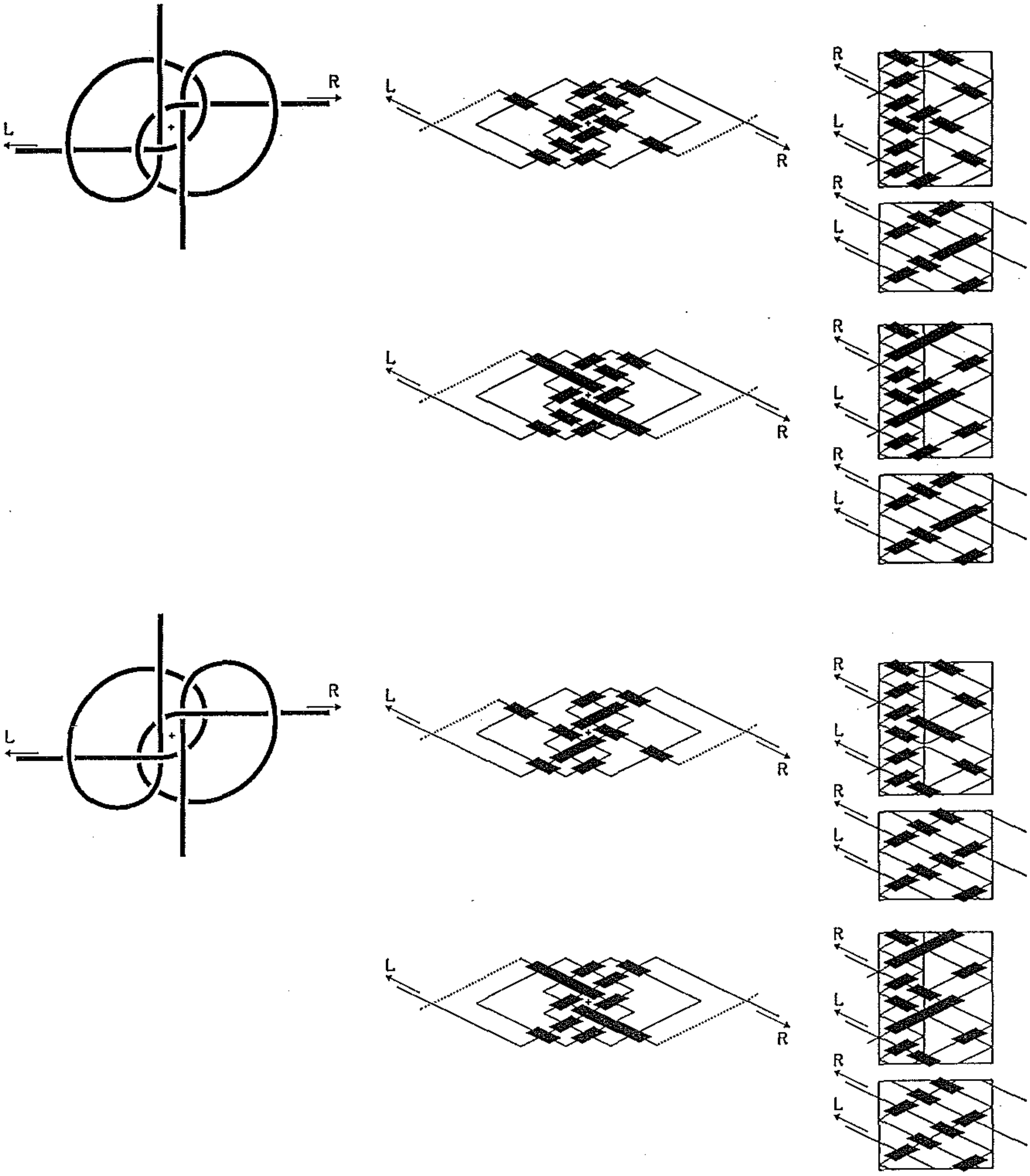


Fig. 376 — The grid-diagrams associated with the lower layout in Fig. 372.

The active ends L and R of the upper layout in Fig. 376 are the inactive ends of the lower layout in Fig. 377. The lower layout in Fig. 377 depicts **Ashley's Bend** (#1452 in Ashley) and **The Footrope Knot** (#783 in Ashley). By 'rolling over' the crown of the upper layout in Fig. 376 we obtain the lower layout in Fig. 377. Refer also to *The Braider*, Issue No. 5, pg. 100. The upper layout in Fig. 376 is the complement of the upper layout in Fig. 373.

The lower layout in Fig. 376 can be rearranged to the complement of **The True-**

Lover's Knot (the complement to#798 in Ashley). The lower layout in Fig. 376 is the complement to the lower layout in Fig. 373.

The upper layout in Fig. 377 depicts #776 & 795 in Ashley (a **Matthew Walker** open ender). The upper layout in Fig. 377 is the complement of the upper layout in Fig. 374.

The lower layout in Fig. 377 depicts **Ashley's Bend** (#1452 in Ashley) and **The Footrope Knot** (#783 in Ashley). The lower layout in Fig. 377 is the complement to the lower layout in Fig. 374.

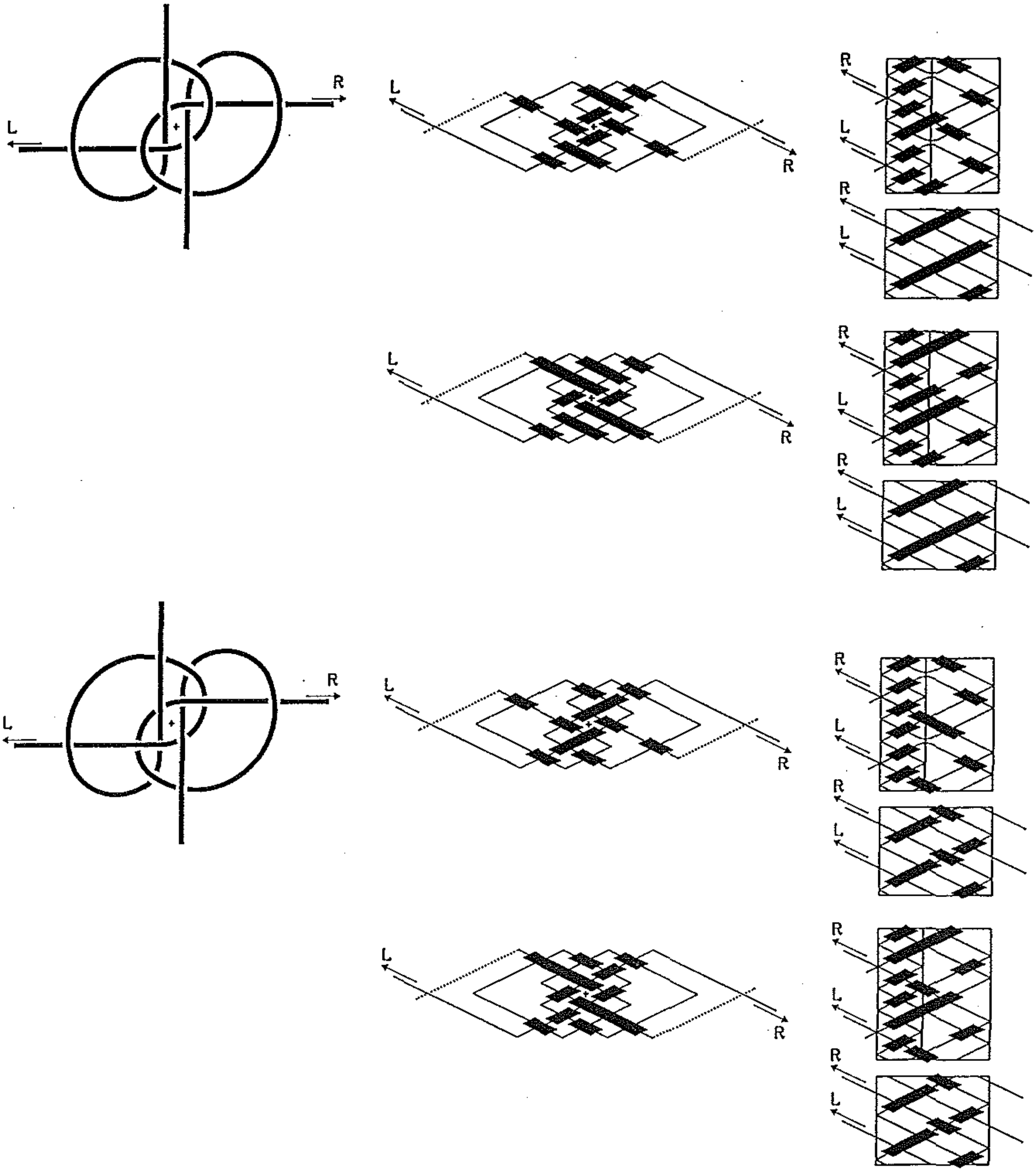


Fig. 377 — The grid-diagrams associated with the lower layout in Fig. 372.

In the previous Figs. 373–377 as well as in the following Figs. 379–381 the crossing-sequences from the ends L and R are identical for the coding arrangements of the two interbraided strings. These two sets of figures form however, with respect to the types of grid-diagram shown, two distinct categories.

The first layout in Fig. 379 depicts **Hunter's Bend** (#1425*A* in Ashley).

The second layout in Fig. 379 results in the left-hand string configuration of Fig. 378.

The third layout in Fig. 379 depicts #1425 in Ashley. This is an excellent bend for flat string.

The final layout in Fig. 379 results in the right-hand string configuration of Fig. 378.

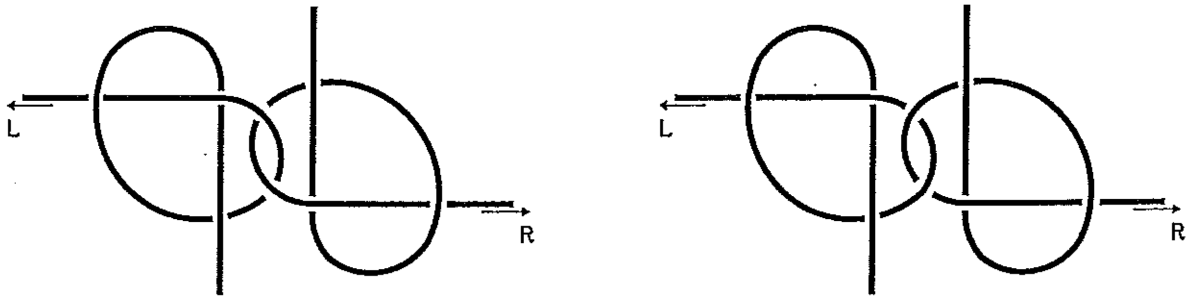


Fig. 378 — The rearranged layouts for the second and fourth layout in Fig. 379.

The everted forms of the layouts in Fig. 380 are depicted in Fig. 381. These everted forms are respectively the complements of the forms in Fig. 379.

Since we are limiting our discussion to braiding material which is symmetric with respect to shape, colour and pattern/texture, we need to concern ourselves only with the various balanced superimposed coding arrangements associated with the second layout from the top in Fig. 371 (note that with these limitations the third layout from the top in Fig. 371 is **lateral equivalent**[†] to the second layout from the top).

We thus have to investigate the eight balanced coding arrangements which are associated with the second layout from the top in Fig. 371. These arrangements are shown in Figs. 382 & 385.

The crossing-sequences from the ends L and R, associated with these coding arrangements, are opposites for each of the two interbraided strings.

The uppermost layout in Fig. 382 depicts **The Zeppelin Bend**.

The two components in the second and third layout from the top in Fig. 382 slide apart.

The lowermost layout in Fig. 382 can be rearranged into the central layout shown in Fig. 383. Although this rearrangement is called in *The Century Guide to Knots*, by Mario Bigon and Guido Regazzoni, pp. 138 & 139, **The Water Knot**[‡], **The Fisherman's Knot**, **The English Knot**, **The Englishman's Knot**, **The True-Lover's Knot**, **The Angler's Knot**, it is quite a different knot to the one by the same names in Ashley (#496 & #1414). The Waterman's Knot in Ashley has been depicted in Fig. 384. Although the string-run of the knot in Fig. 383 can be rearranged into the lowermost string-runs shown in Fig. 383, hence to the type of string-runs shown for the knot in Ashley (Fig. 384), the knot in Ashley (Fig. 384) cannot be rearranged into the type of

---

[†] Refer for more details to Pamphlet No. 8: *Knots — Facts and Fallacies*.

[‡] Note that the name **Water Knot** is used for various knots (see Ashley #296, #343, #1412, #1414). The name **Waterman's Knot** should here be preferred instead (see Ashley #1414).

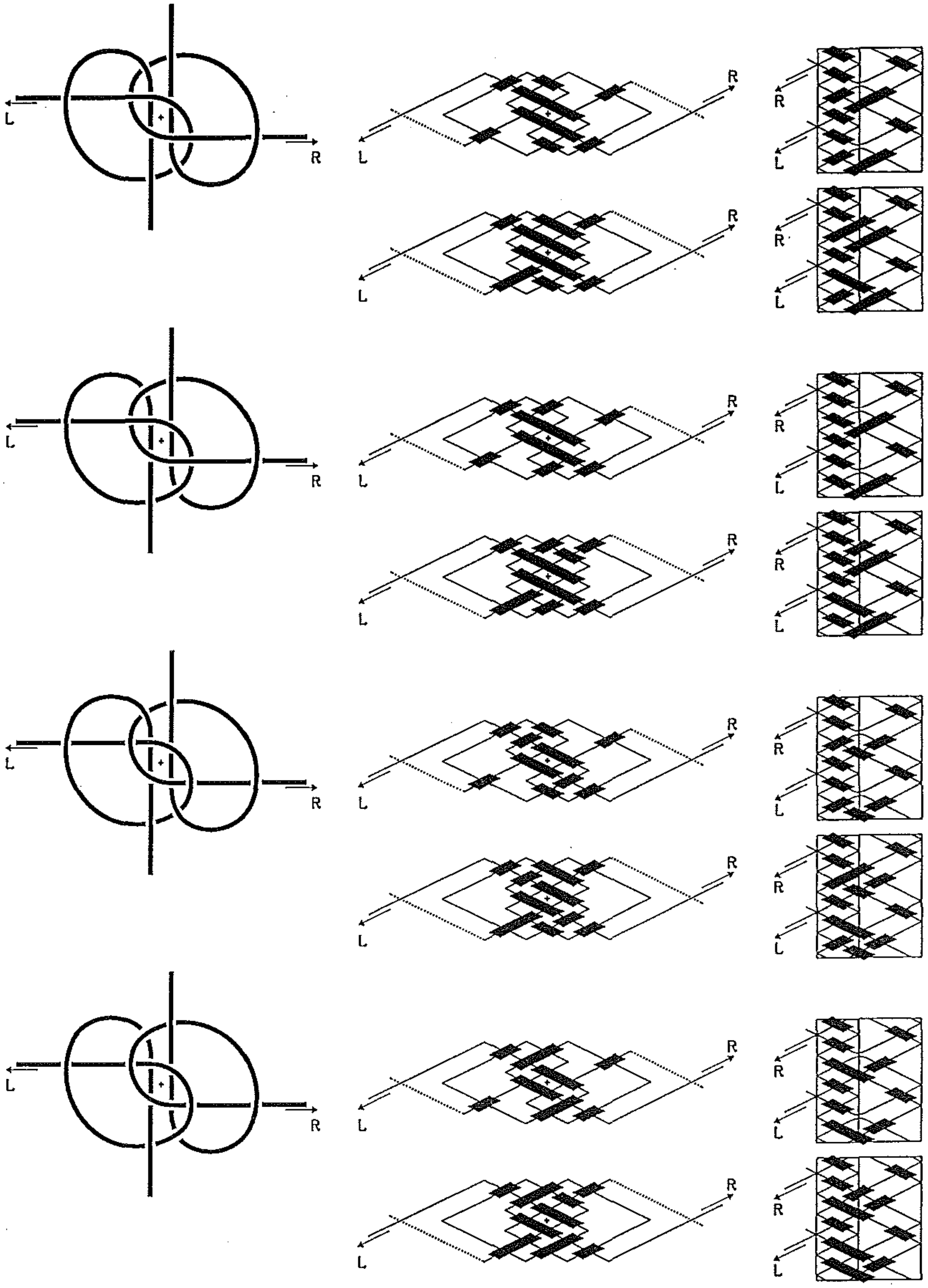


Fig. 379 — Further layouts with identical crossing-sequences for the two components.

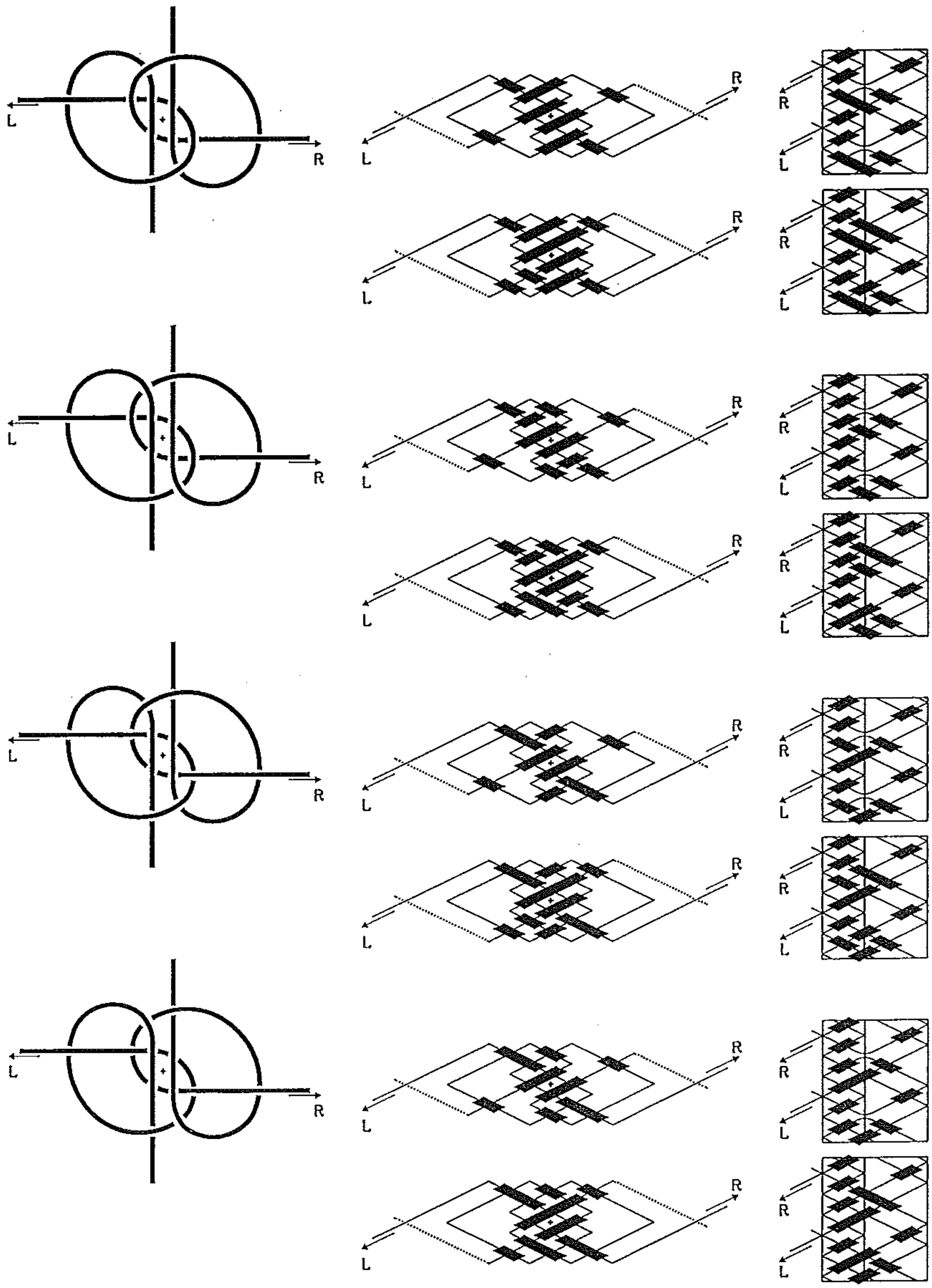


Fig. 380 — Further layouts with identical crossing-sequences for the two components.

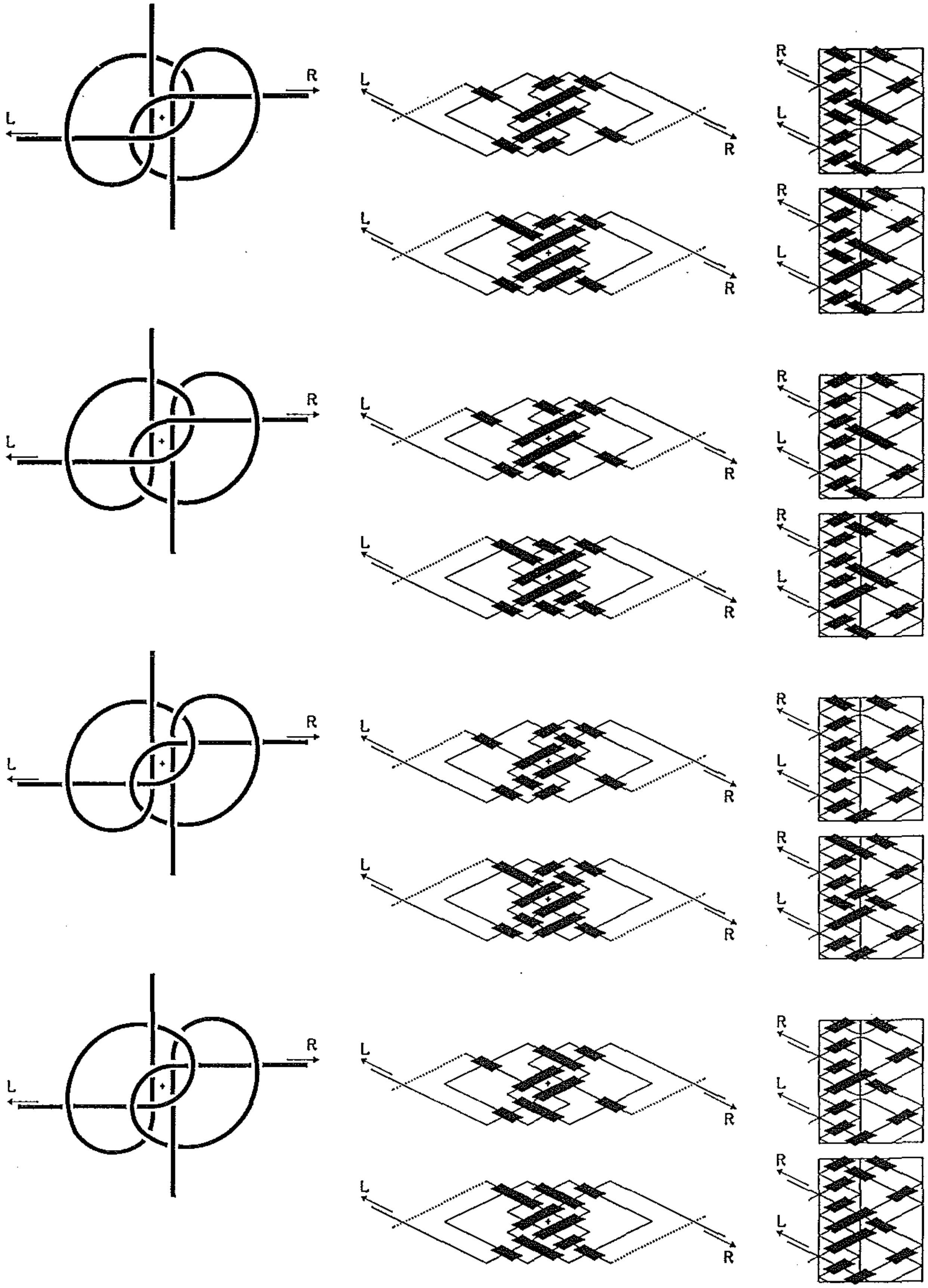


Fig. 381 — The everted forms of the layouts in Fig. 380.

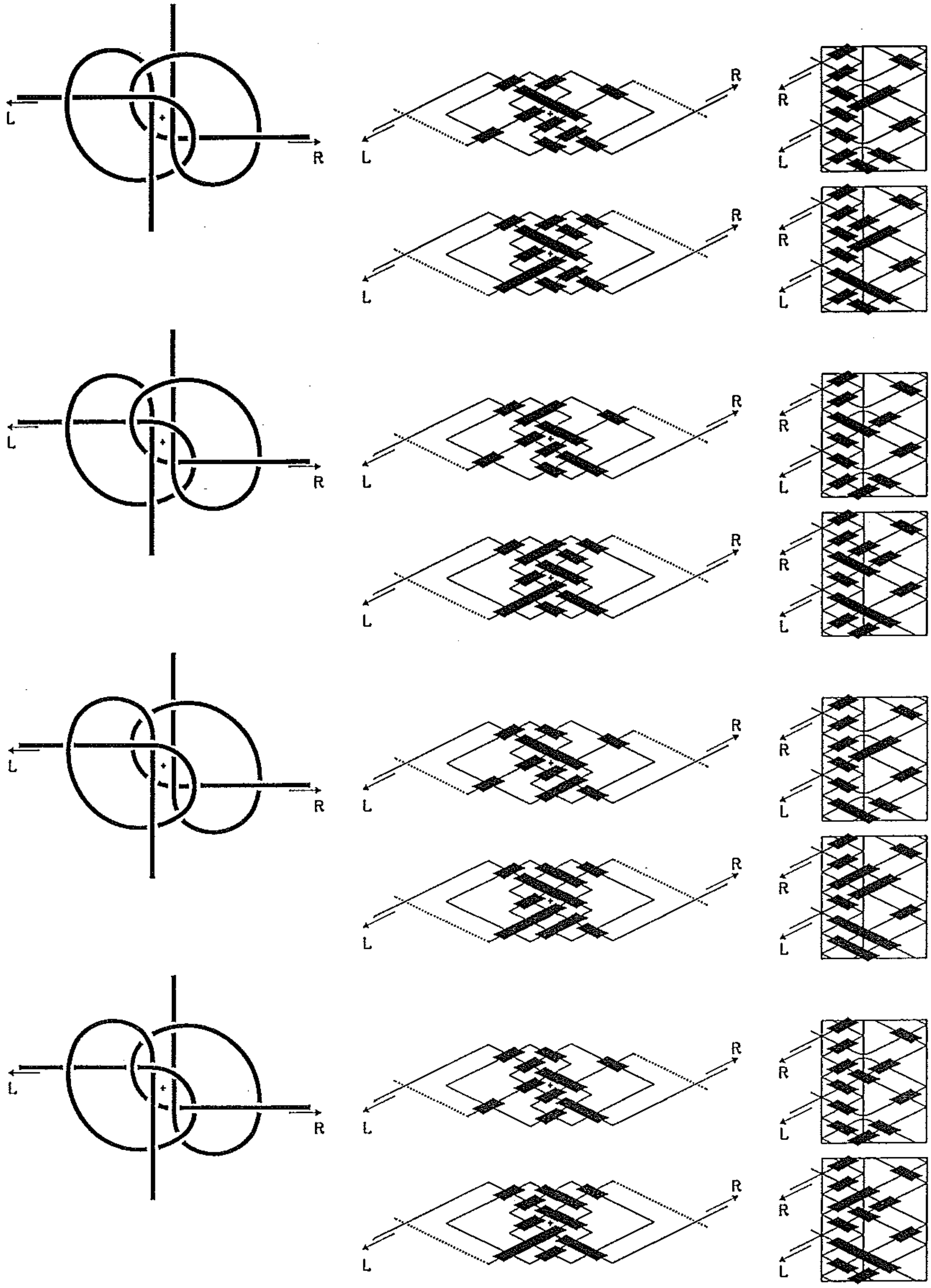


Fig. 382 — Layouts with opposites as crossing-sequences for the two components.

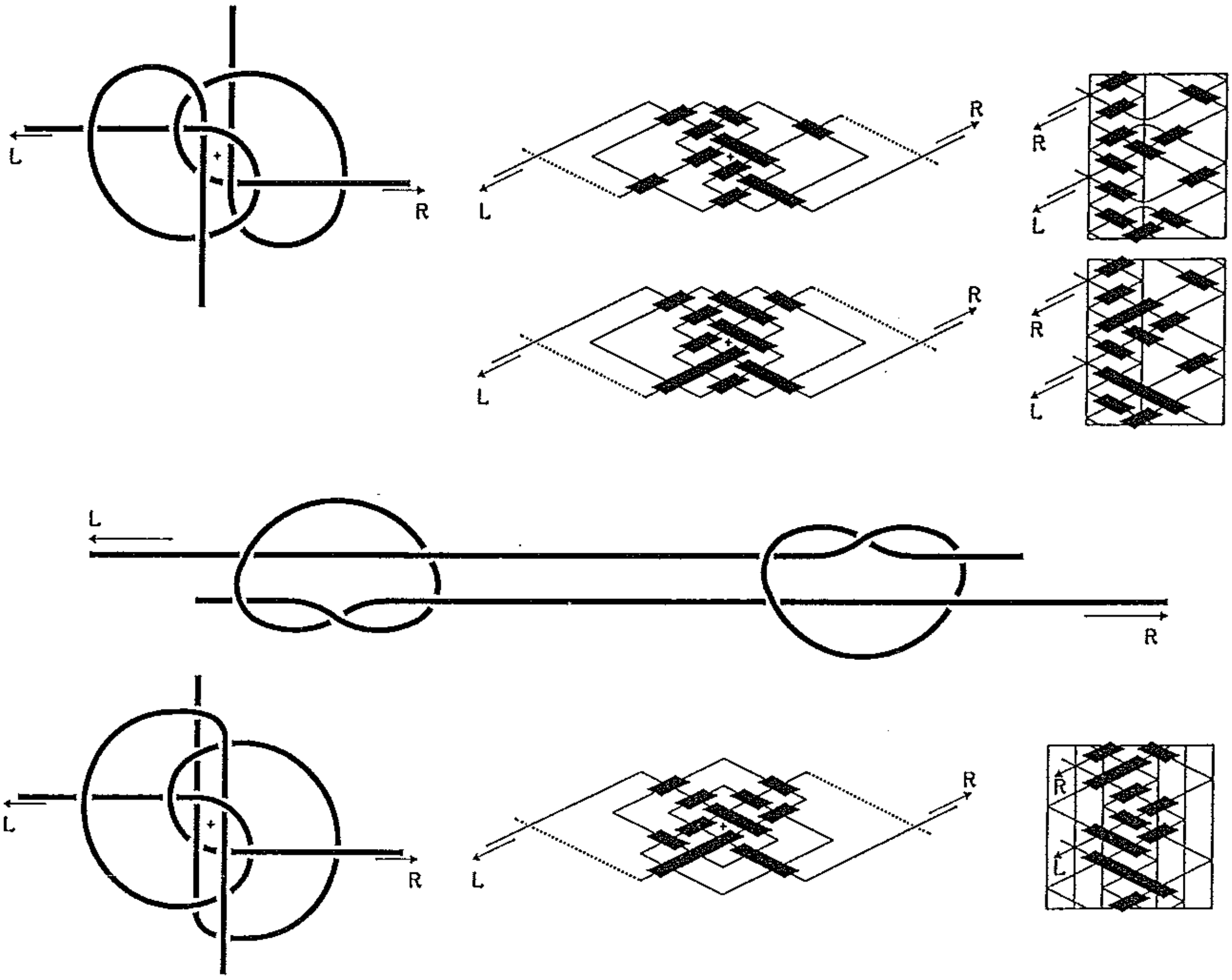


Fig. 383 — The lowermost layout in Fig. 382.

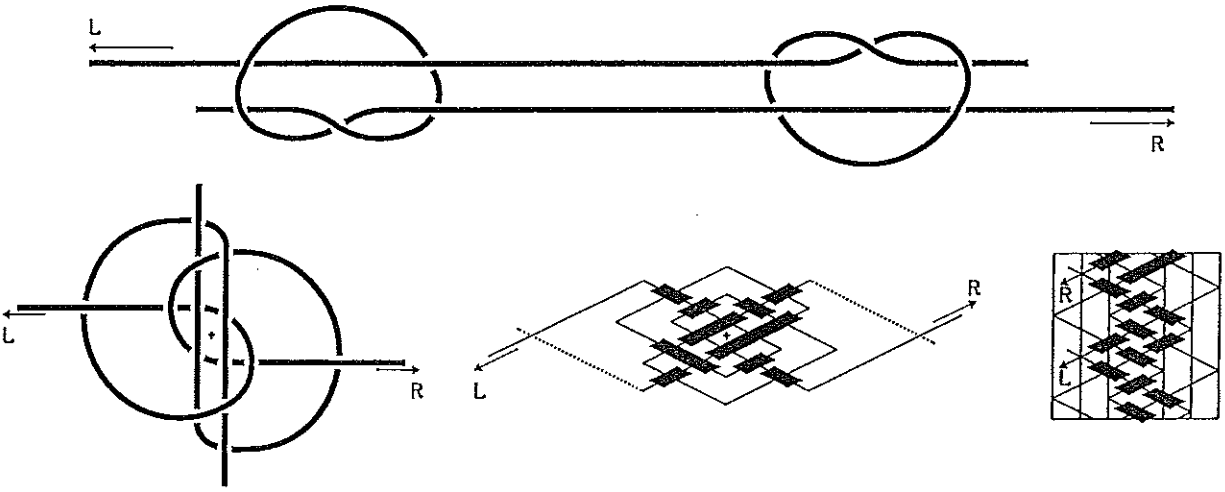


Fig. 384 — The Waterman's Knot #496 & #1414 in Ashley.

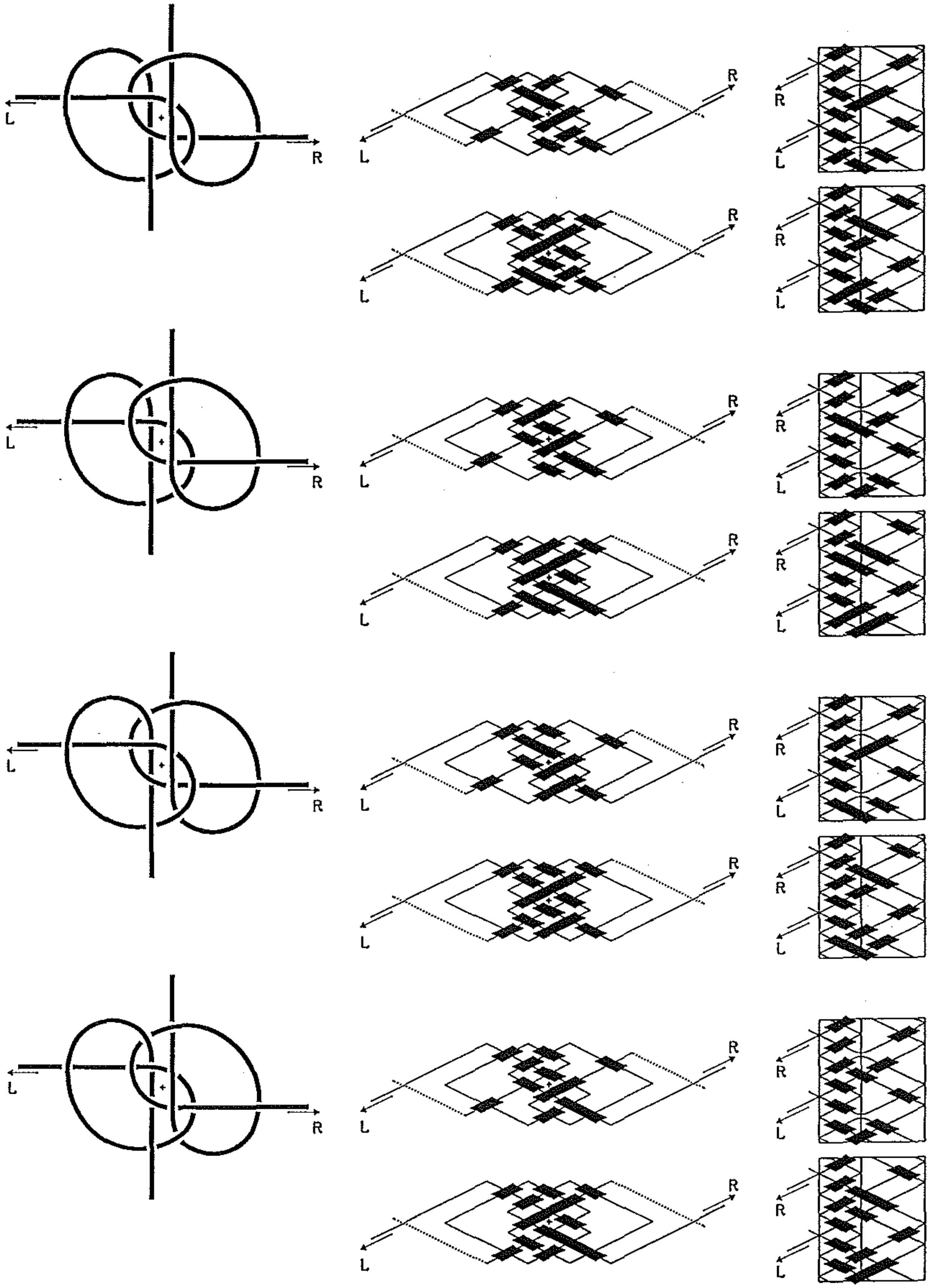


Fig. 385 — Layouts with opposites as crossing-sequences for the two components.

string-runs shown in the upper part of Fig. 383 (refer to pg. 447). Hence, notwithstanding that the Waterman's Knots knots in Ashley and in *The Century Guide to Knots* appear superficially similar, they are quite different. It should be noted that the Waterman's Knot in Ashley has in the rearrangements shown in Fig. 384 **not** a balanced coding, whereas the Waterman's Knot in *The Century Guide to Knots*, shown in Fig. 383, does have a balanced coding. However, the Waterman's Knot in Ashley which does have a balanced coding in the upper string-run of Fig.384 ([L: $u-2o-u-2o-2u-o-u$.] and [R: $u-2o-u-2o-2u-o-u$.]) is a considerably better knot than the Waterman's Knot in *The Century Guide to Knots*, which also has a balanced coding in the central string-run of Fig. 383 ([L: $u-o-u-o-2u-2o-u-o$.] and [R: $o-u-o-u-2o-2u-o-u$.]).

The first two layouts in Fig. 385 are rearrangements of each other and the two components can be slid apart.

The last two layouts in Fig. 385 are also rearrangements of each other. The active ends L and R of these two layouts are the inactive ends of the everted Zeppelin Bend, the everted upper layout in Fig. 382.

This very limited discussion involving a rather simple string run layout should clearly demonstrate the complications which are an inherent part of any classification attempt for knots and braids.

The Zeppelin Bend is also called the Rosendahl Bend by Geoffrey Budworth (*The Knot Book, pg. 129*), and is called the Rosenthal Bend by Brion Toss (*The Rigger's Apprentice, pg. 47*). Who can solve the question who of the two got it right, hence whether it should be Rosendahl Bend or Rosenthal Bend.

In *The Australian Whipmaker*, No. 44, pg. 838, a knot has been depicted by Ron Edwards to whom it was shown by a visitor who apparently called it 'Hunter's Knot'. However, this knot is **not** Hunter's Bend as the leftmost layout in Fig. 386 readily shows. In order to avoid any confusion we shall call this knot here '**The Edwards' Knot**'.

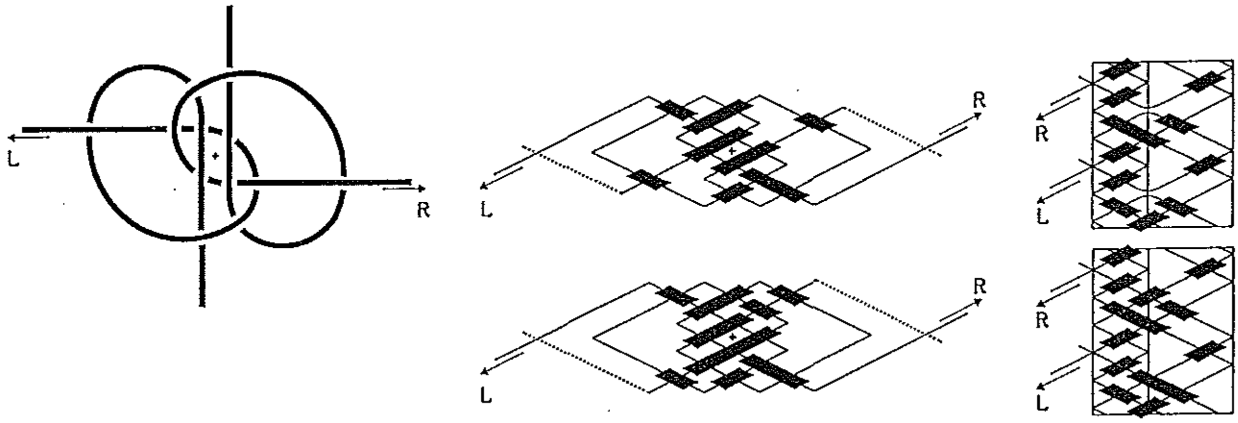


Fig. 386 — The Edwards' Knot.

It should be observed that this knot does not have a balanced coding. It differs in one crossing from the uppermost layout in Fig. 380, or its string-run can be rearranged so that it differs in one crossing (at the same crossing-point) from the third layout in Fig. 380. This knot undoubtedly resulted from erroneously tying Hunter's Bend.

THE BRAIDER

APPENDIX 1999

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A quarterly publication for the braiding artisan

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Knot nomenclature

In *The Braider*, Issues No. 18, pp. 403–404, and No. 20, pp. 446–464, we have already met some of the confusing trends regarding the naming of knots. To complicate matters even further, we see far too often incorrect referencing, or worse still, no referencing at all. A typical case, for example, involves the **Hunter's Bend**, also known as a **Rigger's Bend**.

We mentioned already in Issue No. 20, pg. 464, a naming error associated with the **Zeppelin Bend** by either Geoffrey Budworth (*The Knot Book, pg. 129*) or Brion Toss (*The Rigger's Apprentice, pg. 47*), as well as the error in *The Australian Whipmaker*, No. 44, pg. 838, where the **Edwards' Knot** was called the **Hunter's Knot**. In *The Australian Whipmaker*, No. 47, pp. 892–893, a follow-up article adds a further number of undesirables to the saga of the **Hunter's Bend** and the **Zeppelin Bend**.

Unfortunately, this follow-up article does not give any references, not only as to the name **Rosendahl** (as opposed to **Rosenthal**), but even more seriously also not to its contained remark: *"... I have seen this knot published as a Zeppelin Bend and Hunter's Bend"* (referring to the fourth row of its drawings (depicting the Zeppelin Bend), the central drawing of which depicts the everted-lateral uppermost left diagram in Fig. 382 of *The Braider*, Issue No. 20, pg. 461). There is however a very essential and distinct difference between the Zeppelin Bend and the Hunter's Bend, namely, for braiding material which is symmetric with respect to shape, colour and pattern/texture, the Zeppelin Bend and its complementary form are identical whereas the Hunter's Bend and its complementary form are **not** identical.

The knot depicted in the article's second row of drawings is, contrary to the statement made, **not** [identical to] the Hunter's Bend as illustrated in Ashley (# 1425*A*), but is instead the complementary form of it (note that in Ashley's lowermost drawing the crossing of the black lines must be opposite to that depicted).

At least some of the confusion concerning the Hunter's Bend and the Zeppelin Bend may have been caused by Geoffrey Budworth in his booklet *The Knot Book* (ISBN 0-7160-0704-5). Although the Hunter's Bend in this booklet (pg. 128, Fig. 85 (*A*)) is the same as in Ashley, but with the correct crossing between the thin black lines in Fig. 85 (*B*) as opposed to Ashley, the story on pg. 129 under *poor man's pride* wrongly states: *"It's* [the Hunter's Bend] *really the Rosendahl Bend (otherwise known as the Zeppelin Knot)"*. His **Poor Man's Pride** in Fig. 87 on pg. 131 is in fact the Zeppelin Bend.

Notwithstanding that the Hunter's Bend in Ashley (#1425*A*) and its complementary form (the **complementary Hunter's Bend**) are in fact two different bends (they are not identical!!!), in some publications (*The Century Guide to Knots* (ISBN 0 7126 0089 2) by Mario Bigon and Guido Regazzoni, pg. 131, and *Identifying Knots* (ISBN 1-85348-900-X) by Peter Owen, pg. 55) the complementary Hunter's Bend is called the Hunter's Bend.

Although there is a great deal of confusion concerning the Zeppelin Bend and the Hunter's Bend, or even incorrectly tied Hunter's bends (the Edwards' Knot for example), the Hunter's Bend is certainly stronger than the Zeppelin Bend due to its more gentle string curvature, and since the construction of the Hunter's Bend is for many easier to remember, it is likely that the remarks by Alf Reimers, in *The Australian Whipmaker*

1997 No. 46, pg. 873, correctly do refer to the Hunter's Bend. He states there the following: *"When I was about ten years of age my father showed me how to tie this knot* [the Hunter's Bend]. *That was seventy-three years ago* [1924]. *This was called the No. 8 knot. I guess because it was a beautiful way to join No. 8 gauge wire and was probably used by my forebears and others in the pioneering days of the last century. I have used this knot while fencing all my life."*. Furthermore, the Hunter's Bend and its (everted) complementary form were most likely known and used in ancient times due to their easy to remember construction algorithms.

For the Hunter's Bend we start with the **overhand** knot, hence with the Standing End (S_1) towards the left; the first crossing its Working End (W_1) makes is an **over**-crossing. The second string, with its Standing End (S_2) towards the right makes then with its Working End (W_2) the crossing-movements $2o - 2u - 2o - 2u - o$ (see Fig. 1).

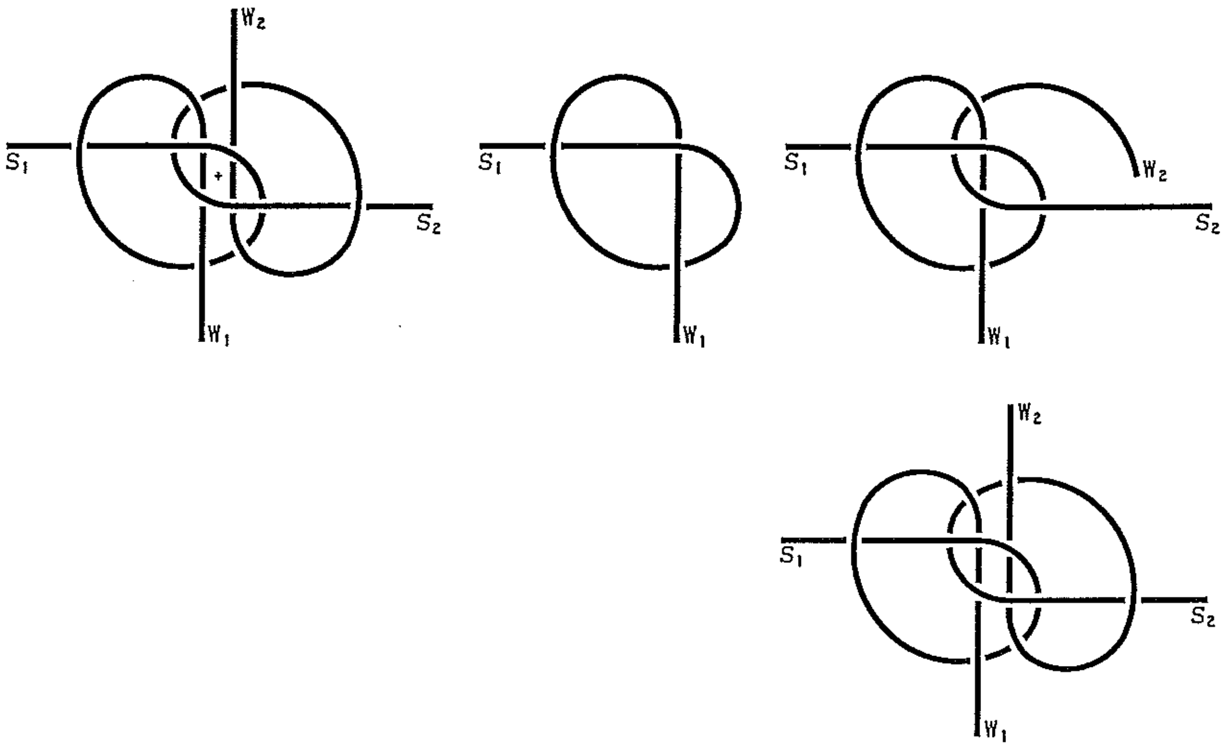

Fig. 1 — The Hunter's Bend and its construction algorithm.

For the (everted) complementary Hunter's Bend we start with the **underhand** knot, hence with the Standing End (S_1) towards the left; the first crossing its Working End (W_1) makes is an **under**-crossing. The second string, with its Standing End (S_2) towards the right makes then with its Working End (W_2) the crossing-movements $2u - 2o - 2u - 2o - u$ (see Fig. 2).

Not only are these tying algorithms very easy to remember (which is very essential for a useful bend), but they contain another very interesting property. Let the second string make the inverse sequence of the above crossing-movements, hence when we start with the **overhand** knot, the second string makes the crossing-movements $o - 2u - 2o - 2u - 2o$, and when we start with the **underhand** knot, the second string makes the crossing-movements $u - 2o - 2u - 2o - 2u$. The resulting knots are then respectively the (everted) Ashley's bend or Footrope knot and the complementary Ashley's bend or Footrope knot. Thus with the easy to remember algorithm-form for the Hunter's bend, we know at the same time the algorithm-form for the Ashley's bend or Footrope knot. Furthermore, here (and only here; see Appendix 1997, pp. *viii–x*) it is where the name **Ashley's bend** comes in handy: the first letter in Ashley is the first letter of the alphabet and the algorithm-form for Ashley's bend begins with 1*o* or 1*u* and the rest of the alternating crossing-movements are in sets of two; the algorithm-form for Hunter's bend begins with 2*o* or 2*u* and the rest of the alternating crossing-movements are as far as possible in sets of two.

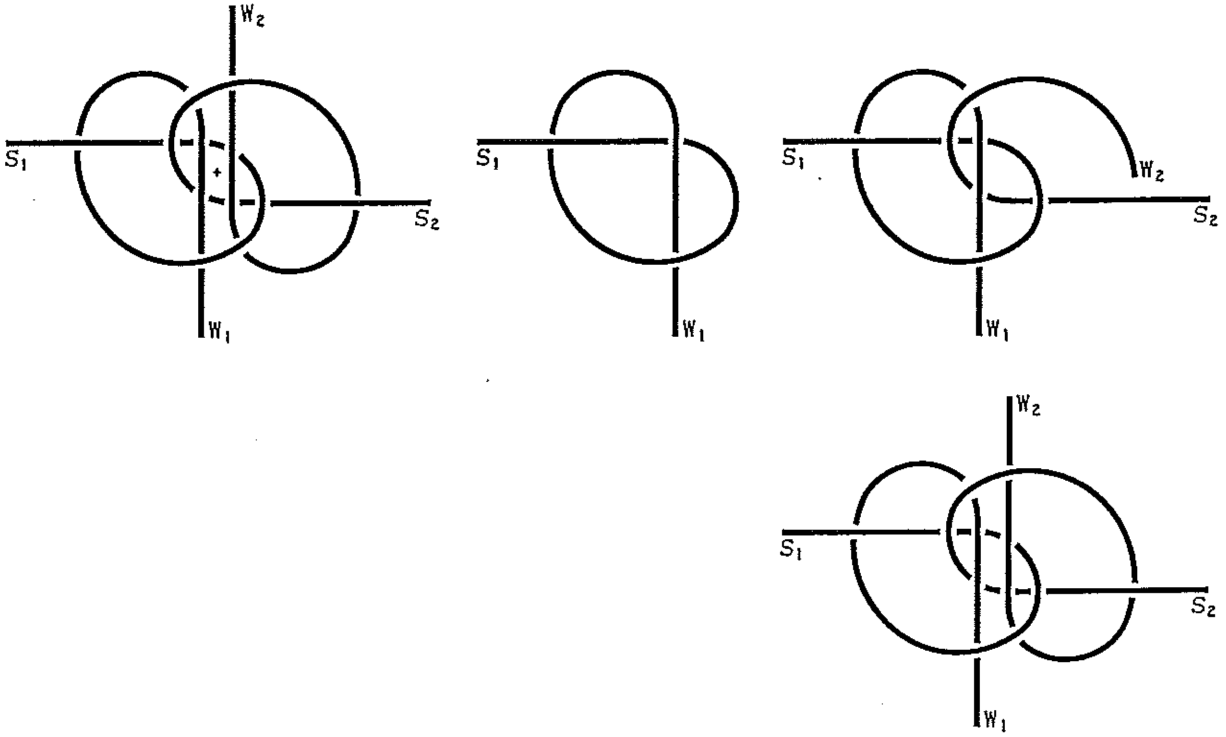

Fig. 2 — The (everted) complementary Hunter's Bend and its construction algorithm.

These tying algorithms may not necessarily be the quickest, but they avoid erroneous tying procedures and are certainly the easiest to remember. That's what counts in practice!!!

Not only was the Hunter's bend rediscovered by Hunter, but from the above it appears logic that also Ashley's bend as such was a rediscovery; logically and hence obviously we may assume that both were known as bends in the distant past.

Names and Terms used in the Braiding World

The average braider doesn't realise that in a knot or braid it is the string-run which is of prime importance and **not** the coding. The coding is independent of the string-run, although we might not be able to superimpose **any** overall coding **pattern** on **any** string-run. Hence the coding is only of minor value, hence of secondary importance only. Sure, in some knots the coding is of more importance than in other knots, but it is in the first place the string-run and **not** the coding which 'classifies' a braid. It should therefore be appreciated that there isn't a knot-name or part of a knot-name that has a 'good definitive meaning', and what is more, there never will be such a thing. Most terms used in the knot-naming game are relative indicators only, nothing more and nothing less. Take for example the term 'Casa' in the name "Casa Knot" :

Tom Hall specifies the 'Casa' Knot as an **over-under coded Regular Knot**. A Regular Knot being a **single string Cylindrical Braid with two parallel circumferential bight-boundaries between which the string-run zigzags**. He couples the term 'Casa' to a **Regular Knot** made from a **single string** which then has an **over-under** coding.

We, on the other hand, did not use such a seemingly more precise and hence much more restrictive coupling procedure, but instead coupled the term 'Casa' to an **over-under coding** instead, hence to the **coding**-form of Tom's 'Casa' Knot. Consequently we could couple the term 'Casa' to an **over-under coded Regular Cylindrical Braid** (referred to as an over-under coded **Regular Knot** if it requires **one essential string** (g.c.d. $(p, b) = 1$), and referred to as a **Semi-Regular Knot** if it requires **more than one essential string** (g.c.d. $(p, b) > 1$)). In fact, we could couple the term 'Casa' to any braidform as long as it had an **over-under coding**.

The seemingly more precise coupling way of Tom Hall is of course, as already mentioned, much more restrictive than ours, and the question arises if such a seemingly more precise way of coupling the term 'Casa' by Tom Hall is of sufficient value to offset its more restrictive nature. The answer, as we shall see, is certainly **no**.

We used the term 'seemingly' above not for nothing, because the term 'Casa' as used by Tom compared to its use by us offers nothing of specific practical value since it still does **not define** the knot used any closer[†] other than its in practical sense rather irrelevant nature of **having to be constructed with a single string**. In fact, Tom's definition is not only in practical sense but even in theoretical sense a rather irrelevant one since an emphasis is placed on the compulsary **single string** property. Hence if, for example, a 7-parts/4-bights over-under coded Regular Knot is made with a single string, then according to Tom's definition the knot is a 'Casa' Knot, but if it is made with say four strings then it is **not** a 'Casa' Knot. **Note, however, that in either case the knot is still a Regular Knot since it requires only one essential string**; apart from its bight-boundary and string-run conditions, the only important parameter for being a Regular Knot is that the g.c.d. (p, b) is equal to 1 (there is no compulsion to make it from a single string!!!).

In Tom Hall's definition of a 'Casa' Knot we used the term **over-under coded**, but Tom uses the term **one-pass** instead. This brings us to the much used, but in general meaningless term **pass**.

The term **pass** is being used for two different knot 'parameters'.

We find the term **pass** being used for a section of the string-run from **somewhere around one bight to somewhere around the next consecutive bight along the string-run**. Since 'somewhere' can be anywhere, a **pass** is undefined and consequently is totally useless as a parameter. It is really astounding how much it is being used by 'braiders' and obviously their accompanying further description has, with a bit of luck, to make it clear what was meant. A **half-cycle** on the other hand has been clearly defined as a section of string-run from **one bight** to the **next consecutive bight** along the string-run, consequently it is very useful as a parameter.

We find the term **pass** also being used for some overall general coding arrangement. Apart from the term **one-pass** (meaning **over-under coding**), the term n-**pass**, where

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[†] The reader is hereby referred to *The Braider*, Issue No. 21, pp. 471-482, to be published in February 2000.

n is a natural number greater than 1, can only be understood by people familiar with some commonly used braiders jargon accompanying the term. In other words, it is **not** unambiguously defined. If we would unambiguously define an n-pass weaving-pattern as an n over-n under coding along the string-run, then, for example, a 2-pass Standard Herringbone Knot doesn't exist, nor does a 2-pass Standard Herringbone Pineapple Knot exist, but a 2-pass Gaucho Knot and a 2-pass Headhunter's Knot do exist. One might ask: *can we give the term n-pass a definition so that it always clearly, hence unambiguously, defines any so-called n-pass weaving-pattern?* The answer is **no**, we cannot; we can only use the term as a rough indicator.

We use many of such rough indicator terms in the world of braiding. Unfortunately some braiders get a hang-up about one or more of such indicator terms and want to see them as the best thing after sliced bread; they often belong to the old-time pattern-braiders who are rusted solid in their braiding tracks.

A lot of nonsense and claptrap has, and still is, being written about braiding: academic nitwits love to see the hypothetical topological knot theory as being relevant to actual knotting and braiding and hence write volumes of claptrap as far as real knots and braids are concerned, the braiders which are solidly anchored in their tracks write volumes with claptrap too. Some of those still love to use enlargement methods during their actual braiding.†

Sure, nice artistic knot and braid drawings look terrific and often mysterious. It often appears that the more mysterious such drawings look, the better. The trouble is that even the very best drawings, or photographs for that matter, even with their associated construction procedures, cannot readily transmit to the reader a clear encompassing picture of the knot or braid. Only the grid-diagram of the knot or braid can do this, and hence it is of the utmost importance to depict also their grid-diagram. Some braiders have started to do this now, but unfortunately most of them don't understand (or don't want to understand in order to be different) why we depict grid-diagrams in the way we do. They should again thoroughly read and try to fully comprehend the reason for that what has been written in *The Braider*, Issue No. 2, pg. 32 line 32 from the top.

Nested Cylindrical Braids — Hunter's Bend

Refer to *The Braider*, Issue No. 19, August 1999 pp. 415 - 422; Issue No. 20, November 1999 pp. 457 - 458, and Fig. 3 below on pg. *vi*.

The string-run of Hunter's Bend may be derived from the string-run of the Nested Cylindrical Braid $(0/2/3)\{1/112\}6$. The general string-run at the left and right bight-edges is as depicted in Fig. 4. Consequently, the value of $A_l = 1$ and the value of $A_r = 3$, while $\mathcal{K}_l = 1$ and $\mathcal{K}_r = 2$.

The string-run of $(0/2/3)\{1/112\}6$ with the ranking-numbers as subscripts attached to the bight-boundary numbers is thus: $(0/2/3)\{1_1/1_1 1_2 2_3\}6$.

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† Enlargement methods are now only of theoretical importance and not any longer in the actual production of braidwork; algorithm diagrams offer a much simpler and direct way in the actual production of braidwork.

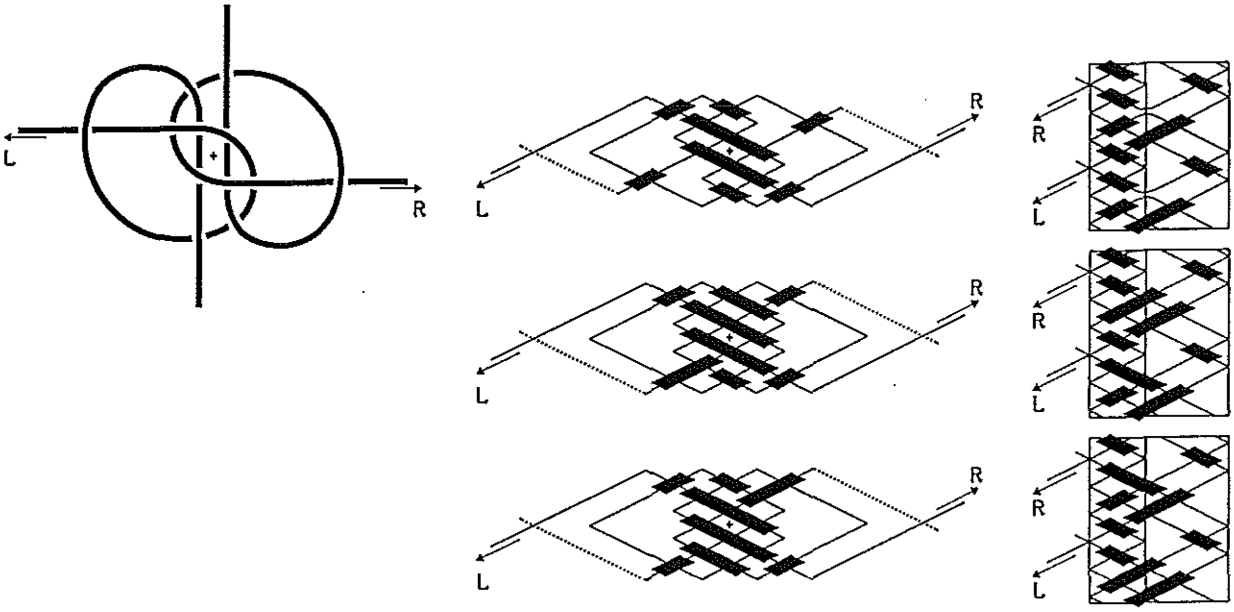


Fig. 3 — Hunter's Bend and its associated Nested Cylindrical Braids.

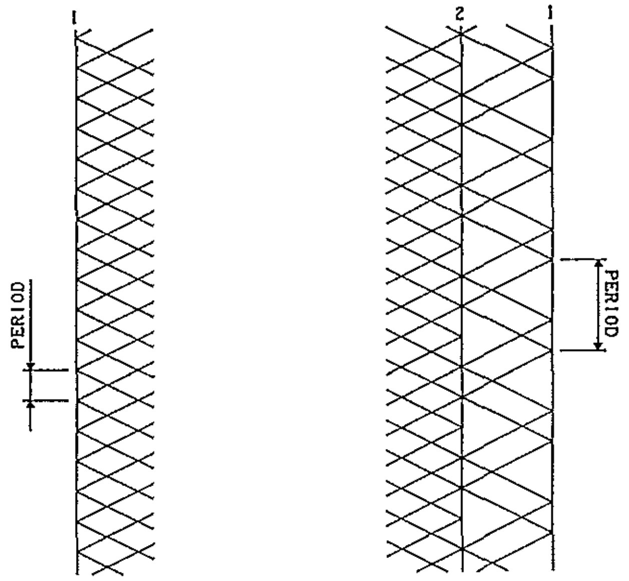


Fig. 4 — The string-run at the left and the right bight-edge.

For this string-run $((0/2/3)\{1_1/1_1 1_2 2_3\}6)$ we thus obtain:

$$A_l = 1\ ;\ A_r = 3\ ;\ d = \text{g.c.d.}\,(A_l\,,\ A_r) = \text{g.c.d.}\,(1\,,\ 3) = 1\,.$$

$$A^{**} = \frac{A_l \cdot A_r}{d} = \frac{1 \cdot 3}{1} = 3\,.$$

$$B_l^* = \text{number of periods at left bight-edge} = 6\,.$$

$$B_r^* = \text{number of periods at right bight-edge} = 2\,.$$

$$B_{total} = A_l B_l^* = A_r B_r^* = A^{**} B^{**} = 3 \cdot 2 = 6\,.$$

$$B^{**} = \frac{B_{total}}{A^{**}} = 2\,.$$

$$\mathcal{K}_l = 1\ ;\ \mathcal{K}_r = 2\,.$$

Hence:

$$\Delta_{l_i} = 0 \quad \text{for } l_i = 1\,. \qquad\qquad \begin{aligned} \Delta_{r_i} &= 3 \quad \text{for } r_i = 1\,. \\ \Delta_{r_i} &= 0 \quad \text{for } r_i = 2\,. \end{aligned}$$

Each lower-left to upper-right half-cycle type in a Nested Cylindrical Braid occurs only once in all the first-return string-runs of such a braid. The lower-left to upper-right half-cycle types can be read from the Nested Cylindrical Braid specification $(0/2/3)\{1_1/1_1 1_2 2_3\}$:

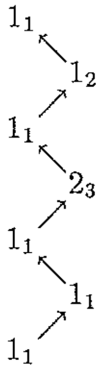
$$\begin{aligned} 1_1 &\longrightarrow 1_1 \\ 1_1 &\longrightarrow 1_2 \\ 1_1 &\longrightarrow 2_3 \end{aligned}$$

Any one of these listed types may be taken as the first lower-left to upper-right half-cycle in the 1st first-return string-run, but normally we take the first listed one. Every lower-left to upper-right half-cycle encountered in this 1st first-return string-run gets deleted from the type-list.

Any one of the remaining types in the type-list may be taken as the first lower-left to upper-right half-cycle in the 2nd first-return string-run, and again every lower-left to upper-right half-cycle encountered in this 2nd first-return string-run gets deleted from the remaining type-list.

This process is carried on till all the lower-left to upper-right half-cycle types have been deleted from the type-list.

For the first-return string-runs we thus obtain:



$$\begin{aligned}
l_{1_{j_l}} = 1_1 \longrightarrow 1_1 = r_{1_{j_r}} \quad &\rightarrow \quad j'_l = |1+0+2+3|_1 = 1 \quad &&\rightarrow \quad l_{2_{j'_l}} = 1_1\,. \\
l_{2_{j'_l}} = 1_1 \longleftarrow 1_1 = r_{1_{j_r}} \quad &\rightarrow \quad j'_r = |1+3+2+0|_3 = 3 \quad &&\rightarrow \quad r_{2_{j'_r}} = 2_3\,. \\
l_{2_{j_l}} = 1_1 \longrightarrow 2_3 = r_{2_{j_r}} \quad &\rightarrow \quad j'_l = |1+0+2+0|_1 = 1 \quad &&\rightarrow \quad l_{3_{j'_l}} = 1_1\,. \\
l_{3_{j'_l}} = 1_1 \longleftarrow 2_3 = r_{2_{j_r}} \quad &\rightarrow \quad j'_r = |3+0+2+0|_3 = 2 \quad &&\rightarrow \quad r_{3_{j'_r}} = 1_2\,. \\
l_{3_{j_l}} = 1_1 \longrightarrow 1_2 = r_{3_{j_r}} \quad &\rightarrow \quad j'_l = |1+0+2+3|_1 = 1 \quad &&\rightarrow \quad l_{4_{j'_l}} = 1_1\,. \\
l_{4_{j'_l}} = 1_1 \longleftarrow 1_2 = r_{3_{j_r}} \quad &\rightarrow \quad j'_r = |2+3+2+0|_3 = 1 \quad &&\rightarrow \quad r_{4_{j'_r}} = 1_1\,.
\end{aligned}$$

$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{3 \cdot 2 + (0+0+0) + (3+0+3)}{3} = 4\,.$
g.c.d. $(P_c\,,\, B^{**}) =$ g.c.d. $(4\,,\,2) = 2\,.$

Hence:

$$P_{total} = \sum P_{component} = 4\,.$$

$$\left.\begin{array}{r}\text{number of} \\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1\,.$$

$$\left.\begin{array}{r}\text{total number of} \\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 2\,.$$

Nested Cylindrical Braids — The Kirsten Knots

Refer for the calculation procedures to *The Braider*, Issue No. 19, August 1999, pp. 415 - 422.

In the Dutch bimonthly knotting publication *Het Knoopeknauwertje*, Issue No. 5, April 1997 pp. 16 - 17, we find the desription of a small braided knot called *The Kirsten Knot* which can be used for covering a small spherical object. It was, however, only a small example of the **Kirsten Knots** designed by Frans Masurel. As usual, the grid-diagram of the knot discussed has not been shown in the above mentioned publication and consequently could not be analysed with the result that its true nature remained hidden (we have depicted its grid-diagram in Fig. 6).

The string-run of the Kirsten Knots is characterized by the left-hand and right-hand periods depicted in Fig. 5. Consequently, the value of A_l and the value of A_r are both 6 for these knots, while $\mathcal{K}_l$ and $\mathcal{K}_r$ are both equal to 3.

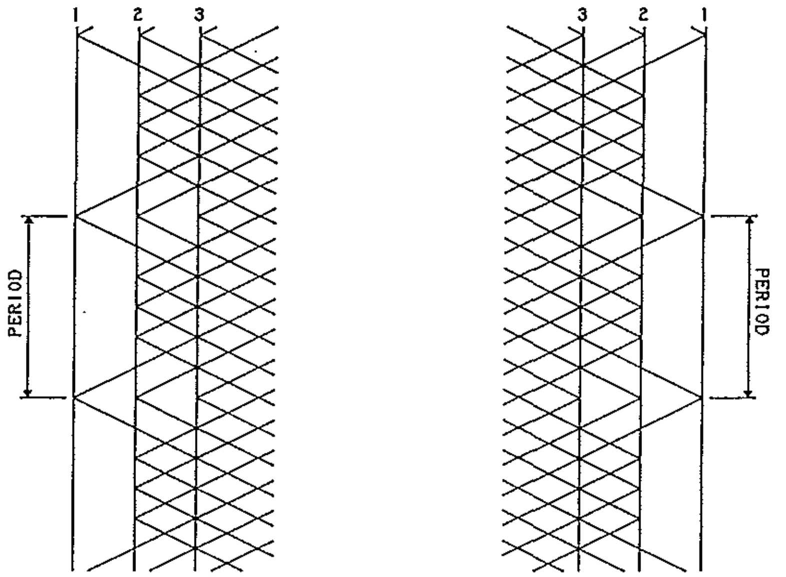


Fig. 5 — The string-run at the left and the right bight-edge of the Kirsten Knots.

The string-run specification of *The Kirsten Knot* is : (22/3/22){122232/222123}6, or with the ranking-numbers as subscripts attached to the bight-boundary numbers:

$$(22/3/22)\{1_1 2_4 2_5 2_6 3_3 2_2 / 2_1 2_2 2_3 1_4 2_5 3_6\}6\,.$$

For the string-run in Fig. 6 we obtain:

$$\begin{aligned}
A_l &= 6\,;\; A_r = 6\,;\; d = \text{g.c.d.}\,(A_l\,,\, A_r) = \text{g.c.d.}\,(6\,,\,6) = 6\,.\\
A^{**} &= \frac{A_l \cdot A_r}{d} = \frac{6 \cdot 6}{6} = 6\,.\\
B_l^* &= \text{number of periods at left bight-edge} = 1\,.\\
B_r^* &= \text{number of periods at right bight-edge} = 1\,.\\
B_{total} &= A_l B_l^* = A_r B_r^* = A^{**} B^{**} = 6 \cdot 1 = 6\,.\\
B^{**} &= \frac{B_{total}}{A^{**}} = 1\,.\\
\mathcal{K}_l &= 3\,;\; \mathcal{K}_r = 3\,.
\end{aligned}$$

Hence:

$$\begin{array}{llll} \Delta_{l_i} = 4 & \text{for } l_i = 1\,. & \Delta_{r_i} = 4 & \text{for } r_i = 1\,. \\ \Delta_{l_i} = 2 & \text{for } l_i = 2\,. & \Delta_{r_i} = 2 & \text{for } r_i = 2\,. \\ \Delta_{l_i} = 0 & \text{for } l_i = 3\,. & \Delta_{r_i} = 0 & \text{for } r_i = 3\,. \end{array}$$

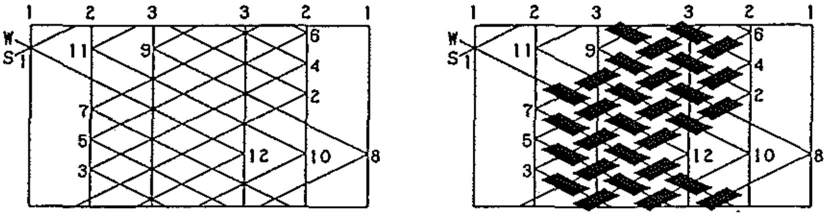


Fig. 6 — The Kirsten Knot..

Each lower-left to upper-right half-cycle type in a Nested Cylindrical Braid occurs only once in all the first-return string-runs of such a braid. The lower-left to upper-right half-cycle types can be read from the Nested Cylindrical Braid specification $(22/3/22)\{1_1 2_4 2_5 2_6 3_3 2_2/2_1 2_2 2_3 1_4 2_5 3_6\}6$:

$$\begin{array}{ll} 1_1 \longrightarrow 2_1 & 2_6 \longrightarrow 1_4 \\ 2_4 \longrightarrow 2_2 & 3_3 \longrightarrow 2_5 \\ 2_5 \longrightarrow 2_3 & 2_2 \longrightarrow 3_6 \end{array}$$

Any one of these listed types may be taken as the first lower-left to upper-right half-cycle in the 1^{st} first-return string-run, but normally we take the first listed one. Every lower-left to upper-right half-cycle encountered in this 1^{st} first-return string-run gets deleted from the type-list.

Any one of the remaining types in the type-list may be taken as the first lower-left to upper-right half-cycle in the 2^{nd} first-return string-run, and again every lower-left to upper-right half-cycle encountered in this 2^{nd} first-return string-run gets deleted from the remaining type-list. This process is carried on till all the lower-left to upper-right half-cycle types have been deleted from the type-list.

For the first-return string-runs we thus obtain:

$$\begin{array}{ll} 1_1 & \\ \nwarrow & \\ & 3_6 \\ \nearrow & \\ 2_2 & \\ \nwarrow & \\ & 2_5 \\ \nearrow & \\ 3_3 & \\ \nwarrow & \\ & 1_4 \\ \nearrow & \\ 2_6 & \\ \nwarrow & \\ & 2_3 \\ \nearrow & \\ 2_5 & \\ \nwarrow & \\ & 2_2 \\ \nearrow & \\ 2_4 & \\ \nwarrow & \\ & 2_1 \\ \nearrow & \\ 1_1 & \end{array}$$

$$\begin{array}{lllll}
l_{1_{j_l}} = 1_1 \longrightarrow 2_1 = r_{1_{j_r}} & \rightarrow & j_l' = |1+4+3+2|_6 = 4 & \rightarrow & l_{2_{j_l'}} = 2_4\,. \\
l_{2_{j_l'}} = 2_4 \longleftarrow 2_1 = r_{1_{j_r}} & \rightarrow & j_r' = |1+2+3+2|_6 = 2 & \rightarrow & r_{2_{j_r'}} = 2_2\,. \\
l_{2_{j_l}} = 2_4 \longrightarrow 2_2 = r_{2_{j_r}} & \rightarrow & j_l' = |4+2+3+2|_6 = 5 & \rightarrow & l_{3_{j_l'}} = 2_5\,. \\
l_{3_{j_l'}} = 2_5 \longleftarrow 2_2 = r_{2_{j_r}} & \rightarrow & j_r' = |2+2+3+2|_6 = 3 & \rightarrow & r_{3_{j_r'}} = 2_3\,. \\
l_{3_{j_l}} = 2_5 \longrightarrow 2_3 = r_{3_{j_r}} & \rightarrow & j_l' = |5+2+3+2|_6 = 6 & \rightarrow & l_{4_{j_l'}} = 2_6\,. \\
l_{4_{j_l'}} = 2_6 \longleftarrow 2_3 = r_{3_{j_r}} & \rightarrow & j_r' = |3+2+3+2|_6 = 4 & \rightarrow & r_{4_{j_r'}} = 1_4\,. \\
l_{4_{j_l}} = 2_6 \longrightarrow 1_4 = r_{4_{j_r}} & \rightarrow & j_l' = |6+2+3+4|_6 = 3 & \rightarrow & l_{5_{j_l'}} = 3_3\,. \\
l_{5_{j_l'}} = 3_3 \longleftarrow 1_4 = r_{4_{j_r}} & \rightarrow & j_r' = |4+4+3+0|_6 = 5 & \rightarrow & r_{5_{j_r'}} = 2_5\,. \\
l_{5_{j_l}} = 3_3 \longrightarrow 2_5 = r_{5_{j_r}} & \rightarrow & j_l' = |3+0+3+2|_6 = 2 & \rightarrow & l_{6_{j_l'}} = 2_2\,. \\
l_{6_{j_l'}} = 2_2 \longleftarrow 2_5 = r_{5_{j_r}} & \rightarrow & j_r' = |5+2+3+2|_6 = 6 & \rightarrow & r_{6_{j_r'}} = 3_6\,. \\
l_{6_{j_l}} = 2_2 \longrightarrow 3_6 = r_{6_{j_r}} & \rightarrow & j_l' = |2+2+3+0|_6 = 1 & \rightarrow & l_{7_{j_l'}} = 1_1\,. \\
l_{7_{j_l'}} = 1_1 \longleftarrow 3_6 = r_{6_{j_r}} & \rightarrow & j_r' = |6+0+3+4|_6 = 1 & \rightarrow & r_{7_{j_r'}} = 2_1\,.
\end{array}$$

$$P_c = \frac{\alpha\cdot x+\sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \frac{6\cdot 3+(4+2+2+2+0+2)+(2+2+2+4+2+0)}{6} = 7\,.$$

$$\text{g.c.d.}\,(P_c\,,\,B^{**}) = \text{g.c.d.}\,(7\,,\,1) = 1\,.$$

Hence:

$$P_{total} = \sum P_{component} = 7\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1\,.$$

Note that as long as $B_l^* = B_r^* \neq$ a multiple of 7, the knot has only one essential string.

From the string-run of the Kirsten Knots at the bight-edges (see Fig. 5) it follows that:

(i.) the number of parts between the left bight-boundaries 1 and 3 is equal to 2,
(ii.) the number of parts between the right bight-boundaries 1 and 3 is equal to 2,
hence the total number of parts of Kirsten Knots is equal to $2+2+x=4+x$.

In *Het Knoopeknauwertje* attention was drawn to the fact that the braiding of its Kirsten Knot begins with braiding a $p/b = 5/4$ over–under coded Regular Knot by braiding the first eight half-cycles. However, since no proper grid-diagram was used, it was, for example, not realised that there were more Kirsten Knots with $B = 6$ which require one essential string only and go through an over–under coded four bight Regular Knot stage. Such Kirsten Knots form the following two sets:

1. The Kirsten Knots with specification $(22/1{+}6n/22)\{1_1 2_4 2_5 2_6 3_3 2_2/3_1 2_2 2_3 2_4 1_5 2_6\}6$ go through the $p/b=(3+4n)/4$ over–under coded Regular Knot stage, where n is a whole number.
2. The Kirsten Knots with specification $(22/3{+}6n/22)\{1_1 2_4 2_5 2_6 3_3 2_2/2_1 2_2 2_3 1_4 2_5 3_6\}6$ go through the $p/b=(5+4n)/4$ over–under coded Regular Knot stage, where n is a whole number.

Nested Cylindrical Braids — The Knot of Brian Walsh

Refer for the calculation procedures to *The Braider*, Issue No. 19, August 1999, pp. 415 - 422.

In the English knotting publication of the IGKT (International Guild of Knot Tyers) *Knotting Matters*, Issue No. 55, March 1997 pp. 30-31, we find the description of a small braided knot under the heading **The Impossible Knot (a 5 lead x 5 bight Turk's Head?)**. The heading is followed by: *From Brian Walsh, Ipswich, England. Found by Geoffrey Budworth in some archives sent to him by Frank Harris.* Then, in Issue No. 57, September 1997 pg. 43, Jesse Coleman, Alabama, U.S.A., shows the planar construction form of this knot with the comment that it is not a Turk's Head. Next, in the Dutch knotting publication *Het Knoopeknauwertje*, Issue No. 12, pp. 22 - 25, Pieter van de Griend shows the construction of this knot as in *Knotting Matters*, Issue No. 55, and its planar construction form as in *Knotting Matters*, Issue No. 57. He compares this knot, for the purpose of covering a sphere, with the *Kirsten Knot* as described in *Het Knoopeknauwertje*, Issue No. 5, pp. 16 - 17.

Again as usual, the grid-diagram of the knot by Brian Walsh has not been shown in any of the above referred to publications and consequently its true nature was not discovered (a weird contraption of its supposed grid-diagram is depicted in *Het Knoopeknauwertje*, Issue No. 12, on pg. 24).

The upper left-hand diagram in Fig. 7 represents the construction layout of the knot in KM No. 55, and the lowermost diagram in Fig. 7 is the planar construction form as depicted in KM No. 57.

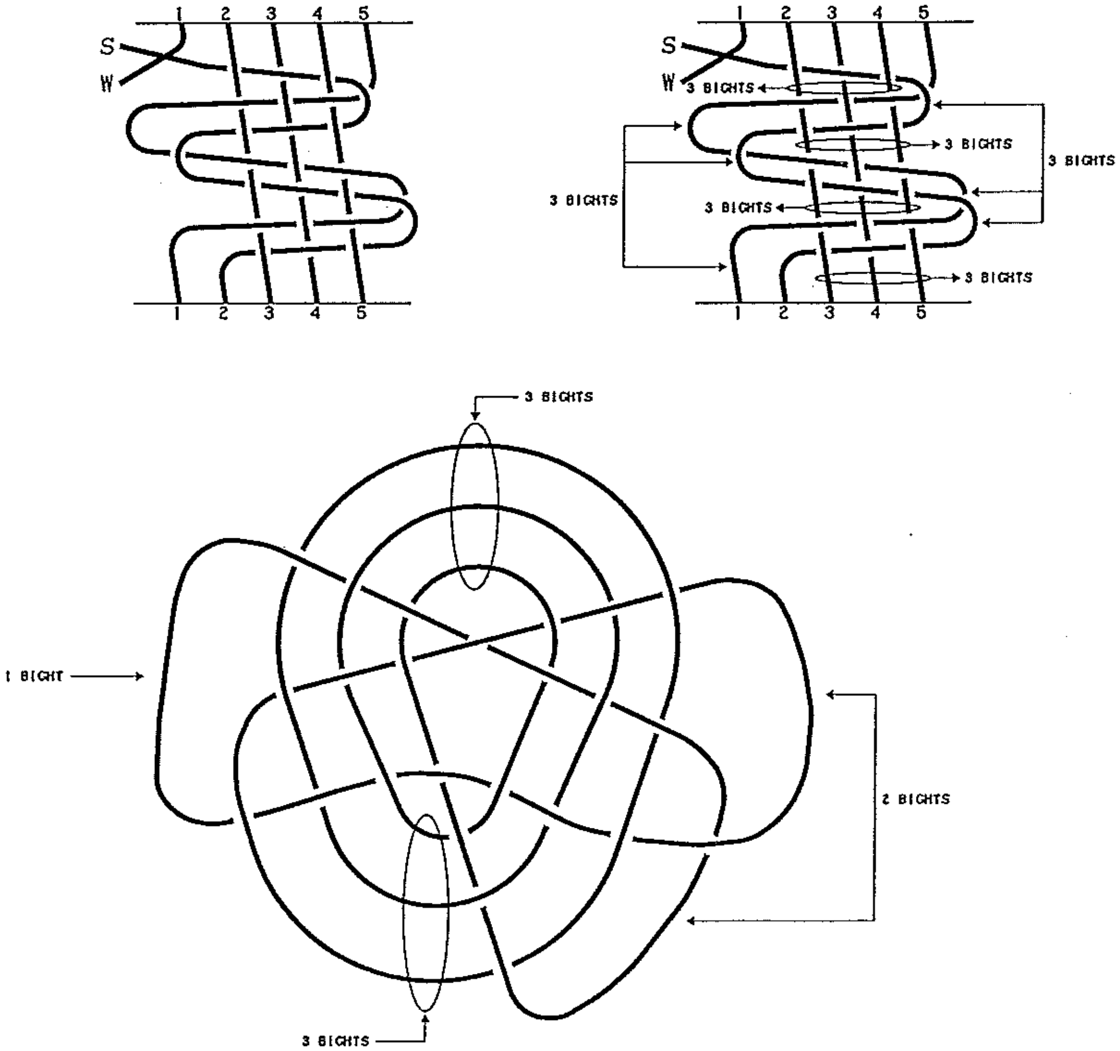


Fig. 7 — The knot by Brian Walsh.

It is interesting to note that none of the above mentioned knot tyers saw that the construction diagrams clearly indicate that the knot by Brian Walsh has **nine** bights instead of five. The formation of the nine left-hand bights and the nine right-hand bights is shown in the upper right-hand construction layout of the knot in Fig. 7. The formation of the nine bights on one of the two bight-edges is also shown in its lower planar construction layout in Fig. 7.

The string-run diagram and grid-diagram of the knot by Brian Walsh in accordance with its construction procedure as described in KM No. 55, hence as indicated by the upper left-hand construction layout in Fig. 7, is depicted in Fig. 8.

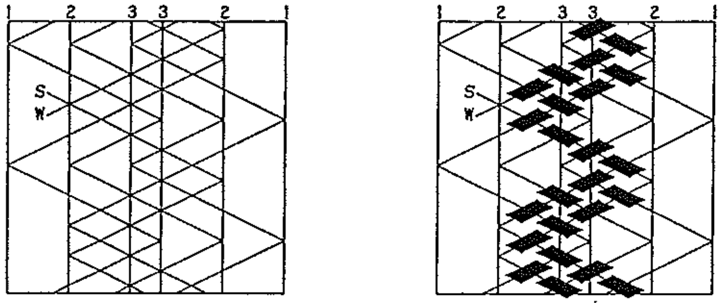


Fig. 8 — String-run and grid-diagram of the knot by Brian Walsh.

The string-run of the knot by Brian Walsh is characterized by the left-hand and right-hand periods depicted in Fig. 9. Consequently, the value of A_l and the value of A_r are both 9 for this knot, while $\mathcal{K}_l$ and $\mathcal{K}_r$ are both equal to 3.

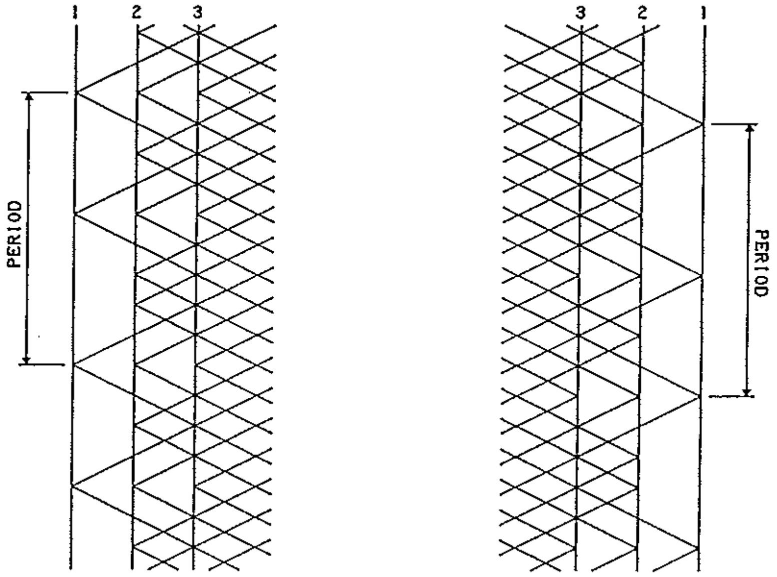


Fig. 9 — The string-run of the knot by Brian Walsh at the left and the right bight-edge.

We can thus depict the string-run and grid-diagram of the knot by Brian Walsh as shown in Fig. 10.

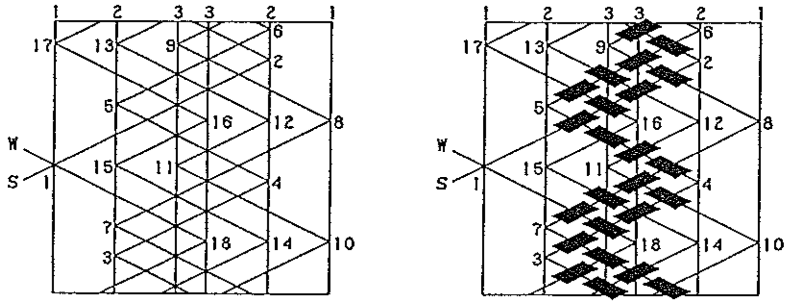


Fig. 10 — String-run and grid-diagram of the knot by Brian Walsh.

The string-run specification of this knot is : (22/1/22){123212232/221232123}9, or with the ranking-numbers as subscripts attached to the bight-boundary numbers: $(22/1/22)\{1_1 2_4 3_7 2_6 1_5 2_8 2_9 3_3 2_2 / 2_1 2_2 1_3 2_4 3_5 2_6 1_7 2_8 3_9\}9$.

For the string-run in Fig. 10 we obtain:

$$
\begin{aligned}
A_l &= 9\,;\ A_r = 9\,;\ d = \text{g.c.d.}(A_l\,,\,A_r) = \text{g.c.d.}(9\,,\,9) = 9\,.\\
A^{**} &= \frac{A_l \cdot A_r}{d} = \frac{9 \cdot 9}{9} = 9\,.\\
B_l^* &= \text{number of periods at left bight-edge} = 1\,.\\
B_r^* &= \text{number of periods at right bight-edge} = 1\,.\\
B_{total} &= A_l B_l^* = A_r B_r^* = A^{**} B^{**} = 9 \cdot 1 = 9\,.\\
B^{**} &= \frac{B_{total}}{A^{**}} = 1\,.\\
\mathcal{K}_l &= 3\,;\ \mathcal{K}_r = 3\,.
\end{aligned}
$$

Hence:

$$
\begin{array}{llll}
\Delta_{l_i} = 4 & \text{for } l_i = 1\,. & \qquad \Delta_{r_i} = 4 & \text{for } r_i = 1\,.\\
\Delta_{l_i} = 2 & \text{for } l_i = 2\,. & \qquad \Delta_{r_i} = 2 & \text{for } r_i = 2\,.\\
\Delta_{l_i} = 0 & \text{for } l_i = 3\,. & \qquad \Delta_{r_i} = 0 & \text{for } r_i = 3\,.
\end{array}
$$

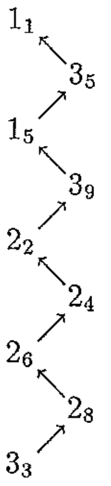
Each lower-left to upper-right half-cycle type in a Nested Cylindrical Braid occurs only once in all the first-return string-runs of such a braid. The lower-left to upper-right half-cycle types can be read from the Nested Cylindrical Braid specification $(22/1/22)\{1_1 2_4 3_7 2_6 1_5 2_8 2_9 3_3 2_2 / 2_1 2_2 1_3 2_4 3_5 2_6 1_7 2_8 3_9\}9$:

$$
\begin{array}{lll}
1_1 \longrightarrow 2_1 & \qquad 2_6 \longrightarrow 2_4 & \qquad 2_9 \longrightarrow 1_7\\
2_4 \longrightarrow 2_2 & \qquad 1_5 \longrightarrow 3_5 & \qquad 3_3 \longrightarrow 2_8\\
3_7 \longrightarrow 1_3 & \qquad 2_8 \longrightarrow 2_6 & \qquad 2_2 \longrightarrow 3_9
\end{array}
$$

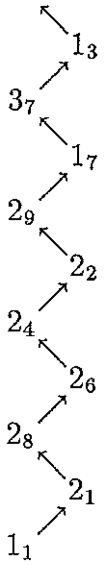
Any one of these listed types may be taken as the first lower-left to upper-right half-cycle in the 1^{st} first-return string-run, but normally we take the first listed one. Every lower-left to upper-right half-cycle encountered in this 1^{st} first-return string-run gets deleted from the type-list.

Any one of the remaining types in the type-list may be taken as the first lower-left to upper-right half-cycle in the 2^{nd} first-return string-run, and again every lower-left to upper-right half-cycle encountered in this 2^{nd} first-return string-run gets deleted from the remaining type-list. This process is carried on till all the lower-left to upper-right half-cycle types have been deleted from the type-list.

For the first-return string-runs we thus obtain:



$$
\begin{array}{llll}
l_{1_{j_l}} = 1_1 \longrightarrow 2_1 = r_{1_{j_r}} & \rightarrow\ j'_l = |1+4+1+2|_9 = 8 & \rightarrow & l_{2_{j'_l}} = 2_8\,.\\
l_{2_{j'_l}} = 2_8 \longleftarrow 2_1 = r_{1_{j_r}} & \rightarrow\ j'_r = |1+2+1+2|_9 = 6 & \rightarrow & r_{2_{j'_r}} = 2_6\,.\\
l_{2_{j_l}} = 2_8 \longrightarrow 2_6 = r_{2_{j_r}} & \rightarrow\ j'_l = |8+2+1+2|_9 = 4 & \rightarrow & l_{3_{j'_l}} = 2_4\,.\\
l_{3_{j'_l}} = 2_4 \longleftarrow 2_6 = r_{2_{j_r}} & \rightarrow\ j'_r = |6+2+1+2|_9 = 2 & \rightarrow & r_{3_{j'_r}} = 2_2\,.\\
l_{3_{j_l}} = 2_4 \longrightarrow 2_2 = r_{3_{j_r}} & \rightarrow\ j'_l = |4+2+1+2|_9 = 9 & \rightarrow & l_{4_{j'_l}} = 2_9\,.\\
l_{4_{j'_l}} = 2_9 \longleftarrow 2_2 = r_{3_{j_r}} & \rightarrow\ j'_r = |2+2+1+2|_9 = 7 & \rightarrow & r_{4_{j'_r}} = 1_7\,.\\
l_{4_{j_l}} = 2_9 \longrightarrow 1_7 = r_{4_{j_r}} & \rightarrow\ j'_l = |9+2+1+4|_9 = 7 & \rightarrow & l_{5_{j'_l}} = 3_7\,.\\
l_{5_{j'_l}} = 3_7 \longleftarrow 1_7 = r_{4_{j_r}} & \rightarrow\ j'_r = |7+4+1+0|_9 = 3 & \rightarrow & r_{5_{j'_r}} = 1_3\,.
\end{array}
$$



$$\begin{array}{lllll}
l_{5_{j_l}} = 3_7 \longrightarrow 1_3 = r_{5_{j_r}} & \rightarrow & j'_l = |7+0+1+4|_9 = 3 & \rightarrow & l_{6_{j'_l}} = 3_3\,. \\
l_{6_{j'_l}} = 3_3 \longleftarrow 1_3 = r_{5_{j_r}} & \rightarrow & j'_r = |3+4+1+0|_9 = 8 & \rightarrow & r_{6_{j'_r}} = 2_8\,. \\
l_{6_{j_l}} = 3_3 \longrightarrow 2_8 = r_{6_{j_r}} & \rightarrow & j'_l = |3+0+1+2|_9 = 6 & \rightarrow & l_{7_{j'_l}} = 2_6\,. \\
l_{7_{j'_l}} = 2_6 \longleftarrow 2_8 = r_{6_{j_r}} & \rightarrow & j'_r = |8+2+1+2|_9 = 4 & \rightarrow & r_{7_{j'_r}} = 2_4\,. \\
l_{7_{j_l}} = 2_6 \longrightarrow 2_4 = r_{7_{j_r}} & \rightarrow & j'_l = |6+2+1+2|_9 = 2 & \rightarrow & l_{8_{j'_l}} = 2_2\,. \\
l_{8_{j'_l}} = 2_2 \longleftarrow 2_4 = r_{7_{j_r}} & \rightarrow & j'_r = |4+2+1+2|_9 = 9 & \rightarrow & r_{8_{j'_r}} = 3_9\,. \\
l_{8_{j_l}} = 2_2 \longrightarrow 3_9 = r_{8_{j_r}} & \rightarrow & j'_l = |2+2+1+0|_9 = 5 & \rightarrow & l_{9_{j'_l}} = 1_5\,. \\
l_{9_{j'_l}} = 1_5 \longleftarrow 3_9 = r_{8_{j_r}} & \rightarrow & j'_r = |9+0+1+4|_9 = 5 & \rightarrow & r_{9_{j'_r}} = 3_5\,. \\
l_{9_{j_l}} = 1_5 \longrightarrow 3_5 = r_{9_{j_r}} & \rightarrow & j'_l = |5+4+1+0|_9 = 1 & \rightarrow & l_{10_{j'_l}} = 1_1\,. \\
l_{10_{j'_l}} = 1_1 \longleftarrow 3_5 = r_{9_{j_r}} & \rightarrow & j'_r = |5+0+1+4|_9 = 1 & \rightarrow & r_{10_{j'_r}} = 2_1\,.
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{9 \cdot 1 + (4+2+2+2+0+0+2+2+4) + (2+2+2+4+4+2+2+0+0)}{9} = 5\,.$$

$\text{g.c.d.}\,(P_c\,,\,B^{**}) = \text{g.c.d.}\,(5\,,\,1) = 1\,.$

Hence:

$$P_{total} = \sum P_{component} = 5\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1\,.$$

Note that as long as $B^*_l = B^*_r \neq$ a multiple of 5, the knot has only **one** essential string.

As with the Kirsten Knot we can generalise the Brian Walsh Knot. From the string-run of the Brian Walsh Knots at the bight-edges (see Fig. 9) it follows that:
(i.) the number of parts between the left bight-boundaries 1 and 3 is equal to 2,
(ii.) the number of parts between the right bight-boundaries 1 and 3 is equal to 2,
hence the total number of parts of Brian Walsh Knots is equal to $2+2+x = 4+x$.

Apparently none of the above mentioned knot tyers (Brian Walsh, Frank Harris, Geoffrey Budworth, Jesse Coleman and Pieter van de Griend) nor any of those associated with the compilation of *Knotting Matters* realised that the construction of the Brian Walsh Knot with $x = 1$ could begin with braiding a $p/b = 3/5$ over–under coded Regular Knot by braiding the first ten half-cycles of the Brian Walsh Knot $(22/1/22)\{1_1 2_4 3_7 2_6 1_5 2_8 2_9 3_3 2_2/2_1 2_2 1_3 2_4 3_5 2_6 1_7 2_8 3_9\}9$. Furthermore it was apparently also not realised that there were more Brian Walsh Knots with $B = 9$ which require one essential string only and go through an over–under coded five bight Regular Knot stage. Such Brian Walsh Knots form the following two sets:

1. The Brian Walsh Knots with specification:
$$(22/9n/22)\{1_1 2_4 3_7 2_6 1_5 2_8 2_9 3_3 2_2/3_1 2_2 1_3 2_4 3_5 2_6 2_7 1_8 2_9\}9$$

go through the $p/b = (2+5n)/5$ over-under coded Regular Knot stage, where n is a whole number.

2. The Brian Walsh Knots with specification:

$$(22/1+9n/22)\{1_1 2_4 3_7 2_6 1_5 2_8 2_9 3_3 2_2/2_1 2_2 1_3 2_4 3_5 2_6 1_7 2_8 3_9\}9$$

go through the $p/b = (3+5n)/5$ over-under coded Regular Knot stage, where n is a whole number.

Many braiders seem to have a hang-up about braids which cover a sphere. Hence it is no wonder that some are going to compare for such purpose the $x = 1$ Brian Walsh Knot with the $x = 3$ Kirsten Knot. It is of course a rather stupid act to do so, not only since the x-values are not the same, but the Brian Walsh Knots and the Kirsten Knots are **cylindrical braids** with more or less dome shaped ends of which only a few can be used to cover to some extent a sphere.

Our Questions and Readers Answers.

Is Mike Hickey (Tom Hall) really the only reader who tries to solve our questions??? He seems to be the only one who discovered that the string-run specification (22/5/22){123/312}12 under (1) of the question on pg. 422 is impossible. He thought that there was a typing error in the specification, however, this specification was purposely given to see if there was anyone who takes the effort trying to answer the questions presented, since we never did receive a solution to any of our questions. Hence congratulations to Mike Hickey who is the very first one to send us a solution to a question!!! The purpose of the questions is to enable the reader to test his/her understanding of the material presented.

ERRATA—*The Braider*

The updated errata list below lists the typing errors which were noted in at least some copies of the following Issues of *The Braider*.

No. 1

pg. 3, line 9 — **it** is
pg. 3, line 10 — **unnecessarily**
pg. 3, line 17 — in order **to** present
pg. 12, line 5 under "An Introduction to Flat Braids" — Four pages are devoted **to**
pg. 16, line 9 under "Reviews" — **instructor**

No. 2

pg. 21, line 2 — delete **that**
pg. 28, line 6 — However, **a** one colour
pg. 38, line 18 under "Reviews" — **they** are first class

No. 3

pg. 42, line 2 — Naturally, **queries** not directly

No. 4

pg. 76, line 5 — **phenomenon**
pg. 81, line 16 — **phenomenon**

No. 5

pg. 87, line 3 from bottom of page — **exclude**
pg. 106, line 11 — **locality**

No. 6

pg. 129, line 15 — **2-colour**

No. 7

No. 8

pg. 158, line 3 — Hence **the** number of bights
pg. 161, line 12 — In Fig. 144 are depicted the **parts-raising** processes

No. 9

No. 10

pg. 213, line 2 — **choose**

No. 11

pg. 232, line 17 — **in general**
pg. 245, line 8 — **in general**

No. 12

pg. 257, line 6 — or $B = \{(2m+2)(\alpha+1)-1\}N-(\alpha+1)$ bights
pg. 257, line 7 — Those with $B = \{(2m+2)(\alpha+1)-1\}N-(\alpha+1)$ bights
pg. 257, line 9 — $b = \{(2m+2)\alpha-1\}N-\alpha$ bights. Those with $B = \{(2m+2)(\alpha+1)-1\}N+(\alpha+1)$ bights
pg. 257, line 10 & 11 — with $p = \{(2m+2)\alpha-1$ parts and $b = \{(2m+2)\alpha-1\}N+\alpha$ bights
pg. 257, line 7 from bottom — delete **thinspace**

No. 13

pg. 276, line 7 — **in general**

No. 14

pg. 308, line 3 — **in general**

No. 15

pg. 338, line 1 — **independent**

No. 16

pg. 347, line 3 — $P = 2m(\alpha + 1) + 1$
pg. 353, line 16 — **screw**
pg. 356, line 3 — **two**
pg. 359, line 5 under Fig. 310 — **boundary**
pg. 361, line 3 from bottom — **Headhunter's**-coded
pg. 366, line 30 — **braided**

No. 17

pg. 369, line 12 — **mathematical**
pg. 372, lines 8, 9, 10, 13 — **positive**
pg. 383, line 13 — Figs. **326–328**
pg. 387, line 2 below Fig. 332 — in **general**
pg. 387, line 13 below Fig. 332 — **cases**

No. 18

pg. 393, the right-hand brace } immediately before the second equal sign is missing in the three lines above the upper .... line, and the right-hand brace } immediately before the third equal sign is missing in the first line below the upper .... line.
pg. 395, line 4 — **additionally**
pg. 396, bottom line — in **general**
pg. 397, line 4 from bottom — in **general**
pg. 412, Fig. 345 immediately below the sixth diagram — change $o - u - o - u$ to $u - o - u - o$.
pg. 413, Fig. 346 leftmost diagram in second row of diagrams: the two non coded crossing points should have been provided with an under coding each for the braiding direction indicated.

No. 19

pg. 419, lines 2 & 3 from bottom — in **general**
pg. 421, line 2 — $(12/19/113)\{1_1 1_2 2_4 3_3 / 1_1 1_2 3_3 1_4 4_5 2_6\}24$
pg. 430, lines 3 & 5 — in **general**

No. 20

pg. 446, line 14 — A few **simple** examples
pg. 446, line 3 from bottom — in **general**
pg. 447, Footnote — **These** are the
pg. 450, last line — delete **thinspace**
pg. 464, line 20 — Brion **T**oss

No. 21

pg. 465, line 16 — in **general**

No. 22

pg. 493, second line under "Question on pg. 479" — **appliqué**

No. 23

pg. 516, line 3 — **categorisation**
pg. 523, line 14 — **Fig. 445**

No. 24

pg. 537, line 2 — **arises**
pg. 537, line 14 — **actual**
pg. 545, line 1 — read as: greater than or equal to 2 and less than p

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THE BRAIDER

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A quarterly publication for the braiding artisan

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A Braid Name Change

Let's kick this new century off by changing the name of a braid that is known by at least two rather ambiguous names. It is the Regular Cylindrical Braid[†] of **one** string with an **over–under** weaving pattern throughout. Initially we defined this braid as a **Turk's Head Knot**[‡] Since the name *Turk's Head* is used by many people for a large number of single and multi string cylindrical braids, it is not a good name for a specific braidform. Later we adopted the name **Casa Knot**, solely for the shortness of the name. This name was extensively used by Tom Hall, for in the purely pragmatic world of braiding this braidform formed, and still forms, the main basis of interwoven braids.[††] It is therefore not surprising that in most publications far too much emphasis is placed on these so-called 'Casa Knots' in the case of interwoven knots.

It should be remembered that for interbraided knots, one knotform is not any more important than any other knotform.

The by us previously adopted names *'Turk's Head Knot'* and *'Casa Knot'* may cause a further problem in that an **over–under coding** throughout a braid not only may be seen as a *'Turk's Head' coding* or *'Casa' coding*, but also as a **column-coding** and **row-coding**. However, an **over–under coding** throughout a braid does in general **not** imply that the braid has a column-coding or a row-coding! The braid in Fig. 387 has an *over–under coding* throughout, its coding is a *column-coding*, but it is **not** a *row-coding* (see arrowed rows).

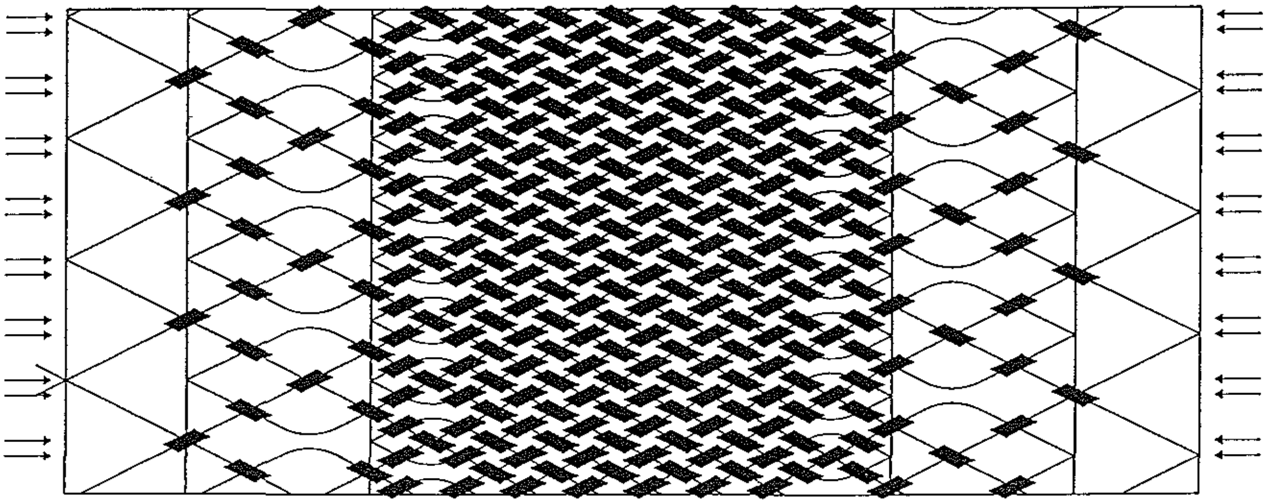


Fig. 387 — Over–under coding throughout; a column-coding, but not a row-coding.

The braid in Fig. 388 has also an *over–under coding* throughout, its coding is a *row-coding*, but it is **not** a *column-coding* (see arrowed columns).

In order to overcome the ambiguity associated with the names *'Turk's Head'* and *'Casa'*, we shall in future denote a braid which has an *over–under coding* throughout, as an **over–under coded** braid. Although the name is somewhat long, there is at least no ambiguity. For example, in future we shall call a 'Turk's Head Knot' or 'Casa Knot' (a single string Regular Cylindrical Braid with an over–under coding throughout) an **over–under coded Regular Cylindrical Knot.** A multi-string Regular Cylindrical Braid with a Casa-coding we shall in future call an **over-under coded Regular Cylindrical Braid.** In this context, the term **Braid** will be used as a general term, hence one or more strings may be required in the construction, while the term **Knot** will indicate that only one string is required in the construction.

[†] For the definition of **Regular Cylindrical Braid**, see *The Braider*, Issue No. 2, pg. 30, and Issue No. 1, pg. 6.

[‡] Refer to *Braiding — Regular Knots; pg. 16.*

[††] Casa is the Spanish word for: house, home, building. In the purely pragmatic world of braiding, a 'Casa Knot' forms the foundation knot of nearly all interwoven knots, and in that world many interwoven knots do consist of interbraided 'Casa Knots' — in interwoven knots, 'Casa Knots' house other knots, or are used in building them.

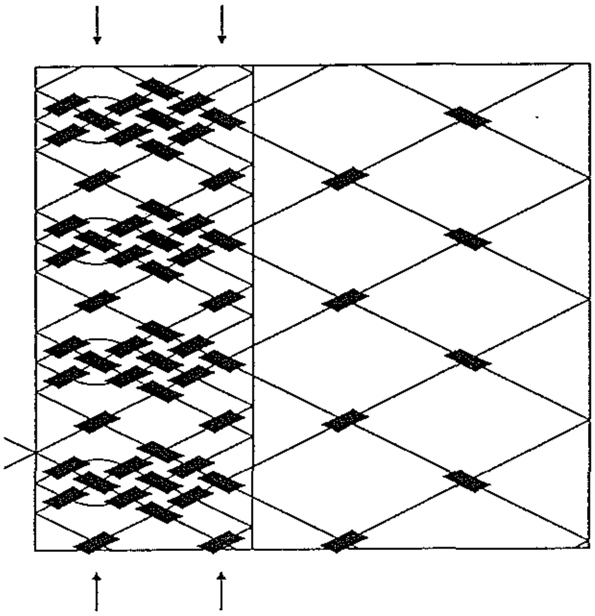


Fig. 388 — Over–under coding throughout; a row-coding, but not a column-coding.

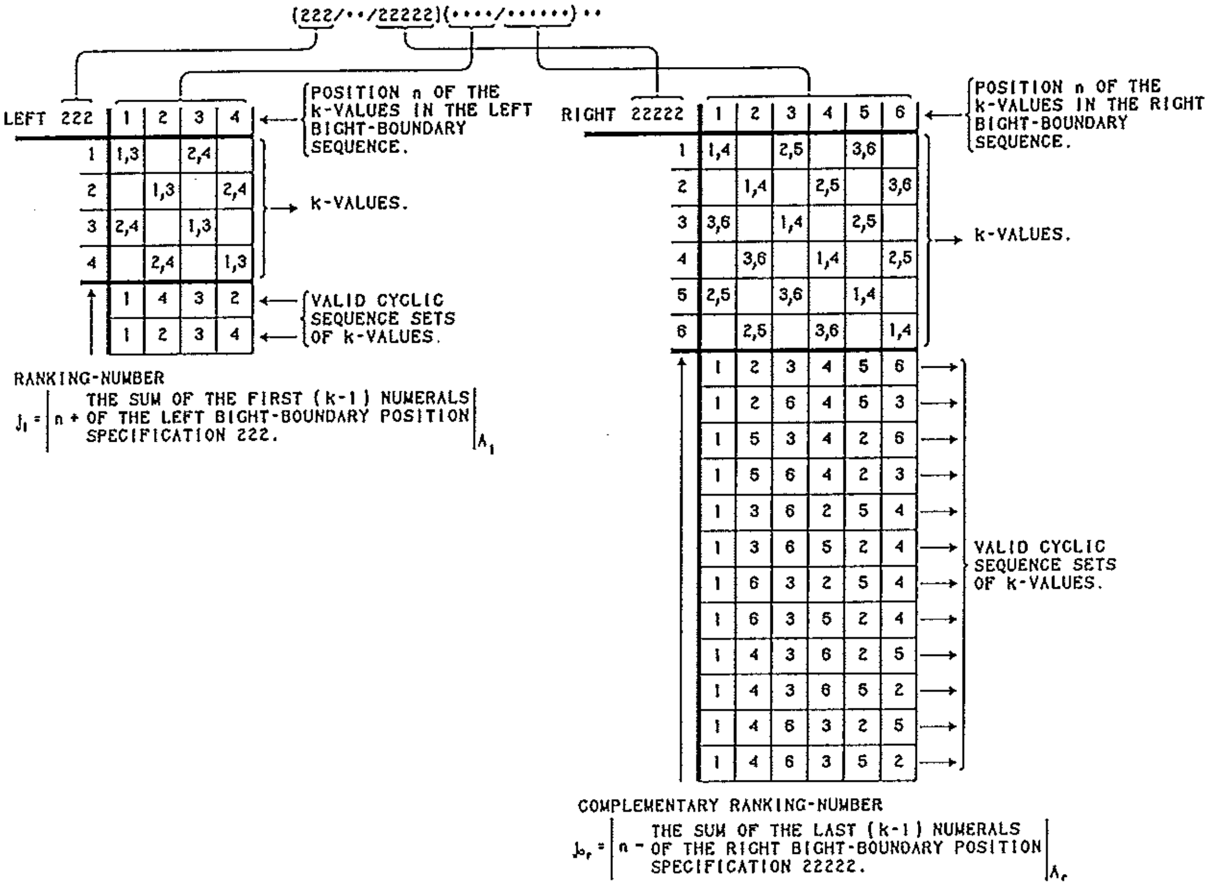
Nested Cylindrical Braids

In the previous Issue of *The Braider*, No. 20, pp. 437-441 we have seen that a set number $(\eta_l \cdot \eta_r \cdot d)$ of possible braid-types is associated with a left and right bight-boundary position specification pair. Under these possible braid-types there are η_l left bight-boundary arrangements and η_r right bight-boundary arrangements.

One of the most often encountered left and right bight-boundary position specifications is $222\cdots$ with $\mathcal{K}_l = A_l$ and $\mathcal{K}_r = A_r$ (the well-known 'asymmetric Pineapple knots' have such a left and right bight-boundary position specification with $\mathcal{K}_l = A_l$ and $\mathcal{K}_r = A_r$). Hence let's have a look at an example of such a left and right bight-boundary specification and see which left and right bight-boundary arrangements are associated with them.

Let the left bight-boundary specification be 222 and the right bight-boundary specification be 22222. Hence $\mathcal{K}_l = 4 = A_l$ and $\mathcal{K}_r = 6 = A_r$. The calculation of the left and right valid cyclic sequence sets of k-values is shown in Fig. 389. These calculations show that there are $\eta_l = 2$ valid left cyclic sequence sets of k-values and $\eta_r = 12$ valid right cyclic sequence sets of k-values. Since the g.c.d.$(A_l\,,\,A_r) =$ g.c.d.$(4\,,\,6) = 2 = d$, the number of possible braid-types is $\eta_l \cdot \eta_r \cdot d = 2 \cdot 12 \cdot 2 = 48$.

The $\eta_l = 2$ valid left cyclic sequence sets of k-values give us the 2 left bight-boundary arrangements shown in Fig. 390, while the 12 right bight-boundary arrangements associated with the $\eta_r = 12$ valid right cyclic sequence sets of k-values are depicted in Figs. 391 & 392.


(222/••/22222)(••••/••••••)••
POSITION n OF THE K-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.
K-VALUES.
VALID CYCLIC SEQUENCE SETS OF K-VALUES.
POSITION n OF THE K-VALUES IN THE RIGHT BIGHT-BOUNDARY SEQUENCE.
K-VALUES.
VALID CYCLIC SEQUENCE SETS OF K-VALUES.

LEFT 222	1	2	3	4
1	1,3		2,4	
2		1,3		2,4
3	2,4		1,3	
4		2,4		1,3
	1	4	3	2
	1	2	3	4

RANKING-NUMBER

$$J_l = \left| n + \text{THE SUM OF THE FIRST } (k-1) \text{ NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 222.} \right|_{A_l}$$

RIGHT 22222	1	2	3	4	5	6
1	1,4		2,5		3,6	
2		1,4		2,5		3,6
3	3,6		1,4		2,5	
4		3,6		1,4		2,5
5	2,5		3,6		1,4	
6		2,5		3,6		1,4
	1	2	3	4	5	6
	1	2	6	4	5	3
	1	5	3	4	2	6
	1	5	6	4	2	3
	1	3	6	2	5	4
	1	3	6	5	2	4
	1	6	3	2	5	4
	1	6	3	5	2	4
	1	4	3	6	2	5
	1	4	3	6	5	2
	1	4	6	3	2	5
	1	4	6	3	5	2

COMPLEMENTARY RANKING-NUMBER

$$J_r = \left| n - \text{THE SUM OF THE LAST } (k-1) \text{ NUMERALS OF THE RIGHT BIGHT-BOUNDARY POSITION SPECIFICATION 22222.} \right|_{A_r}$$

Fig. 389 — The calculation of the left and right valid cyclic sequence sets of k-values.

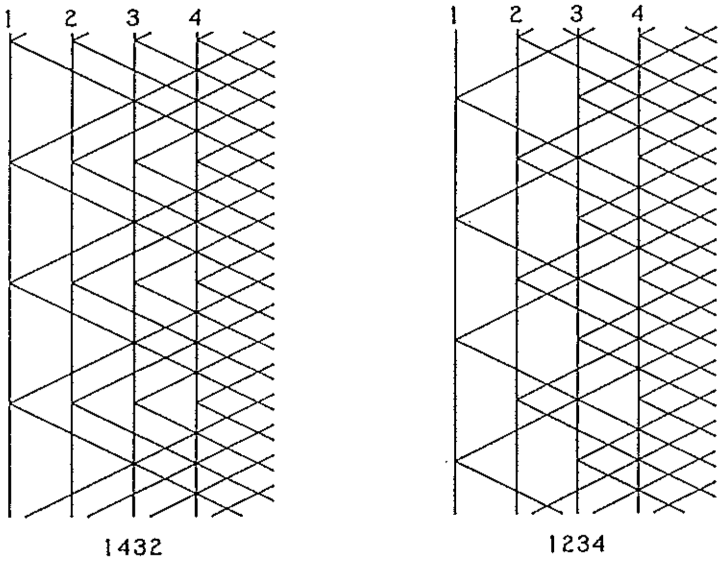
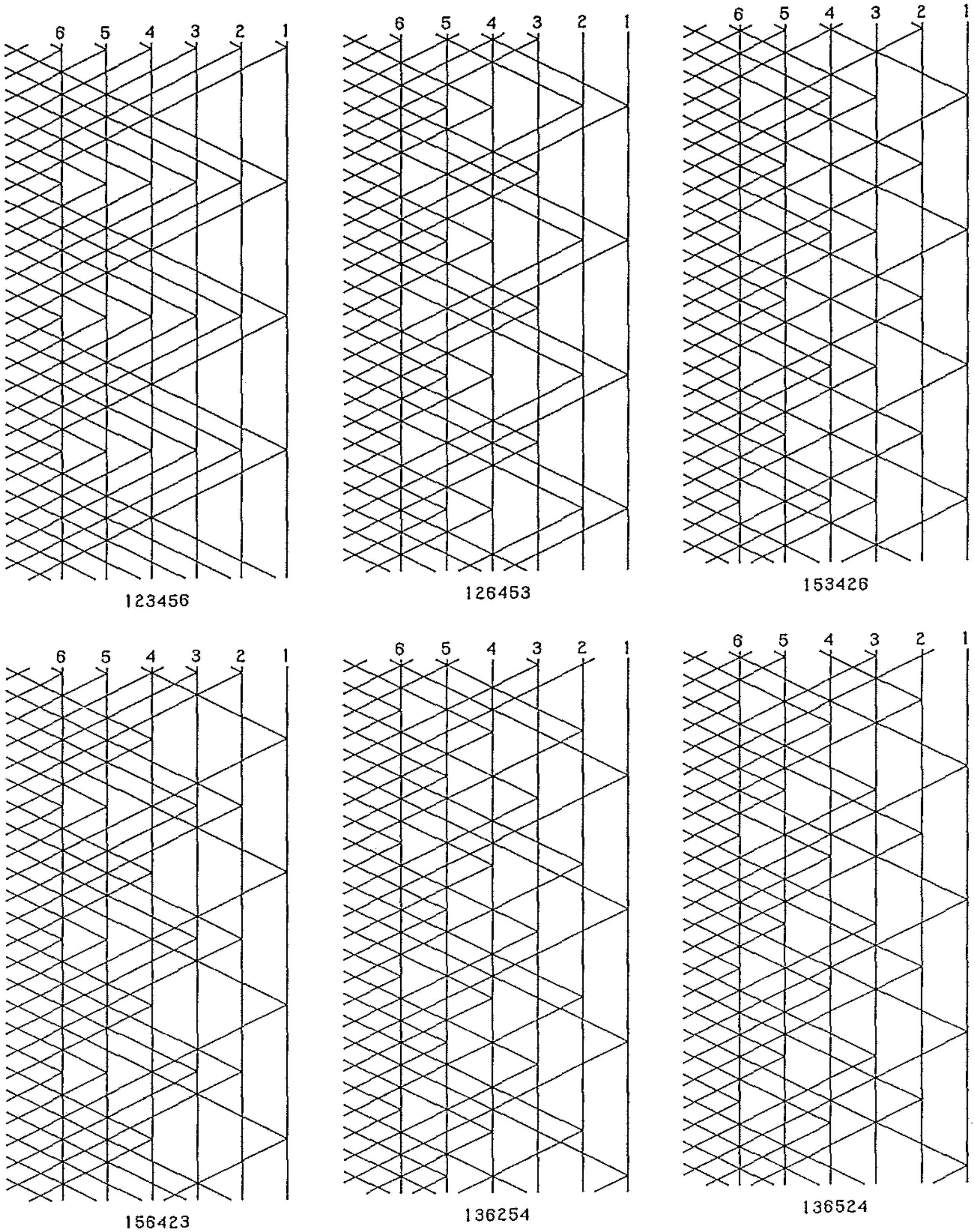


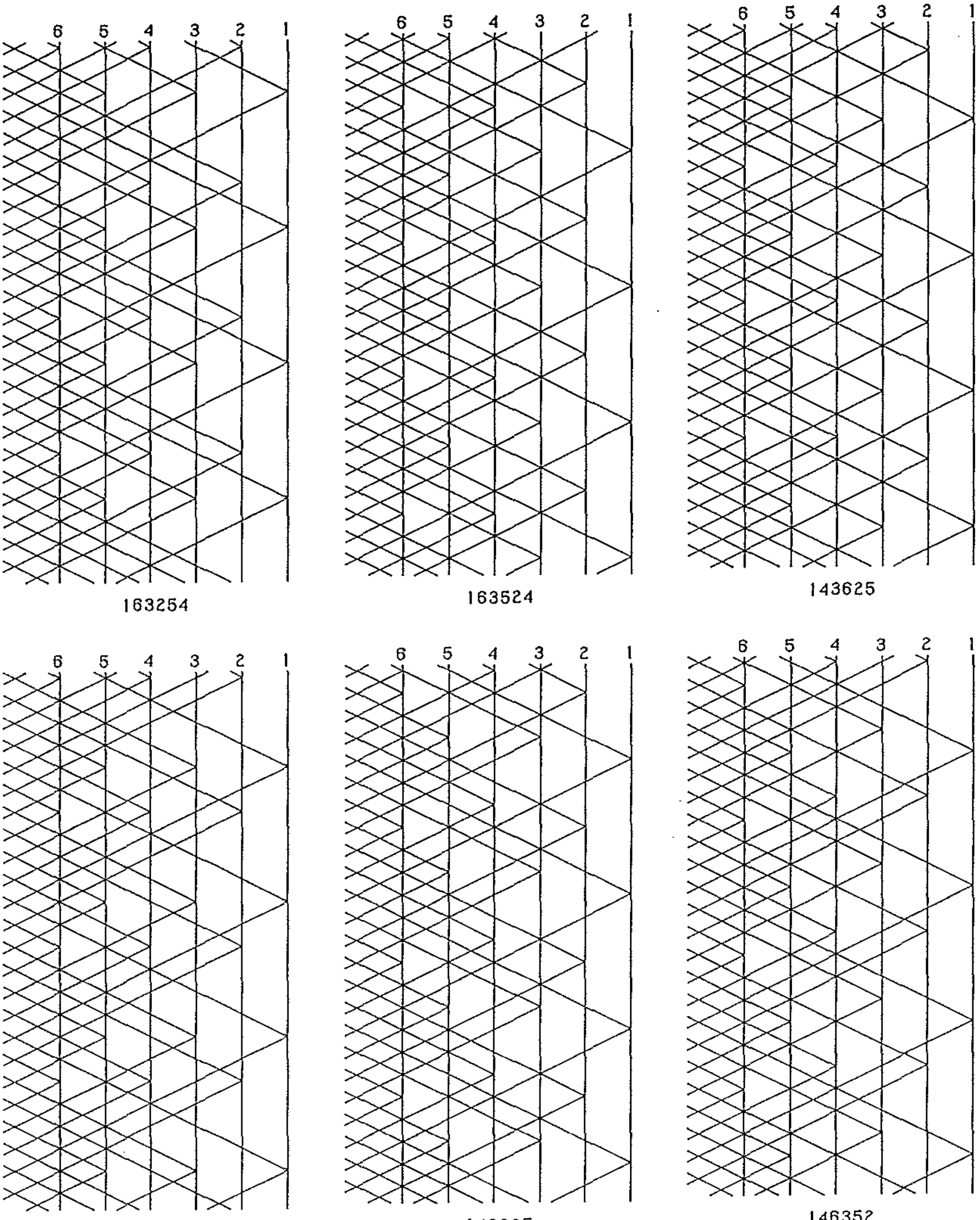
Fig. 390 — The $\eta_l = 2$ possible left bight-boundary arrangements.



123456 126453 153426

156423 136254 136524

Fig. 391 — Together with Fig. 392: the $\eta_r = 12$ possible right bight-boundary arrangements.



163254

163524

143625

143652

146325

146352

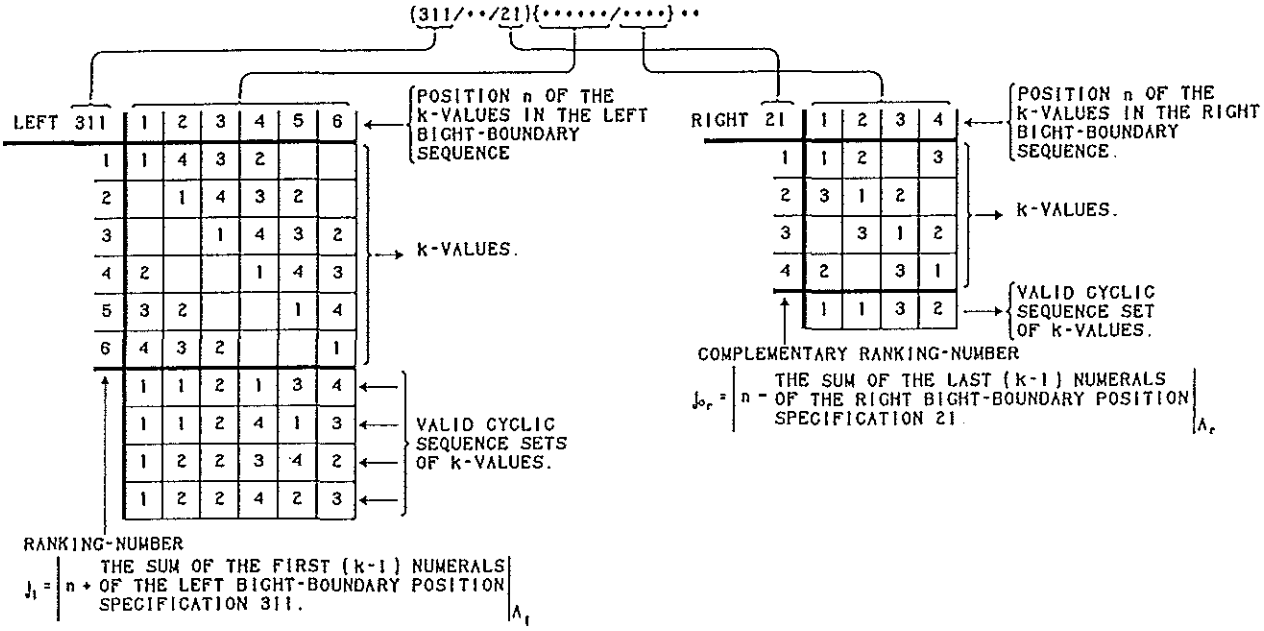
Fig. 392 — Together with Fig. 391: the $\eta_r = 12$ possible right bight-boundary arrangements.

The 'asymmetric Pineapple knots' with $A_l = 4$ and $A_r = 6$, hence with the bight-boundary position specification $(222/\cdots/22222)$, have the string-run specifications:

(1). $(222/\cdots/22222)\{1432/123456\}\cdots$.
(2). $(222/\cdots/22222)\{1432/234561\}\cdots$.

Although in the foregoing we calculated the left valid sequence sets and the right valid sequence sets in their respective ways, it is more convenient to obtain the right valid sequence sets from its corresponding left valid sequence sets. By rotating the string-run diagram through an angle of 180 deg so that the right bight-edge becomes the left bight-edge (and the left bight-edge becomes the right bight-edge) we can calculate for this new left bight-edge the left valid sequence sets. The corresponding right valid sequence sets (those associated with the original right bight-edge) are then the left valid sequence sets read in reverse cyclic order. Let's illustrate this with an example.

In Fig. 362 (see *The Braider*, Issue No. 20, pg. 439) we have shown the calculation of the right valid sequence sets (those associated with the right bight-boundary position specification 113 and the right nesting-number $A_r = 6$). The corresponding left bight-boundary position specification becomes then 311 (read the right bight-boundary position specification in reverse order) with the corresponding left nesting-number $A_l = 6$. The calculation of the associated left valid sequence sets is shown in Fig. 393. Note that the right valid sequence sets associated with the right bight-boundary position specification 113 and the right nesting-number $A_r = 6$, are readily obtained from the left valid sequence sets, associated with the left bight-boundary position specification 311 and the left nesting-number $A_l = 6$, by reading these sequence sets in reverse cyclic order.



LEFT 311	1	2	3	4	5	6
1	1	4	3	2		
2		1	4	3	2	
3			1	4	3	2
4	2			1	4	3
5	3	2			1	4
6	4	3	2			1
	1	1	2	1	3	4
	1	1	2	4	1	3
	1	2	2	3	4	2
	1	2	2	4	2	3

RIGHT 21	1	2	3	4
1	1	2		3
2	3	1	2	
3		3	1	2
4	2		3	1
	1	1	3	2

(311/••/21){••••••/••••}••
POSITION n OF THE K-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE
K-VALUES.
VALID CYCLIC SEQUENCE SETS OF K-VALUES.
RANKING-NUMBER
J₁ = | n + THE SUM OF THE FIRST (K-1) NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 311. |A₁
POSITION n OF THE K-VALUES IN THE RIGHT BIGHT-BOUNDARY SEQUENCE.
K-VALUES.
VALID CYCLIC SEQUENCE SET OF K-VALUES.
COMPLEMENTARY RANKING-NUMBER
J₀ᵣ = | n - THE SUM OF THE LAST (K-1) NUMERALS OF THE RIGHT BIGHT-BOUNDARY POSITION SPECIFICATION 21 |Aᵣ

Fig. 393 — The string-run specification $(311/**/21)\{******/****\}**$.

Similarly, the right valid sequence set associated with the right bight-boundary position specification 21 and the right nesting number $A_r = 4$, may be obtained from the left valid sequence set, associated with the left bight-boundary position specification 12 and the left nesting-number $A_l = 4$, by reading this sequence set in reverse cyclic order.

THE BRAIDER'S NOTEBOOK

A few years ago we read in a publication on braiding the statement: *"Almost all of the knots braiders use are tied from two bight Casa Knots."*. For certain 'braiders' this may well be true, but it is also true that such 'braiders' cannot really be regarded to be braiders. Thus, with regards the above mentioned statement, it will be instructive to examine the **over–under coded Regular Knots** (Casa Knots) in some greater detail.

Let's first see how various people might 'classify' the **over–under coded Regular Knots**, braided from a flat string which has for each opposite face pair identical faces in size, shape, texture and colour.

Some people might think that there are only two types, respectively depicted in Fig. 394 and Fig. 395, of **over–under coded Regular Knots** for the above specified string conditions.

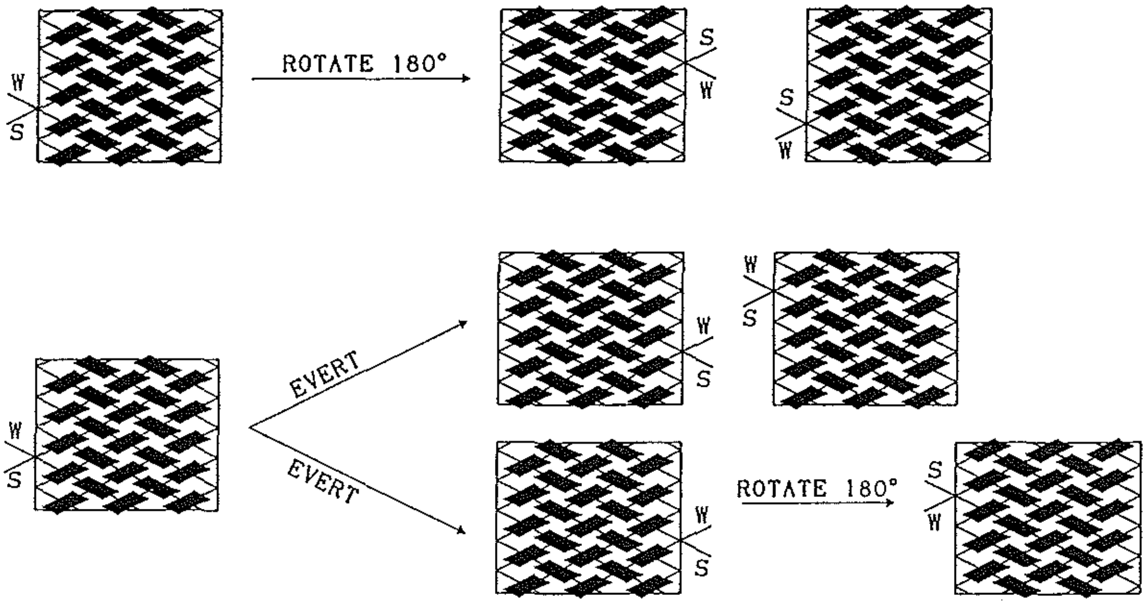


Fig. 394 — The type with an **even** number of parts.

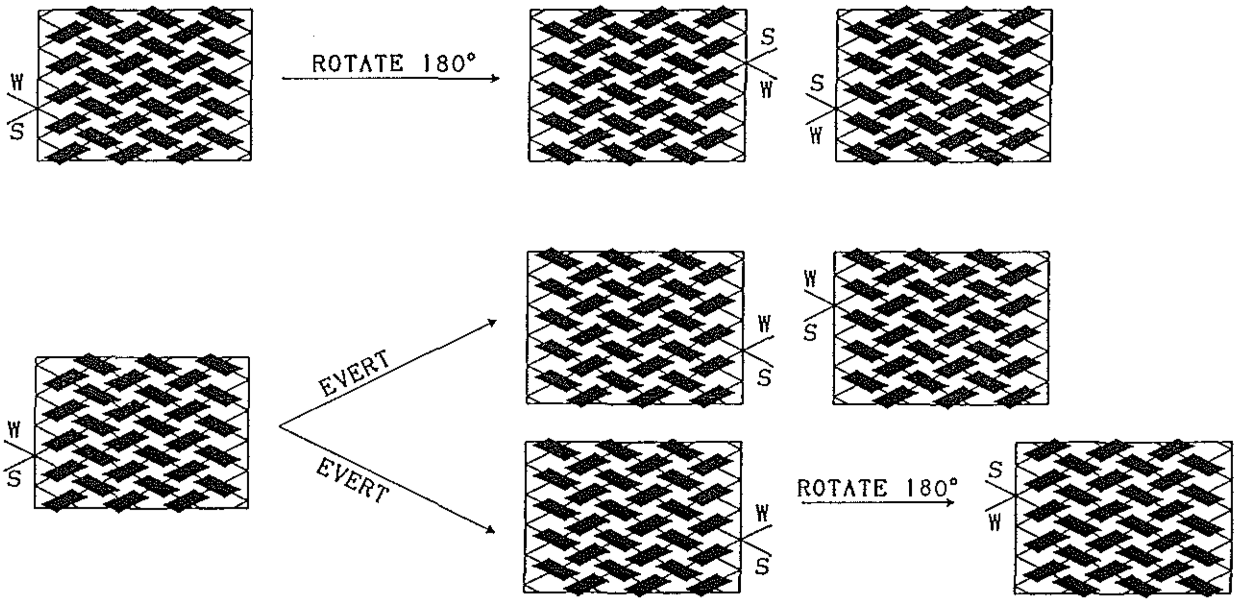


Fig. 395 — The type with an **odd** number of parts.

g.c.d. (parts , bights) = 1 , an **even** number of parts can only have an **odd** number of bights). Hence we can consider four types: two with respectively the complementary weaving-patterns, an **even** number of parts and an **odd** number of bights; and two with respectively an **odd** or an **even** number of bights and an **odd** number of parts. These four types are then illustrated in respectively Figs. 394 , 396 , 395 & 397.

Note that by only taking into account the possible parity relationships between the parts and the bights, one would also have obtained three types: one type for an **even** number of parts with an **odd** number of bights and two types for an **odd** number of parts with respectively an **odd** or **even** number of bights. This three-type 'classification' is of course quite different to the previous three-type 'classification' outlined before on pg. 472.

Let's consider the three possible parity-pairs associated with the parts/bights numbers:

$$p/b = \text{even}/\text{odd} = e/o\,,$$
$$p/b = \text{odd}/\text{odd} = o/o\,,$$
$$p/b = \text{odd}/\text{even} = o/e\,.$$

Each of these three parity-pairs occupies **four** different positions in the RKT (Regular Knot Tree). For the raising of the associated over–under coded Regular Knots to bigger over–under coded Regular Knots, each position is associated with **six** enlargement types, two of these are respectively an **even** multiple of first order enlargements in accordance with Enlargement Method I and an **even** multiple of first order enlargements in accordance with Enlargement Method II, while four of these are each a combination of a higher order (**odd** or **even**) enlargement followed by a first order enlargement both of the same Enlargement Method. The paths in the RKT of these six enlargement types are depicted in Fig. 398, where left of the thick vertical line are depicted the three types associated with Enlargement Method I, and where right of the thick vertical line are depicted the three types associated with Enlargement Method II (for $n =$ **odd** a higher **even** order, and for $n =$ **even** a higher **odd** order).

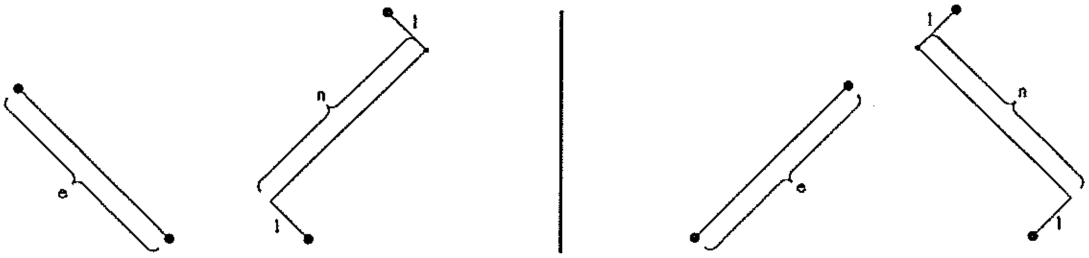


Fig. 398 — The **six** enlargement types.

Thus for each of the three parity-pairs $p/b = e/o$, $p/b = o/o$ & $p/b = o/e$ we obtain, with the four positions of the parity-pair in the RKT, **twenty four** enlargement types. With the four positions of each parity-pair in the RKT we can 'classify' the over–under coded Regular Knots into $4 \times 4 = 16$ types with each type having 6 raising procedures.

The 24 enlargement types associated with the parity-pair $p/b = e/o$ are depicted in Fig. 399; the 24 enlargement types associated with the parity-pair $p/b = o/o$ are depicted in Fig. 400; and The 24 enlargement types associated with the parity-pair $p/b = o/e$ are depicted in Fig. 401.

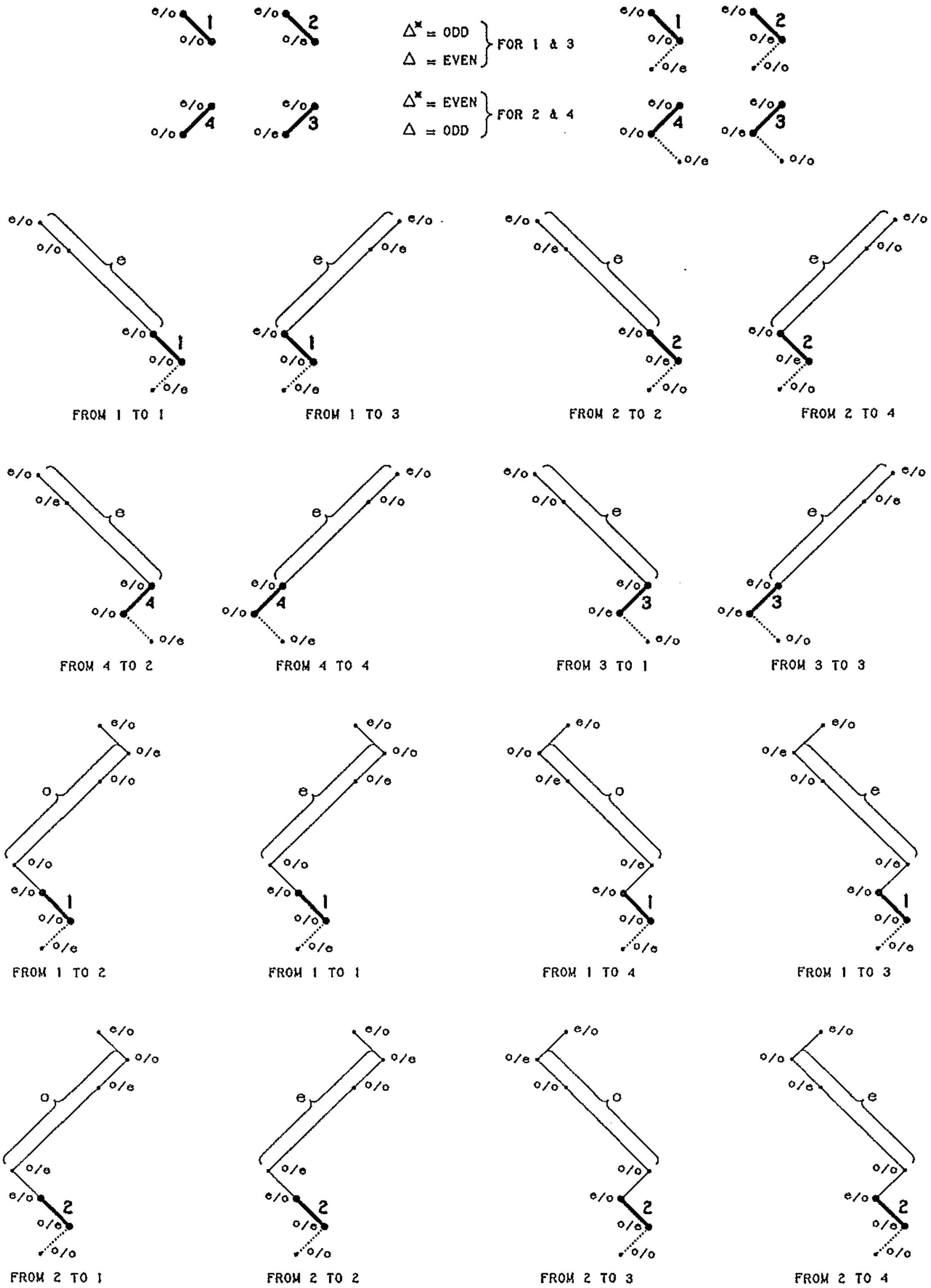
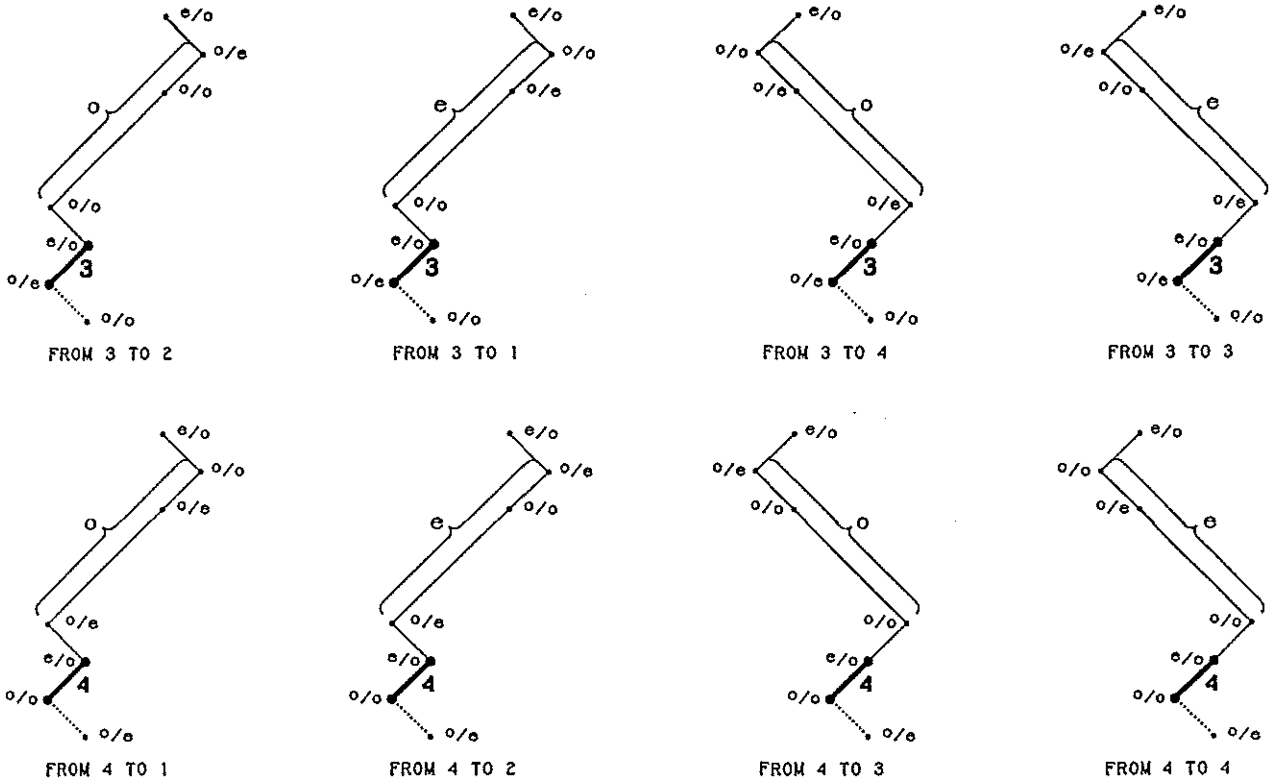


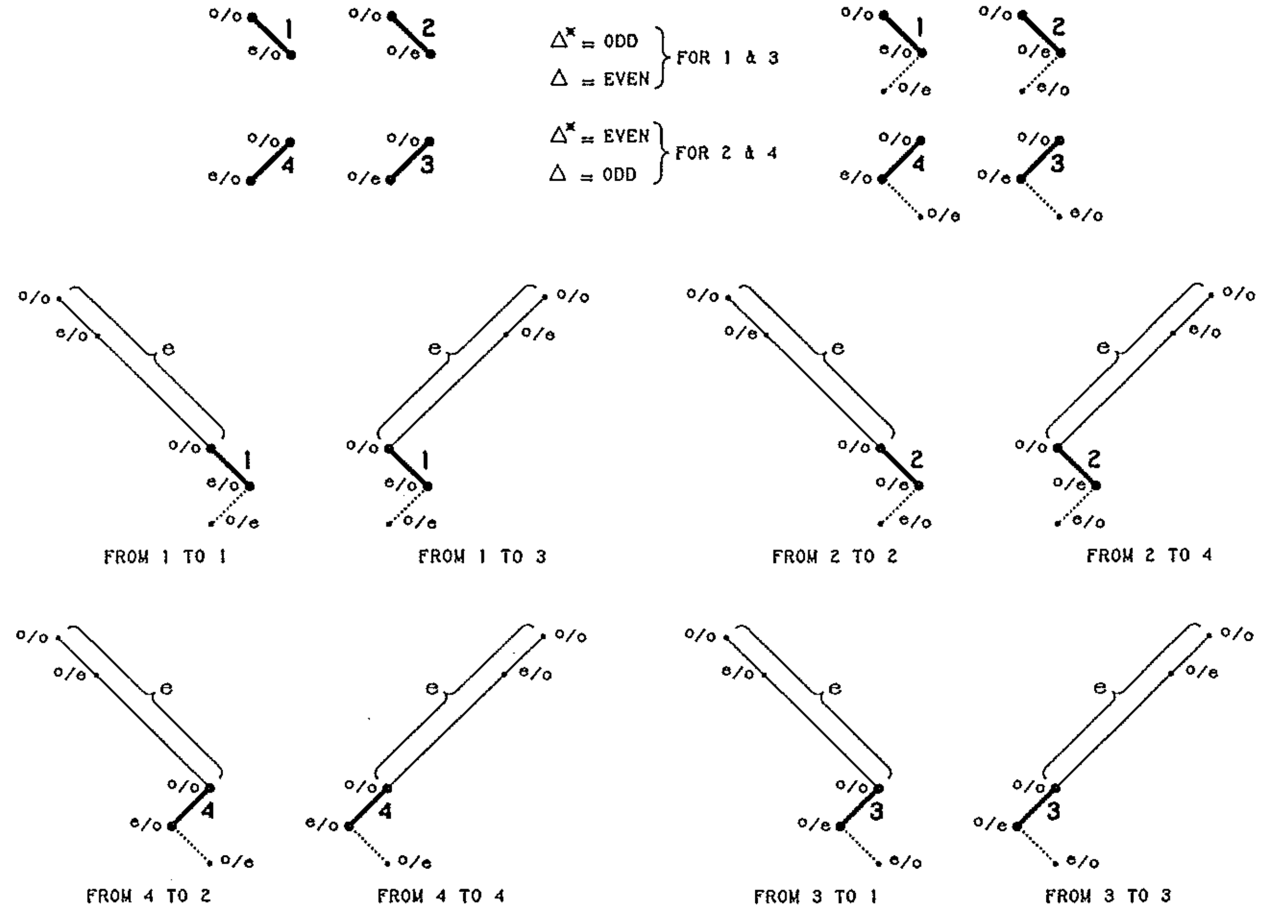
Fig. 399 — $p/b = e/o$, and its 24 enlargement types.



FROM 3 TO 2 FROM 3 TO 1 FROM 3 TO 4 FROM 3 TO 3

FROM 4 TO 1 FROM 4 TO 2 FROM 4 TO 3 FROM 4 TO 4

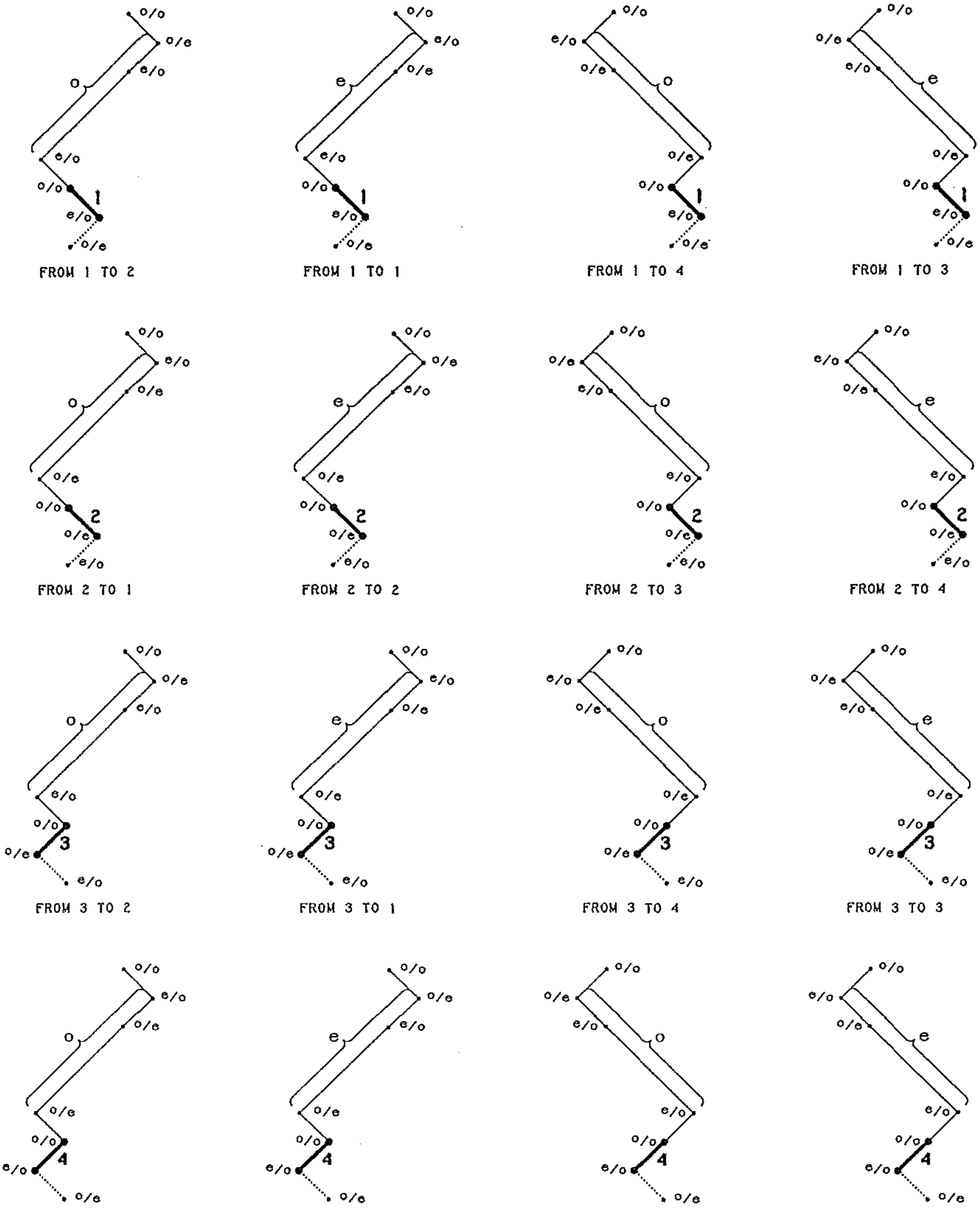
Fig. 399 continued — $p/b = e/o$, and its 24 enlargement types.



FROM 1 TO 1 FROM 1 TO 3 FROM 2 TO 2 FROM 2 TO 4

FROM 4 TO 2 FROM 4 TO 4 FROM 3 TO 1 FROM 3 TO 3

Fig. 400 — $p/b = o/o$, and its 24 enlargement types.



FROM 1 TO 2 FROM 1 TO 1 FROM 1 TO 4 FROM 1 TO 3

FROM 2 TO 1 FROM 2 TO 2 FROM 2 TO 3 FROM 2 TO 4

FROM 3 TO 2 FROM 3 TO 1 FROM 3 TO 4 FROM 3 TO 3

FROM 4 TO 1 FROM 4 TO 2 FROM 4 TO 3 FROM 4 TO 4

Fig. 400 continued — $p/b = o/o$, and its 24 enlargement types.

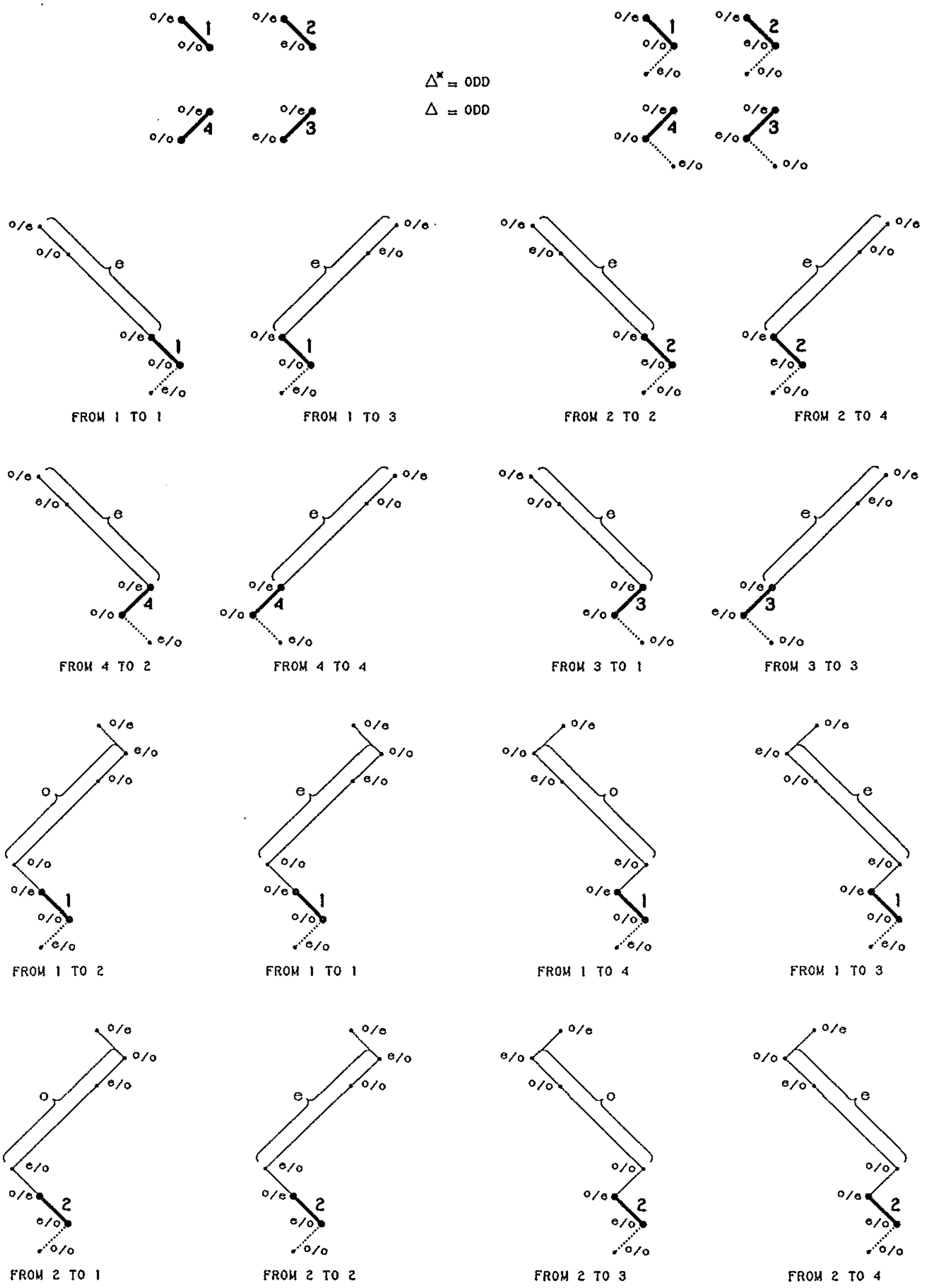


Fig. 401 — $p/b = o/e$, and its 24 enlargement types.

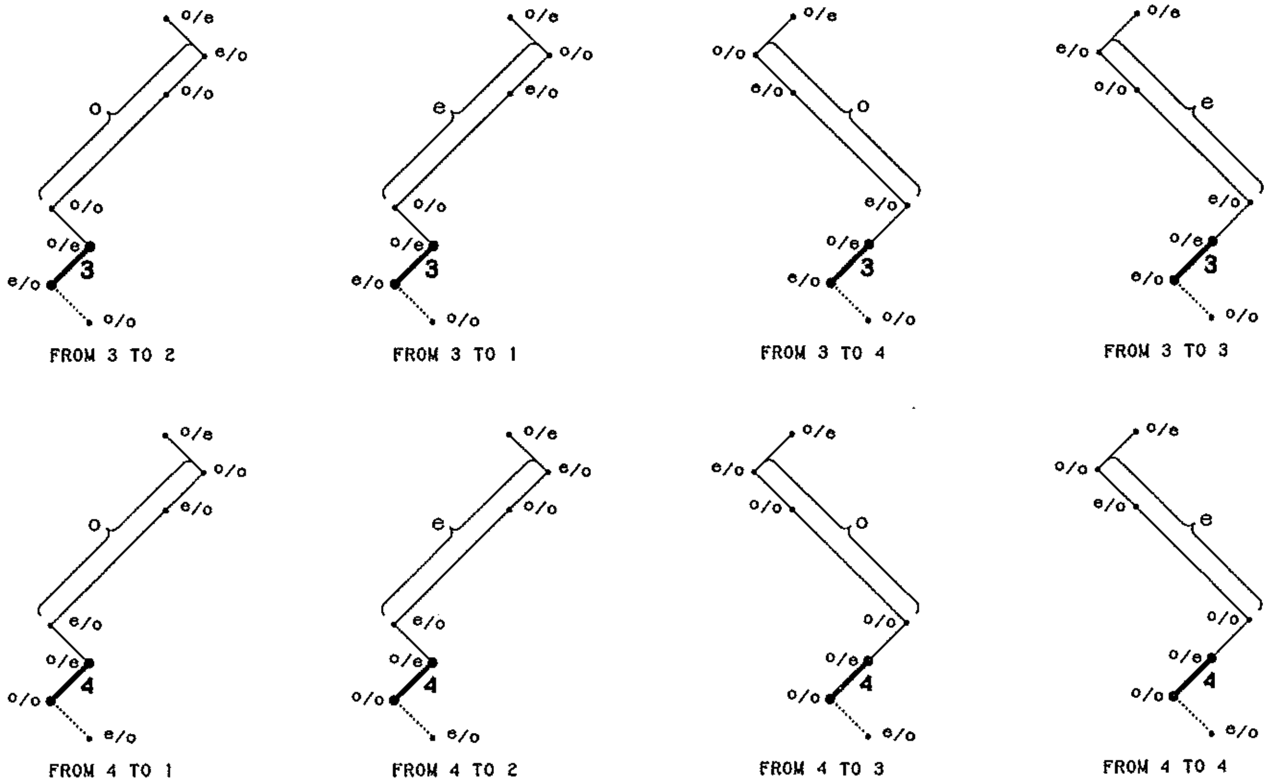


Fig. 401 continued — $p/b = o/e$, and its 24 enlargement types.

We note from these enlargement types that the parity-pair of a over–under coded Regular Knot is an **invariant**, hence when $p/b = e/o$, then any over–under coded Regular Knot raised from it also has $p/b = e/o$. Similarly, when $p/b = o/o$, then any over–under coded Regular Knot raised from it also has $p/b = o/o$, and when $p/b = o/e$, then any over–under coded Regular Knot raised from it also has $p/b = o/e$.

⋆⋆ Prove in two much easier and quicker ways that a p/b parity-pair is an invariant in the enlargement from one over–under coded Regular Knot to another bigger over–under coded Regular Knot.

Fig. 402 depicts the various relationships between p/b, the colour of the pattern (the weaving-pattern), the p/b parity-pairs, and the parities of Δ^* & Δ.

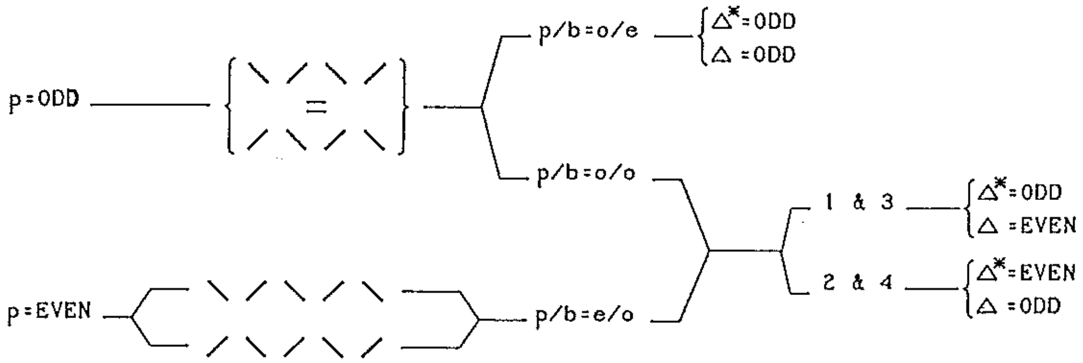


Fig. 402 — The relationships between the important parameters.

From the relationships depicted in Fig. 402 it follows that one must at least ‘classify’ the over–under coded Regular Knots into **seven** types, although a full ‘classification’

within the over–under coded Regular Knots requires **sixteen** types as we have seen earlier.

Even the 'classification' of the simple over–under coded Regular Knots **within** their own restricted area of **over–under coded Regular Knots** is no doubt more extensive than most classification-proponents could imagine. If we would also try to cover in their 'classification' their properties **outside** their own, above indicated, restricted area, then the complications are enormous.

⋆⋆ *Name, as an example, an area which would complicate their 'classification' (hint: we have met this area in some previous Issues of The Braider).*

Let's now return to the statement mentioned in the beginning (see pg. 471). Since a p/b parity-pair is an invariant in the enlargement from one over–under coded Regular Knot (Casa Knot) to another bigger over–under coded Regular Knot (Casa Knot) and since by using over–under coded Regular Knots which almost all have been enlarged (raised) from 2-bight over–under coded Regular Knots, it follows that almost all over–under coded Regular Knots used by those so-called braiders have an **even** number of bights, hence with the p/b parity-pair o/e. Let's therefore see what proportion of over–under coded Regular Knots are apparently rarely, if ever, used by those people.

First we examine the distribution of each p/b parity-pair.

In Fig. 403 is depicted a general section of the RKT, starting near the bottom with a node which has a $p/b = o/o$ parity-pair. The uppermost two levels give the two cyclic p/b parity-pair sequences in the levels of the RKT. These two cyclic p/b parity-pair sequences in the levels of the RKT are diagrammetically depicted by the right-hand diagram. By reading the parity-pairs of a level from **left** to **right** we have the cyclic sequences:

$$\text{Odd levels: } o/e - e/o - o/o\,,$$
$$\text{Even levels: } o/o - e/o - o/e\,.$$

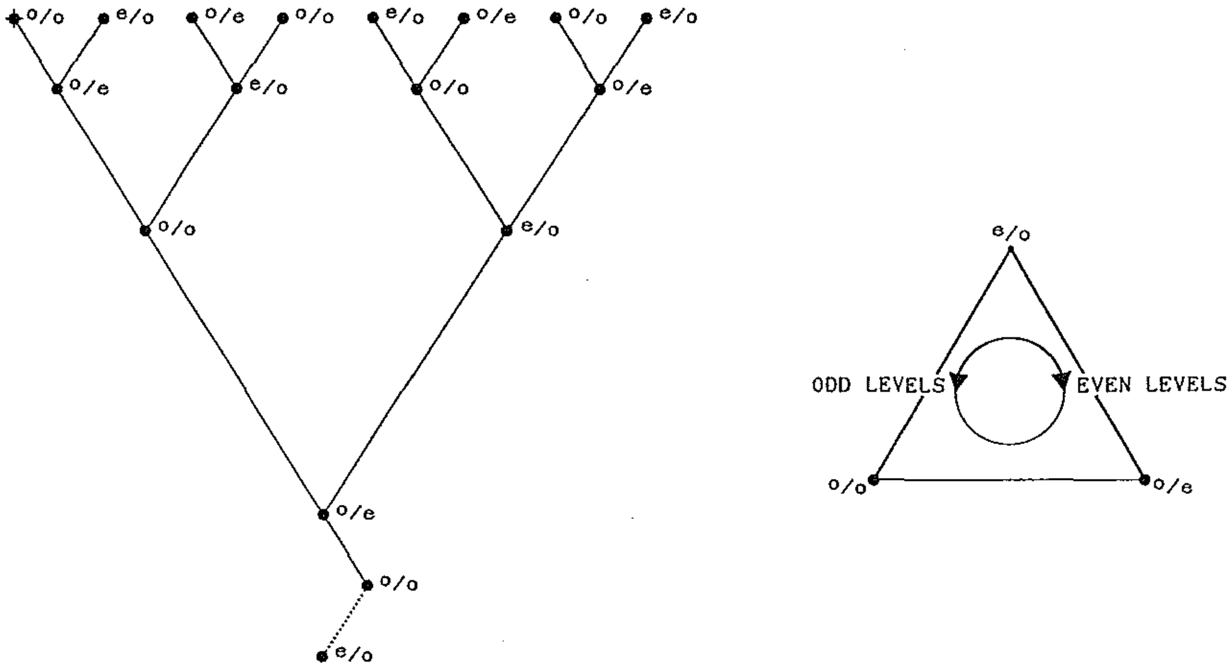


Fig. 403 — The p/b parity-pair sequences in the levels of the RKT.

The RKT up to and including level 7 has been shown in Fig. 404. In Fig. 405 the p/b-values in Fig. 404 have been replaced by their corresponding parity-pairs.

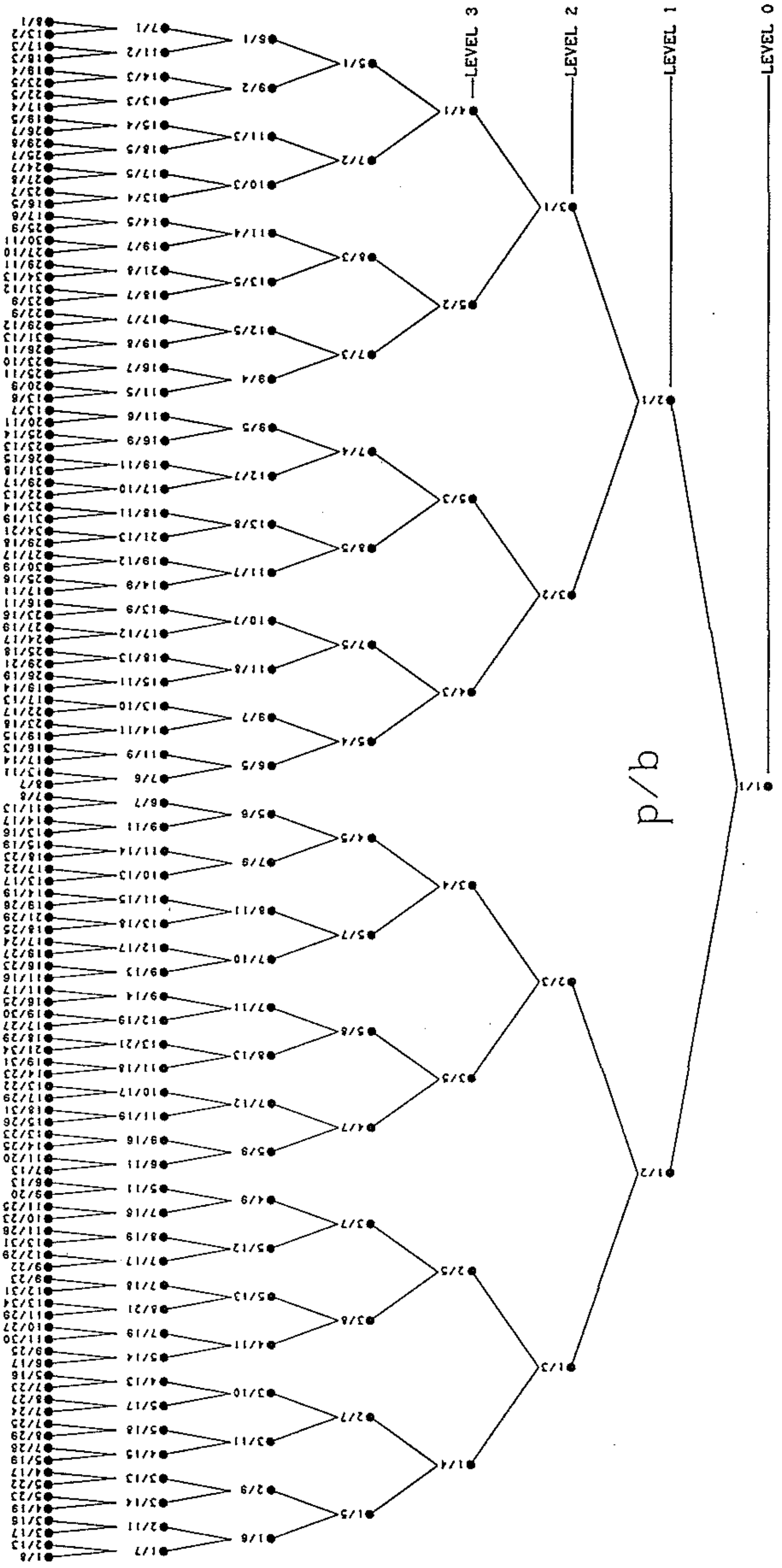


Fig. 404 — The RKT up to and including level 7.

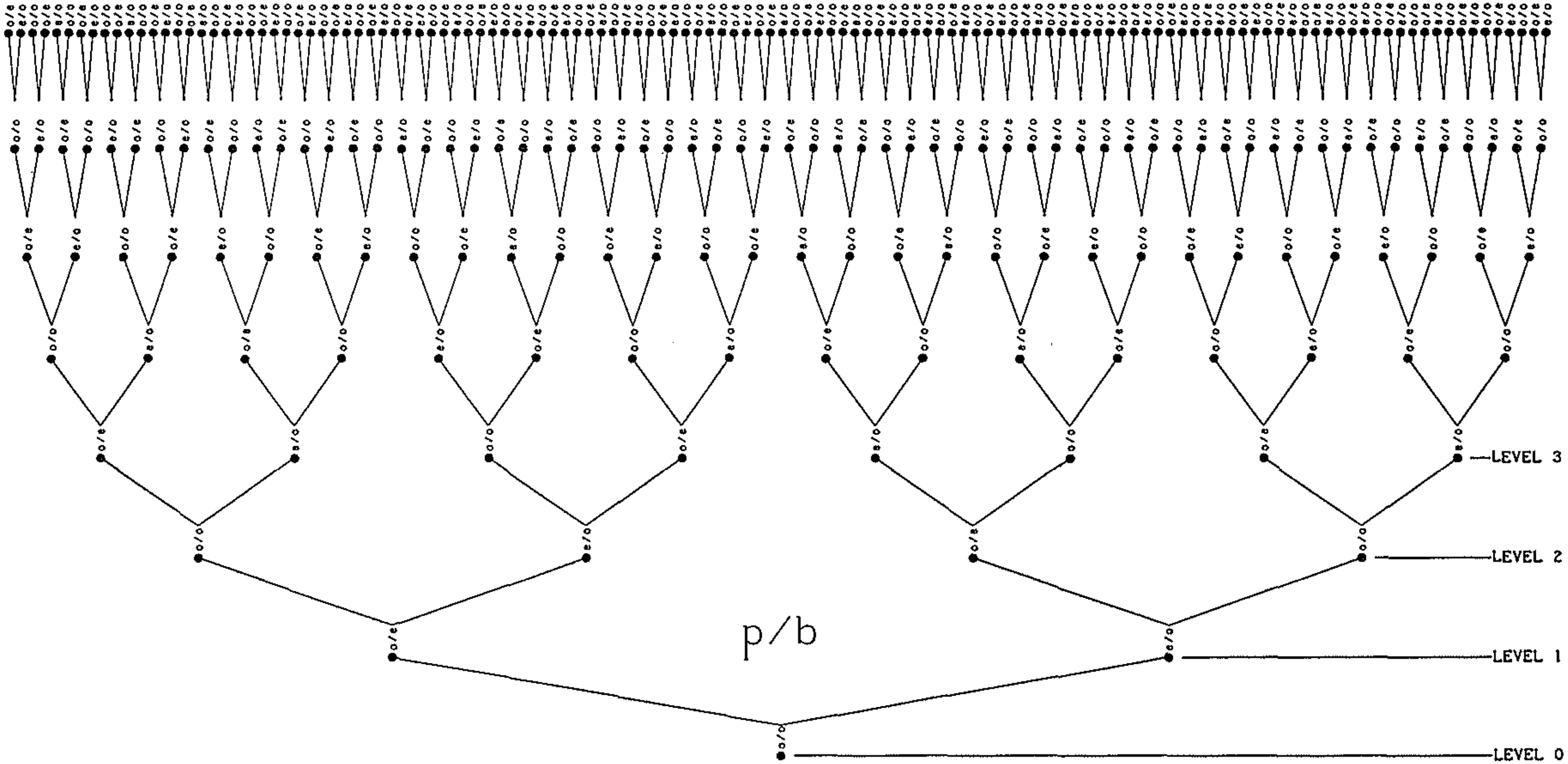


Fig. 405 — The p/b parity-pairs in the RKT up to and including level 7.

The level i in the RKT has 2^i nodes (parity-pairs), hence for $i = \mathbf{odd}$ we obtain:

number of o/e's is: $\lfloor\frac{2^i-1}{3}\rfloor + 1 = \lceil\frac{2^i}{3}\rceil$,
number of e/o's is: $\lfloor\frac{2^i-2}{3}\rfloor + 1 = \lceil\frac{2^i}{3}\rceil$,
number of o/o's is: $\lfloor\frac{2^i-3}{3}\rfloor + 1 = \lfloor\frac{2^i}{3}\rfloor$.

For $i = \mathbf{even}$ we obtain:

number of o/o's is: $\lfloor\frac{2^i-1}{3}\rfloor + 1 = \lceil\frac{2^i}{3}\rceil$,
number of e/o's is: $\lfloor\frac{2^i-2}{3}\rfloor + 1 = \lfloor\frac{2^i}{3}\rfloor$,
number of o/e's is: $\lfloor\frac{2^i-3}{3}\rfloor + 1 = \lfloor\frac{2^i}{3}\rfloor$.

Example:

For level 5:

number of o/e's is: $\lfloor\frac{2^i-1}{3}\rfloor + 1 = \lfloor\frac{32-1}{3}\rfloor + 1 = 11$,
number of e/o's is: $\lfloor\frac{2^i-2}{3}\rfloor + 1 = \lfloor\frac{32-2}{3}\rfloor + 1 = 11$,
number of o/o's is: $\lfloor\frac{2^i-3}{3}\rfloor + 1 = \lfloor\frac{32-3}{3}\rfloor + 1 = 10$.

For level 6:

number of o/o's is: $\lfloor\frac{2^i-1}{3}\rfloor + 1 = \lfloor\frac{64-1}{3}\rfloor + 1 = 22$,
number of e/o's is: $\lfloor\frac{2^i-2}{3}\rfloor + 1 = \lfloor\frac{64-2}{3}\rfloor + 1 = 21$,
number of o/e's is: $\lfloor\frac{2^i-3}{3}\rfloor + 1 = \lfloor\frac{64-3}{3}\rfloor + 1 = 21$.

Since the 1-part, the 2-parts column-coded Regular Knots and the 3-parts over–under coded Regular Knots as well as the 1-bight and the 2-bights over–under coded Regular Knots can, for the raising of over–under coded Regular Knots, be deducted from each level[†], we obtain:

For $i = \mathbf{odd}$:

number of o/e's is: $\lfloor\frac{2^i-1}{3}\rfloor - 2 = \lfloor\frac{2^i}{3}\rfloor - 2$,
number of e/o's is: $\lfloor\frac{2^i-2}{3}\rfloor - 1 = \lfloor\frac{2^i}{3}\rfloor - 1$,
number of o/o's is: $\lfloor\frac{2^i-3}{3}\rfloor = \lfloor\frac{2^i}{3}\rfloor - 1$.

For $i = \mathbf{even}$:

number of o/o's is: $\lfloor\frac{2^i-1}{3}\rfloor - 2 = \lfloor\frac{2^i}{3}\rfloor - 2$,
number of e/o's is: $\lfloor\frac{2^i-2}{3}\rfloor = \lfloor\frac{2^i}{3}\rfloor - 1$,
number of o/e's is: $\lfloor\frac{2^i-3}{3}\rfloor - 1 = \lfloor\frac{2^i}{3}\rfloor - 2$.

It will be evident that just over 2/3 of the over–under coded Regular Knots will be out of bounds to those who only would use the over–under coded Regular Knots which can be enlarged from 2-bights over–under coded Regular Knots. It will thus be obvious that in general the statement: *"Almost all of the knots braiders use are tied from two bight Casa Knots."* is certainly incorrect.

[†] The 3-parts over–under coded Regular Knots may be raised from the $p/b = 3/1$ and the $p/b = 3/2$ over–under coded Regular Knots as discussed in *The Braider*, Appendix 1996, pp. *iii*–*v*.

Integrated Braids

Sometimes we require long flat braids with a buttonhole or buttonholes at one end and a button Knot at the other end. An Example of the end with the buttonholes is shown in Fig. 406 for a six, respectively an eight part over–under coded Regular Flat Braid. This is the end at which braiding begins.

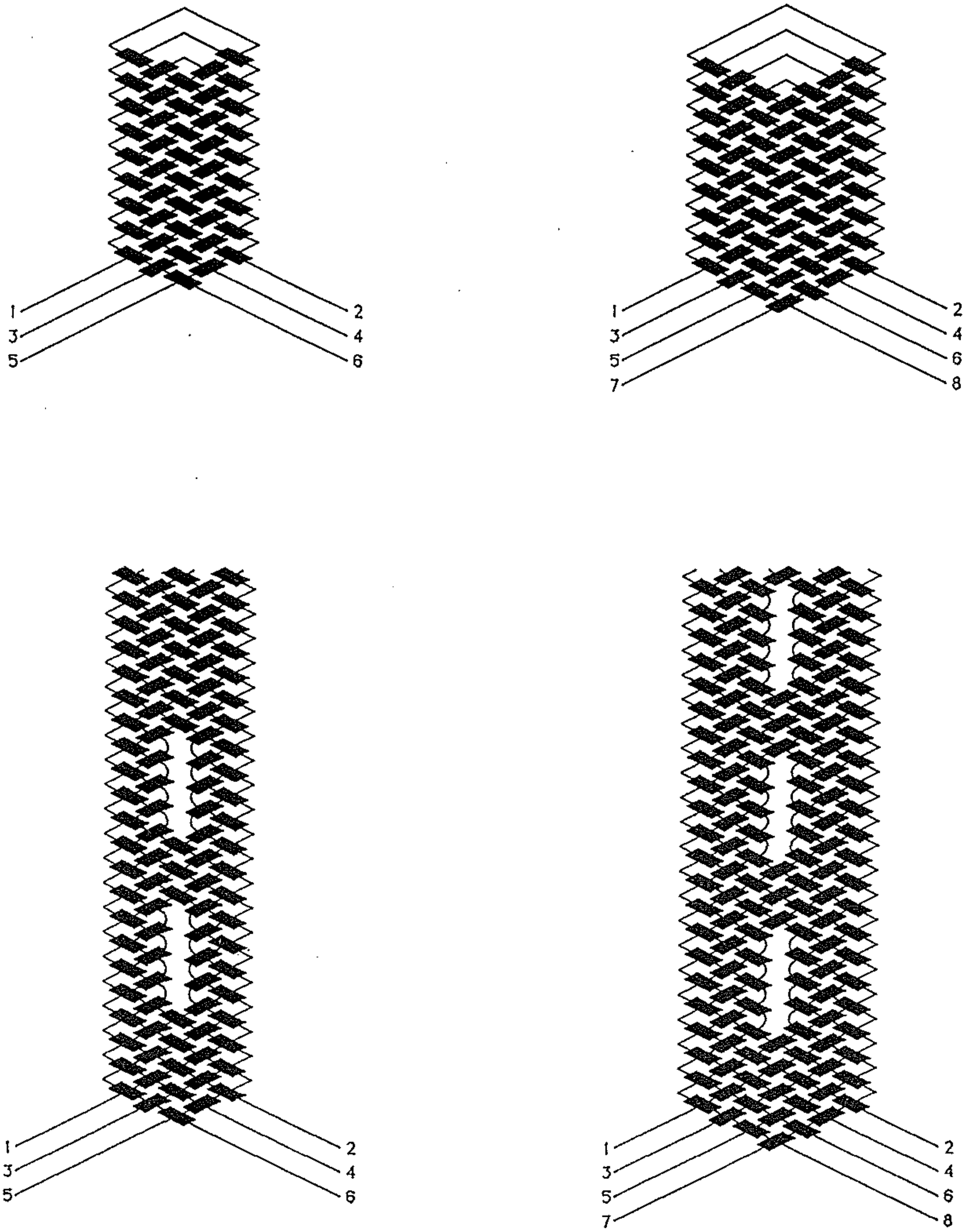


Fig. 406 — The start of the braid with the buttonholes.

At the other end of the flat braid (the end which will receive the button), the flat braid goes over into a round braid. For both the 6-part and the 8-part Regular Flat Braid, the round braid is a four-string over–under Round Braid. After braiding the round braid for a short distance, as indicated in Fig. 407, we braid over it the foundation knot of the button knot. In the case of the 6-part Regular Flat Braid, the foundation knot is constructed with the string-ends *A* and *B*, while in the case of the 8-part Regular Flat Braid, the foundation knot is constructed with the string-ends *A*, *B*, *C* and *D*.

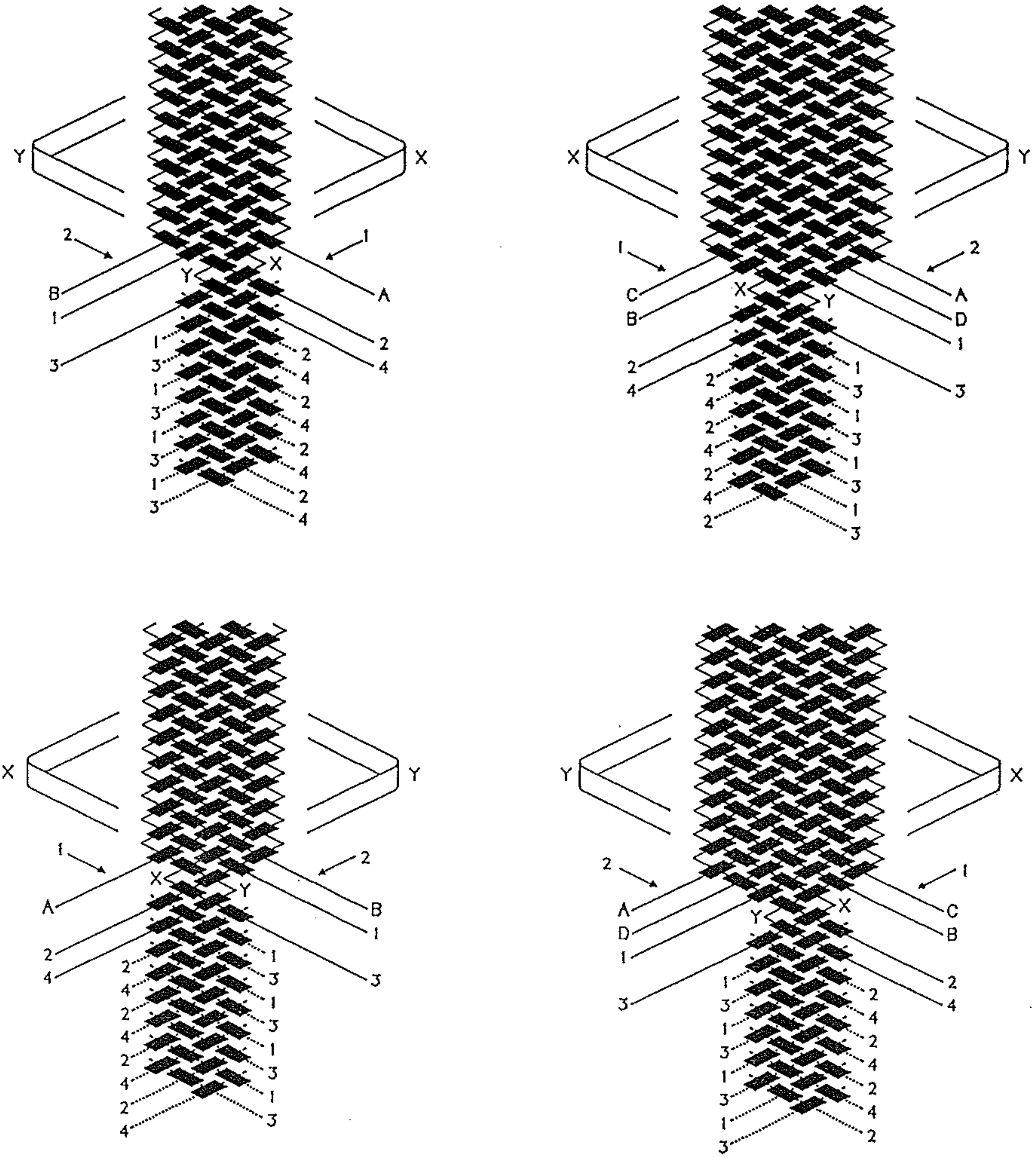


Fig. 407 — The transition from flat braid to round braid.

For the upper 6-part Regular Flat Braid in Fig. 407, the construction of the foundation knot is shown in Fig. 408, while for the lower 6-part Regular Flat Braid in Fig. 407, the construction of the foundation knot is shown in Fig. 409. Thus for Fig. 408, turn string *A* in the upper left diagram of Fig. 407 to the left, and for Fig. 409, turn string *A* in the lower left diagram of Fig. 407 to the right.

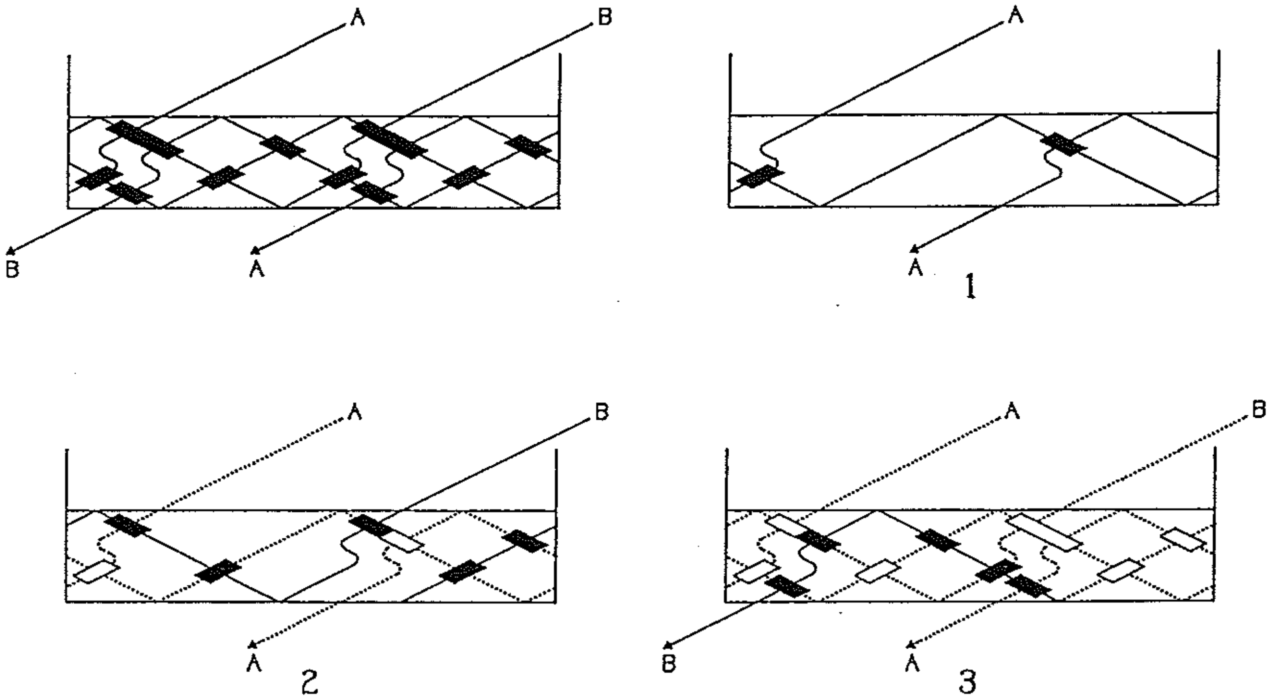


Fig. 408 — The foundation knot for the upper 6-part Regular Flat Braid in Fig. 407.

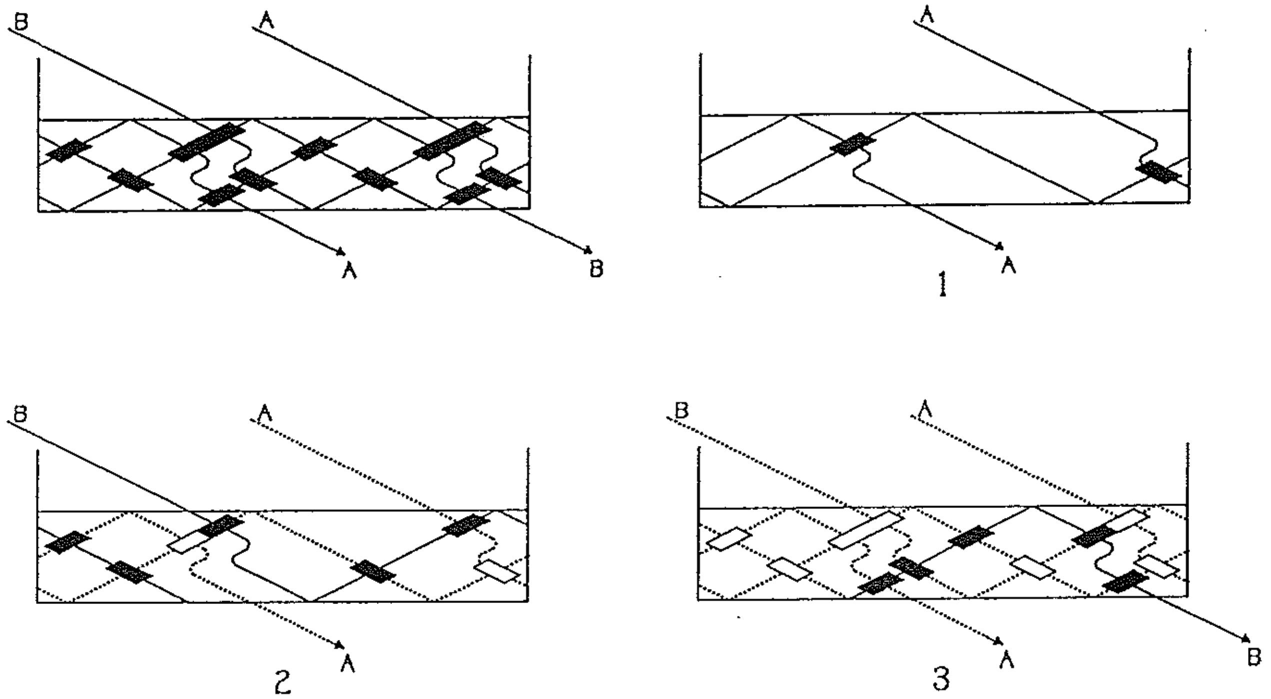
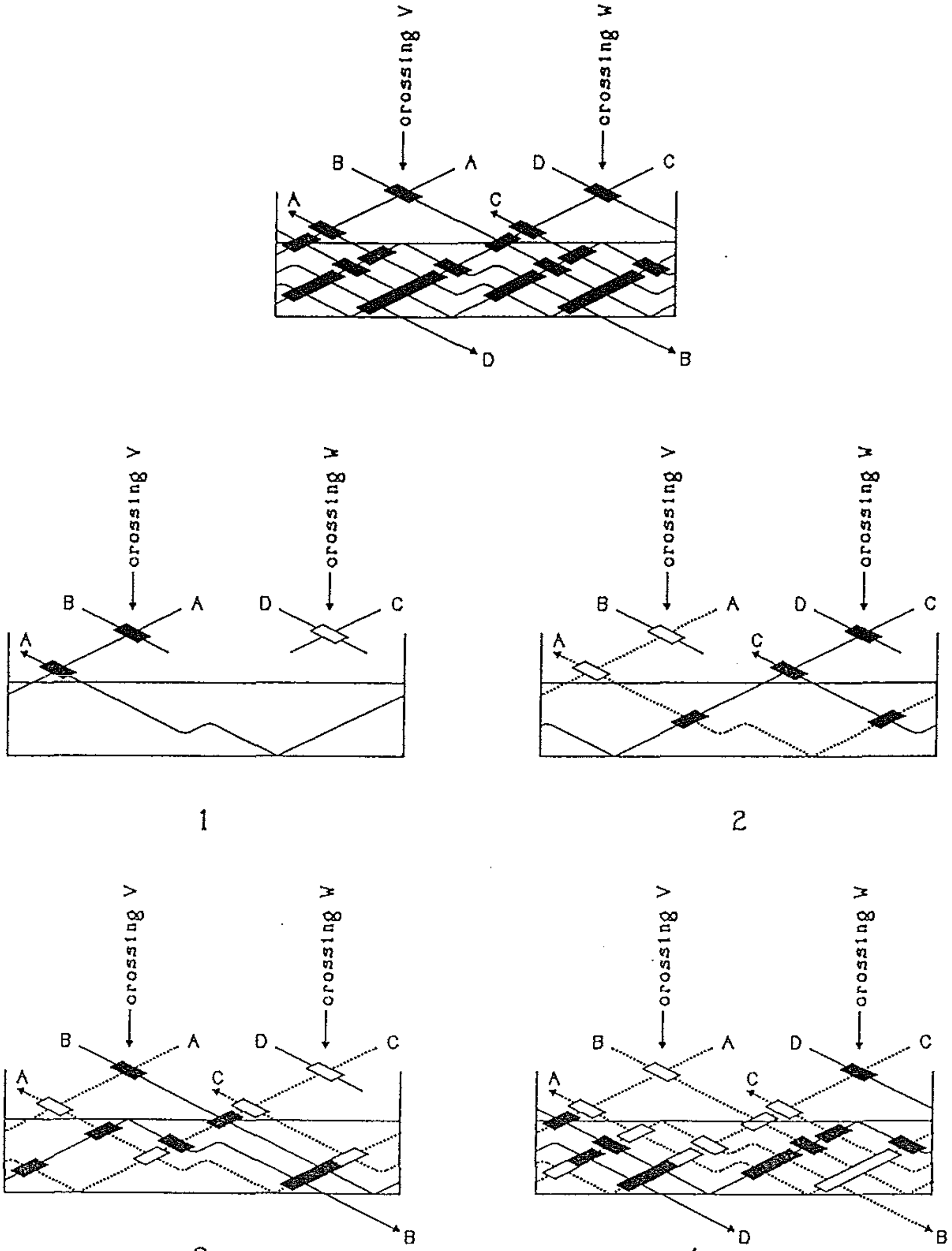


Fig. 409 — The foundation knot for the lower 6-part Regular Flat Braid in Fig. 407.

For the upper 8-part Regular Flat Braid in Fig. 407, the construction of the foundation knot is shown in Fig. 410. Here we turn string *A* to the left and string *B* to the right **over** string *A* to form crossing *V* on the front; then we turn string *C* to the right and string *D* to the left **under** string *C* to form crossing *W* on the back. After the foundation knot has been braided, turn *A* and *C* to lower left and turn *B* and *D* to upper right. Tie down with a constrictor knot around them and cut the strings flush with the upper and lower edge of the knot. Unbraid the four-string round braid as far as the foundation knot.



1

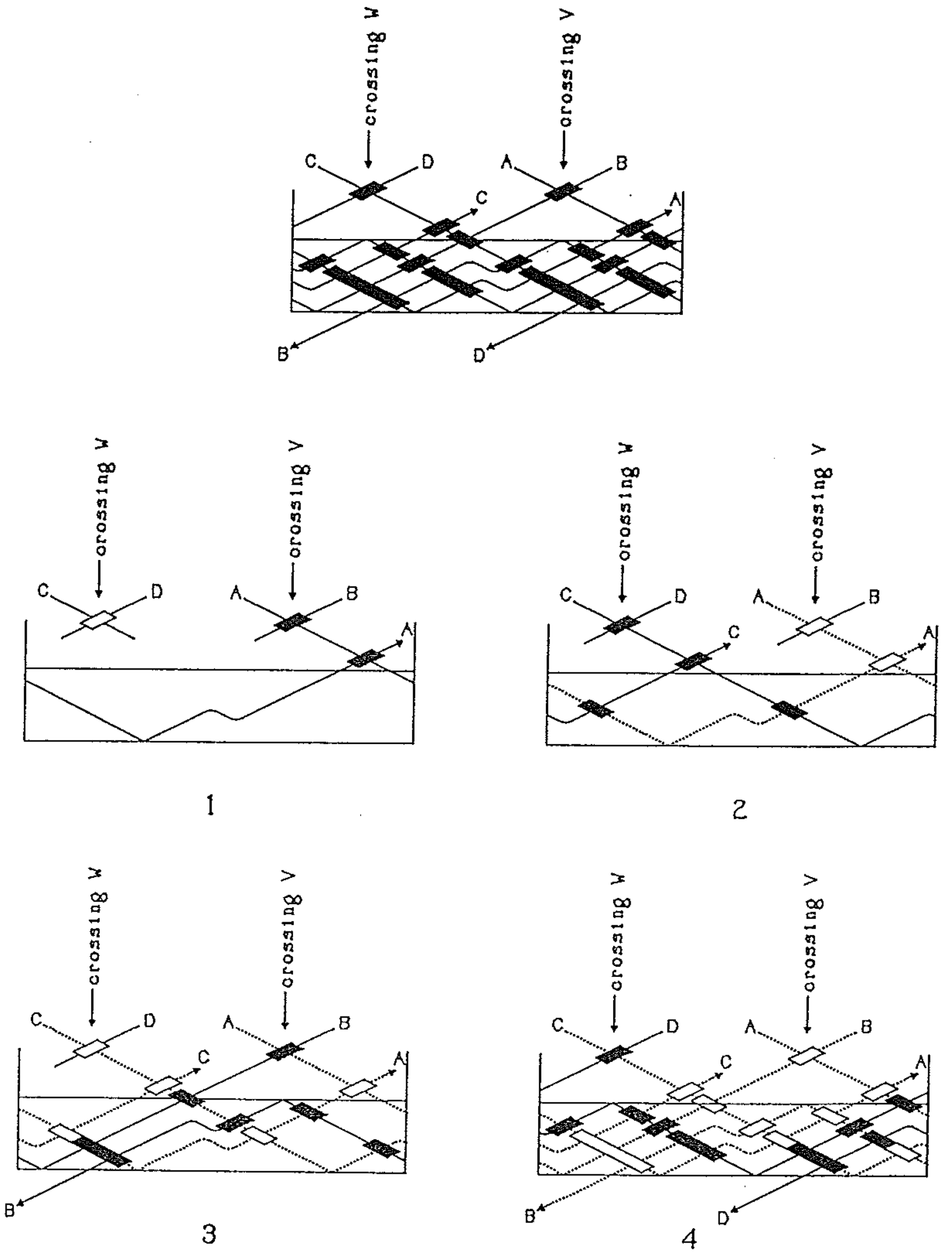
2

3

4

Fig. 410 — The foundation knot for the upper 8-part Regular Flat Braid in Fig. 407.

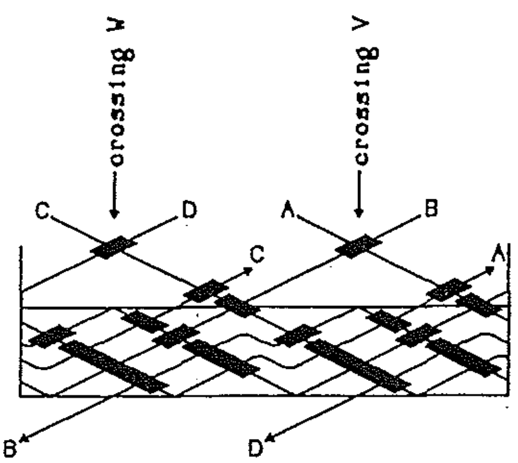
For the lower 8-part Regular Flat Braid in Fig. 407, the construction of the foundation knot is shown in Fig. 411. Here we turn string A to the right and string B to the left over string A to form crossing V on the front; then we turn string C to the left and string D to the right **under** string C to form crossing W on the back. After the foundation knot has been braided, turn A and C to lower right and turn B and D to upper left. Tie down with a constrictor knot around them and cut the strings flush with the upper and lower edge of the knot. Unbraid the four-string round braid as far as the foundation knot.



1 2 3 4

Fig. 411 — The foundation knot for the lower 8-part Regular Flat Braid in Fig. 407.

Fig. 412 shows an alternative braiding sequence for the foundation knot in Fig. 411.



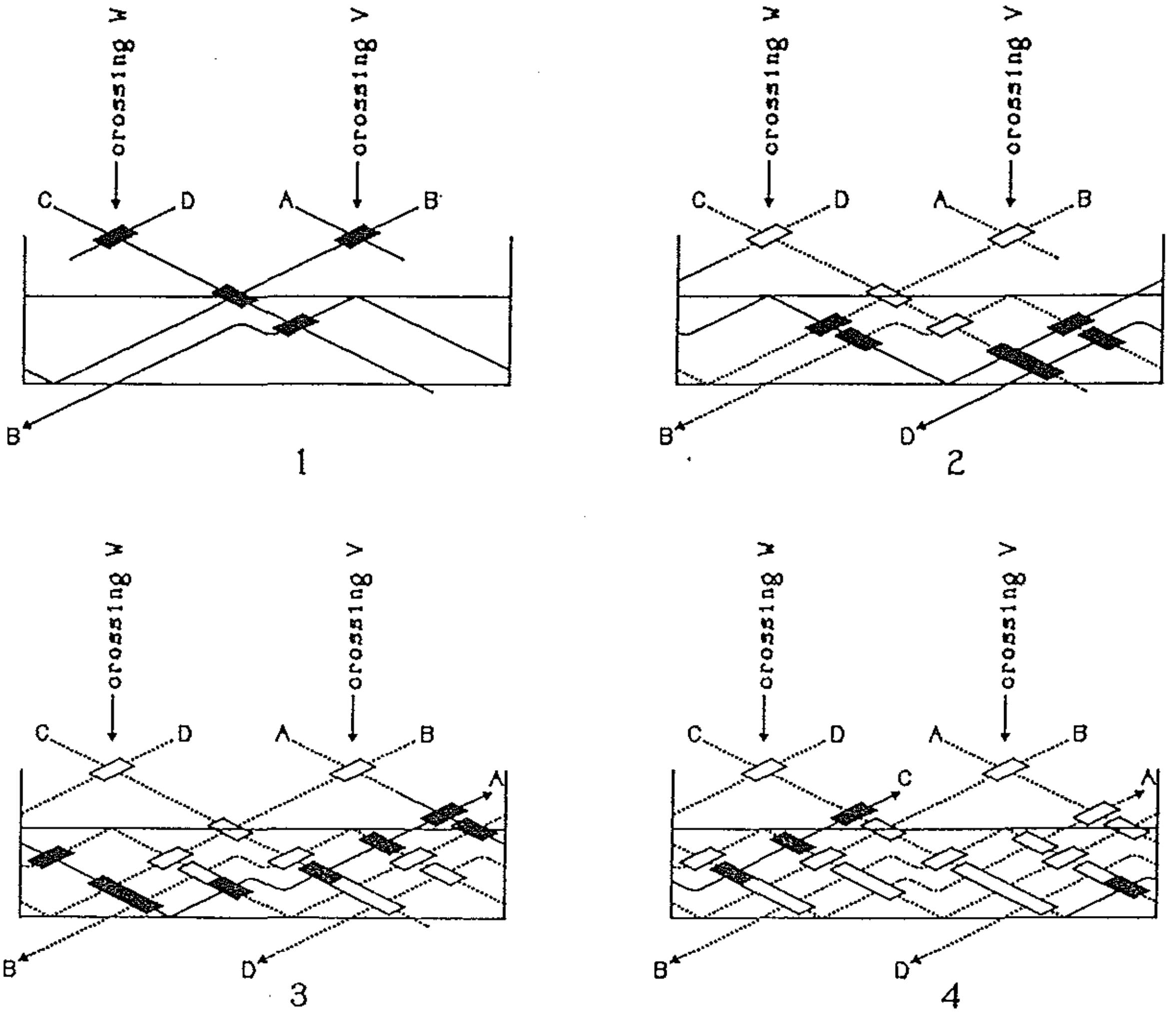


Fig. 412 — The foundation knot for the lower 8-part Regular Flat Braid in Fig. 407.

A similar alternative braiding sequence may be employed for the foundation knot in Fig. 410.

The final integrated Gaucho button knot, braided with the string-ends of the four-string round braid, is depicted in Fig. 413 and Fig. 414.

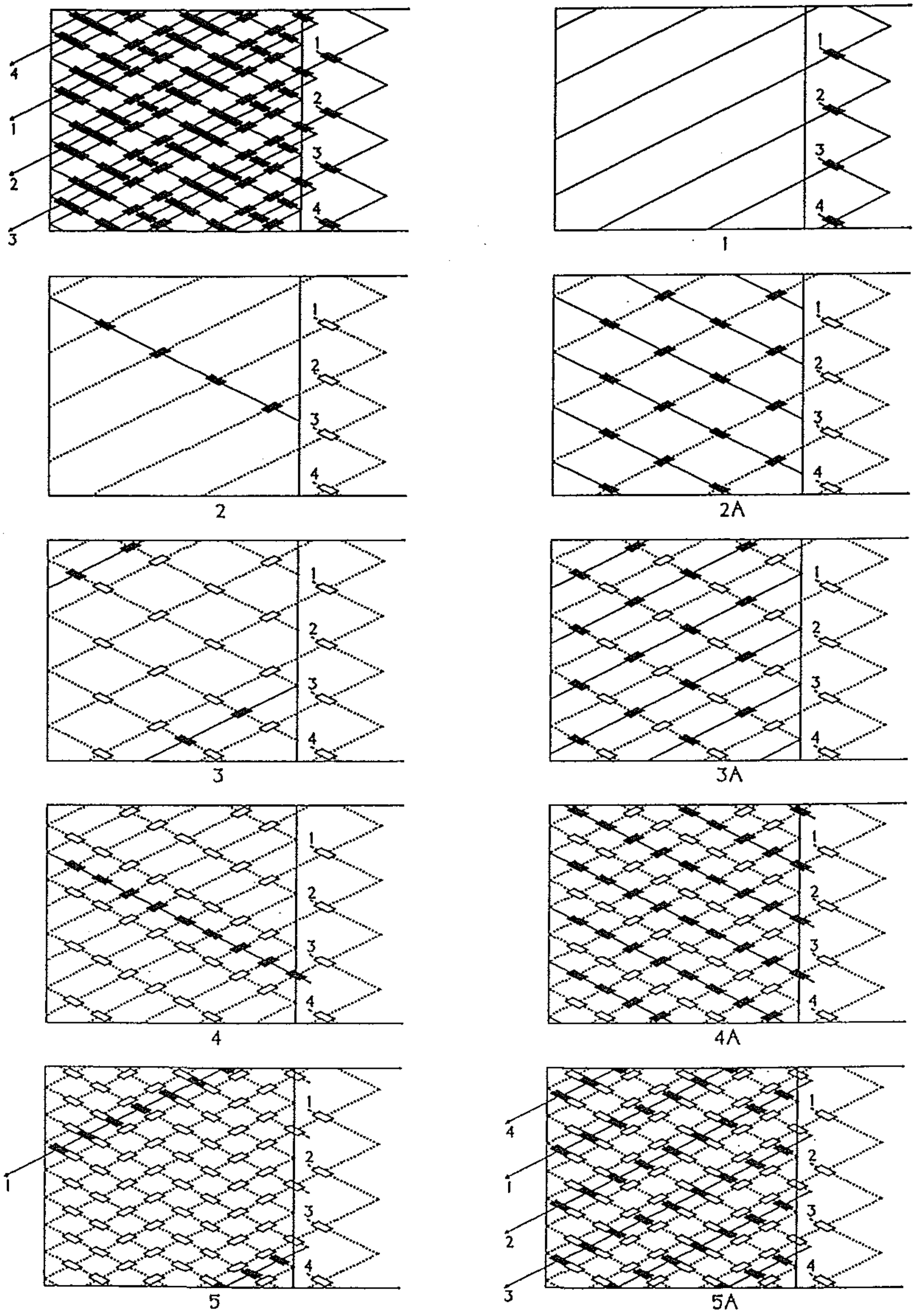
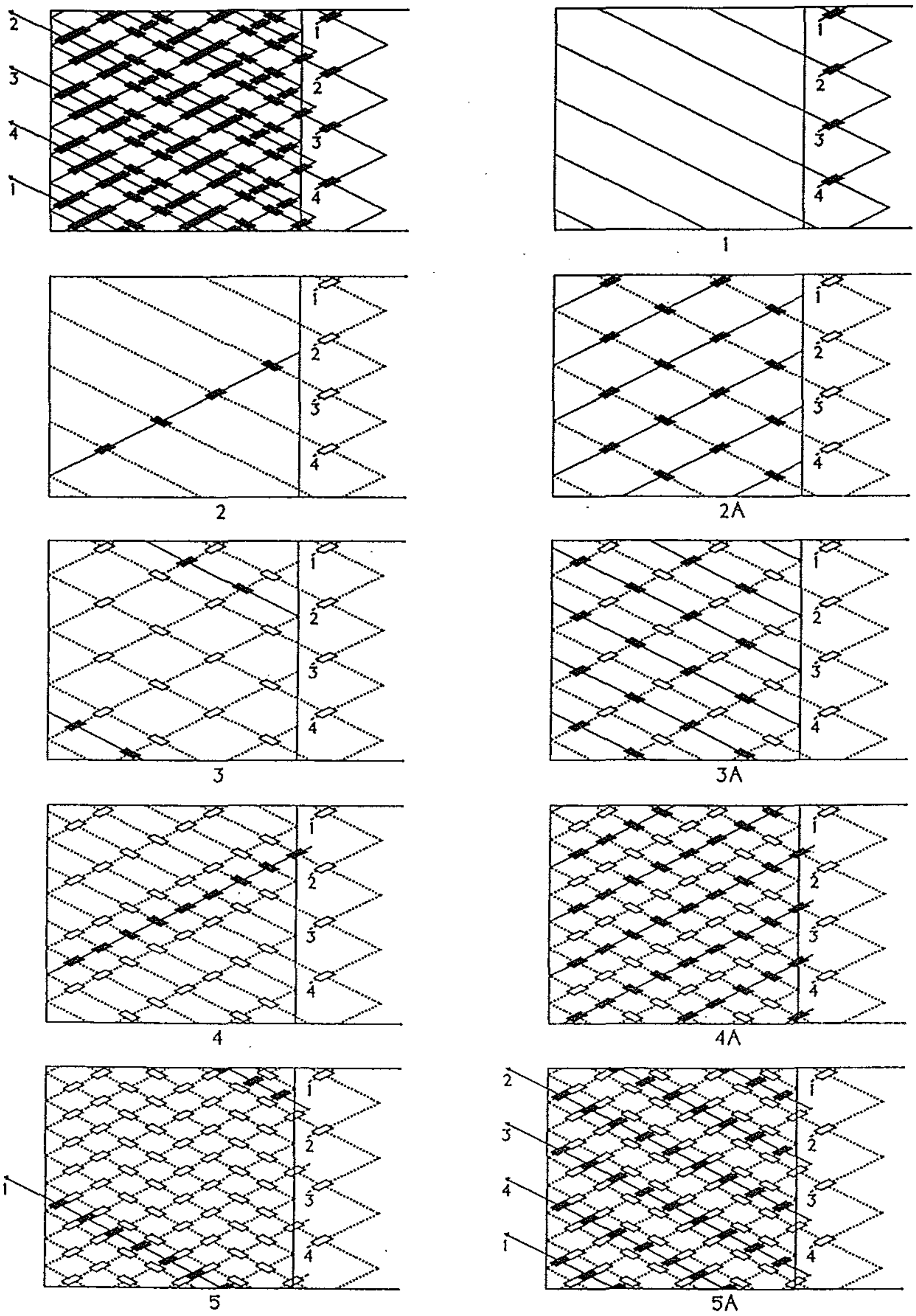


Fig. 413 — The final integrated Gaucho button knot.



1

2

2A

3

3A

4

4A

5

5A

Fig. 414 — The final integrated Gaucho button knot.

THE BRAIDER

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A quarterly publication for the braiding artisan

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Solutions to the Questions in Issue No. 21

Question on pg. 478.

(1.) Let the Regular Knot p/b have an over–under coding and let's raise this knot to a bigger over–under coded Regular Knot P/B.

Say we give the over–under coded Regular Knot p/b an $(n+1)$ order enlargement[†] which results in an additional $2x$ half-cycles ($x = |\Delta b|_b + nb$ additional bights). In order to obtain an enlarged Regular Knot with an over–under coding, we must ensure that adjacent to each additional half-cycle there comes a further additional half-cycle. Hence in order to obtain an over–under coding for the enlarged Regular Knot P/B, we must give the over–under coded Regular Knot p/b an $(n+1)$ order enlargement followed by a first order enlargement of the same Enlargement Method. This is illustrated in Fig. 415 (Method I at the left of the thick vertical line, and Method II at the right of the thick vertical line). We see that the parities of p and P are the same, and that the parities of b and B are the same, hence the p/b parity-pair is an invariant.

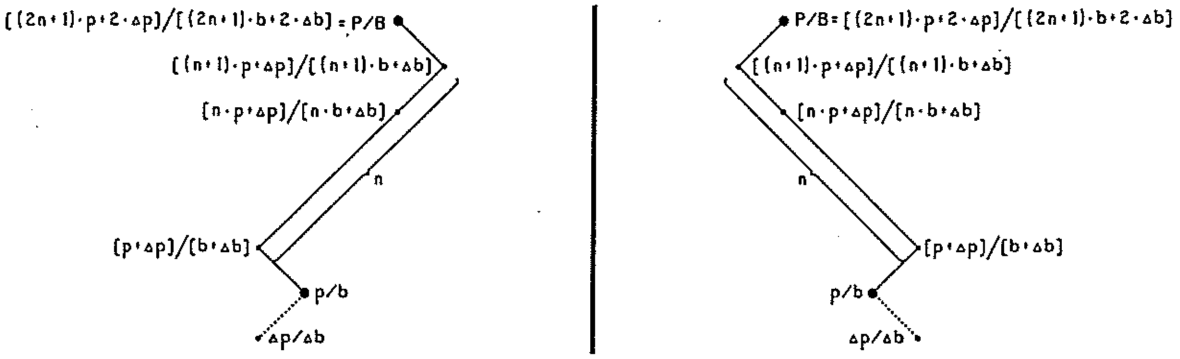

[(2n+1)·p+2·Δp]/[(2n+1)·b+2·Δb] = P/B
[(n+1)·p+Δp]/[(n+1)·b+Δb]
[n·p+Δp]/[n·b+Δb]
n
[p+Δp]/[b+Δb]
p/b
Δp/Δb
P/B=[(2n+1)·p+2·Δp]/[(2n+1)·b+2·Δb]
[(n+1)·p+Δp]/[(n+1)·b+Δb]
[n·p+Δp]/[n·b+Δb]
n
[p+Δp]/[b+Δb]
p/b
Δp/Δb

Fig. 415 — An $(n+1)$ order followed by a first order enlargement, both same Method.

A special case arises when $n = 0$ for the $(n+1)$ order enlargement. In this case we have two successive first order enlagements. By repeating this process a number of times we obtain the situation depicted in Fig. 416. Also here is of course the p/b parity-pair an invariant.

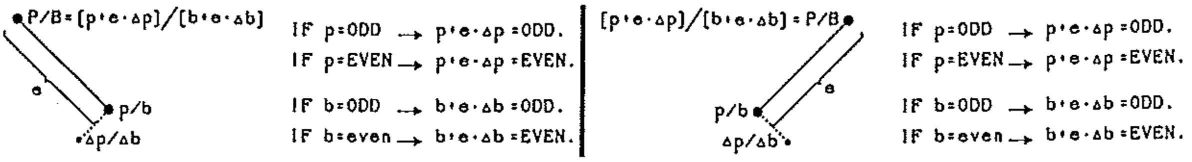


Fig. 416 — An even number (e) fundamental enlargements.

(2.) *Method I:* $p \cdot \Delta b = b \cdot \Delta p + 1$.

$$|\Delta b|_b = \frac{b \cdot |\Delta p|_p + 1}{p},$$

$$|\Delta p|_p = \frac{p \cdot |\Delta b|_b - 1}{b},$$

[†] See *The Braider*, Issue No. 6, pg. 109.

$$\begin{aligned}
\Delta b &= \frac{b(|\Delta p|_p + n\cdot p)+1}{p} = |\Delta b|_b + n\cdot b\,,\\
\Delta p &= \frac{p(|\Delta b|_b + n\cdot b)-1}{b} = |\Delta p|_p + n\cdot p\,,\\
p^* &= p + \Delta p = (n+1)p + |\Delta p|_p\,,\\
b^* &= b + \Delta b = (n+1)b + |\Delta b|_b\,,\\
\Delta b^* &= \Delta b = |\Delta b|_b + n\cdot b\,,
\end{aligned}$$

$$\text{since } |\Delta b|_b + n\cdot b < b^* \quad\to\quad |\Delta b|_b + n\cdot b = |\Delta b^*|_{b^*}\,,$$

$$\begin{aligned}
\Delta p^* &= \frac{p^*\cdot\Delta b^* - 1}{b^*}\\
&= \frac{p^*(|\Delta b|_b + n\cdot b)-1}{|\Delta b|_b + (n+1)b}\\
&= \frac{\{(n+1)p + |\Delta p|_p\}(|\Delta b|_b + n\cdot b)-1}{|\Delta b|_b + (n+1)b}\\
&= \frac{|\Delta b|_b\cdot n\cdot p + |\Delta b|_b\cdot p + |\Delta p|_p\cdot|\Delta b|_b + (n+1)n\cdot p\cdot b + |\Delta p|_p\cdot n\cdot b - 1}{|\Delta b|_b + (n+1)b}\,,
\end{aligned}$$

with $p\cdot|\Delta b|_b = b\cdot|\Delta p|_p + 1 \;\to$

$$\begin{aligned}
\Delta p^* &= \frac{|\Delta b|_b\cdot n\cdot p + |\Delta p|_p\cdot b + |\Delta p|_p\cdot|\Delta b|_b + (n+1)n\cdot p\cdot b + |\Delta p|_p\cdot n\cdot b}{|\Delta b|_b + (n+1)b}\\
&= \frac{\{|\Delta b|_b + (n+1)b\}(|\Delta p|_p + n\cdot p)}{|\Delta b|_b + (n+1)b} = |\Delta p|_p + n\cdot p\,,
\end{aligned}$$

$$\text{since } |\Delta p|_p + n\cdot p < p^* \;\to\quad \Delta p^* = |\Delta p|_p + n\cdot p = \Delta p = |\Delta p^*|_{p^*}\,.$$

Method II: $p\cdot\Delta b = b\cdot\Delta p - 1$.

$$\begin{aligned}
|\Delta b|_b &= \frac{b\cdot|\Delta p|_p - 1}{p}\,,\\
|\Delta p|_p &= \frac{p\cdot|\Delta b|_b + 1}{b}\,,
\end{aligned}$$

$$\begin{aligned}
\Delta b &= \frac{b(|\Delta p|_p + n\cdot p)-1}{p} = |\Delta b|_b + n\cdot b\,,\\
\Delta p &= \frac{p(|\Delta b|_b + n\cdot b)+1}{b} = |\Delta p|_p + n\cdot p\,,\\
p^* &= p + \Delta p = (n+1)p + |\Delta p|_p\,,\\
b^* &= b + \Delta b = (n+1)b + |\Delta b|_b\,,\\
\Delta b^* &= \Delta b = |\Delta b|_b + n\cdot b\,,
\end{aligned}$$

$$\text{since } |\Delta b|_b + n\cdot b < b^* \quad\to\quad |\Delta b|_b + n\cdot b = |\Delta b^*|_{b^*}\,,$$

$$
\begin{aligned}
\Delta p^* &= \frac{p^* \cdot \Delta b^* + 1}{b^*} \\
&= \frac{p^*(|\Delta b|_b + n \cdot b) + 1}{|\Delta b|_b + (n+1)b} \\
&= \frac{\{(n+1)p + |\Delta p|_p\}(|\Delta b|_b + n \cdot b) + 1}{|\Delta b|_b + (n+1)b} \\
&= \frac{|\Delta b|_b \cdot n \cdot p + |\Delta b|_b \cdot p + |\Delta p|_p \cdot |\Delta b|_b + (n+1)n \cdot p \cdot b + |\Delta p|_p \cdot n \cdot b + 1}{|\Delta b|_b + (n+1)b},
\end{aligned}
$$

with $p \cdot |\Delta b|_b = b \cdot |\Delta p|_p - 1 \;\rightarrow$

$$
\begin{aligned}
\Delta p^* &= \frac{|\Delta b|_b \cdot n \cdot p + |\Delta p|_p \cdot b + |\Delta p|_p \cdot |\Delta b|_b + (n+1)n \cdot p \cdot b + |\Delta p|_p \cdot n \cdot b}{|\Delta b|_b + (n+1)b} \\
&= \frac{\{|\Delta b|_b + (n+1)b\}(|\Delta p|_p + n \cdot p)}{|\Delta b|_b + (n+1)b} = |\Delta p|_p + n \cdot p,
\end{aligned}
$$

$$\text{since } |\Delta p|_p + n \cdot p < p^* \rightarrow \quad \Delta p^* = |\Delta p|_p + n \cdot p = \Delta p = |\Delta p^*|_{p^*}.$$

Thus if $\Delta b^* = \Delta b$, then also $\Delta p^* = \Delta p$. Hence when raising an over–under coded Regular Knot to a bigger over–under coded Regular Knot the p/b parity-pair is an invariant. Furthermore, when the first stage in the raising process is an $(n+1)$ order enlargement, then the second stage in the raising process must be a first order enlargement of the same Method. The special case in which the first stage is a first order enlargement ($n = 0$) is of course contained in the general case.

Question on pg. 479.

An area which would complicate the 'classification' of over–under coded Regular Knots is, for example, in **appliquè**. Here a one-string Regular Cylindrical Braid does not necessarily remain a one-string braid (see *The Braider*, Issue No. 15, pp. 344 – 346).

Regular Cylindrical Bihelix Braids
with an
Interbraided Overlay

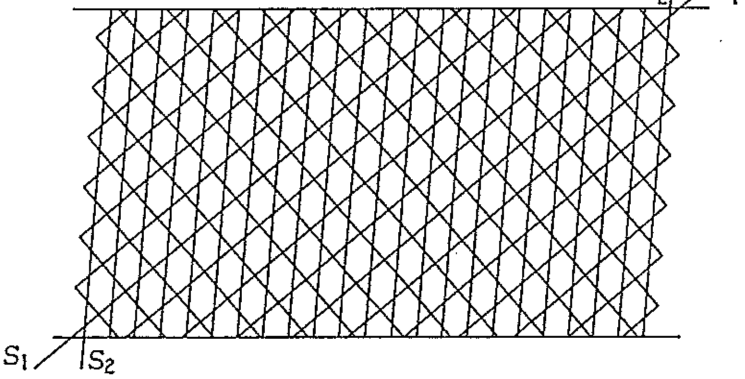


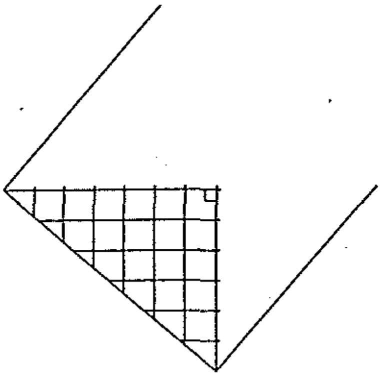
Fig. 417 — A string-run example of a Regular Cylindrical Bihelix Braid with an Interbraided Overlay.

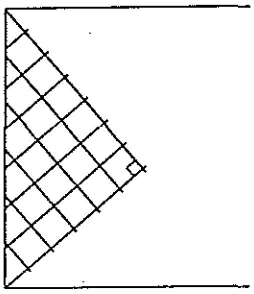
The [7, 6] Prime Regular Cylindrical Bihelix Braid[†] $S_1 \longrightarrow W_1$ depicted in Fig. 417 has $N^* = 144$ crossings in its string-run. Its equivalent extended Premature Regular Cylindrical Braid has $b = 7$ and $p = 24$ with $\delta = 6$, hence is the $p/b = 25/7$ Premature Regular Cylindrical Knot. The Interbraided Overlay is the string-run $S_2 \longrightarrow W_2$.

In Fig. 418 are indicated the first 17 single string Prime Regular Cylindrical Bihelix Braids (the Prime Regular Cylindrical Bihelix Knots). Those whose equivalent Premature Regular Cylindrical Braid has $\delta = 0$ or $\delta = n = 6$, hence which are equivalent Premature Regular Cylindrical Knots are generally the quickest and easiest to braid. Neglecting the trivial $p = 1$ with $\delta = 0$, their p-values with $\delta = 0$ can readily be found with the formulae:

$$\left.\begin{matrix} p = 2kn + 1 \\ p = 2kn + 2 \end{matrix}\right\} \quad \text{where } k = 1, 2, 3, \cdots; \quad p \text{ and } b = n + 1 \text{ to be coprime.}$$

⋆⋆ *Show how these formulae can readily be derived.*





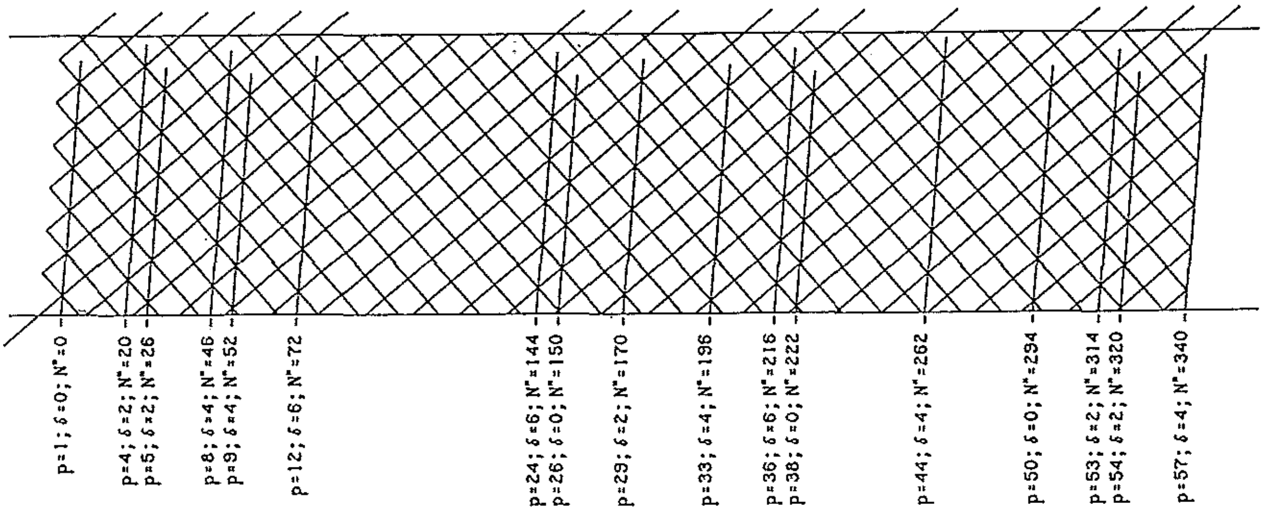


Fig. 418 — Prime Regular Cylindrical Bihelix Knots in the [7, 6] Bihelix Round Braid.

Note that a Premature Regular Cylindrical Braid with $p = j$ and $\delta = n$ is identical to the Premature Regular Cylindrical Braid with $p = j + 1$ and $\delta = 0$.

The string-run diagrams of the first ten [7, 6] Prime Regular Cylindrical Bihelix Knots are depicted in Fig. 419, while the table in Fig. 420 gives the particulars of the first 84 Prime Regular Cylindrical Bihelix Braids. Note that the entry $p = 13$, $\delta = 6$

[†] Refer to *The Braider*, Issue No. 17, pp. 378–392, and Issue No. 18, pp. 393–401.

also occurs as entry $p = 14$, $\delta = 0$; the entry $p = 24$, $\delta = 6$ also occurs as entry $p = 25$, $\delta = 0$; the entry $p = 36$, $\delta = 6$ also occurs as entry $p = 37$, $\delta = 0$; the entry $p = 48$, $\delta = 6$ also occurs as entry $p = 49$, $\delta = 0$; the entry $p = 60$, $\delta = 6$ also occurs as entry $p = 61$, $\delta = 0$; the entry $p = 72$, $\delta = 6$ also occurs as entry $p = 73$, $\delta = 0$.

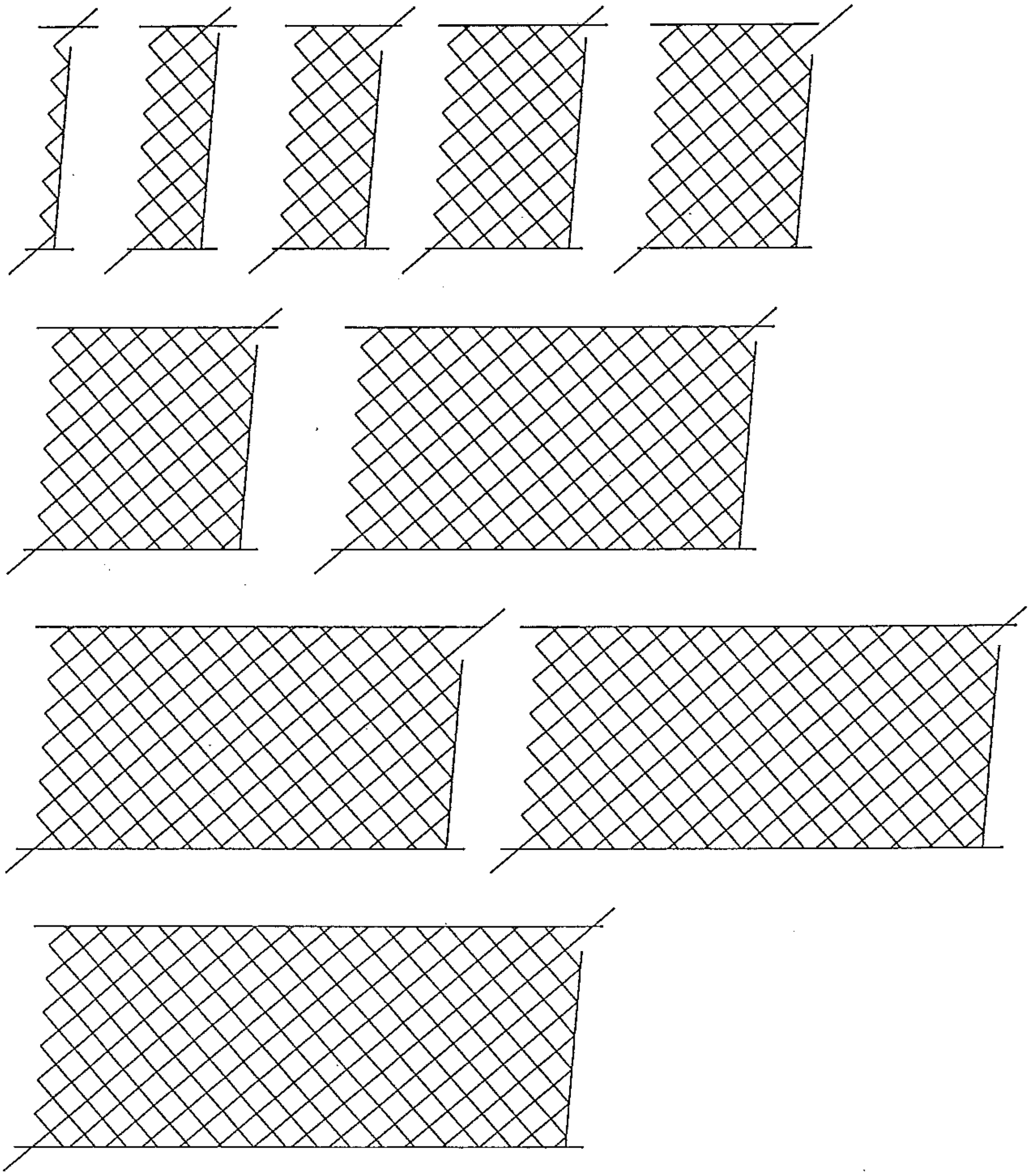


Fig. 419 — The first ten Prime Regular Cylindrical Bihelix Knots.

When we superimpose a **Matthew Walker** coding on the string-run of a Prime Regular Cylindrical Bihelix Knot, the braiding of any Prime Regular Cylindrical Bihelix Knot by means of its equivalent extended Premature Regular Cylindrical Knot becomes of course a very simple matter. Next we can then interbraid the Matthew Walker coded Prime Regular Cylindrical Bihelix Knot with an under–over coded overlay. An example is shown in Fig. 421. In this example, the Matthew Walker coded Prime Regular

Cylindrical Bihelix Knot (S_1) has $N^* = 144$ crossings and is equivalent to a Matthew Walker coded Premature Regular Knot with $p = 25$ ($\delta = 0$). It is interbraided with the under–over coded overlay (S_2) (see also Fig. 422).

PRIME REGULAR CYLINDRICAL BIHELIX BRAIDS NUMBER OF CROSSINGS N*	PREMATURE REGULAR CYLINDRICAL BRAIDS b=7		
	p	δ	NUMBER OF ESSENTIAL STRINGS
0	1	0	1
7	2	1	2
13	3	1	2
20	4	2	1
26	5	2	1
33	6	3	4
39	7	3	4
46	8	4	1
52	9	4	1
59	10	5	2
65	11	5	2
72	12	6	1
78	13	6	7
78	14	0	7
79	14	1	6
85	15	1	2
92	16	2	3
98	17	2	3
105	18	3	2
111	19	3	2
118	20	4	5
124	21	4	3
131	22	5	2
137	23	5	2
144	24	6	1
144	25	0	1
150	26	0	1
157	27	1	2
163	28	1	6
164	28	2	5
170	29	2	1
177	30	3	2
183	31	3	4
190	32	4	3
196	33	4	1
203	34	5	6
209	35	5	2
216	36	6	1
216	37	0	1
222	38	0	1
229	39	1	2
235	40	1	2
242	41	2	3
248	42	2	5
249	42	3	4

PRIME REGULAR CYLINDRICAL BIHELIX BRAIDS NUMBER OF CROSSINGS N*	PREMATURE REGULAR CYLINDRICAL BRAIDS b=7		
	p	δ	NUMBER OF ESSENTIAL STRINGS
255	43	3	2
262	44	4	1
268	45	4	3
275	46	5	2
281	47	5	2
288	48	6	7
288	49	0	7
294	50	0	1
301	51	1	2
307	52	1	2
314	53	2	1
320	54	2	1
327	55	3	4
333	56	3	4
334	56	4	3
340	57	4	1
347	58	5	2
353	59	5	2
360	60	6	1
360	61	0	1
366	62	0	1
373	63	1	6
379	64	1	2
386	65	2	3
392	66	2	3
399	67	3	2
405	68	3	2
412	69	4	5
418	70	4	3
419	70	5	2
425	71	5	2
432	72	6	1
432	73	0	1
438	74	0	1
445	75	1	2
451	76	1	2
458	77	2	5
464	78	2	1
471	79	3	2
477	80	3	4
484	81	4	3
490	82	4	1
497	83	5	6
503	84	5	2
504	85	0	1

Fig. 420 — The first 84 Prime Regular Cylindrical Bihelix Braids.

The interbraided overlay can of course be braided with the string of the Prime Regular Cylindrical Bihelix Knot; the overall braid consists then of one string. This has been depicted in Fig. 423, the end W_1 can finally be worked away over W_2. The equivalent $p/b = 25/7$ Matthew Walker coded Premature Regular Knot with its interbraided under–over coded overlay is shown in Fig. 425.

A distance of $2w$ (twice the string width) is maintained between the adjacent strands as shown in Fig. 424.

This braid can make, for example, an attractive covering knot for a quirt handle.

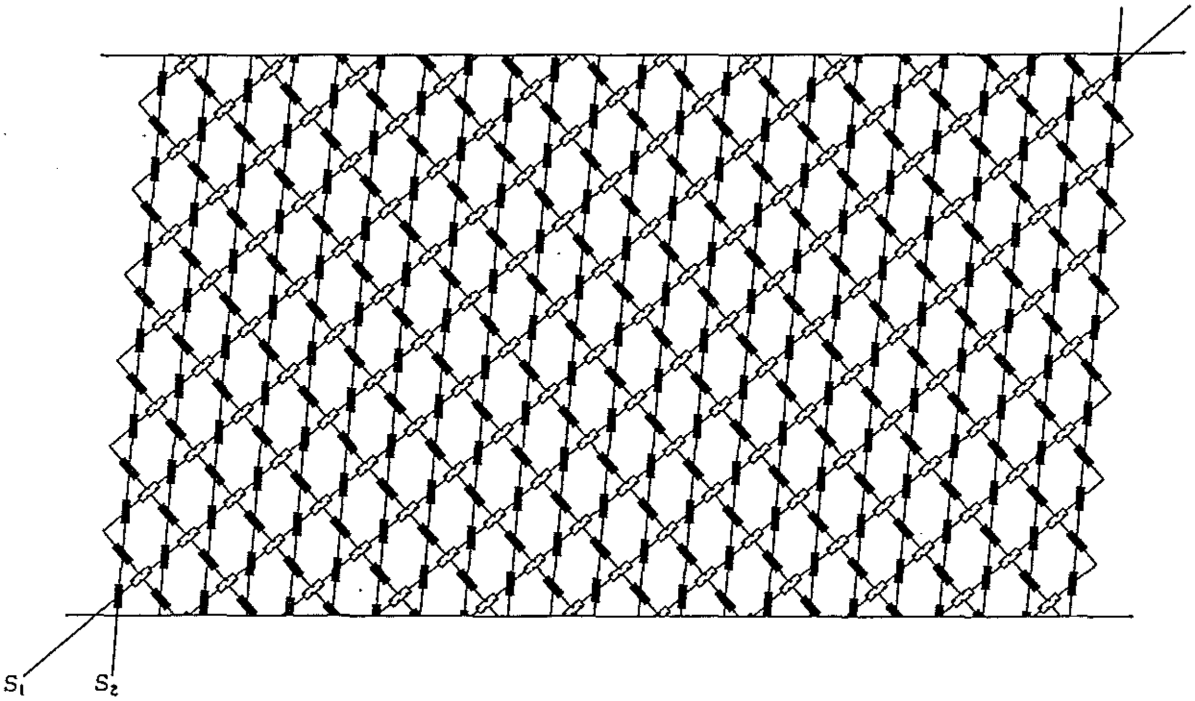


Fig. 421 — A Matthew Walker coded [7, 6] Prime Regular Cylindrical Bihelix Knot with an Interbraided under–over coded Overlay.

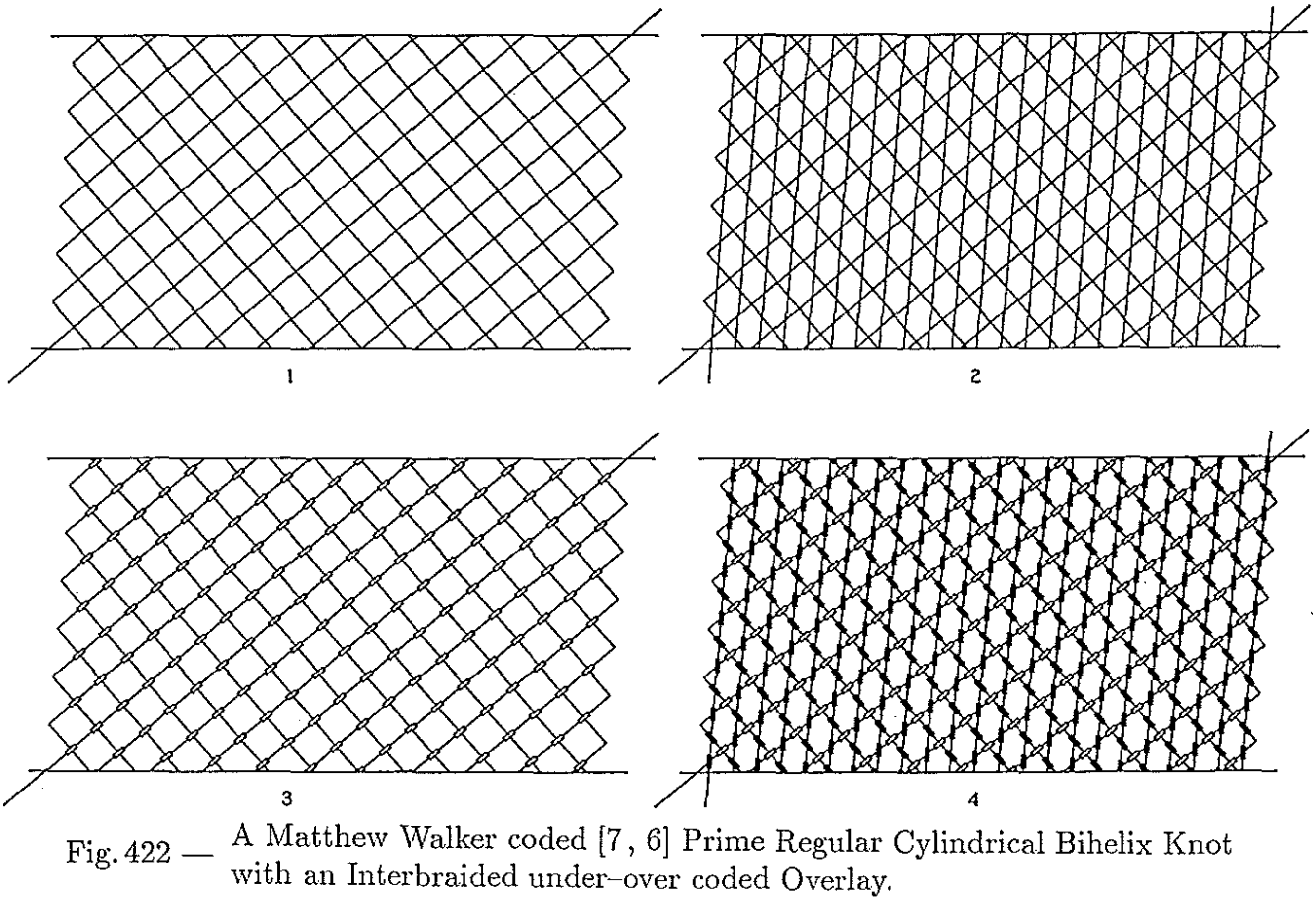


Fig. 422 — A Matthew Walker coded [7, 6] Prime Regular Cylindrical Bihelix Knot with an Interbraided under–over coded Overlay.

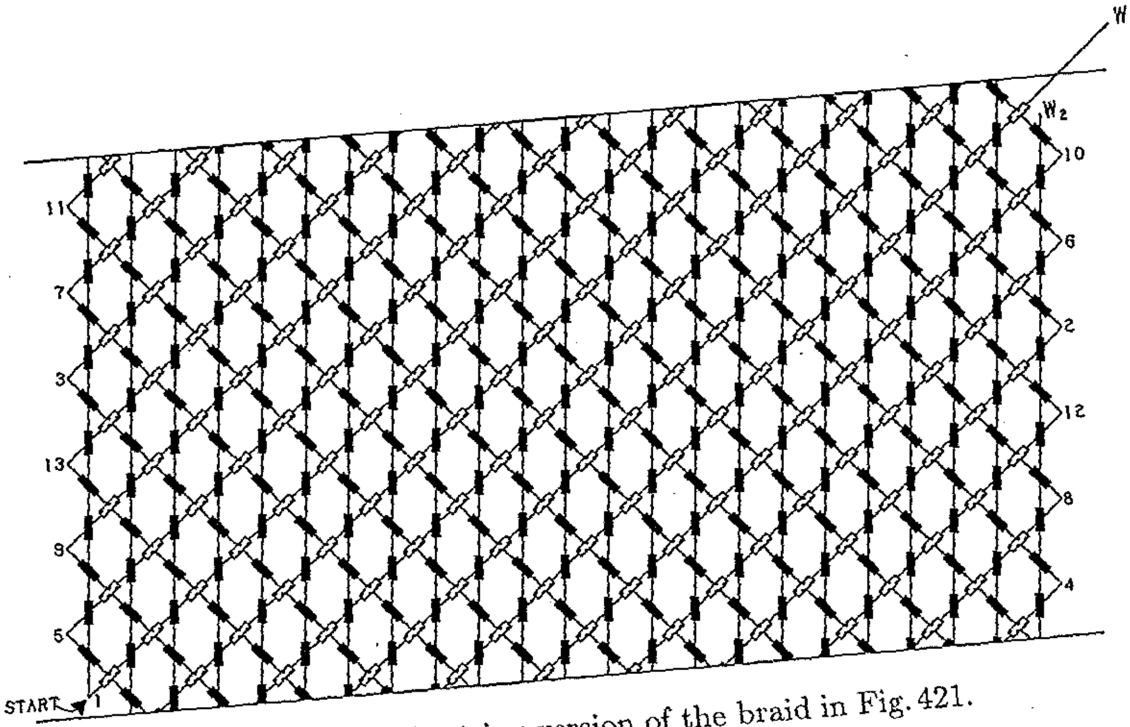


Fig. 423 — The single string version of the braid in Fig. 421.

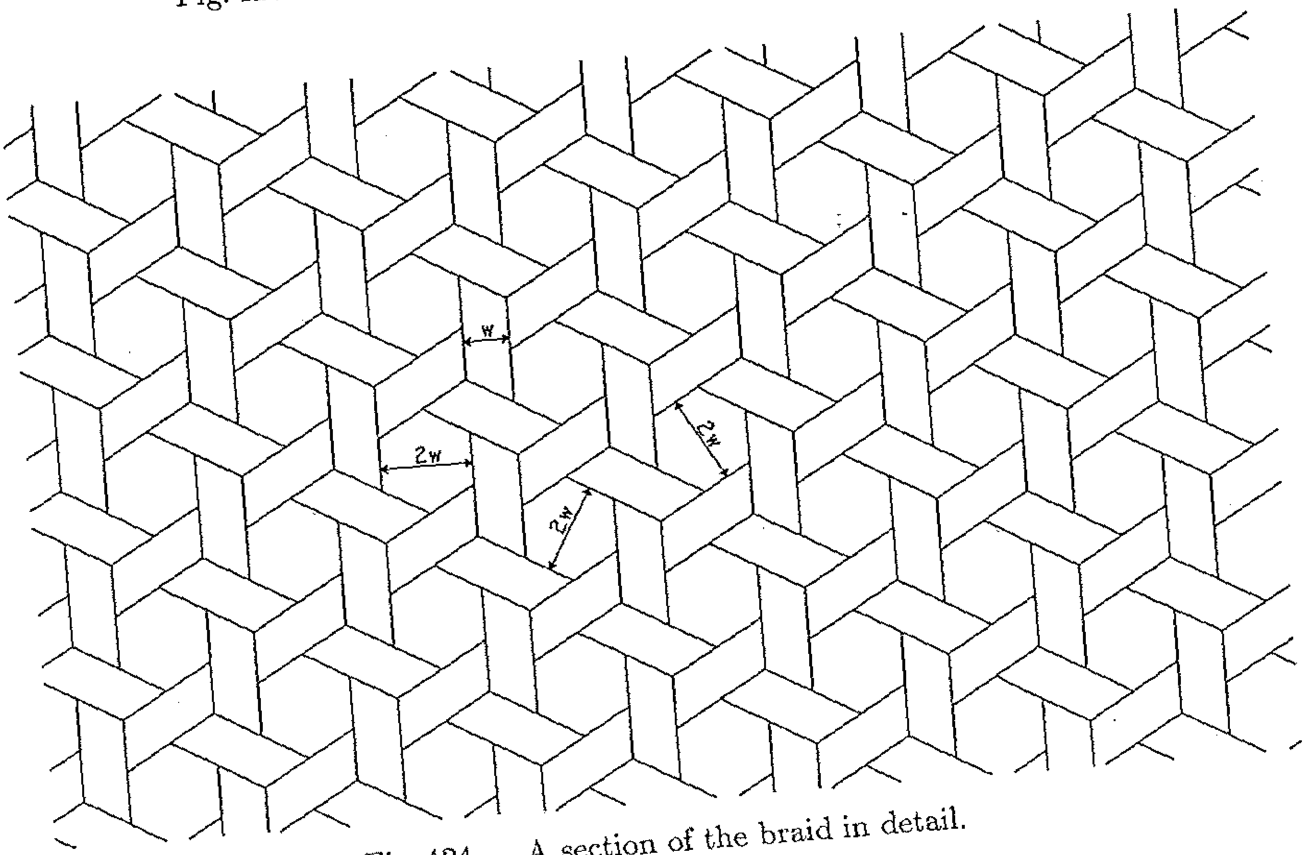


Fig. 424 — A section of the braid in detail.

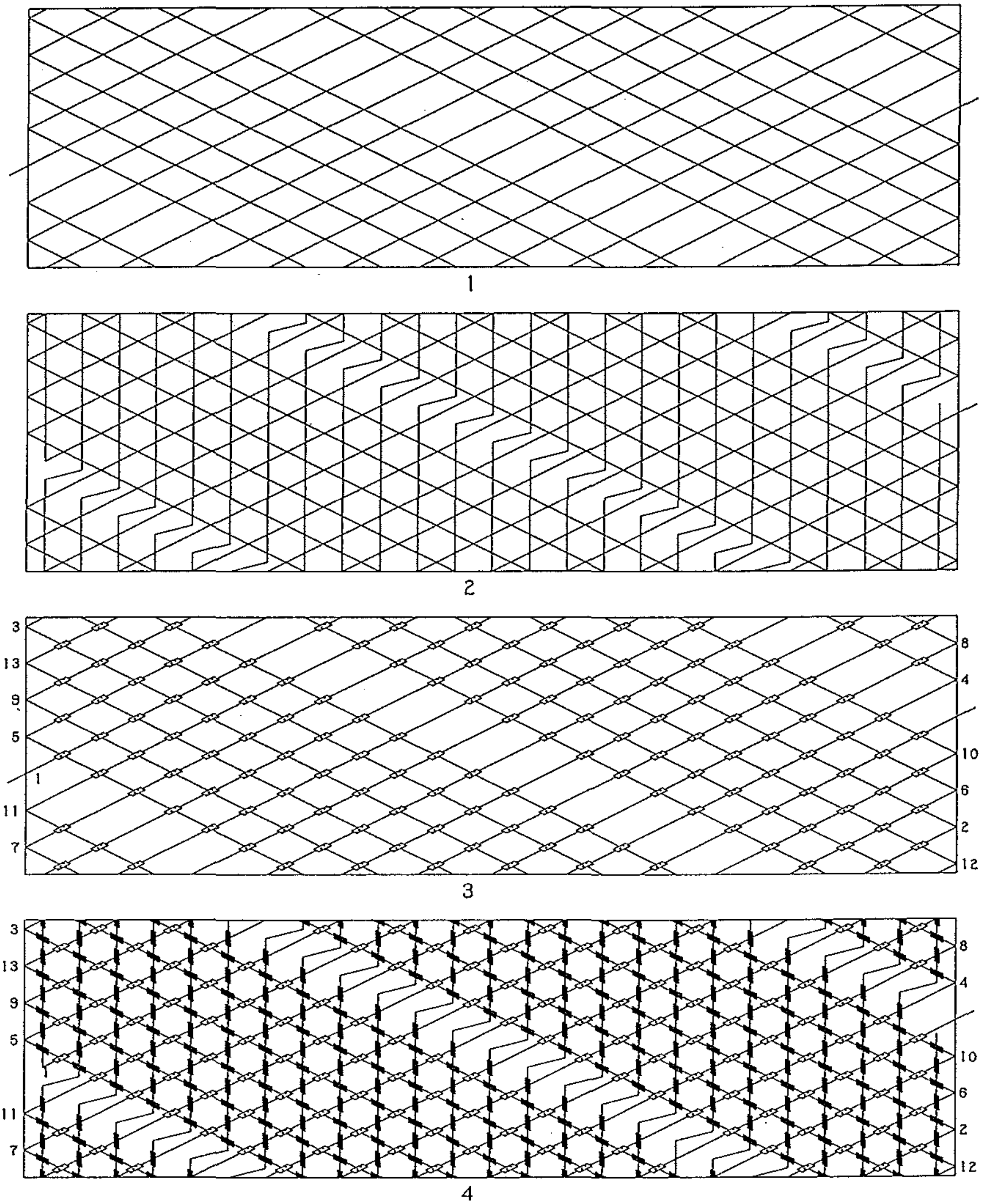


Fig. 425 — The equivalent $p/b = 25/7$ Matthew Walker coded Premature Regular Knot with its Interbraided under–over coded Overlay.

When thin lace is used for braiding the $[n{+}1\,,\,n]$, and $[n\,,\,n{+}1]$, Regular Cylindrical Bihelix Braids ($\sigma = 1$) with interbraided under–over coded Overlay, a useful formula is:

$$2n+1 = \frac{0.73C}{w},$$

where C is the circumference of the object to be covered. For the $[n+1\,,\,n]$, or the $[n\,,\,n+1]$, Regular Cylindrical Bihelix Braid ($\sigma = 1$), the value of w can then be made such that $2n+1$ is very near an odd positive integer.

Fig. 426 depicts an example of a Matthew Walker coded 2-string Regular Cylindrical Bihelix Braid with an Interbraided under–over coded Overlay.

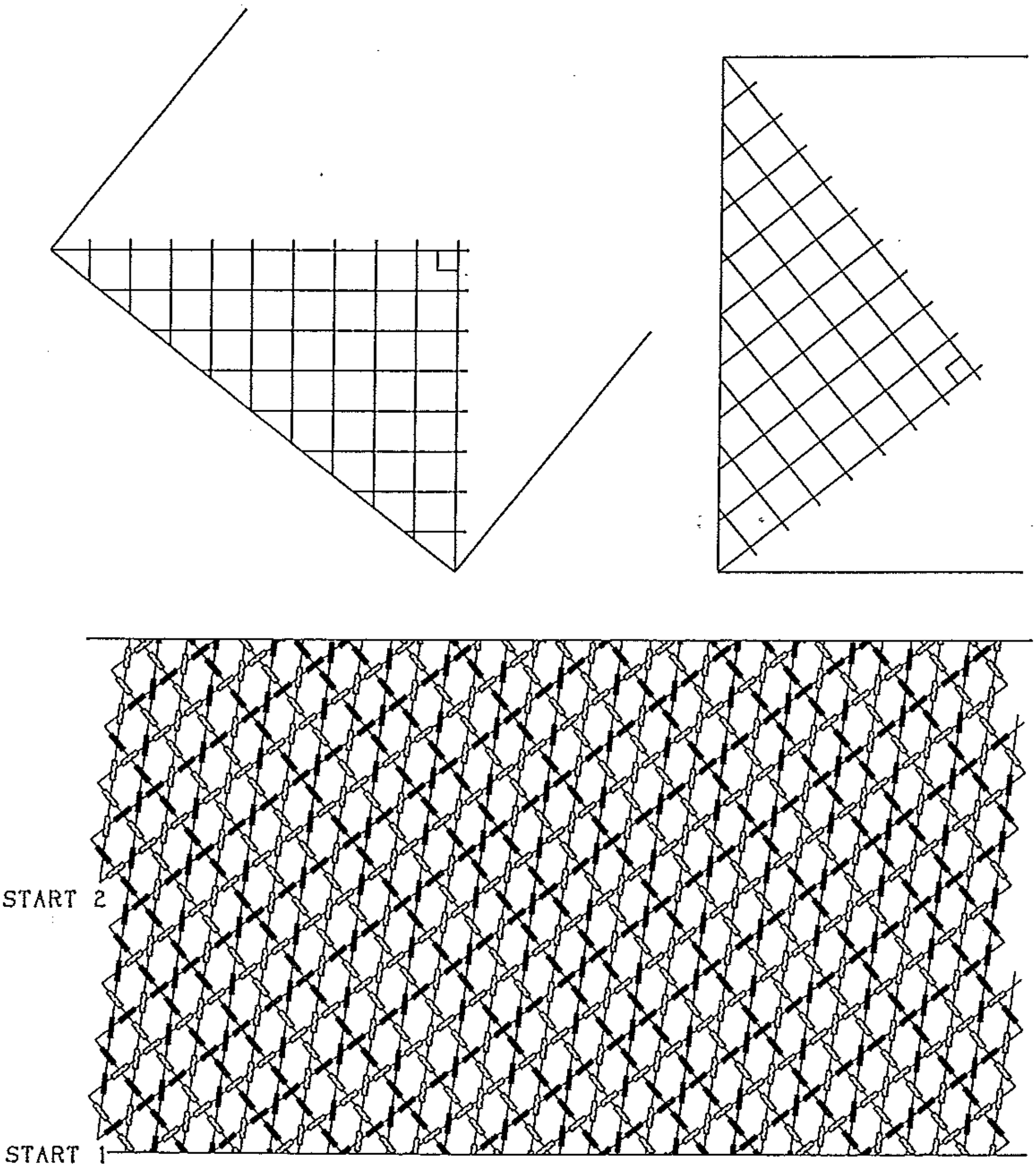


Fig. 426 — A Matthew Walker coded two string [10, 8] Regular Cylindrical Bihelix Braid ($\sigma = 2$) with Interbraided under–over coded Overlay.

Since for the same braiding material the distortion of the hexagons depends on the slant of the bight-boundary lines, this distortion is for an $[n^* + 2\,, n^*]$ Regular Cylindrical Bihelix Braid the same as for an $[n + 1\,, n]$ Regular Cylindrical Bihelix Braid when $n^* = 2n$. Thus for bigger diameter braids, the Matthew Walker coded $[n^* + 2\,, n^*]$ Regular Cylindrical Bihelix Braids with an interbraided under–over coded overlay provide us with an opportunity to create some interesting colour patterns.

Nested Cylindrical Braids

In the previous Issue of *The Braider*, No. 21, pp. 466 – 470 we have discussed how a right-hand valid sequence set is obtained from its corresponding left-hand valid sequence set. We also mentioned that the most often encountered left and right bight-boundary position specifications were $222\cdots$ with $\mathcal{K}_l = A_l$ and $\mathcal{K}_r = A_r$. Since in practice we generally limit A_l and/or A_r to 7, we have shown for $2 \leq A_l \leq 7$ in Figs. 427 – 432 the left valid cyclic sequence sets of k-values and their associated left bight-boundary arrangements.

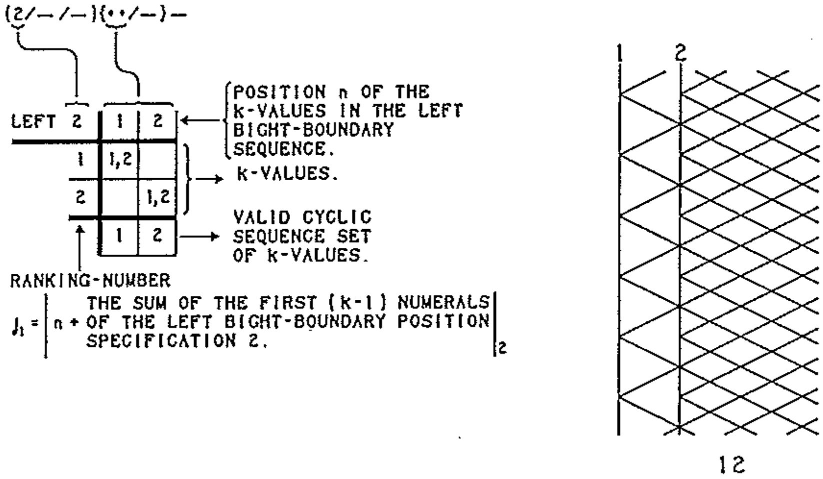

(2/–/–)(••/–)–
LEFT 2
1
2
1
2
1,2
1,2
1
2
POSITION n OF THE K-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.
K-VALUES.
VALID CYCLIC SEQUENCE SET OF K-VALUES.
RANKING-NUMBER
J₁ = | n + THE SUM OF THE FIRST (K-1) NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 2. | 2
1
2
12

Fig. 427 — Left bight-boundary specification 2 with $\mathcal{K}_l = A_l = 2$.

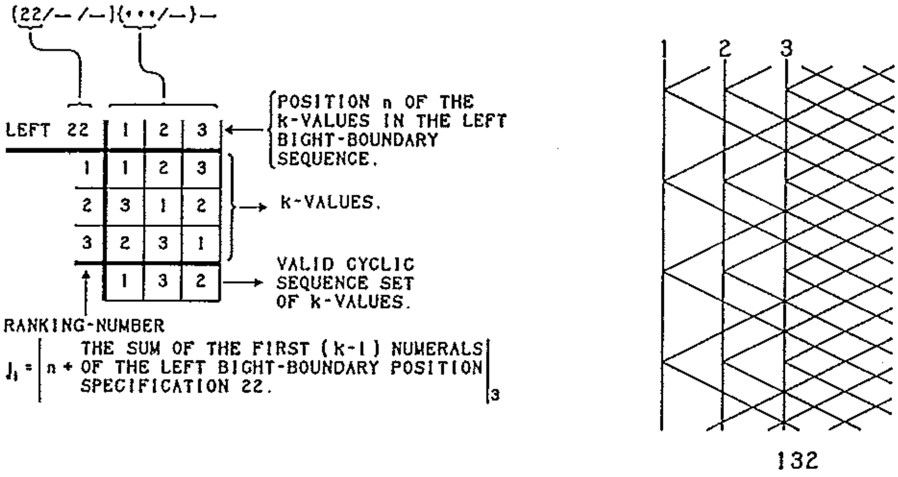
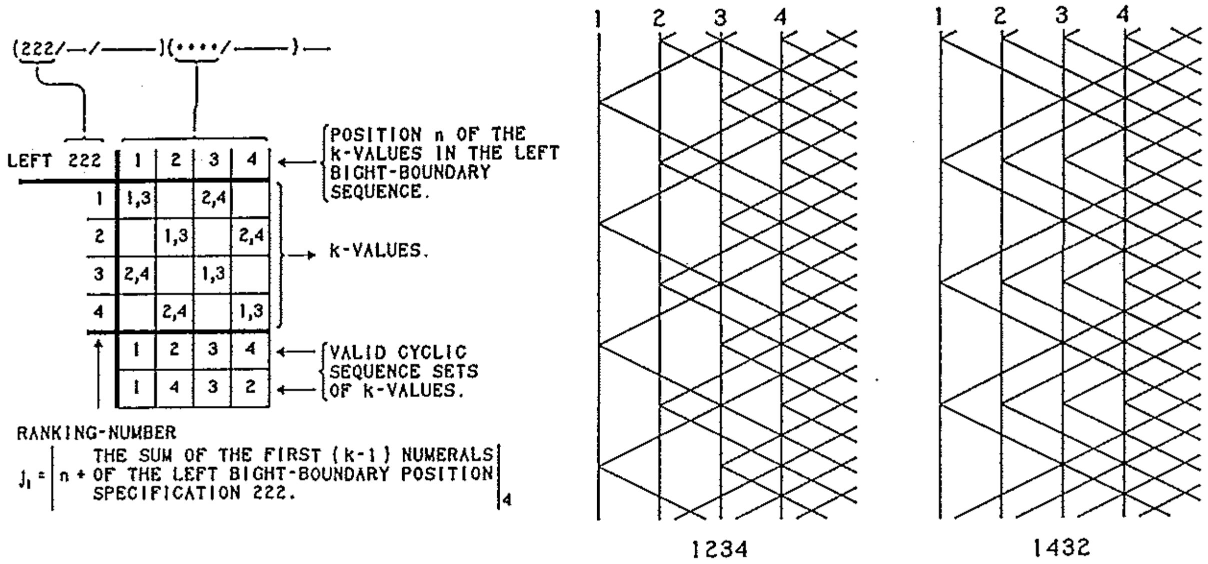

(22/–/–)(•••/–)–
LEFT 22
1
2
3
1
2
3
1
2
3
3
1
2
2
3
1
1
3
2
POSITION n OF THE K-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.
K-VALUES.
VALID CYCLIC SEQUENCE SET OF K-VALUES.
RANKING-NUMBER
J₁ = | n + THE SUM OF THE FIRST (K-1) NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 22. | 3
1
2
3
132

Fig. 428 — Left bight-boundary specification 22 with $\mathcal{K}_l = A_l = 3$.


(222/—/——)(••••/——)—
POSITION n OF THE k-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.
k-VALUES.
VALID CYCLIC SEQUENCE SETS OF k-VALUES.
RANKING-NUMBER
J_l = | n + THE SUM OF THE FIRST (k-1) NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 222. |_4
1
2
3
4
1234
1432

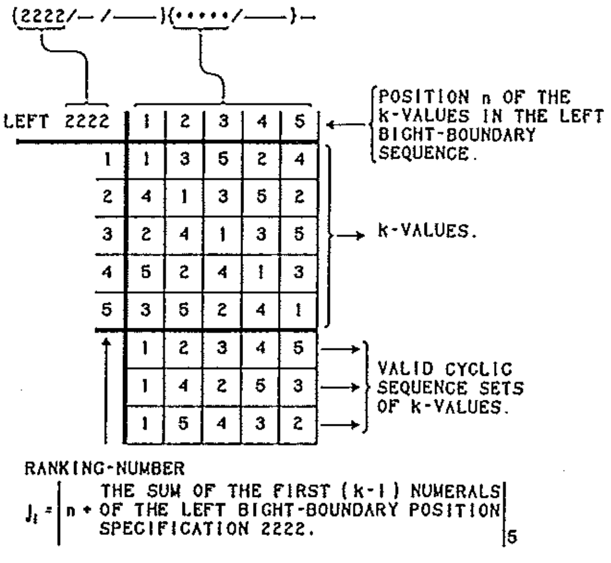
LEFT 222	1	2	3	4
1	1,3		2,4	
2		1,3		2,4
3	2,4		1,3	
4		2,4		1,3
	1	2	3	4
	1	4	3	2

Fig. 429 — Left bight-boundary specification 222 with $\mathcal{K}_l = A_l = 4$.

The more commonly encountered Nested Cylindrical Braids which have the bight-boundary position specifications $222\cdots$ with $\mathcal{K}_l = A_l$ and $\mathcal{K}_r = A_r$ are those which have the left sequence set $1A_l(A_l-1)(A_l-2)(A_l-3)\cdots432$ and the right sequence set $k|k+1|_{A_r}|k+2|_{A_r}|k+3|_{A_r}\cdots A_r123\cdots|k-2|_{A_r}|k-1|_{A_r}$, where $1 \leq k \leq A_r$. They are the **Asymmetric Regular Nested Cylindrical Braids** when $A_r \neq A_l$, and they are the **Regular Nested Cylindrical Braids** when $A_r = A_l = A$. Thus the **Regular Nested Cylindrical Braids** have left sequence set $1A(A-1)(A-2)(A-3)\cdots432$ and the right sequence set $k|k+1|_A|k+2|_A|k+3|_A\cdots A123\cdots|k-2|_A|k-1|_A$, where $1 \leq k \leq A$. We shall discuss first the Regular Nested Cylindrical Braids and then the Asymmetric Regular Nested Cylindrical Braids.

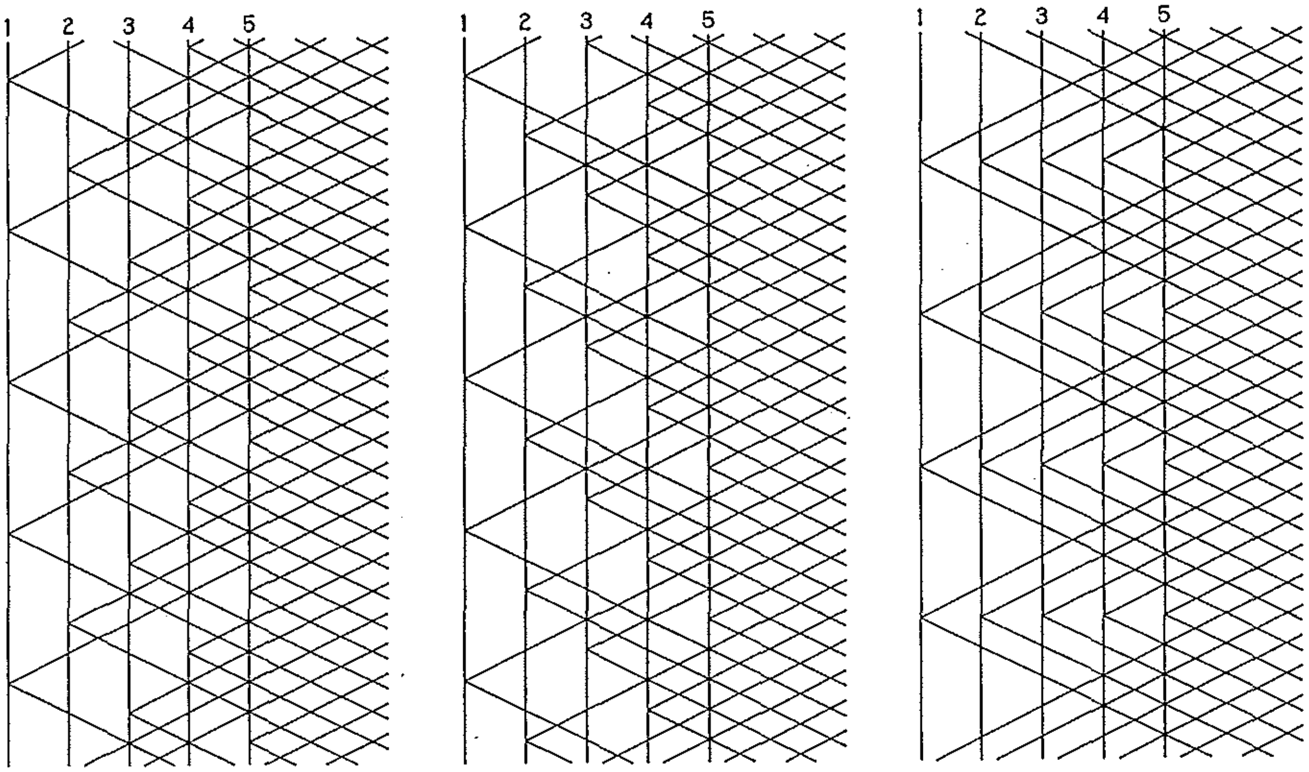
The Regular Nested Cylindrical Braids:

Since the bight-boundary position specification for both the left and right bight-edges are the same and equal to $222\cdots$ with $\mathcal{K}_l = A_l = \mathcal{K}_r = A_r = A$ the formulae on pg. 417 can be simplified considerably.


(2222/—/——)(•••••/——)—
POSITION n OF THE k-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.
k-VALUES.
VALID CYCLIC SEQUENCE SETS OF k-VALUES.
RANKING-NUMBER
J_l = | n + THE SUM OF THE FIRST (k-1) NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 2222. |_5

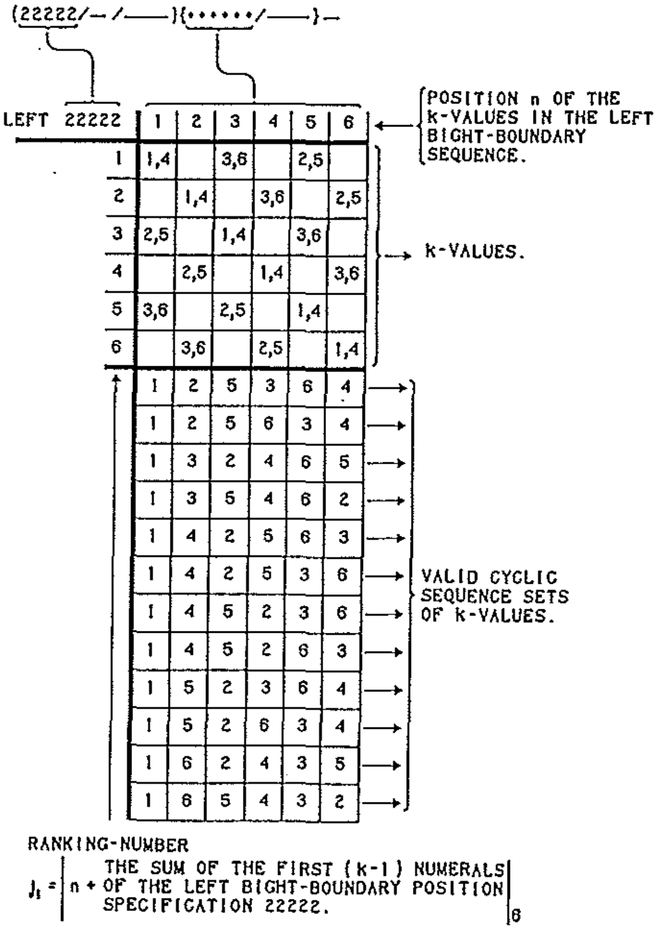
LEFT 2222	1	2	3	4	5
1	1	3	5	2	4
2	4	1	3	5	2
3	2	4	1	3	5
4	5	2	4	1	3
5	3	5	2	4	1
	1	2	3	4	5
	1	4	2	5	3
	1	5	4	3	2

Fig. 430 — Left bight-boundary specification 2222 with $\mathcal{K}_l = A_l = 5$.



12345 14253 15432

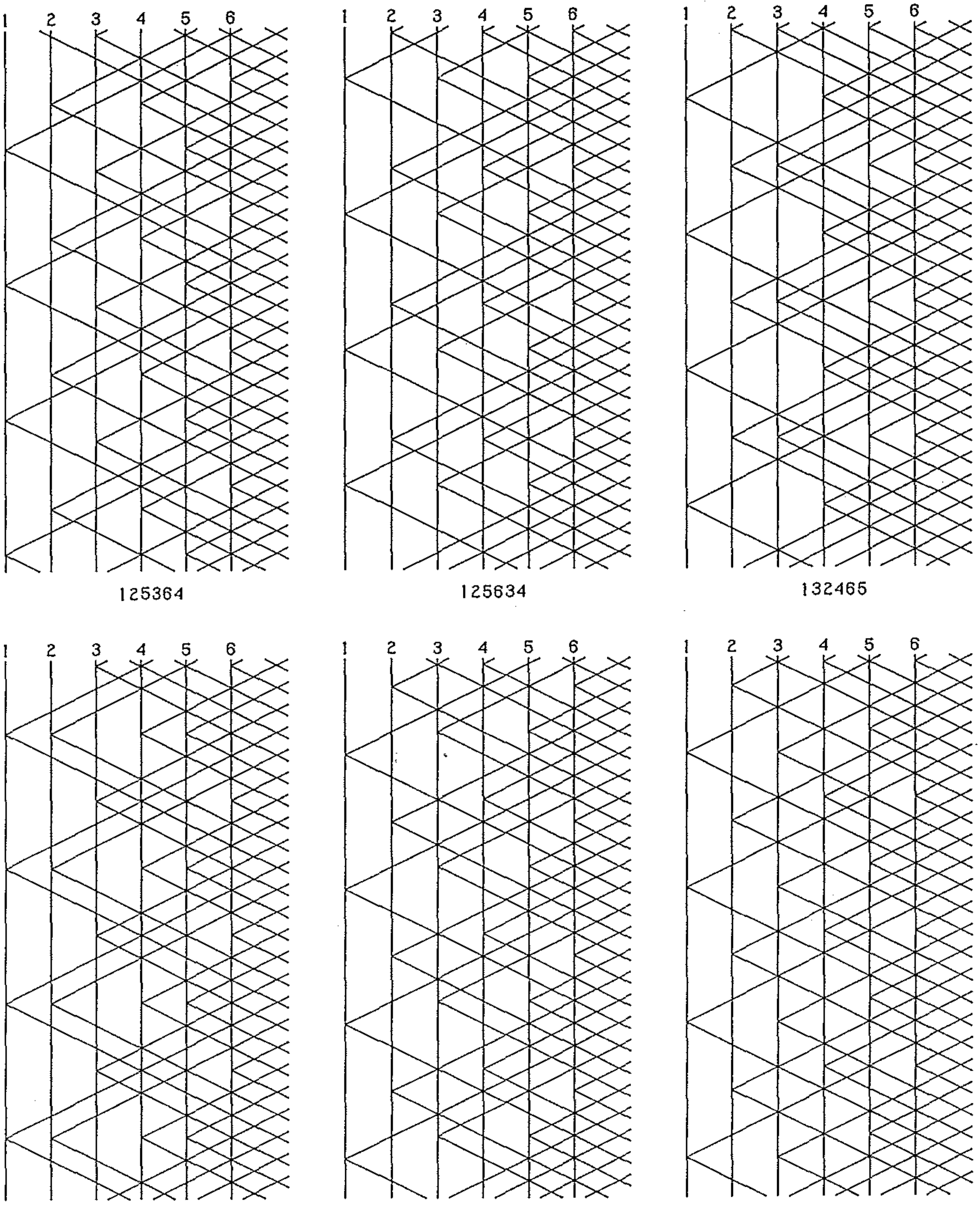
Fig. 430 cont.


(22222/—/——)(••••••/——)—
POSITION n OF THE k-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.
k-VALUES.
VALID CYCLIC SEQUENCE SETS OF k-VALUES.
RANKING-NUMBER

LEFT 22222	1	2	3	4	5	6
1	1,4		3,6		2,5	
2		1,4		3,6		2,5
3	2,5		1,4		3,6	
4		2,5		1,4		3,6
5	3,6		2,5		1,4	
6		3,6		2,5		1,4
	1	2	5	3	6	4
	1	2	5	6	3	4
	1	3	2	4	6	5
	1	3	5	4	6	2
	1	4	2	5	6	3
	1	4	2	5	3	6
	1	4	5	2	3	6
	1	4	5	2	6	3
	1	5	2	3	6	4
	1	5	2	6	3	4
	1	6	2	4	3	5
	1	6	5	4	3	2

$J_l = \left| n + \text{THE SUM OF THE FIRST } (k-1) \text{ NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 22222.} \right|_6$

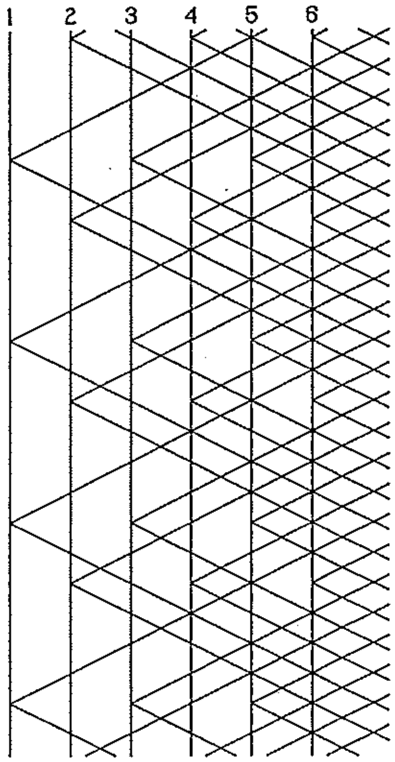
Fig. 431 — Left bight-boundary specification 22222 with $K_l = A_l = 6$.



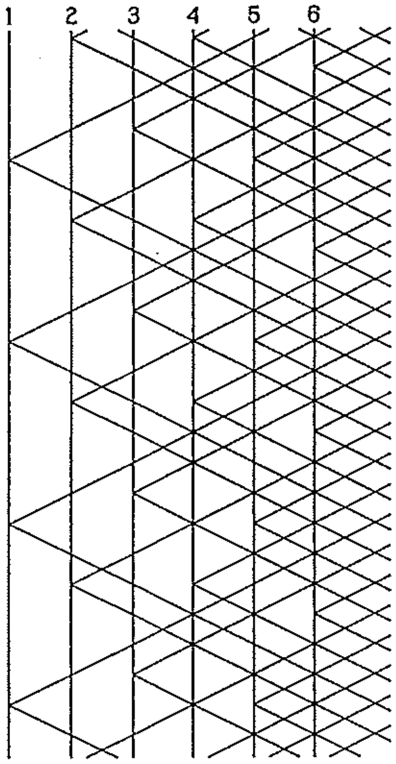
125364 125634 132465

135462 142563 142536

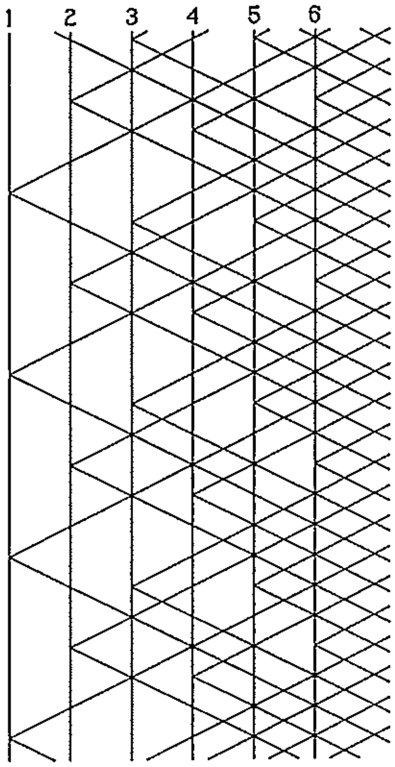
Fig. 431 cont.



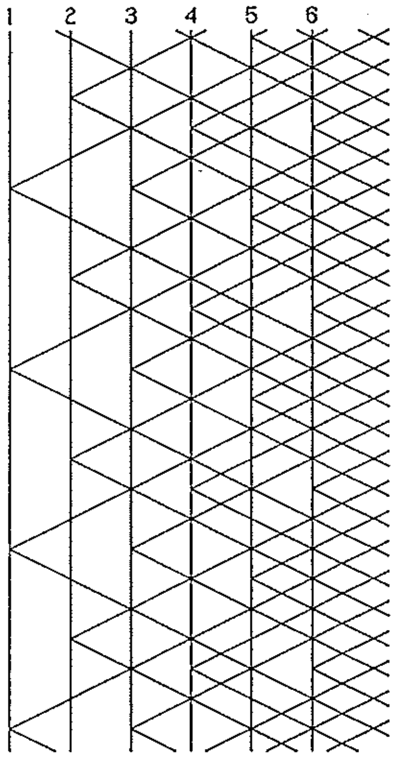
145236



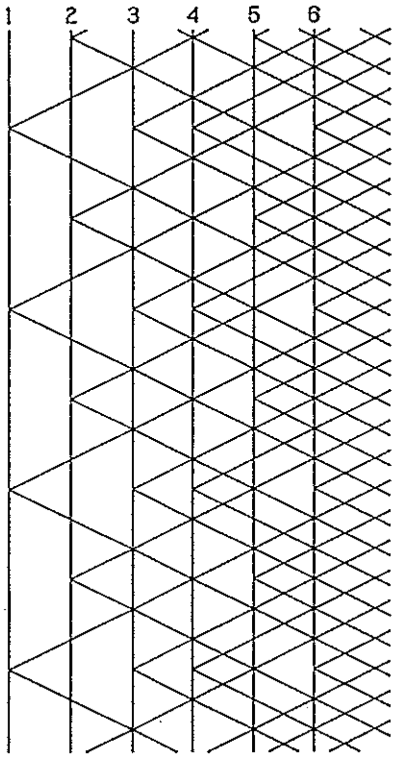
145263



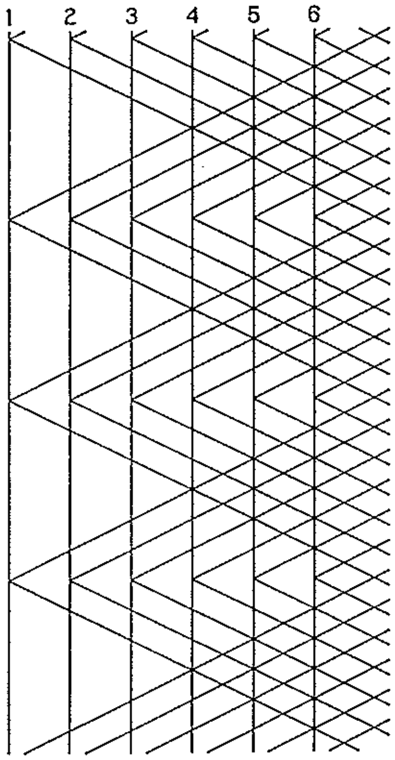
152364



152634



162435



165432

Fig. 431 cont.

The formulae (see pg. 417):

$$\Delta_{l_i} = \text{sum of the last } (\mathcal{K}_l - l_i) \text{ numerals in left bight-boundary position specification sequence.}$$

$$\Delta_{l_{i+1}} = \text{sum of the last } (\mathcal{K}_l - l_{i+1}) \text{ numerals in left bight-boundary position specification sequence.}$$

$$\Delta_{r_i} = \text{sum of the first } (\mathcal{K}_r - r_i) \text{ numerals in right bight-boundary position specification sequence.}$$

$$j_l' = |j_l + \Delta_{l_i} + x + \Delta_{r_i}|_{A_l}\,.$$

$$j_r' = \left|j_r + \Delta_{r_i} + x + \Delta_{l_{i+1}}\right|_{A_r}\,.$$

modify to:

$$\Delta_{l_i} = 2(A - l_i)\,.$$

$$\Delta_{l_{i+1}} = 2(A - l_{i+1})\,.$$

$$\Delta_{r_i} = 2(A - r_i)\,.$$

$$j_l' = |j_l + x - 2(l_i + r_i)|_A\,.$$

$$j_r' = |j_r + x - 2(r_i + l_{i+1})|_A\,.$$

(222222/–/——)(•••••••/——)–

POSITION n OF THE K-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.

LEFT 222222	1	2	3	4	5	6	7
1	1	4	7	3	6	2	5
2	5	1	4	7	3	6	2
3	2	5	1	4	7	3	6
4	6	2	5	1	4	7	3
5	3	6	2	5	1	4	7
6	7	3	6	2	5	1	4
7	4	7	3	6	2	5	1
	1	2	3	4	5	6	7
	1	2	3	5	7	6	4
	1	2	3	7	5	4	6
	1	2	4	6	5	3	7
	1	2	6	4	3	5	7
	1	3	2	7	4	5	6
	1	3	4	5	2	7	6
	1	3	5	4	2	6	7
	1	3	5	6	7	4	2
	1	3	5	7	2	4	6
	1	5	2	6	3	7	4
	1	5	3	2	4	6	7
	1	5	6	7	2	4	3
	1	6	3	4	5	7	2
	1	6	4	2	7	5	3
	1	6	5	7	2	3	4
	1	7	2	4	5	6	3
	1	7	5	2	3	4	6
	1	7	6	5	4	3	2

K-VALUES

VALID CYCLIC SEQUENCE SETS OF K-VALUES.

RANKING-NUMBER

$$J_l = \left| n + \begin{array}{l}\text{THE SUM OF THE FIRST } (k-1) \text{ NUMERALS}\\ \text{OF THE LEFT BIGHT-BOUNDARY POSITION}\\ \text{SPECIFICATION 222222.}\end{array} \right|_7$$

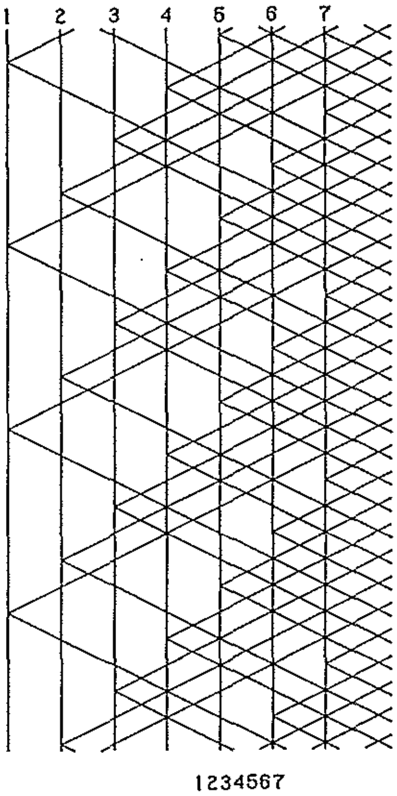
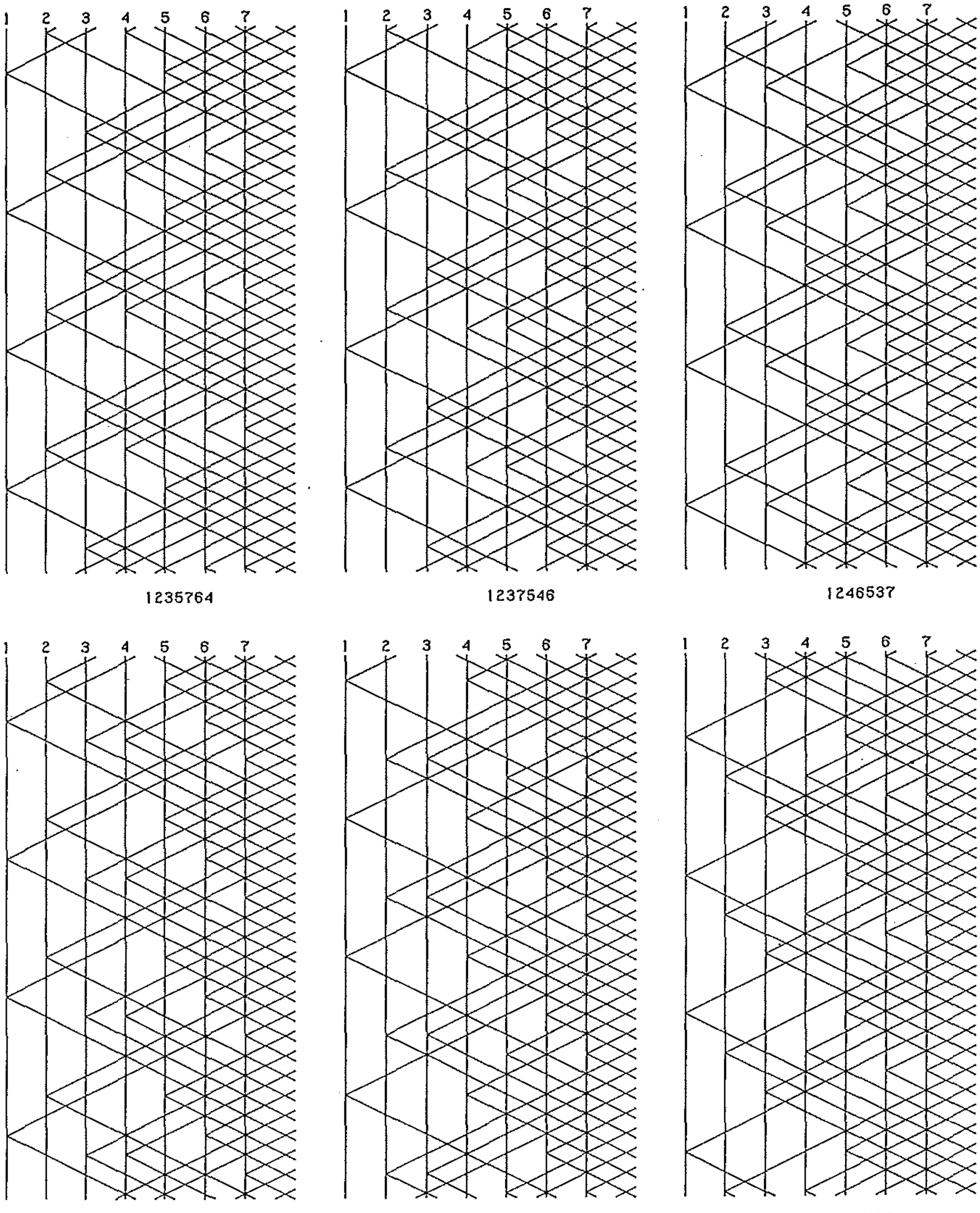


Fig. 432 — Left bight-boundary specification 222222 with $\mathcal{K}_l = A_l = 7$.



1235764

1237546

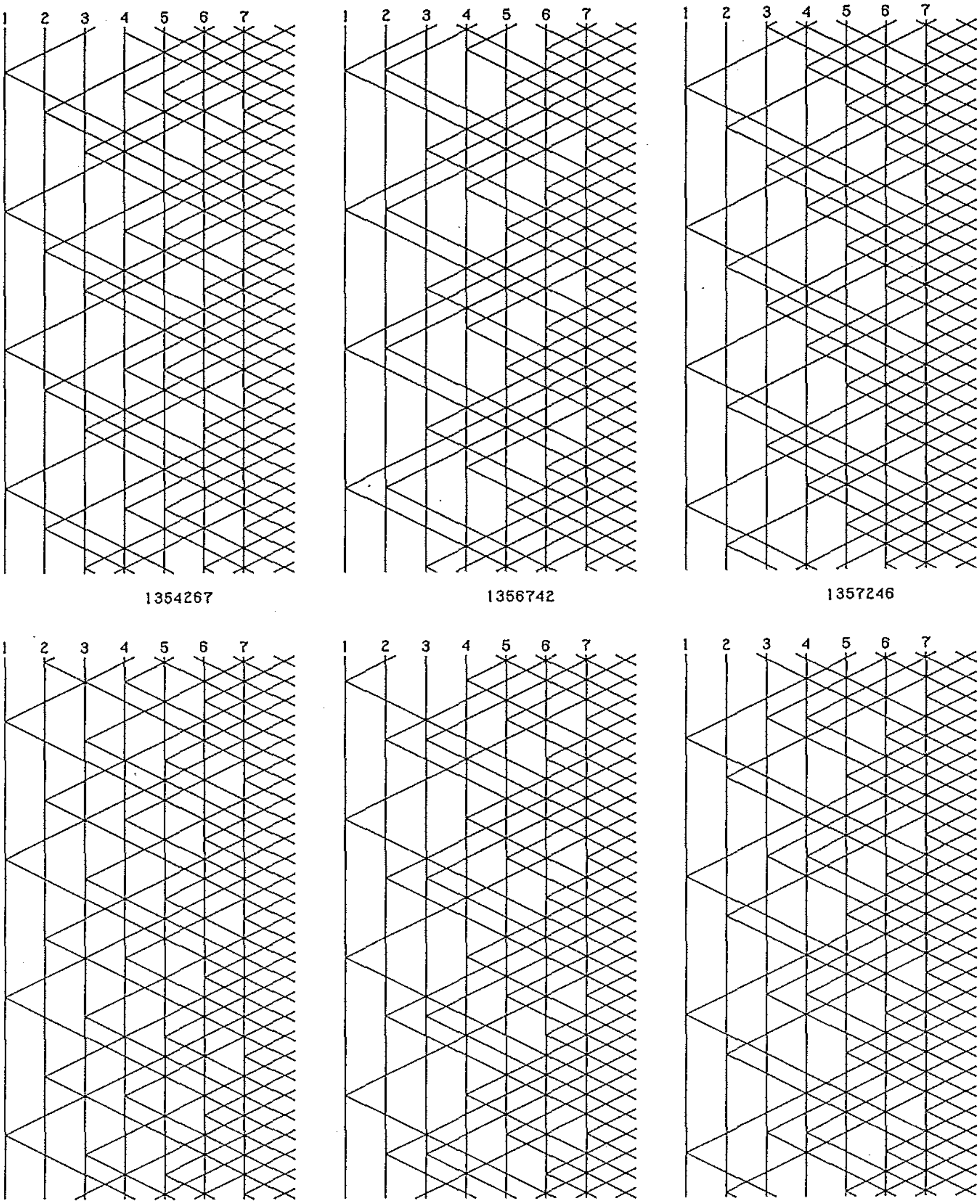
1246537

1264357

1327456

1345276

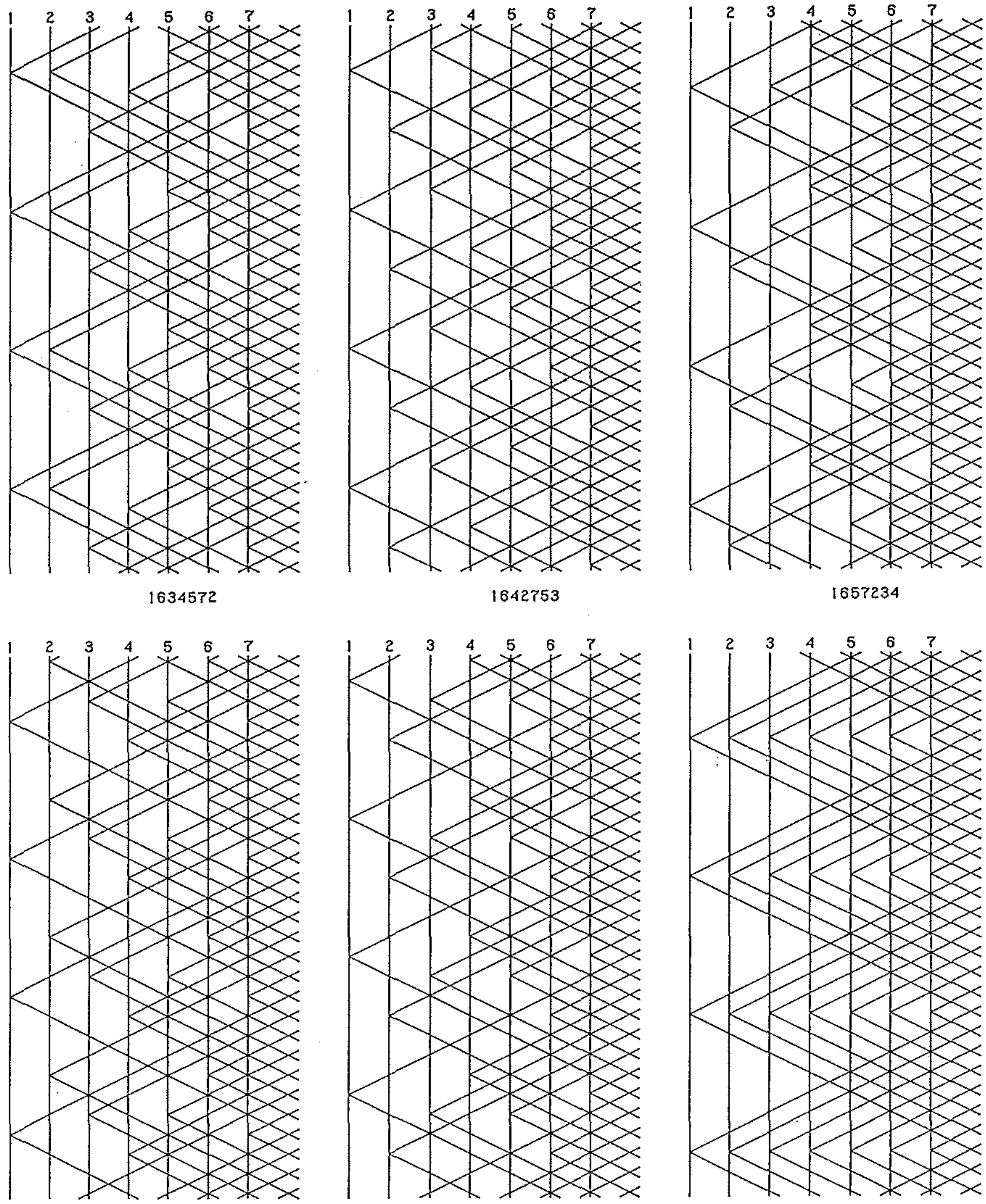
Fig. 432 cont.



1354267 1356742 1357246

1526374 1532467 1567243

Fig. 432 cont.



1634572 1642753 1657234

1724563 1752346 1765432

Fig. 432 cont.

Hence due to the left sequence set $1A(A-1)(A-2)(A-3)\cdots 432$ and the right sequence set $k|k+1|_A|k+2|_A|k+3|_A\cdots A123\cdots|k-2|_A|k-1|_A$, where $1 \leq k \leq A$ there is no need to carry the ranking-numbers, and consequently we obtain for a general left and right cycle (see Fig. 433):

$$
\begin{aligned}
l_{i+1} &= |l_i + x - 2(l_i + r_i)|_A\,. \\
r_{i+1} &= |r_i + x - 2(r_i + l_{i+1})|_A\,.
\end{aligned}
$$

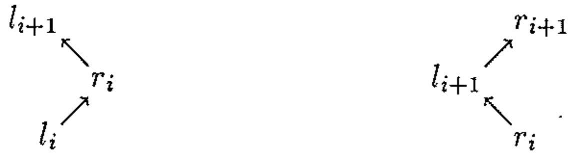


Fig. 433 — A general left and right cycle associated with the Regular Nested Cylindrical Braids.

The further formulae on pg. 417 modify to:

$$
\begin{aligned}
B_l^* &= B_r^* = B^*\,. \\
B_{total} &= A_l B_l^* = A_r B_r^* = A^{**}B^{**} = AB^*\,. \\
d &= \text{g.c.d.}(A_l\,,\, A_r) = \text{g.c.d.}(A\,,\, A) = A\,. \\
A^{**} &= A\,. \\
B^{**} &= B^*\,. \\
\alpha &= \text{number of bights in first-return string-run}\,. \\
P_{component} &= 4\alpha + \frac{\alpha \cdot x - 2\sum(l_i + r_i)}{A}\,. \\
P_{total} &= \sum P_{component}\,.
\end{aligned}
$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs}\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{sub-components}\\ \text{in a component}\end{array}\right\} = \text{g.c.d.}(P_{component}\,, B^*)\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components}\,.$$

Example:

The Regular Nested Cylindrical Braid specification is:
(22222/8/22222){165432/561234}24.
The string-run of this braid is shown in Fig. 434.

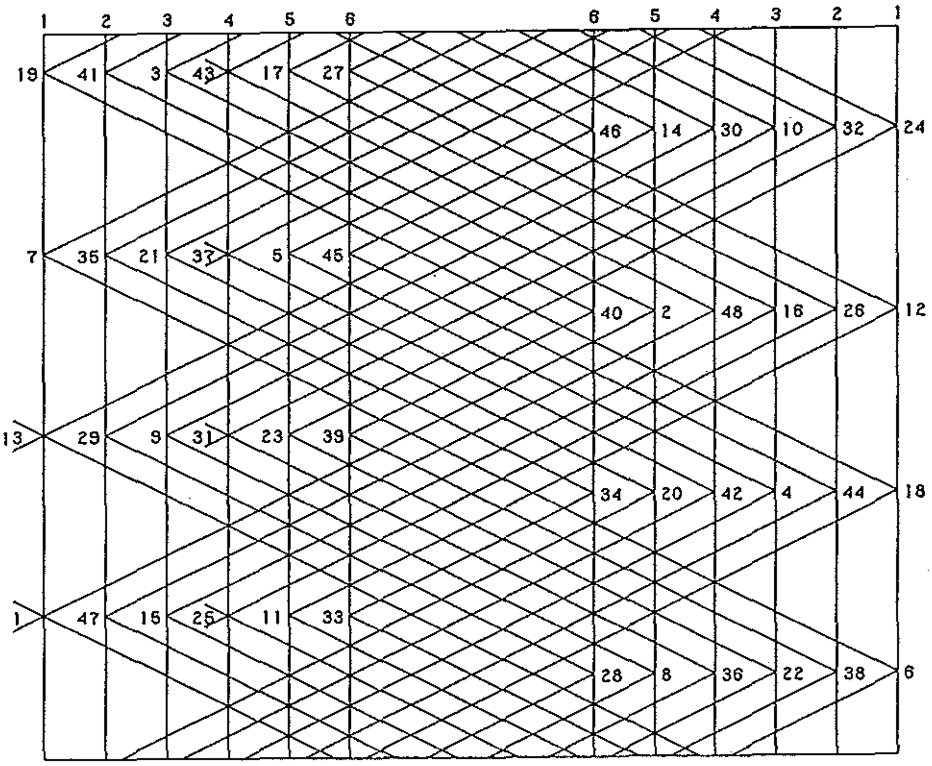


Fig. 434 — The string-run diagram of (22222/8/22222){165432/561234}24.

From the Regular Nested Cylindrical Braid specification we obtain:

$$A = 6 .$$

$$x = 8 .$$

$$d = A = 6 .$$

$$B_{total} = AB^{*} = 24 .$$

$$B^{*} = \frac{B_{total}}{A} = \frac{24}{6} = 4 .$$

$$\mathcal{K}_l = \mathcal{K}_r = \mathcal{K} = A = 6 .$$

From the given Regular Nested Cylindrical Braid specification we can read the lower-left to upper-right half-cycle types:

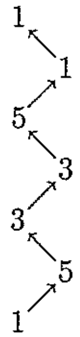
$$\begin{array}{lcl} 1 \longrightarrow 5 & \qquad & 4 \longrightarrow 2 \\ 6 \longrightarrow 6 & & 3 \longrightarrow 3 \\ 5 \longrightarrow 1 & & 2 \longrightarrow 4 \end{array}$$

Anyone of these listed types may be taken as the first lower-left to upper-right half-cycle in the 1^{st} first-return string-run, but normally we take the first listed one. Every lower-left to upper-right half-cycle encountered in this 1^{st} first-return string-run gets deleted from the type-list.

Anyone of the remaining types in the type-list may be taken as the first lower-left to upper-right half-cycle in the 2^{nd} first-return string-run, and again every lower-left to upper-right half-cycle encountered in this 2^{nd} first-return string-run gets deleted from the remaining type-list.

This process is carried on till all the lower-left to upper-right half-cycle types have been deleted from the type-list.

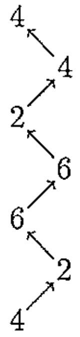
For the first-return string-runs we thus obtain:



$$\begin{array}{lll}
l_1 = 1 \longrightarrow 5 = r_1 & \rightarrow & l_2 = |1+8-2(1+5)|_6 = 3\,. \\
l_2 = 3 \longleftarrow 5 = r_1 & \rightarrow & r_2 = |5+8-2(5+3)|_6 = 3\,. \\
l_2 = 3 \longrightarrow 3 = r_2 & \rightarrow & l_3 = |3+8-2(3+3)|_6 = 5\,. \\
l_3 = 5 \longleftarrow 3 = r_2 & \rightarrow & r_3 = |3+8-2(3+5)|_6 = 1\,. \\
l_3 = 5 \longrightarrow 1 = r_3 & \rightarrow & l_4 = |5+8-2(5+1)|_6 = 1\,. \\
l_4 = 1 \longleftarrow 1 = r_3 & \rightarrow & r_4 = |1+8-2(1+1)|_6 = 5\,.
\end{array}$$

$$P_c = 4\alpha + \frac{\alpha x - 2\sum(l_i + r_i)}{A} = 4\cdot 3 + \frac{3\cdot 8 - 2\{(1+3+5)+(5+3+1)\}}{6} = 10\,.$$

g.c.d. $(P_c\,,\,B^*) =$ g.c.d. $(10\,,\,4) = 2\,.$



$$\begin{array}{lll}
l_1 = 4 \longrightarrow 2 = r_1 & \rightarrow & l_2 = |4+8-2(4+2)|_6 = 6\,. \\
l_2 = 6 \longleftarrow 2 = r_1 & \rightarrow & r_2 = |2+8-2(2+6)|_6 = 6\,. \\
l_2 = 6 \longrightarrow 6 = r_2 & \rightarrow & l_3 = |6+8-2(6+6)|_6 = 2\,. \\
l_3 = 2 \longleftarrow 6 = r_2 & \rightarrow & r_3 = |6+8-2(6+2)|_6 = 4\,. \\
l_3 = 2 \longrightarrow 4 = r_3 & \rightarrow & l_4 = |2+8-2(2+4)|_6 = 4\,. \\
l_4 = 4 \longleftarrow 4 = r_3 & \rightarrow & r_4 = |4+8-2(4+4)|_6 = 2\,.
\end{array}$$

$$P_c = 4\alpha + \frac{\alpha x - 2\sum(l_i + r_i)}{A} = 4\cdot 3 + \frac{3\cdot 8 - 2\{(4+6+2)+(2+6+4)\}}{6} = 8\,.$$

g.c.d. $(P_c\,,\,B^*) =$ g.c.d. $(8\,,\,4) = 4\,.$

Hence:

$$P_{total} = \sum P_{component} = 10 + 8 = 18\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 2\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 2 + 4 = 6\,.$$

⋆⋆ *The* **Standard Regular Nested Cylindrical Braids** *(g.c.d.* $(P_c\,,\,B^*) = 1$*), and the* **Semi-Standard Regular Nested Cylindrical Braids** *(g.c.d.* $(P_c\,,\,B^*) \neq 1$*) have a number of Components equal to* A*. For the Regular Nested Cylindrical Braids with their respective left and right sequence sets* $1A(A-1)(A-2)(A-3)\cdots 432$ *and* $k|k+1|_A|k+2|_A|k+3|_A\cdots A123\cdots|k-2|_A|k-1|_A$*, where* $1 \leq k \leq A$*, we know that when for the first half-cycle, starting at the left bight-boundary* $l_1 = 1$*, the right bight-boundary* r_1 *is known, the left and right bight-boundaries of all the left to right half-cycle types are known. How do we calculate* r_1 *for the Standard and Semi-Standard Regular Nested Cylindrical Braids, and what do we observe about their string-runs?*

THE BRAIDER

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for
the braiding artisan

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Solutions to the Questions in Issue No. 22

Question on pg. 494.

A Premature Regular Cylindrical Braid ($\delta = 0$) with b bights and p parts has $b(p-1)-(p-1) = (b-1)(p-1)$ crossings. When $b = n+1$, then $(b-1)(p-1) = n(p-1)$. When the string-run of this Premature Regular Cylindrical Braid is equivalent to the string-run of a Prime Regular Cylindrical Bihelix Braid, the number of crossings in such a Prime Regular Cylindrical Bihelix Braid must be a multiple of n. From the formulae on pp. 393 – 394 which express the sequential number of crossings in $[(n+1), n]$ Prime Regular Cylindrical Bihelix Braids we observe that those $[(n+1), n]$ Prime Regular Cylindrical Bihelix Braids in which the number of crossings are equal to:

$$\begin{gathered} 2n^2\,, \\ 2n^2 + n\,, \\ 4n^2\,, \\ 4n^2 + n\,, \\ 6n^2\,, \\ 6n^2 + n\,, \\ 8n^2\,, \\ 8n^2 + n\,, \\ \vdots \end{gathered}$$

fulfil this requirement. Hence the string-run of a Premature Regular Cylindrical Braid in which the number of bights is equal to $b = n + 1$ and the number of parts equal to $p = 2kn + 1$ or $p = 2kn + 2$, where $k = 1, 2, 3, \cdots$, is identical to the string-run of an $[(n+1), n]$ Prime Regular Cylindrical Bihelix Braid in which the number of crossings is equal to $2kn^2$ respectively $2kn^2 + n$. When b and p are coprime, hence g.c.d. $(b, p) = 1$, the Premature Regular Cylindrical Braid, and hence its equivalent Prime Regular Cylindrical Bihelix Braid, is a single string braid.

Question on pg. 512.

Since for the Standard Regular Nested Cylindrical Braids and the Semi-Standard Regular Nested Cylindrical Braids the number of Components is equal to A, it follows that $\alpha = 1$. Hence $l_{i+1} = l_i$ and consequently a first-return string-run is completed.

Since $l_{i+1} = |l_i + x - 2(l_i + r_i)|_A$ and hence $l_{i+1} = l_i + x - 2(l_i + r_i) + nA$, where n is an integer:

$$\begin{aligned} l_{i+1} = l_i \quad \longrightarrow \qquad 0 &= x - 2(l_i + r_i) + nA\,, \\ 2r_i &= x - 2l_i + nA\,, \\ r_i &= \frac{x + nA}{2} - l_i\,, \end{aligned}$$

but since $1 \le r_i \le A$, we obtain :

$$r_i = \left| \frac{x + nA}{2} - l_i \right|_A .$$

For $l_i = l_1 = 1$:

$$x = \text{even} \begin{cases} A = \text{odd} & \longrightarrow \qquad r_i = r_1 = \left| \dfrac{x}{2} - 1 \right|_A . \\ A = \text{even} & \longrightarrow \qquad r_i = r_1 = \left| \dfrac{x}{2} - 1 \right|_A \quad \text{and} \quad r_i = r_1 = \left| \dfrac{x + A}{2} - 1 \right|_A . \end{cases}$$

$$x = \text{odd} \begin{cases} A = \text{odd} & \longrightarrow \quad r_i = r_1 = \left|\dfrac{x+A}{2} - 1\right|_A . \\ A = \text{even} & \longrightarrow \quad \text{not possible.} \end{cases}$$

For the Standard Regular Nested Cylindrical Braids and the Semi-Standard Regular Nested Cylindrical Braids the shortest vertical distance (the distance along a bight-edge) in bight-units between the apex of a left-hand nest of bights and the apex of a right-hand nest of bights is equal to:

$$\begin{aligned} & \left|\frac{2(A-l_1) + x + 2(A-r_1)}{2}\right|_A \\ = & \left|\frac{x - 2(l_1 + r_1)}{2}\right|_A \\ = & \left|\frac{x - 2(1 + r_1)}{2}\right|_A \\ = & \left|\frac{x}{2} - (1 + r_1)\right|_A \end{aligned}$$

Hence $\left|\frac{x}{2} - (1 + r_1)\right|_A = 0$ for:

$$x = \text{even} \begin{cases} A = \text{odd} & \longrightarrow \quad r_i = r_1 = \left|\dfrac{x}{2} - 1\right|_A . \\ A = \text{even} & \longrightarrow \quad r_i = r_1 = \left|\dfrac{x}{2} - 1\right|_A . \end{cases}$$

$$x = \text{odd} \begin{cases} A = \text{odd} & \longrightarrow \quad \text{not possible.} \\ A = \text{even} & \longrightarrow \quad \text{not possible.} \end{cases}$$

And $\left|\frac{x}{2} - (1 + r_1)\right|_A = \frac{A}{2}$ for:

$$x = \text{even} \begin{cases} A = \text{odd} & \longrightarrow \quad \text{not possible.} \\ A = \text{even} & \longrightarrow \quad r_i = r_1 = \left|\dfrac{x+A}{2} - 1\right|_A . \end{cases}$$

$$x = \text{odd} \begin{cases} A = \text{odd} & \longrightarrow \quad r_i = r_1 = \left|\dfrac{x+A}{2} - 1\right|_A . \\ A = \text{even} & \longrightarrow \quad \text{not possible.} \end{cases}$$

There are thus for the Standard Regular Nested Cylindrical Braids and the Semi-Standard Regular Nested Cylindrical Braids two types of string-run as far as the apex positions of the left-hand nests of bights relative to the apex position of the right-hand nests of bights are concerned: one in which the apex positions of the right-hand nests of bights line up with the apex positions of the left-hand nests of bights, and one in which the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights (see Figs. 435 & 436 respectively). To the latter type belong, for example, the string-runs of the well-known **Standard Herringbone Pineapple Knots**, more commonly known as the Pineapple Knots, and the **Semi-Standard Herringbone Pineapple Knots**.

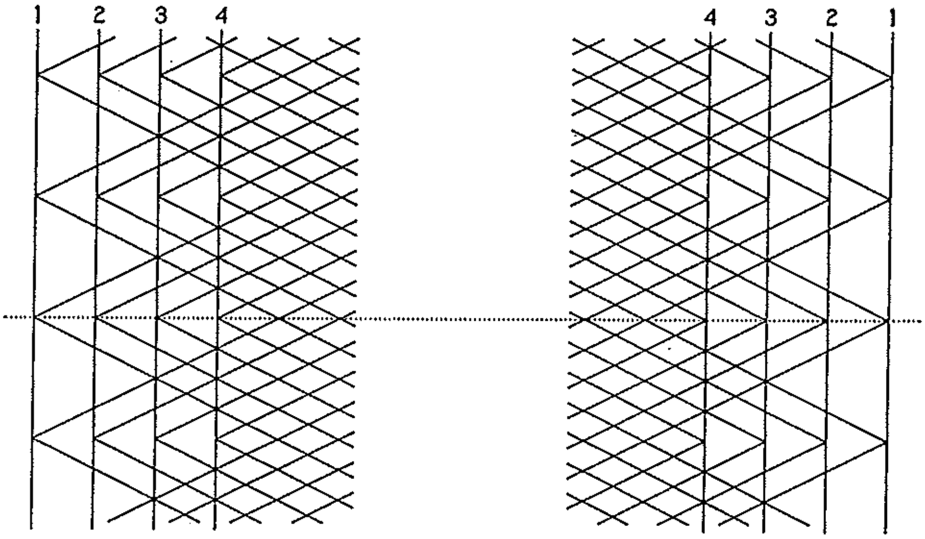


Fig. 435 — The case $\left|\frac{x}{2}-(1+r_1)\right|_A = 0$.

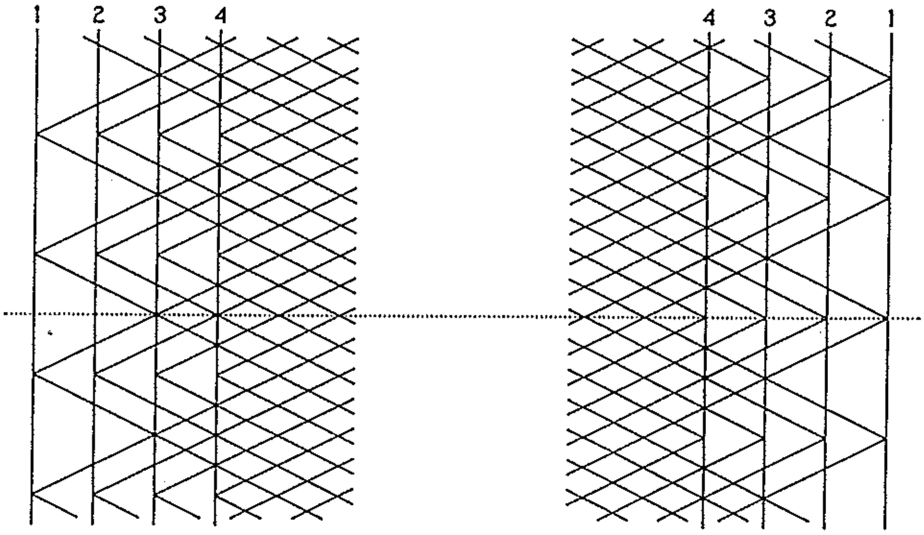


Fig. 436 — The case $\left|\frac{x}{2}-(1+r_1)\right|_A = \frac{A}{2}$.

Column-coded Regular Cylindrical Braids
with a
# balanced coding.

In *The Braider*, Issue No. 21, pp. 471 – 482, we discussed the 'classification' of the **over–under coded Regular Knots**, and in Issue No. 9, pp. 178 – 187 we discussed their 'categorisation' which can be regarded as a further refinement of their 'classification'. The reader, no doubt, has become aware that even for the most simple Regular Knots, the over–under coded type, a classification becomes a rather complicated affair. Since our knowledge about **Braids** in general is infinitely small, any attempt in setting up a proper classification system amounts to a futile exercise as we have mentioned in earlier Issues of *The Braider*. Hence we like to stress once again that our discussions about classifications and categorisation must be seen as restricted and isolated snippets in this immensely complicated field. These snippets should however not only make us appreciate the enormous difficulties associated with a classification system, but should also give us some insight about relationships which are often quite remarkable.

Since Column-coded Regular Cylindrical Braids are in fact Column-coded Regular Flat Braids in the form of a cylinder, the results in this discussion apply of course to Column-coded Regular Flat Braids also. Of special interest to braiders are those Column-coded Regular Cylindrical Braids and Column-coded Regular Flat Braids which have a balanced (symmetric) coding relative to the (circumferential) centre-line of the braid since they have the most pleasing appearance.

The tables depicting the balanced Column-coding types are shown in Fig. 437 for $2 < p \leq 10$ and in Fig. 438 for $11 \leq p \leq 12$. In these tables the c-value indicates the number of intersection-columns in the central region with identically coded crossings. The coding patterns coupled by a brace are each others complement. The c-value can only be equal to zero when p is odd (there is no central intersection-column), in which case each coding pattern is identical to its complement (we assume that the braiding-material is a flat string which has for each opposite face pair identical faces in size, shape, texture and colour).

For $p =$ **even** $= 2n$, where n is a natural number[†], there are a total of 2^n balanced coding types.

The number of balanced coding types for $c = 2m - 1$, where $1 \leq m \leq n$, is:

$$\begin{array}{rcl} \text{for} \quad 1 \leq m < n & \longrightarrow & 2^{n-m}\,. \\ \text{for} \quad m = n & \longrightarrow & 2\,. \end{array}$$

For $p =$ **odd** $= 2n + 1$, where n is a natural number, there are a total of $3 \times 2^{n-1}$ balanced coding types.

The number of balanced coding types for $c = 2m$, where $0 \leq m \leq n$, is:

$$\begin{array}{rcl} \text{for} \quad m = 0 & \longrightarrow & 2^{n-1}\,. \\ \text{for} \quad 1 \leq m < n & \longrightarrow & 2^{n-m}\,. \\ \text{for} \quad m = n & \longrightarrow & 2\,. \end{array}$$

Let's look at the position of a few well known knots:

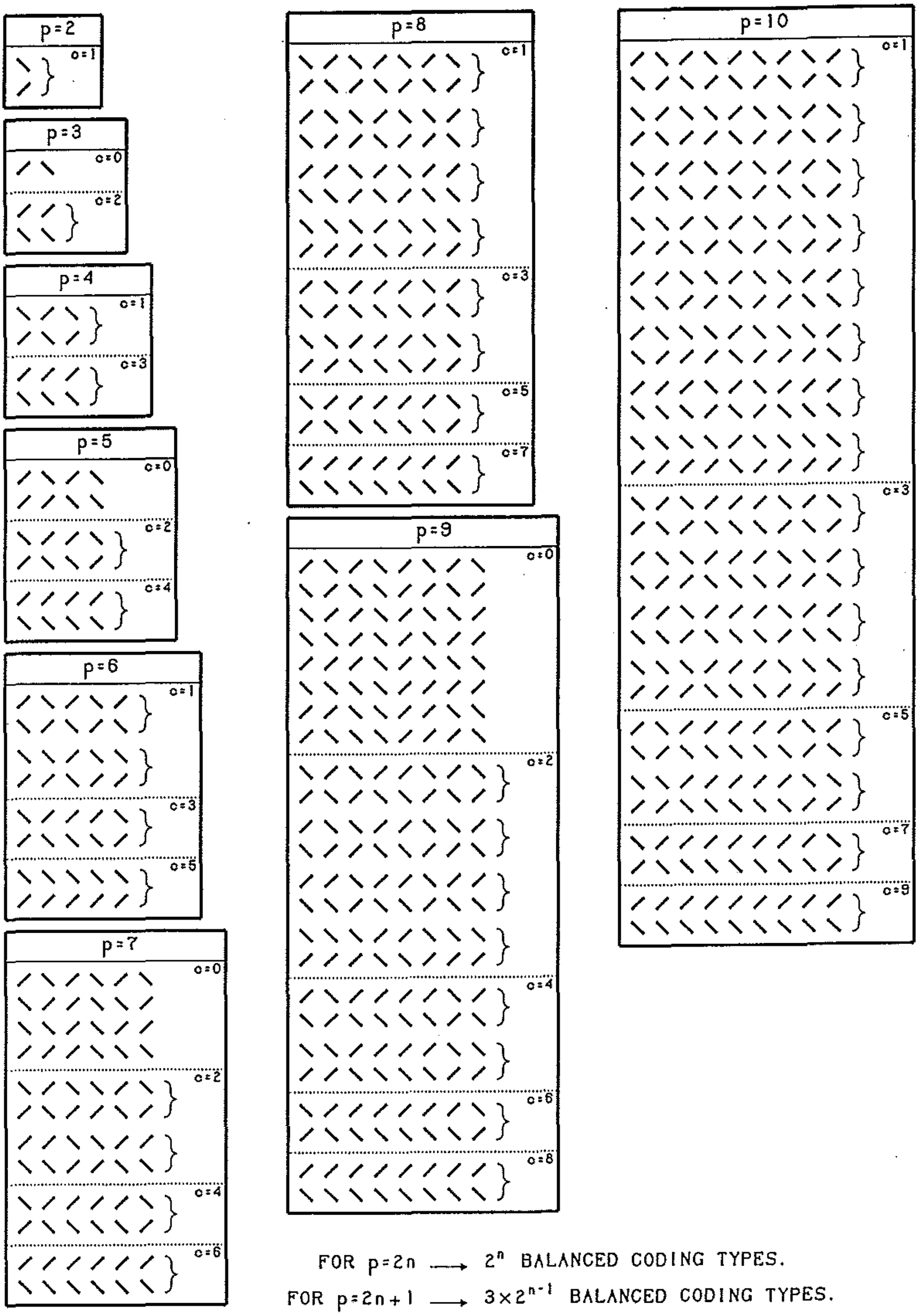
The 2-pass Spanish Ring Knots come under $p = 5 \mid c = 0$. The 3-pass Spanish Ring Knots come under $p = 7 \mid c = 0$. The 4-pass Spanish Ring Knots come under $p = 9 \mid c = 0$. And so on.

The 2-pass Gaucho Knots come under $p = 9 \mid c = 0$; $p = 13 \mid c = 0$; $\cdots$. The 3-pass Gaucho Knots come under $p = 13 \mid c = 0$; $p = 19 \mid c = 0$; $\cdots$. The 4-pass Gaucho Knots come under $p = 17 \mid c = 0$; $p = 25 \mid c = 0$; $\cdots$. And so on.

The 2-pass Headhunter's Knots come under $p = 7 \mid c = 2$; $p = 11 \mid c = 2$; $\cdots$. The 3-pass Headhunter's Knots come under $p = 10 \mid c = 3$; $p = 16 \mid c = 3$; $\cdots$. The 4-pass Headhunter's Knots come under $p = 13 \mid c = 4$; $p = 21 \mid c = 4$; $\cdots$. And so on.

Note that there are two types for each Headhunter's Knot entry, but that there is only one type for each Spanish Ring Knot and each Gaucho Knot entry.

[†] **Natural numbers**, or **positive integers** are the numbers $1, 2, 3, \cdots$.



FOR $p=2n \longrightarrow 2^n$ BALANCED CODING TYPES.
FOR $p=2n+1 \longrightarrow 3\times2^{n-1}$ BALANCED CODING TYPES.
n=POSITIVE INTEGER.

Fig. 437 — The balanced Column-coded Regular Cylindrical Braids for $2 \leq p \leq 10$.

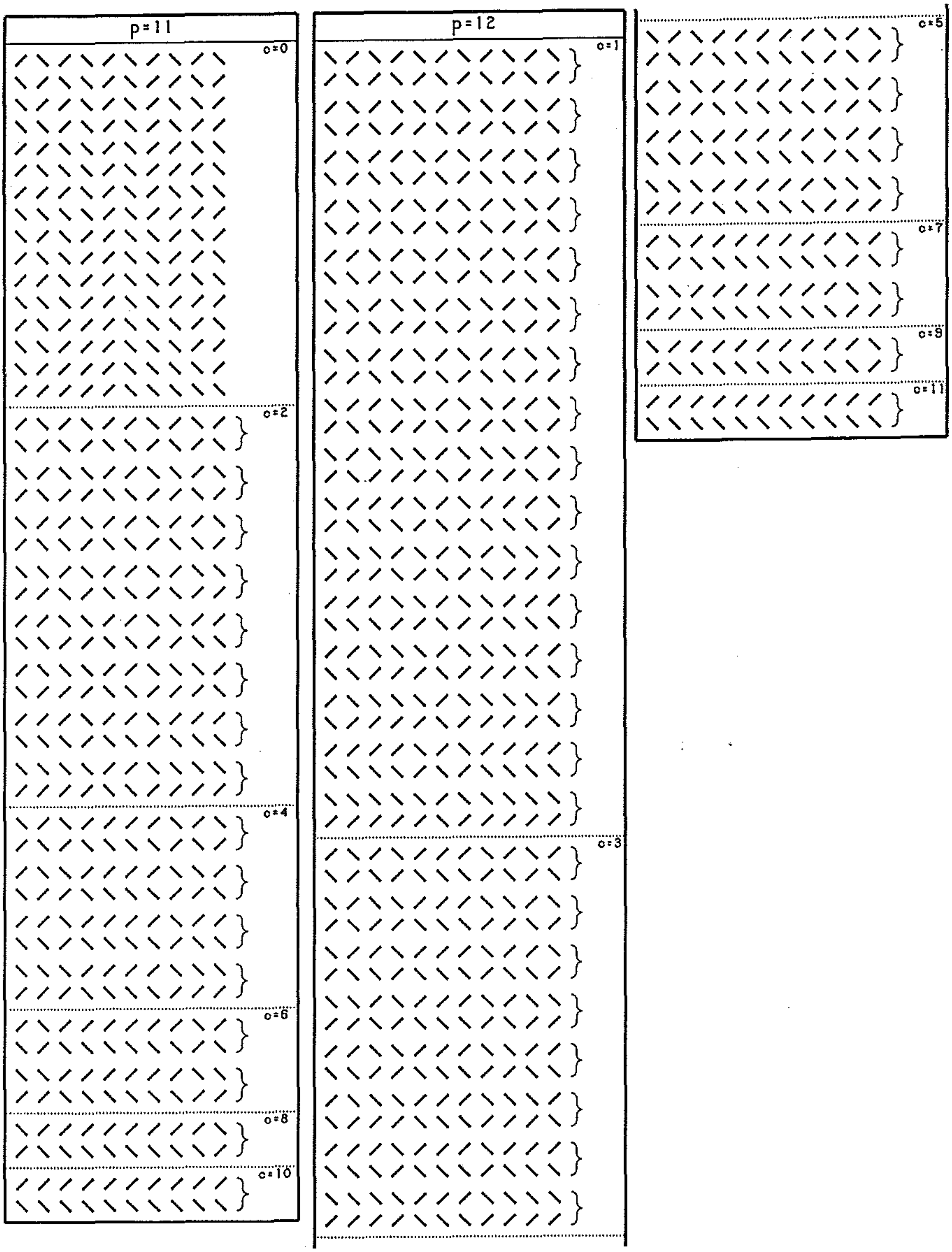


Fig. 438 — The balanced Column-coded Regular Cylindrical Braids for $11 \leq p \leq 12$.

Nested Cylindrical Braids

The number of Components is equal to A for the Standard Regular Nested Cylindrical Braids (g.c.d. $(P_c, B^*) = 1$) and Semi-Standard Regular Nested Cylindrical Braids (g.c.d. $(P_c, B^*) \neq 1$). For these braids there are two types of string-run with respect to the relative apex positions of the left-hand and the right-hand nests of bights: one in which the apex positions of the right-hand and the left-hand nests of bights line up exactly, and one in which the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights (see this Issue, pp. 513 – 515). The following Example illustrates the there obtained results:

Example 1:

Let $x = 10$ and $A = 7$. Then:

$$r_1 = \left|\tfrac{x}{2} - 1\right|_A = 4\,.$$

The associated string-run diagram is depicted in Fig. 439.

Let $x = 10$ and $A = 6$. Then:

(1) $$r_1 = \left|\tfrac{x}{2} - 1\right|_A = 4\,.$$

The associated string-run diagram is depicted at the left in Fig. 440.

(2) $$r_1 = \left|\tfrac{x+A}{2} - 1\right|_A = 1\,.$$

The associated string-run diagram is depicted at the right in Fig. 440.

Let $x = 11$ and $A = 5$. Then:

$$r_1 = \left|\tfrac{x+A}{2} - 1\right|_A = 2\,.$$

The associated string-run diagram is depicted in Fig. 441.

Note that for the Nested Cylindrical Braids the first-return string-runs do not change with $x = |x|_{\text{l.c.m.}(A_l, A_r)} + n \cdot \text{l.c.m.}\,(A_l, A_r)$, where n is a whole number.[†] Hence for the Standard and Semi-Standard Regular Nested Cylindrical Braids the first-return string-runs do not change with $x = |x|_A + nA$, while the first-return string-runs and their type do not change with $x = |x|_{2A} + 2nA$.

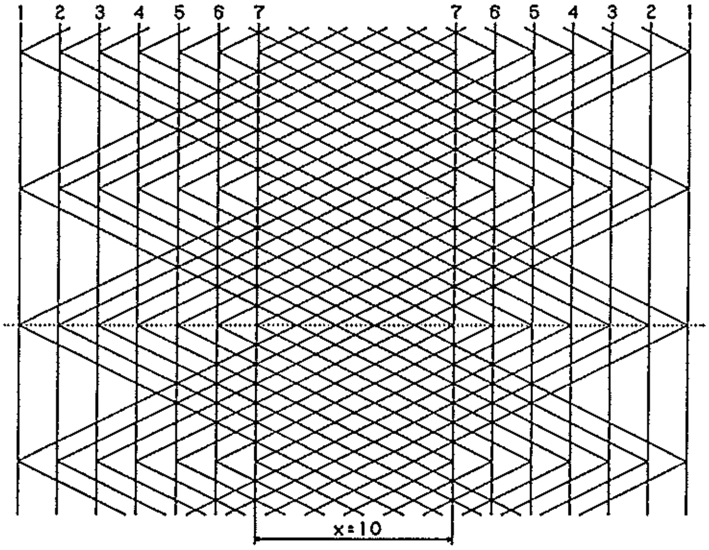


Fig. 439 — $x = 10$ and $A = 7$.

[†] l.c.m. (A_l, A_r) stands for the lowest common multiple of A_l and A_r.
Whole numbers are the numbers $0, 1, 2, 3, \cdots$.

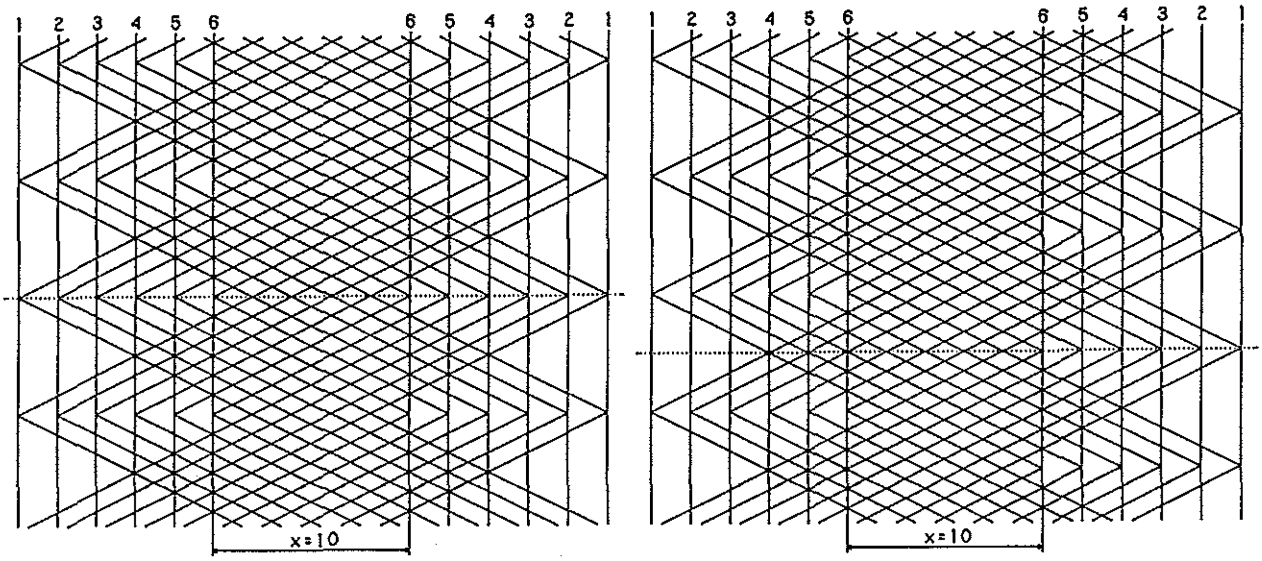


Fig. 440 — $x = 10$ and $A = 6$.

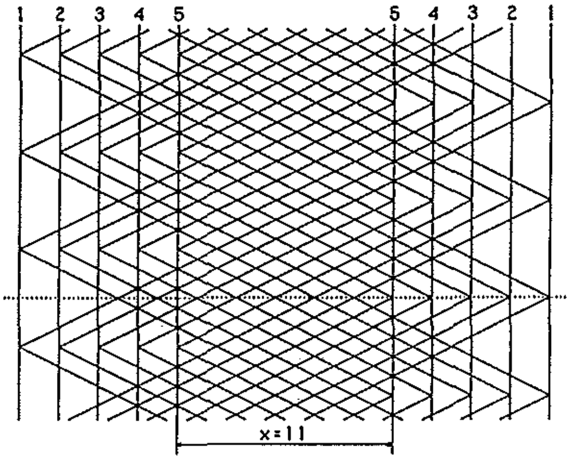


Fig. 441 — $x = 11$ and $A = 5$.

Since $\alpha = 1$ for the Standard and Semi-Standard Regular Nested Cylindrical Braids, we obtain $P_c = 4 + \frac{x-2(l_i+r_i)}{A}$.

When $r_1 = A$, hence when $x = 2 + nA$, where n is a whole number, we have the special case in which all A Components have the same number of parts $P_c = n + 2$. The associated lower-left to upper-right half-cycle types are tabulated in Fig. 442.

$l_i \longrightarrow r_i$	$l_i + r_i$
$1 \longrightarrow A$	$A+1$
$A \longrightarrow 1$	$A+1$
$A-1 \longrightarrow 2$	$A+1$
$A-2 \longrightarrow 3$	$A+1$
$\vdots$	$\vdots$
$4 \longrightarrow A-3$	$A+1$
$3 \longrightarrow A-2$	$A+1$
$2 \longrightarrow A-1$	$A+1$

Fig. 442 — The lower-left to upper-right half-cycle types when $r_1 = A$.

When n = odd, the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights, and when n = even, the apex positions of the right-hand nests of bights line exactly up with the apex positions of the left-hand nests of bights.

When $r_1 = k$, where $1 \leq k < A$, hence where with $x = 2k + 2 + (2n - 1)A$, where n is a whole number, the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights, and where with $x = 2k + 2 + 2nA$, where n is a whole number, the apex positions of the right-hand nests of bights line exactly up with the apex positions of the left-hand nests of bights, there are two different P_c values:
$(A-k)$ Components with $P_c = 2+\frac{x-2(k+1)}{A}$, and k Components with $P_c' = 4+\frac{x-2(k+1)}{A}$.

The associated lower-left to upper-right half-cycle types are tabulated in Fig. 443.

$l_i \longrightarrow r_i$	$l_i + r_i$
$1 \longrightarrow k$	$k+1$
$A \longrightarrow k+1$	$A+k+1$
$A-1 \longrightarrow k+2$	$A+k+1$
$A-2 \longrightarrow k+3$	$A+k+1$
$\vdots$	$\vdots$
$A-(z-1) \longrightarrow k+z$	$A+k+1$
$\vdots$	$\vdots$
$k+2 \longrightarrow A-1$	$A+k+1$
$k+1 \longrightarrow A$	$A+k+1$
$k \longrightarrow 1$	$k+1$
$k-1 \longrightarrow 2$	$k+1$
$k-2 \longrightarrow 3$	$k+1$
$\vdots$	$\vdots$
$4 \longrightarrow k-3$	$k+1$
$3 \longrightarrow k-2$	$k+1$
$2 \longrightarrow k-1$	$k+1$

Fig. 443 — The lower-left to upper-right half-cycle types when $r_1 = k$.

When the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights: $x = 2k + 2 + (2n - 1)A$ with $P_c = 1 + 2n$ for $(A - k)$ Components and $P_c' = 3 + 2n$ for k Components, where n is a whole number. Thus $n = \frac{x+A-2k-2}{2A}$, and the components have an **odd** number of parts which differ by 2 for the two Component types.

When the apex positions of the right-hand nests of bights line up with the apex positions of the left-hand nests of bights: $x = 2k + 2 + 2nA$ with $P_c = 2 + 2n$ for $(A - k)$ Components and $P_c' = 4 + 2n$ for k Components, where n is a whole number. Thus $n = \frac{x-2k-2}{2A}$, and the components have an **even** number of parts which differ by 2 for the two Component types.

Example 2:

Let $x = 10$ and $A = 7$. Then: (see also Fig. 439 on pg. 519)
Since x = even and A = odd, the apex positions of the right-hand nests of bights

line up with the apex positions of the left-hand nests of bights (see pg. 514).

$$r_i = r_1 = k = \left|\frac{x}{2} - 1\right|_A = \left|\frac{10}{2} - 1\right|_7 = |4|_7 = 4\,.$$

$$n = \frac{x - 2k - 2}{2A} = \frac{10 - 2 \times 4 - 2}{2 \times 7} = \frac{0}{14} = 0\,.$$

$$P_c = 2 + 2n = 2\,, \quad \text{number of Component} = A - k = 7 - 4 = 3\,.$$

$$P_c' = 4 + 2n = 4\,, \quad \text{number of Component} = k = 4\,.$$

$$P_{total} = 3 \times 2 + 4 \times 4 = 22 = x + 2A - 2\,.$$

Example 3:

Let $x = 10$ and $A = 6$. Then: (see also Fig. 440 on pg. 520)

Since $x =$ even and $A =$ even, the apex positions of the right-hand nests of bights line up with the apex positions of the left-hand nests of bights when: (see pg. 514)

$$r_i = r_1 = k = \left|\frac{x}{2} - 1\right|_A = \left|\frac{10}{2} - 1\right|_6 = |4|_6 = 4\,.$$

$$\text{Then :} \quad n = \frac{x - 2k - 2}{2A} = \frac{10 - 2 \times 4 - 2}{2 \times 6} = \frac{0}{12} = 0\,.$$

$$P_c = 2 + 2n = 2\,, \quad \text{number of Component} = A - k = 6 - 4 = 2\,.$$

$$P_c' = 4 + 2n = 4\,, \quad \text{number of Component} = k = 4\,.$$

$$P_{total} = 2 \times 2 + 4 \times 4 = 20 = x + 2A - 2\,.$$

The apex positions of the right-hand nests of bights fall midway between the apex positions of the left-hand nests of bights when: (see pg. 514)

$$r_i = r_1 = k = \left|\frac{x + A}{2} - 1\right|_A = \left|\frac{10 + 6}{2} - 1\right|_6 = |7|_6 = 1\,.$$

$$\text{Then :} \quad n = \frac{x + A - 2k - 2}{2A} = \frac{10 + 6 - 2 \times 1 - 2}{2 \times 6} = \frac{12}{12} = 1\,.$$

$$P_c = 1 + 2n = 3\,, \quad \text{number of Component} = A - k = 6 - 1 = 5\,.$$

$$P_c' = 3 + 2n = 5\,, \quad \text{number of Component} = k = 1\,.$$

$$P_{total} = 5 \times 3 + 1 \times 5 = 20 = x + 2A - 2\,.$$

Example 4:

Let $x = 11$ and $A = 5$. Then: (see also Fig. 441 on pg. 520)

Since $x =$ odd and $A =$ odd, the apex positions of the right-hand nests of bights fall midway between the apex positions of the left-hand nests of bights (see pg. 514).

$$r_i = r_1 = k = \left|\frac{x + A}{2} - 1\right|_A = \left|\frac{11 + 5}{2} - 1\right|_5 = |7|_5 = 2\,.$$

$$n = \frac{x + A - 2k - 2}{2A} = \frac{11 + 5 - 2 \times 2 - 2}{2 \times 5} = \frac{10}{10} = 1\,.$$

$$P_c = 1 + 2n = 3\,, \quad \text{number of Component} = A - k = 5 - 2 = 3\,.$$

$$P_c' = 3 + 2n = 5\,, \quad \text{number of Component} = k = 2\,.$$

$$P_{total} = 3 \times 3 + 2 \times 5 = 19 = x + 2A - 2\,.$$

Instead of starting with values for x and A, we can also start with values for P_{total} and A and then calculate the value of x from $P_{total} = x + 2A - 2$, hence with the formula $x = P_{total} + 2 - 2A$. However, in practice it is generally more convenient to start with values for x and A, since x measures the cylindrical length of the braid and A is a measure for its domed ends.

As we already mentioned on pg. 514, the well-known **Standard Herringbone Pineapple Knots** and the **Semi-Standard Herringbone Pineapple Knots** have a string-run in which the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights. The essential coding for these knots is depicted in Fig. 444.

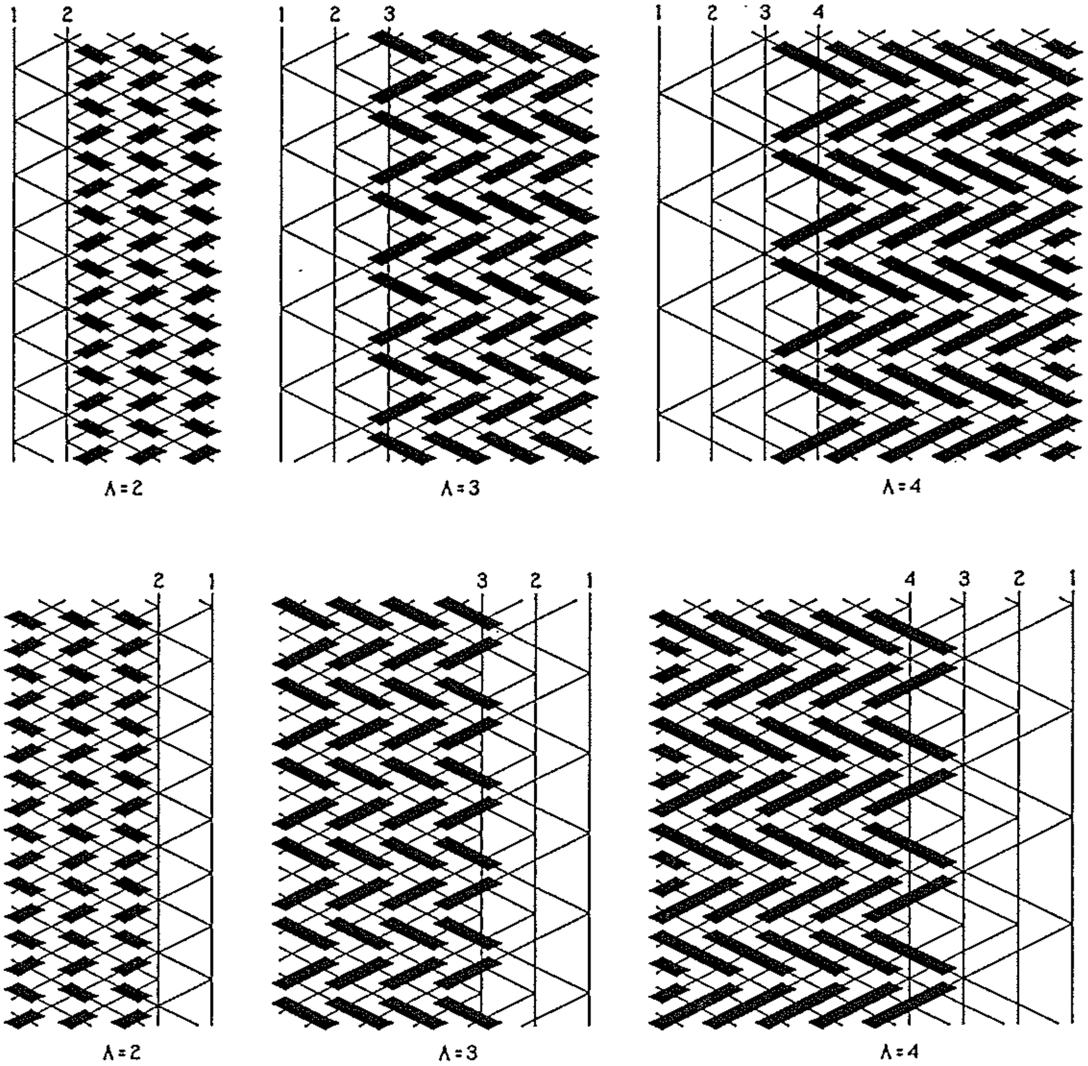


Fig. 444 — The essential coding.

The upper row of diagrams shows the essential coding at the left-hand bight-edge, while the lower row of diagrams shows the essential coding at the right-hand bight-edge. The rows which have not yet received a coding may be coded so as to obtain either the upper two rows of diagrams or the lower two rows of diagrams in Fig. 445. However, when the braiding-material is a flat string with opposite face pairs identical in size, shape, texture and colour, it should be noted that the upper two rows of diagrams are identical to the lower two rows of diagrams (turn one set of diagrams through an angle of 180° about an axis perpendicular to the paper in order to obtain the other set).

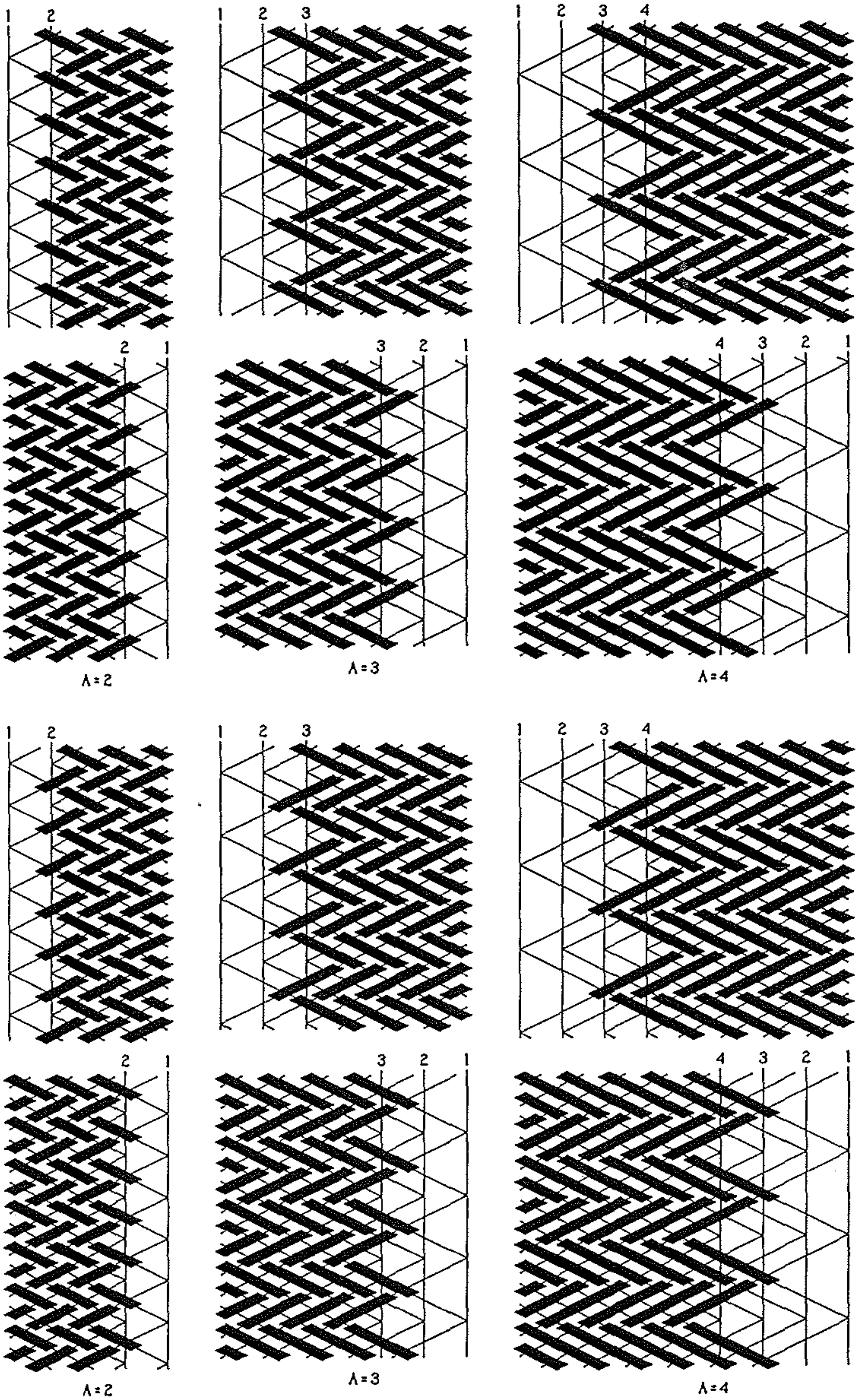


Fig. 445 — The Herringbone-coding.

Fig. 446 shows in relation to the left-hand and right-hand bight-edges once again for the Standard and Semi-Standard Herringbone Pineapple Knots the two identical herringbone coding types. Note that although the coding at the right-hand and left-hand bight-edges are each others mirror image (lateral inversions), they are not lateral equivalent.

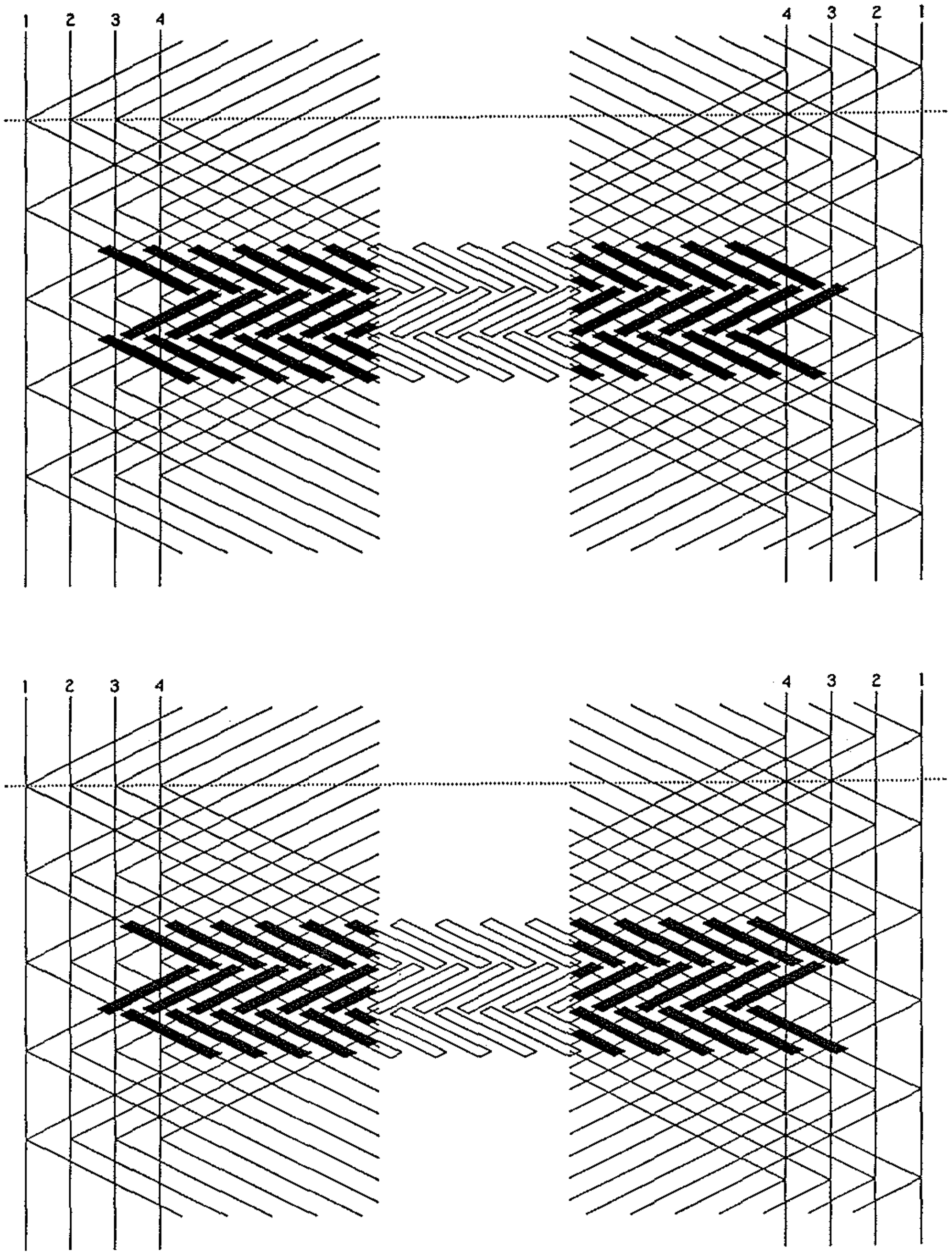


Fig. 446 — The Standard and Semi-Standard Herringbone Pineapple Knots.

Since there is thus only one general herringbone coding-form for nesting-number A, it is sufficient to consider the herringbone coding-form depicted in the diagrams of the upper two rows in Fig. 445, hence the herringbone coding-form depicted in the upper diagram of Fig. 446.

A Standard and a Semi-Standard A-pass Herringbone Pineapple Knot respectively consists of A interbraided over–under coded Regular and Semi-Regular Knots. When $l_1 = 1$ and $r_1 = A$, the interbraided knots are identical with the same odd number of parts each. When $l_1 = 1$ and $r_1 = k < A$, the interbraided knots have an odd number of parts each: $(A-k)$ with $p = (2m-1)$ parts each and k with $p = (2m+1)$ parts each, where m is a natural number.

A lower-left to upper-right half-cycle running from l_i to r_i has the coding-sequence:

$$(l_i)u - Ao - Au - \cdots - Au - Ao - (r_i - 1)u\,,$$

in which each set of the p sets of crossings, except the last set $(r_i - 1)u$, has one crossing belonging to the string-run of the knot between the bight-boundaries l_i and r_i.

A lower-right to upper-left half-cycle running from r_i to l_i has the coding-sequence:

$$(r_i)u - Ao - Au - \cdots - Au - Ao - (l_i - 1)u\,,$$

in which each set of the p sets of crossings, except the last set $(l_i - 1)u$, has one crossing belonging to the string-run of the knot between the bight-boundaries l_i and r_i.

Hence the reference half-cycle sequences, consequently the first lower-left to upper-right half-cycle sequence, are

from lower-left l_i to upper-right r_i :

$$(l_i - 1)u - (A-1)o - (A-1)u - \cdots - (A-1)u - (A-1)o - (r_i - 1)u\,.$$

from lower-right r_i to upper-left l_i :

$$(r_i - 1)u - (A-1)o - (A-1)u - \cdots - (A-1)u - (A-1)o - (l_i - 1)u\,.$$

It will now be evident that for the Standard and Semi-Standard Herringbone Pineapple Knots we can, for the determination of the half-cycle algorithms of an interbraided over–under coded Regular respectively Semi-Regular Knot, employ, with a small modification, their algorithm diagrams.

The general form of such an algorithm diagram for an interbraided over–under coded Regular Knot between the bight-boundaries l_i and r_i is as follows:

$$\begin{array}{ccccccccc}
\longrightarrow & l_i - 1 & A-1 & A-1 & \ldots & A-1 & A-1 & A-1 & r_i - 1 \\
0 & \star & \star & \star & \ldots & \star & \star & \star & \\
/ & \backslash & / & \backslash & \ldots & / & \backslash & / & \backslash \\
 & \star & \star & \star & \ldots & \star & \star & \star & 0 \\
l_i - 1 & A-1 & A-1 & A-1 & \ldots & A-1 & A-1 & r_i - 1 & \longleftarrow
\end{array}$$

The positions of the stars are occupied by the i-values of the complementary bight-number scheme associated with the knot to be interbraided. The value above an upper star, or below a lower star, increases by 1 when its associated i-value is applicable to the half-cycle concerned. The last entry for the lower-left to upper-right half-cycles, $(r_i - 1)u$, remains the same for all of them. Similarly for all the lower-right to upper-left half-cycles, the last entry $(l_i - 1)u$ remains the same.

Example 5:

Let $A = 5$; $B^* = 4$; $x = 23$. Then for a Standard or Semi-Standard Herringbone Pineapple Knot :

$$\text{For } l_1 = 1 \quad \longrightarrow \quad r_1 = \left|\frac{x+A}{2} - 1\right|_A = \left|\frac{23+5}{2} - 1\right|_5 = 3 = k\,.$$

$$n = \frac{x + A - 2k - 2}{2A} = \frac{23 + 5 - 2 \times 3 - 2}{2 \times 5} = 2\,,\text{hence :}$$

$$P_c = 1 + 2n = 1 + 2 \times 2 = 5 \quad \text{for } A - k = 5 - 3 = 2 \text{ Components.}$$

$$P_c' = 3 + 2n = 3 + 2 \times 2 = 7 \quad \text{for } k = 3 \text{ Components.}$$

Since:

$$\text{g.c.d.}\,(P_c, B^*) = \text{g.c.d.}\,(5,4) = 1 \ \text{ and}$$
$$\text{g.c.d.}\,(P'_c, B^*) = \text{g.c.d.}\,(7,4) = 1\,,$$

the string-run is that of a Standard Herringbone Pineapple Knot (one essential string for each Component). The string-run diagram and the grid-diagram are depicted in Fig. 447.

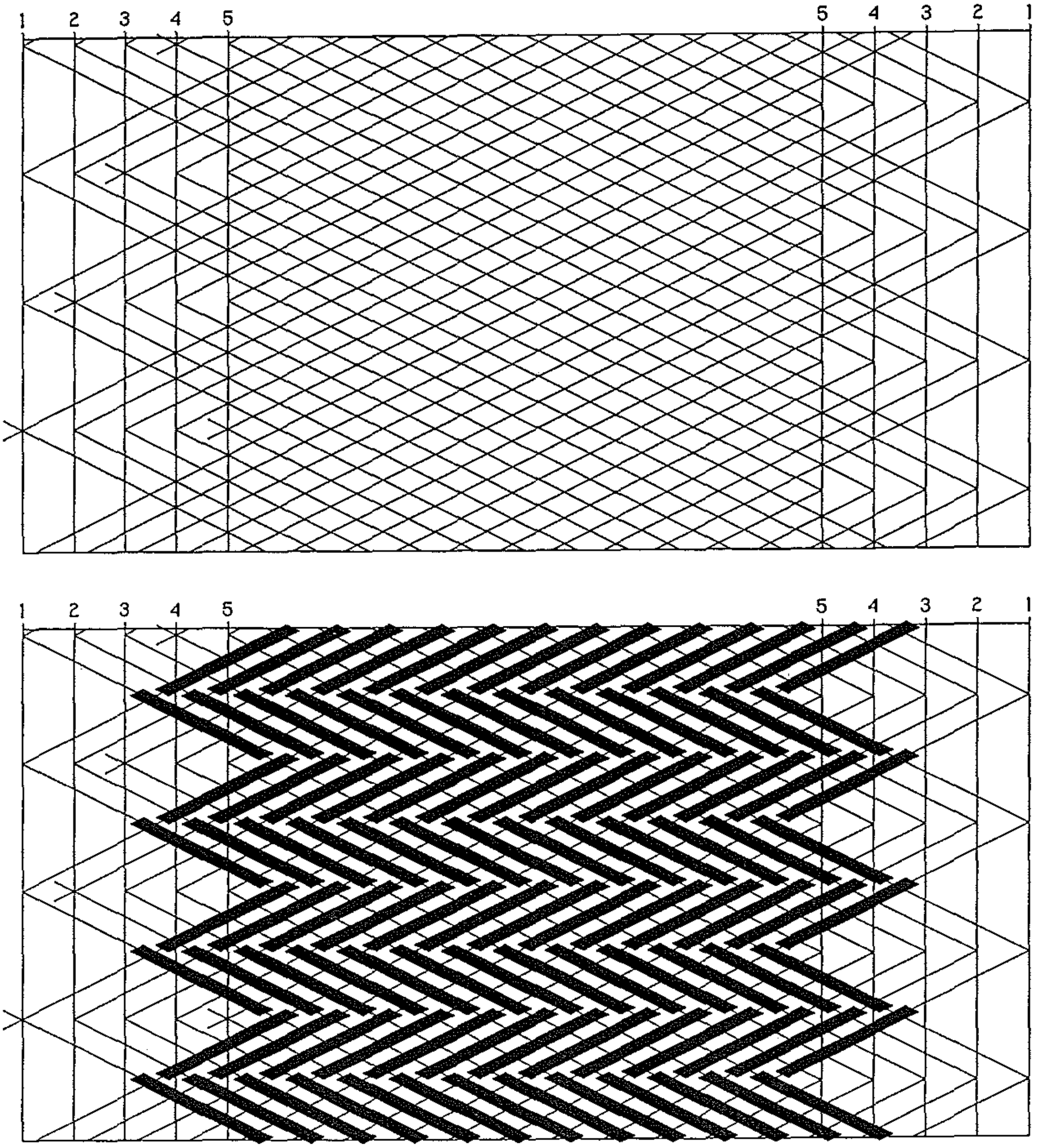


Fig. 447 — A 5-pass Standard Herringbone Pineapple Knot with $x = 23$ and $B^* = 4$.

This knot can be braided in 120 different ways[†]. We shall discuss two of these 120 different ways, randomly selected, in this example.

[†] An A-pass Standard Herringbone Pineapple Knot can be braided in $A!$ different ways. $A!$, pronounced as **A factorial**, stands for $1\times2\times3\times4\times\cdots\times(A-2)\times(A-1)\times A$.

(1). Say that we first braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 1 and its right bight-boundary 3. For this knot $p/b = 7/4$, with $A = 1$ and $l_i = 1$; $r_i = 1$. Its algorithm diagram is thus:

$$\begin{array}{cccccccc} \longrightarrow & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 2 & \\ / & \backslash & / & \backslash & / & \backslash & / & \backslash \\ & 2 & 1 & 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \longleftarrow \end{array}$$

Its half-cycle algorithms are therefore:

half-cycle 1		:	$L \longrightarrow R$	Free Run.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	o .
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	o .
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - o - u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$u - o - u$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$u - 2o - u - o$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$u - 2o - u - o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - o - u - o - u - o$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 2 and its right bight-boundary 2. For this knot $p/b = 7/4$, with $A = 2$ and $l_i = 2$; $r_i = 1$. Its algorithm diagram is thus:

$$\begin{array}{cccccccc} \longrightarrow & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 2 & \\ / & \backslash & / & \backslash & / & \backslash & / & \backslash \\ & 2 & 1 & 0 & 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & \longleftarrow \end{array}$$

Its half-cycle algorithms are therefore:

half-cycle 1		:	$L \longrightarrow R$	$u - o - u - o - u - o$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$o - u - 2o - u - o - u$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$u - o - u - 2o - u - o$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - o - u - 2o - 2u - o - u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$2u - o - u - 2o - 2u - o$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$u - 2o - u - 2o - 2u - 2o - u$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$2u - 2o - u - 2o - 2u - 2o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - 2o - 2u - 2o - 2u - 2o - u$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 3 and its right bight-boundary 1. For this knot $p/b = 7/4$, with $A = 3$ and $l_i = 3$; $r_i = 1$. Its algorithm diagram is thus:

$$\begin{array}{cccccccc} \longrightarrow & 2 & 2 & 2 & 2 & 2 & 2 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 2 & \\ / & \backslash & / & \backslash & / & \backslash & / & \backslash \\ & 2 & 1 & 0 & 3 & 2 & 1 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 & 0 & \longleftarrow \end{array}$$

Its half-cycle algorithms are therefore:

half-cycle 1 : $L \longrightarrow R$ $2u - 2o - 2u - 2o - 2u - 2o$.
half-cycle 2 $i = 0$: $L \longleftarrow R$ $2o - 2u - 3o - 2u - 2o - 2u$.
half-cycle 3 $i = 0$: $L \longrightarrow R$ $2u - 2o - 2u - 3o - 2u - 2o$.
half-cycle 4 $i = 1$: $L \longleftarrow R$ $u - 2o - 2u - 3o - 3u - 2o - 2u$.
half-cycle 5 $i = 1$: $L \longrightarrow R$ $3u - 2o - 2u - 3o - 3u - 2o$.
half-cycle 6 $i = 2$: $L \longleftarrow R$ $u - 3o - 2u - 3o - 3u - 3o - 2u$.
half-cycle 7 $i = 2$: $L \longrightarrow R$ $3u - 3o - 2u - 3o - 3u - 3o$.
half-cycle 8 $i = 3$: $L \longleftarrow R$ $u - 3o - 3u - 3o - 3u - 3o - 2u$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 4 and its right bight-boundary 5. For this knot $p/b = 5/4$, with $A = 4$ and $l_i = 4$; $r_i = 4$. Its algorithm diagram is thus:

$$\begin{array}{ccccccc}
\longrightarrow & 3 & 3 & 3 & 3 & 3 & \\
0 & 3 & 2 & 1 & 0 & & \\
/ & \backslash & / & \backslash & / & \backslash & \\
 & 0 & 1 & 2 & 3 & 0 & \\
3 & 3 & 3 & 3 & 3 & \longleftarrow &
\end{array}$$

Its half-cycle algorithms are therefore:

half-cycle 1 : $L \longrightarrow R$ $3u - 3o - 3u - 3o - 3u$.
half-cycle 2 $i = 0$: $L \longleftarrow R$ $3u - 3o - 3u - 4o - 3u$.
half-cycle 3 $i = 0$: $L \longrightarrow R$ $3u - 3o - 3u - 4o - 3u$.
half-cycle 4 $i = 1$: $L \longleftarrow R$ $3u - 3o - 4u - 4o - 3u$.
half-cycle 5 $i = 1$: $L \longrightarrow R$ $3u - 3o - 4u - 4o - 3u$.
half-cycle 6 $i = 2$: $L \longleftarrow R$ $3u - 4o - 4u - 4o - 3u$.
half-cycle 7 $i = 2$: $L \longrightarrow R$ $3u - 4o - 4u - 4o - 3u$.
half-cycle 8 $i = 3$: $L \longleftarrow R$ $4u - 4o - 4u - 4o - 3u$.

Finally we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 5 and its right bight-boundary 4. For this knot $p/b = 5/4$, with $A = 5$ and $l_i = 5$; $r_i = 4$. Its algorithm diagram is thus:

$$\begin{array}{ccccccc}
\longrightarrow & 4 & 4 & 4 & 4 & 3 & \\
0 & 3 & 2 & 1 & 0 & & \\
/ & \backslash & / & \backslash & / & \backslash & \\
 & 0 & 1 & 2 & 3 & 0 & \\
4 & 4 & 4 & 4 & 3 & \longleftarrow &
\end{array}$$

Its half-cycle algorithms are therefore:

half-cycle 1 : $L \longrightarrow R$ $4u - 4o - 4u - 4o - 3u$.
half-cycle 2 $i = 0$: $L \longleftarrow R$ $3u - 4o - 4u - 5o - 4u$.
half-cycle 3 $i = 0$: $L \longrightarrow R$ $4u - 4o - 4u - 5o - 3u$.
half-cycle 4 $i = 1$: $L \longleftarrow R$ $3u - 4o - 5u - 5o - 4u$.
half-cycle 5 $i = 1$: $L \longrightarrow R$ $4u - 4o - 5u - 5o - 3u$.
half-cycle 6 $i = 2$: $L \longleftarrow R$ $3u - 5o - 5u - 5o - 4u$.
half-cycle 7 $i = 2$: $L \longrightarrow R$ $4u - 5o - 5u - 5o - 3u$.
half-cycle 8 $i = 3$: $L \longleftarrow R$ $4u - 5o - 5u - 5o - 4u$.

The 5-pass Standard Herringbone Pineapple Knot has now been completed.

Instead of using the five algorithm diagrams, one for each interbraided over–under coded Regular Knot, we could have used a general algorithm diagram for the $p/b = 7/4$

over–under coded Regular Knots, and a general algorithm diagram for the $p/b = 5/4$ over–under coded Regular Knots. These general algorithm diagrams are respectively:

$$\begin{array}{cccccccc}
\longrightarrow & l_i - 1 & A-1 & A-1 & A-1 & A-1 & A-1 & r_i - 1 \\
0 & 1 & 2 & 3 & 0 & 1 & 2 & \\
/ & \backslash & / & \backslash & / & \backslash & / & \backslash \\
 & 2 & 1 & 0 & 3 & 2 & 1 & 0 \\
l_i - 1 & A-1 & A-1 & A-1 & A-1 & A-1 & r_i - 1 & \longleftarrow
\end{array}$$

$$\begin{array}{cccccc}
\longrightarrow & l_i - 1 & A-1 & A-1 & A-1 & r_i - 1 \\
0 & 3 & 2 & 1 & 0 & \\
/ & \backslash & / & \backslash & / & \backslash \\
 & 0 & 1 & 2 & 3 & 0 \\
l_i - 1 & A-1 & A-1 & A-1 & r_i - 1 & \longleftarrow
\end{array}$$

(2). Lets use these general algorithm diagrams, and say that we first braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 4 and its right bight-boundary 5. For this knot $p/b = 5/4$, with $A = 1$ and $l_i = 1$; $r_i = 1$. Hence we obtain the following half-cycle algorithms from its associated general algorithm diagram:

half-cycle 1		:	$L \longrightarrow R$	Free Run.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	o.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	o.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - o$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$u - o$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$o - u - o$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$o - u - o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - o - u - o$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 2 and its right bight-boundary 2. For this knot $p/b = 7/4$, with $A = 2$ and $l_i = 1$; $r_i = 1$. Hence from its associated general algorithm diagram we obtain the following half-cycle algorithms:

half-cycle 1		:	$L \longrightarrow R$	$o - u - o - u - o$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$o - u - 2o - u - o$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$o - u - 2o - u - o$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - o - u - 2o - 2u - o$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$u - o - u - 2o - 2u - o$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$u - 2o - u - 2o - 2u - 2o$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$u - 2o - u - 2o - 2u - 2o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - 2o - 2u - 2o - 2u - 2o$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 5 and its right bight-boundary 4. For this knot $p/b = 5/4$, with $A = 3$ and $l_i = 3$; $r_i = 2$. Hence we obtain the following half-cycle algorithms from its associated general algorithm diagram:

half-cycle 1		:	$L \longrightarrow R$	$2u - 2o - 2u - 2o - u$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$u - 2o - 2u - 3o - 2u$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$2u - 2o - 2u - 3o - u$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - 2o - 3u - 3o - 2u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$2u - 2o - 3u - 3o - u$.

half-cycle 6 $i = 2$: $L \longleftarrow R$ $u - 3o - 3u - 3o - 2u$.
half-cycle 7 $i = 2$: $L \longrightarrow R$ $2u - 3o - 3u - 3o - u$.
half-cycle 8 $i = 3$: $L \longleftarrow R$ $2u - 3o - 3u - 3o - 2u$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 1 and its right bight-boundary 3. For this knot $p/b = 7/4$, with $A = 4$ and $l_i = 1$; $r_i = 2$. Hence from its associated general algorithm diagram we obtain the following half-cycle algorithms:

half-cycle 1 : $L \longrightarrow R$ $3o - 3u - 3o - 3u - 3o - u$.
half-cycle 2 $i = 0$: $L \longleftarrow R$ $u - 3o - 3u - 4o - 3u - 3o$.
half-cycle 3 $i = 0$: $L \longrightarrow R$ $3o - 3u - 4o - 3u - 3o - u$.
half-cycle 4 $i = 1$: $L \longleftarrow R$ $2u - 3o - 3u - 4o - 4u - 3o$.
half-cycle 5 $i = 1$: $L \longrightarrow R$ $u - 3o - 3u - 4o - 4u - 3o - u$.
half-cycle 6 $i = 2$: $L \longleftarrow R$ $2u - 4o - 3u - 4o - 4u - 4o$.
half-cycle 7 $i = 2$: $L \longrightarrow R$ $u - 4o - 3u - 4o - 4u - 4o - u$.
half-cycle 8 $i = 3$: $L \longleftarrow R$ $2u - 4o - 4u - 4o - 4u - 4o$.

Finally we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 3 and its right bight-boundary 1. For this knot $p/b = 7/4$, with $A = 5$ and $l_i = 3$; $r_i = 1$. Hence from its associated general algorithm diagram we obtain the following half-cycle algorithms:

half-cycle 1 : $L \longrightarrow R$ $2u - 4o - 4u - 4o - 4u - 4o$.
half-cycle 2 $i = 0$: $L \longleftarrow R$ $4o - 4u - 5o - 4u - 4o - 2u$.
half-cycle 3 $i = 0$: $L \longrightarrow R$ $2u - 4o - 4u - 5o - 4u - 4o$.
half-cycle 4 $i = 1$: $L \longleftarrow R$ $u - 4o - 4u - 5o - 5u - 4o - 2u$.
half-cycle 5 $i = 1$: $L \longrightarrow R$ $3u - 4o - 4u - 5o - 5u - 4o$.
half-cycle 6 $i = 2$: $L \longleftarrow R$ $u - 5o - 4u - 5o - 5u - 5o - 2u$.
half-cycle 7 $i = 2$: $L \longrightarrow R$ $3u - 5o - 4u - 5o - 5u - 5o$.
half-cycle 8 $i = 3$: $L \longleftarrow R$ $u - 5o - 5u - 5o - 5u - 5o - 2u$.

The 5-pass Standard Herringbone Pineapple Knot has now been completed.

From a general algorithm diagram for an interbraided over–under coded Regular Knot, we can compile its associated general half-cycle algorithm table. This has been done in *Book 4/1, Braiding — Standard Herringbone Pineapple Knots* for B^*-values ranging from 3 to 7 inclusive, paired with P_c-values ranging from 3 to 13 inclusive. Especially the novice might find using those tables easier.

A Grant Knot

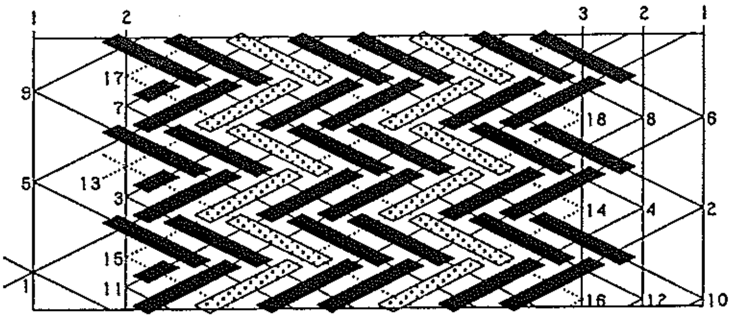


Fig. 448 — (3/15/22){122/123}9.

The Grant Knot in Fig. 448 is an interweave of a Perfect Pineapple Knot and an over–under coded Regular Knot. For the numerals of the left sequence set 122 we obtain with the left bight-boundary position specification 3 and $A_l = 3$ the following ranking-numbers j :[†]

$n = 1$ for numeral $k = 1$, hence its ranking-number $j = |1 + 0|_3 = 1$.
$n = 2$ for numeral $k = 2$, hence its ranking-number $j = |2 + 3|_3 = 2$.
$n = 3$ for numeral $k = 2$, hence its ranking-number $j = |3 + 3|_3 = 0 = 3$.

Hence with the ranking-numbers the Nested Cylindrical Braid specification becomes:

$$(3/15/22)\{1_1 2_2 2_3 / 1_1 2_2 3_3\}9 .$$

From this specification we read the lower-left to upper-right half-cycle types:

$$1_1 \longrightarrow 1_1$$
$$2_2 \longrightarrow 2_2$$
$$2_3 \longrightarrow 3_3$$

Furthermore with $\mathcal{K}_l = 2$ and $\mathcal{K}_r = 3$:

$\Delta_{l_i} = 3$ for $l_i = 1$. $\qquad \Delta_{r_i} = 4$ for $r_i = 1$.
$\Delta_{l_i} = 0$ for $l_i = 2$. $\qquad \Delta_{r_i} = 2$ for $r_i = 2$.
$\qquad\qquad\qquad\qquad \Delta_{r_i} = 0$ for $r_i = 3$.

For the first-return string-runs we thus obtain:

$$\begin{tikzcd}
1_1 & \\
& 2_2 \arrow[ul] \\
2_2 \arrow[ur] & \\
& 1_1 \arrow[ul] \\
1_1 \arrow[ur] &
\end{tikzcd}$$

$$\begin{array}{llllll}
l_{1_{j_l}} = 1_1 \longrightarrow 1_1 = r_{1_{j_r}} & \rightarrow & j'_l = |1 + 3 + 15 + 4|_3 = 2 & \rightarrow & l_{2_{j'_l}} = 2_2 . \\
l_{2_{j'_l}} = 2_2 \longleftarrow 1_1 = r_{1_{j_r}} & \rightarrow & j'_r = |1 + 4 + 15 + 0|_3 = 2 & \rightarrow & r_{2_{j'_r}} = 2_2 . \\
l_{2_{j_l}} = 2_2 \longrightarrow 2_2 = r_{2_{j_r}} & \rightarrow & j'_l = |2 + 0 + 15 + 2|_3 = 1 & \rightarrow & l_{3_{j'_l}} = 1_1 . \\
l_{3_{j'_l}} = 1_1 \longleftarrow 2_2 = r_{2_{j_r}} & \rightarrow & j'_r = |2 + 2 + 15 + 3|_3 = 1 & \rightarrow & r_{3_{j'_r}} = 1_1 .
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum (\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{2 \cdot 15 + (3 + 0) + (4 + 2)}{3} = 13 .$$

g.c.d. $(P_c , B^{**}) =$ g.c.d. $(13 , 3) = 1$.

$$\begin{tikzcd}
2_3 & \\
& 3_3 \arrow[ul] \\
2_3 \arrow[ur] &
\end{tikzcd}$$

$$\begin{array}{llllll}
l_{1_{j_l}} = 2_3 \longrightarrow 3_3 = r_{1_{j_r}} & \rightarrow & j'_l = |3 + 0 + 15 + 0|_3 = 3 & \rightarrow & l_{2_{j'_l}} = 2_3 . \\
l_{2_{j'_l}} = 2_3 \longleftarrow 3_3 = r_{1_{j_r}} & \rightarrow & j'_r = |3 + 0 + 15 + 0|_3 = 3 & \rightarrow & r_{2_{j'_r}} = 3_3 .
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum (\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 15 + (0) + (0)}{3} = 5 .$$

g.c.d. $(P_c , B^{**}) =$ g.c.d. $(5 , 3) = 1$.

Hence:

$$P_{total} = \sum P_{component} = 13 + 5 = 18 .$$

[†] For the formulae used in the following calculations refer to *The Braider*, Issue No. 19, pp. 415–422.

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 2\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1 + 1 = 2\,.$$

1^{st} component; 1^{st} colour; between bight-boundaries 1 & 2 (left) and 1 & 2 (right).

1. $L \longrightarrow R$: Free Run.
2. $R \longrightarrow L$: $o - u$.
3. $L \longrightarrow R$: u.
4. $R \longrightarrow L$: $u - 2o$.
5. $L \longrightarrow R$: $u - 2o - u$.
6. $R \longrightarrow L$: $2u - 3o - u$.
7. $L \longrightarrow R$: $2o - 3u$.
8. $R \longrightarrow L$: $o - 2u - o - u - 2o$.
9. $L \longrightarrow R$: $o - 2u - o - u - 2o - u$.
10. $R \longrightarrow L$: $2o - 2u - o - 2u - 2o - u$.
11. $L \longrightarrow R$: $2u - 2o - u - 2o - 2u$.
12. $R \longrightarrow L$: $u - 2o - 2u - 2o - 2u - 2o$.

2^{nd} component; 2^{nd} colour; between bight-boundaries 2 (left) and 3 (right).

13. $L \longrightarrow R$: $2u - 2o - 2u - 2o - 2u$.
14. $R \longrightarrow L$: $2u - 2o - 2u - 3o - 2u$.
15. $L \longrightarrow R$: $2u - 2o - 2u - 3o - 2u$.
16. $R \longrightarrow L$: $2u - 3o - 2u - 3o - 3u$.
17. $L \longrightarrow R$: $2u - 3o - 2u - 3o - 3u$.
18. $R \longrightarrow L$: $2u - 3o - 3u - 3o - 3u$.

A Pineapple Knot

The Pineapple Knot in Fig. 449 is an interweave of two Perfect Pineapple Knots.

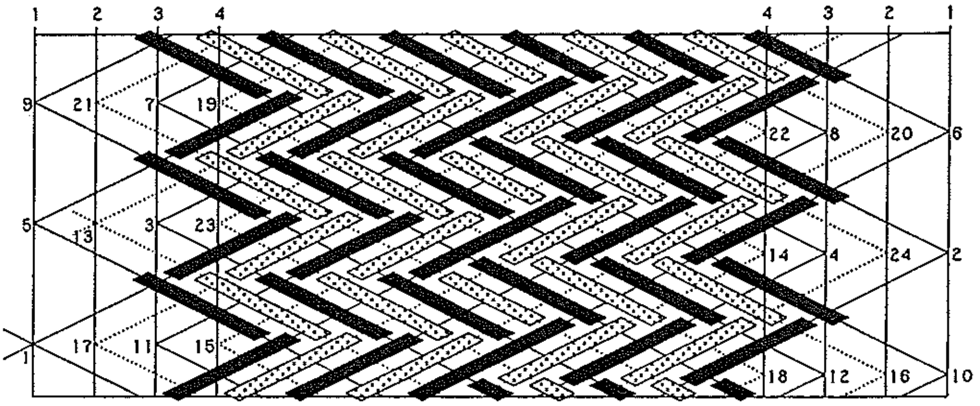


Fig. 449 — (222/18/222){1432/1234}12.

This is a Regular Nested Cylindrical Braid, and as we have seen in *The Braider*, Issue No. 22, pg. 510, we don't require the ranking-numbers. We can therefore use the simplified formulae on pg. 510.

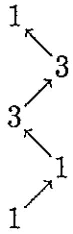
From the Regular Nested Cylindrical Braid specification we obtain:

$$
\begin{aligned}
A &= 4\,. \\
x &= 18\,. \\
d &= A = 4\,. \\
B_{total} &= AB^* = 12\,. \\
B^* &= \frac{B_{total}}{A} = \frac{12}{4} = 3\,. \\
\mathcal{K}_l &= \mathcal{K}_r = \mathcal{K} = A = 4\,.
\end{aligned}
$$

From the given Regular Nested Cylindrical Braid specification we can read the lower-left to upper-right half-cycle types:

$$
\begin{array}{cc}
1 \longrightarrow 1 & 3 \longrightarrow 3 \\
4 \longrightarrow 2 & 2 \longrightarrow 4
\end{array}
$$

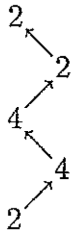
For the first-return string-runs we thus obtain:



$$
\begin{array}{lll}
l_1 = 1 \longrightarrow 1 = r_1 & \rightarrow & l_2 = |1 + 18 - 2(1+1)|_4 = 3\,. \\
l_2 = 3 \longleftarrow 1 = r_1 & \rightarrow & r_2 = |1 + 18 - 2(1+3)|_4 = 3\,. \\
l_2 = 3 \longrightarrow 3 = r_2 & \rightarrow & l_3 = |3 + 18 - 2(3+3)|_4 = 1\,. \\
l_3 = 1 \longleftarrow 3 = r_2 & \rightarrow & r_3 = |3 + 18 - 2(3+1)|_4 = 1\,.
\end{array}
$$

$$P_c = 4\alpha + \frac{\alpha x - 2\sum(l_i + r_i)}{A} = 4 \cdot 2 + \frac{2 \cdot 18 - 2\{(1+3) + (1+3)\}}{4} = 13\,.$$

g.c.d. $(P_c\,, B^*) =$ g.c.d. $(13\,, 3) = 1\,.$



$$
\begin{array}{lll}
l_1 = 2 \longrightarrow 4 = r_1 & \rightarrow & l_2 = |2 + 18 - 2(2+4)|_4 = 4\,. \\
l_2 = 4 \longleftarrow 4 = r_1 & \rightarrow & r_2 = |4 + 18 - 2(4+4)|_4 = 2\,. \\
l_2 = 4 \longrightarrow 2 = r_2 & \rightarrow & l_3 = |4 + 18 - 2(4+2)|_4 = 2\,. \\
l_3 = 2 \longleftarrow 2 = r_2 & \rightarrow & r_3 = |2 + 18 - 2(2+2)|_4 = 4\,.
\end{array}
$$

$$P_c = 4\alpha + \frac{\alpha x - 2\sum(l_i + r_i)}{A} = 4 \cdot 2 + \frac{2 \cdot 18 - 2\{(2+4) + (4+2)\}}{4} = 11\,.$$

g.c.d. $(P_c\,, B^*) =$ g.c.d. $(11\,, 3) = 1\,.$

Hence:

$$P_{total} = \sum P_{component} = 13 + 11 = 24\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 2\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1 + 1 = 2\,.$$

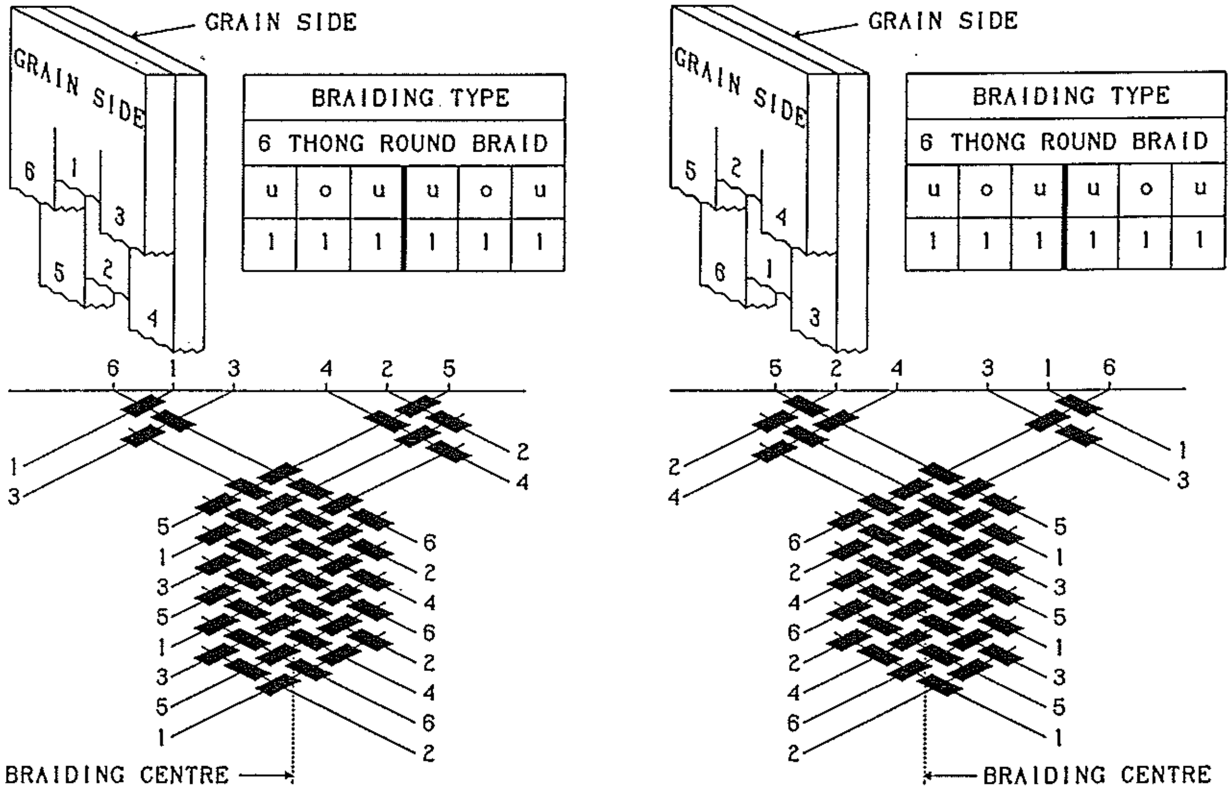
1st component; 1st colour; between bight-boundaries 1 & 3 (left) and 1 & 3 (right).

1. $L \longrightarrow R$: Free Run.
2. $R \longrightarrow L$: $o - u$.
3. $L \longrightarrow R$: u.
4. $R \longrightarrow L$: $u - 2o$.
5. $L \longrightarrow R$: $u - 2o - u$.
6. $R \longrightarrow L$: $2u - 3o - u$.
7. $L \longrightarrow R$: $2o - 3u$.
8. $R \longrightarrow L$: $o - 2u - o - u - 2o$.
9. $L \longrightarrow R$: $o - 2u - o - u - 2o - u$.
10. $R \longrightarrow L$: $2o - 2u - o - 2u - 2o - u$.
11. $L \longrightarrow R$: $2u - 2o - u - 2o - 2u$.
12. $R \longrightarrow L$: $u - 2o - 2u - 2o - 2u - 2o$.

2nd component; 2nd colour; between bight-boundaries 2 & 4 (left) and 2 & 4 (right).

13. $L \longrightarrow R$: $u - 2o - 3u - 2o - 2u$.
14. $R \longrightarrow L$: $2u - 2o - u - 3o - 2u$.
15. $L \longrightarrow R$: $2u - 2o - 4u - 2o - u$.
16. $R \longrightarrow L$: $u - 3o - 2u - 2o - 3u - 2o - u$.
17. $L \longrightarrow R$: $u - 2o - 4u - 3o - 2u$.
18. $R \longrightarrow L$: $2u - 3o - 2u - 3o - 3u$.
19. $L \longrightarrow R$: $4u - 2o - 4u - 4o - u$.
20. $R \longrightarrow L$: $u - 4o - 3u - 2o - 4u - 3o - u$.
21. $L \longrightarrow R$: $2u - 3o - 4u - 4o - 3u$.
22. $R \longrightarrow L$: $3u - 3o - 3u - 4o - 3u$.
23. $L \longrightarrow R$: $4u - 4o - 4u - 4o - 2u$.
24. $R \longrightarrow L$: $u - 4o - 4u - 3o - 4u - 4o - u$.

Some starts for a 6 thong Round Braid



BRAIDING TYPE					
6 THONG ROUND BRAID					
u	o	u	u	o	u
1	1	1	1	1	1

BRAIDING TYPE					
6 THONG ROUND BRAID					
u	o	u	u	o	u
1	1	1	1	1	1

Fig. 450 — A start for a 6 thong Round Braid.

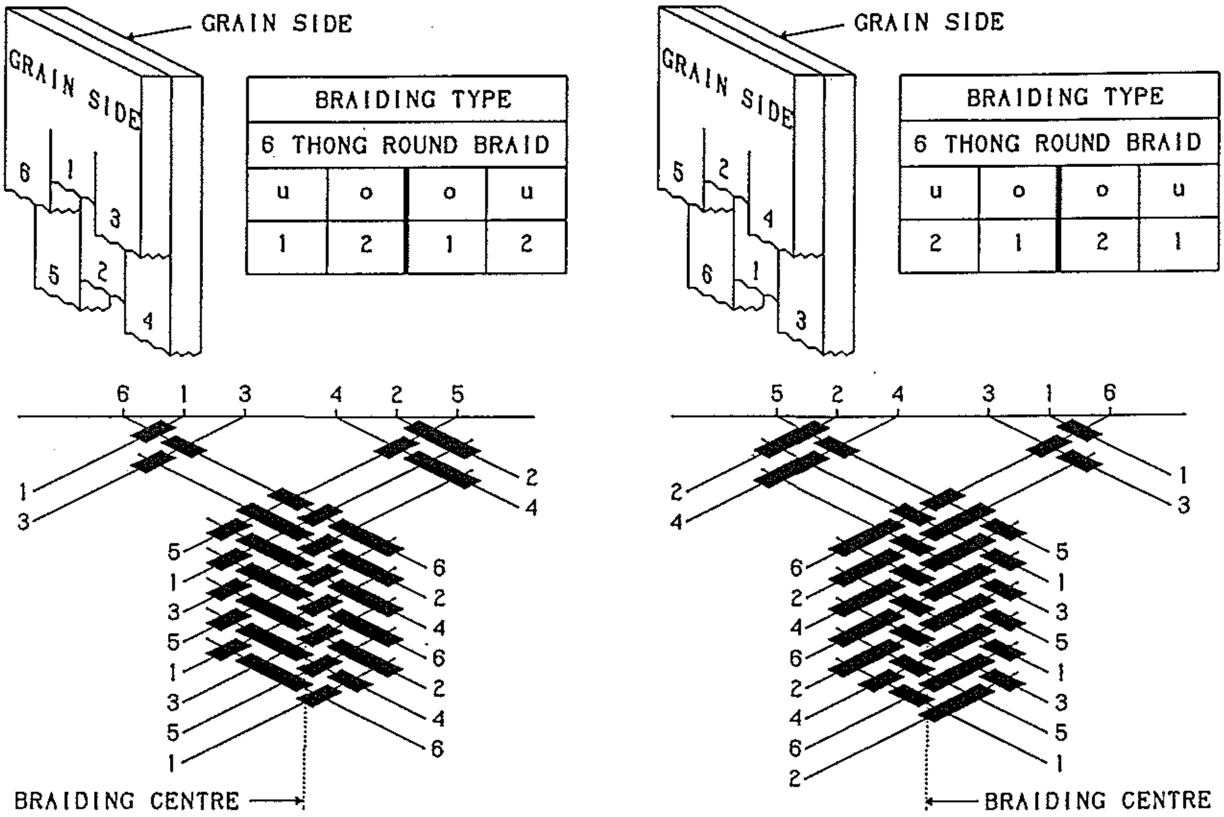


Fig. 451 — A start for a 6 thong Round Braid.

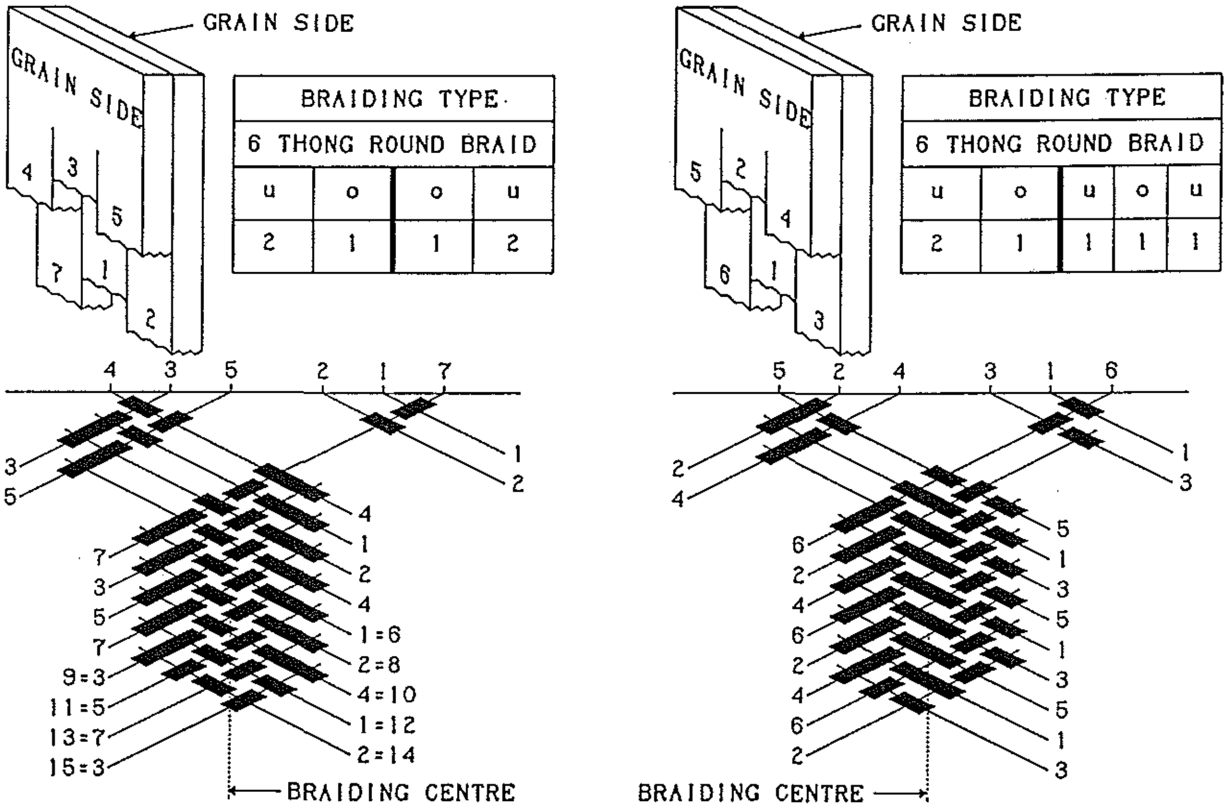


Fig. 452 — A start for a 6 thong Round Braid.

THE BRAIDER

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for
the braiding artisan

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Solving Knotty problems

Most braiders will no doubt have encountered braiding instructions which were, judging from the obtained result, somehow incorrect. The question then arrises to what was incorrect in the given braiding recipe and what it should have been instead. Sometimes, due to various possible reasons, a part of the construction steps might be missing. Nevertheless, since it is impossibe to show in a sketch every part of the braid, and although a sketch of the final braid and sketches of various intermediate construction stages might be given, one often wonders what kind of braid really is associated with the given braiding recipe. Even the final braid in hand might not clearly show what it really is, and hence it will not necessarily be able to suggest better alternatives. Only a grid-diagram enables us to depict every part of a braid, hence a grid-diagram of a finished braid will clearly tell us what kind of braid is involved, and it will also clearly suggest better alternatives if such are possible. In order to avoid a possible waste of materials and time, it is important to translate a braiding recipe into its associated grid-diagram before actually braiding commences.

In our discussion here, we shall present an example in which, due to us not having a complete photocopy of the braiding recipe, the initial construction stages were missing. Furthermore, the instructions are in Spanish, a language we are not familiar with. The photocopied instructions we received are shown on pp. 538 – 541; they come from the book: *Trenzas Gauchas*, by Mario A. Lopez Osornio.

As we see from these pages, the instructions are provided with a great number of sketches, although at least some of them lack in detail. As so often is the case, Lopez Osornio does not work in exact half-cycles, which makes the instructions without the sketches virtually impossible to follow. Our available braiding recipe does not indicate the number of parts and the number of bights of the finished knot. Although the final knot appears, from a casual observation of the final sketches, to be a Column-coded Regular Knot (1 string Column-coded Regular Cylindrical Braid), we cannot be sure about that, and as we will see in our discussion, it is in fact not such a braid.

In Fig. Nº 242 the *armadura* (= foundation knot) is completed, and from it we can see that it is an over–under coded Regular Knot having 5 parts, but since we are missing the sketches and instructions before Fig. Nº 239, we cannot tell how many bights this knot has. It is, however, essential to know the number of bights before we can go on, and hence we have somehow to find this out.

The first step is to employ the general layout of the algorithm diagram for a Regular Knot which has 5-parts and b-bights (see Fig. 453 on page 542). From the sequence of sketches in Fig. Nº 242, Fig. Nº 241, Fig. Nº 240 and Fig. Nº 239, we can deduce the half-cycle algorithms of the last seven half-cycles:

half-cycle 1		Free run.
⋮		⋮
half-cycle $2b-6$	(even numbered half-cycle)	2 crossings, hence 2 i-values involved.
half-cycle $2b-5$	(odd numbered half-cycle)	2 crossings, hence 2 i-values involved.
half-cycle $2b-4$	(even numbered half-cycle)	3 crossings, hence 3 i-values involved.
half-cycle $2b-3$	(odd numbered half-cycle)	3 crossings, hence 3 i-values involved.
half-cycle $2b-2$	(even numbered half-cycle)	4 crossings, hence 4 i-values involved.
half-cycle $2b-1$	(odd numbered half-cycle)	4 crossings, hence 4 i-values involved.
half-cycle $2b$	(even numbered half-cycle)	4 crossings, hence 4 i-values involved.

Fig. Nº 239. — Sobre 4. y bajo 1. la guía marcada con una (M).

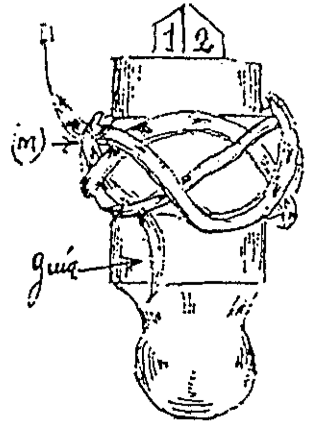


Fig. Nº 239

Fig. 241. — Al bajar bajo 1 y sobre 1. bajo 1 (la guía), sobre 1 y bajo 1 (borde inferior).

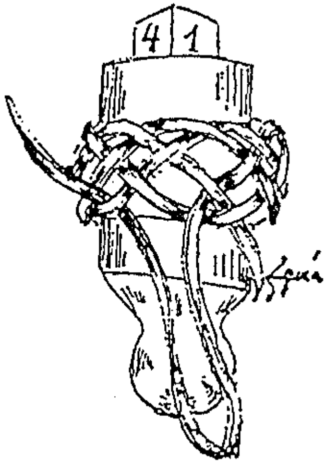


Fig. Nº 241

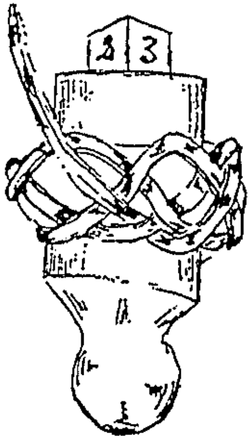


Fig. Nº 240

Fig. 240. — De arriba a abajo: Sobre dos. Esta última pasada forma el borde inferior: bajo 1. Subiendo sobre 2.

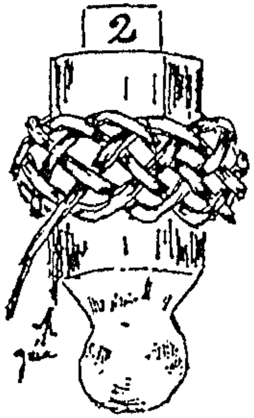


Fig. Nº 242

Fig. 242. — Subiendo: sobre 1, bajo 1, sobre 1 y bajo 1 (borde superior), bajando: sobre 1, bajo 1, sobre 1, llegando con esta pasada al fin de la *armadura*.

Primera maniobra. — Formación de pares.

Fig. Nº 243. — Bajo 1. Al dar esta puntada, la brida comienza a correr a la par de la guía y a la derecha, como podrá observarse: Sobre 1, bajo 1 y sobre 1 y bajo 2. (Al hacer estas dos últimas pasadas, el tiento que teje formó una arista del borde superior).

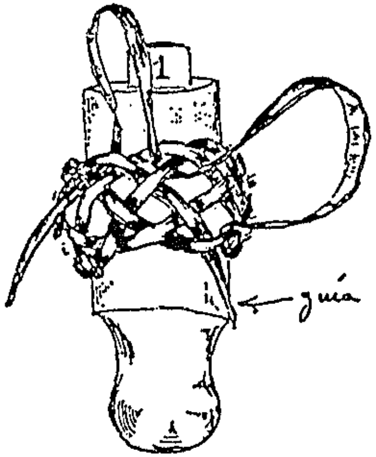


Fig. Nº 243

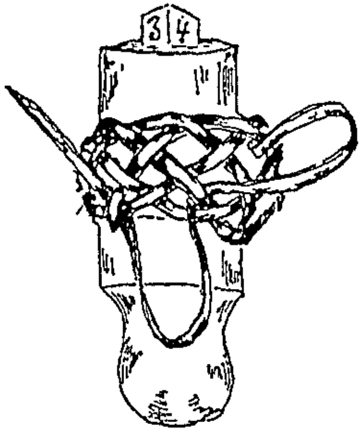


Fig. Nº 244

Fig. Nº 244. — Sobre 1, bajo 1, sobre 1 y bajo 2, formando el borde inferior.

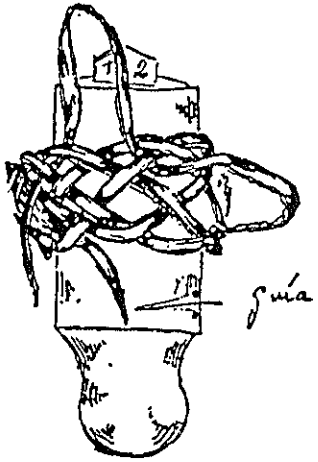


Fig. Nº 245

Fig. Nº 245. — Sobre 1, bajo 1, sobre 1, bajo 2, formando el borde superior.

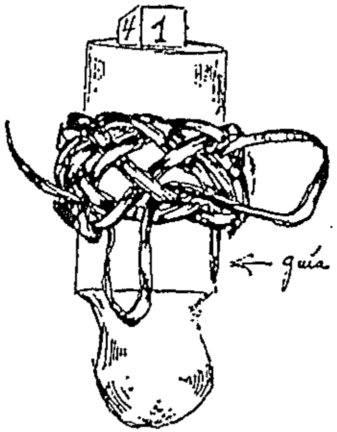


Fig. Nº 246

Fig. Nº 246. — Sobre 2, bajo 1, sobre 1 y bajo 2, formando el borde inferior.

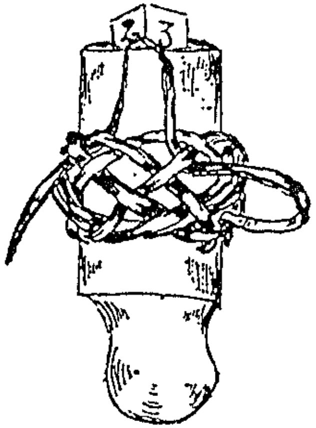


Fig. Nº 247

Fig. Nº 247. — Sobre 2, bajo 1, sobre 1 y bajo 2 (borde superior).

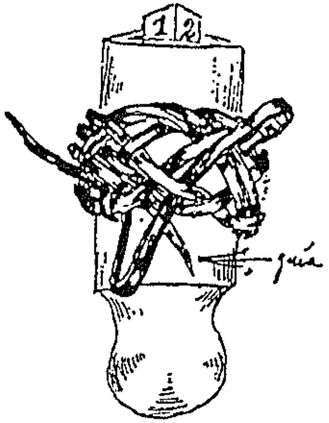


Fig. Nº 248

Fig. Nº 248. — Sobre 2, bajo 1, sobre 2, bajo 2 (borde inferior).

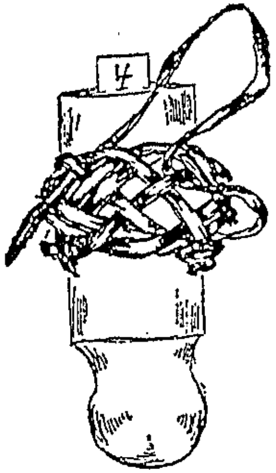


Fig. Nº 249

Fig. Nº 249. — Sobre 2, bajo 1, sobre 2, bajo 2 (borde superior).

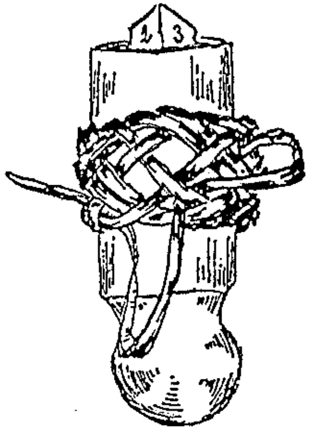


Fig. Nº 250

Fig. Nº 250. — Sobre 2, bajo 1, sobre 2, bajo 2 (borde inferior).

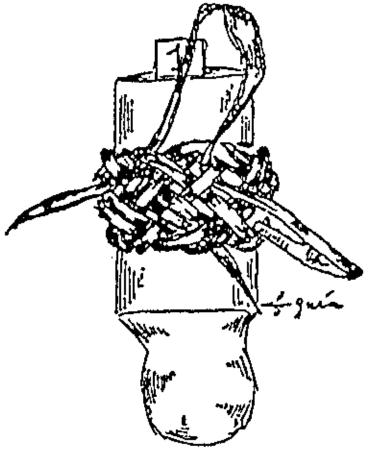


Fig. Nº 251

Fig. Nº 251. — Sobre 2, bajo 1, sobre 2, bajo 3 (borde superior).

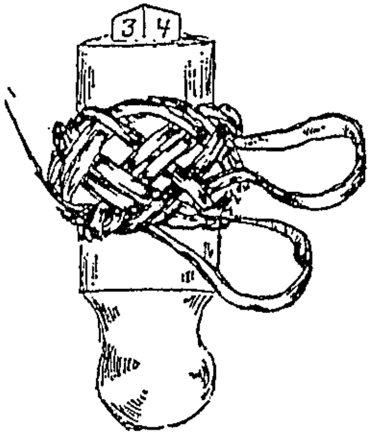


Fig. Nº 252

Fig. Nº 252. — Sobre 2, bajo 1, sobre 2, bajo 3 (borde inferior).

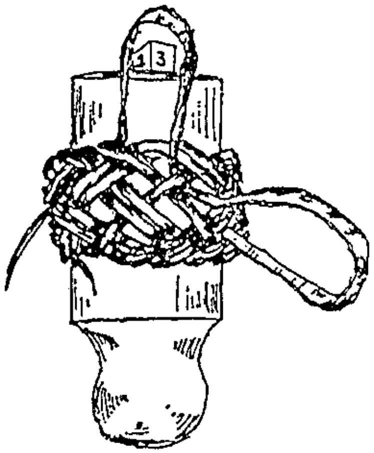


Fig. Nº 253

Fig. Nº 253. — Sobre 2, bajo 1, sobre 2, bajo 3 (borde superior).

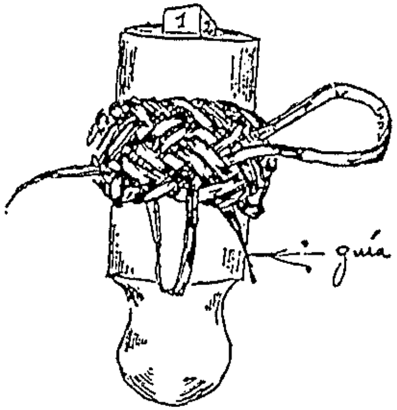


Fig. Nº 254

Fig. Nº 254. — Sobre 2, bajo 2, sobre 2, bajo 3 (borde inferior).

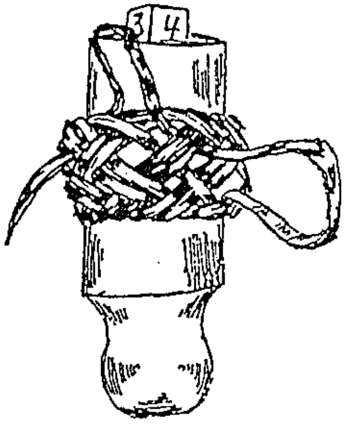


Fig. Nº 255

Fig. Nº 255. — Sobre 2, bajo 2, sobre 2, bajo 3 (borde superior).

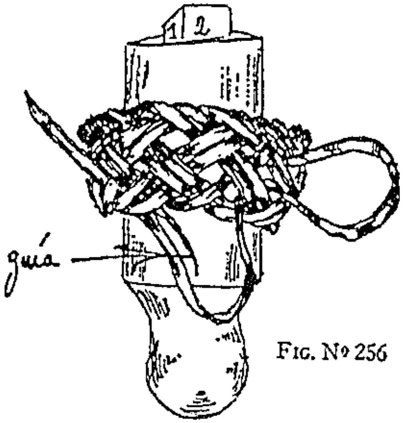


Fig. Nº 256

Fig. Nº 256. — Sobre 2, bajo 2, sobre 2 (borde inferior). Quedando con esto terminada la primer maniobra, es decir la formación de pares. De ahora en adelante la brida comienza a correr a la izquierda de la guía, bajo 3 (borde inferior).

RETEJIDO: 2ª maniobra. — Separación de pares.

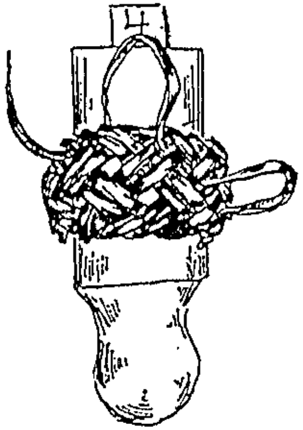


Fig. Nº 257

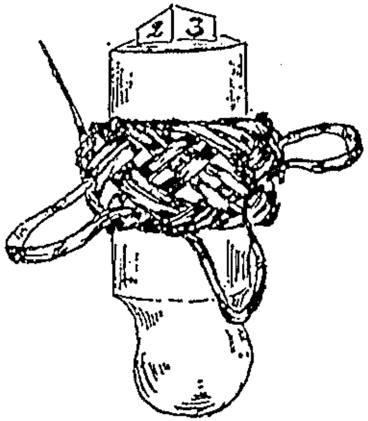


Fig. Nº 258

Fig. Nº 257. — Ya en la anterior figura hemos visto que la última pasada comenzaba separando los pares, las siguientes manipulaciones harán lo mismo, es decir, cada puntada, va separando siempre los tientos que corren juntos ya sea por un extremo o por otro o por los dos a la vez. Sobre 2, bajo 2, sobre 3, bajo 3 (borde superior).

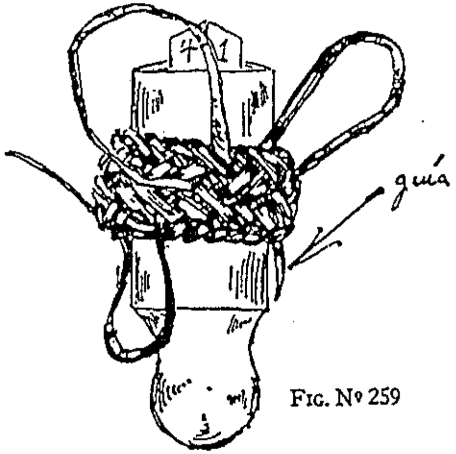
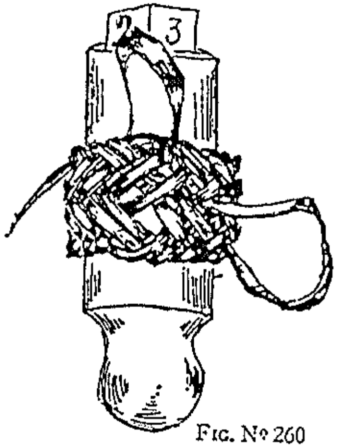


Fig. Nº 259

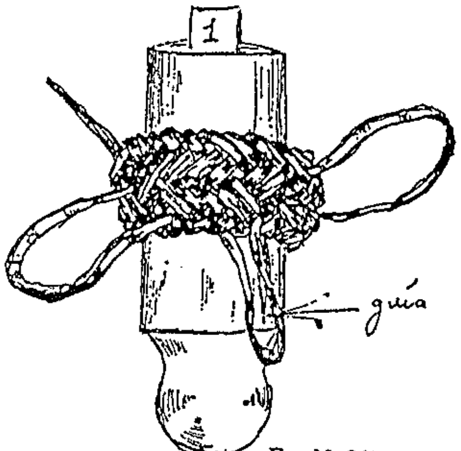
Fig. Nº 258. — Sobre 2, bajo 2, sobre 3, bajo 3 (borde inferior), sobre 2, bajo 2.

Fig. Nº 259. — Sobre 3, bajo 3 (borde superior), sobre 3, bajo 2, sobre 3 y bajo 3 (borde inferior).



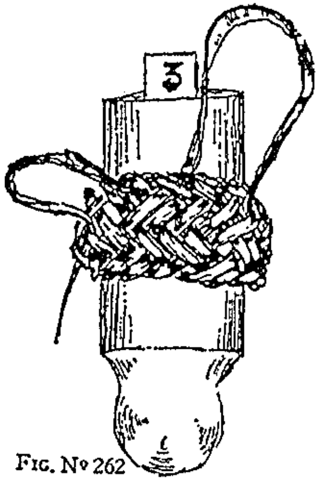
Fic. Nº 260

Fig. Nº 260. — Sobre 3, bajo 2, sobre 3, bajo 3 (borde superior).



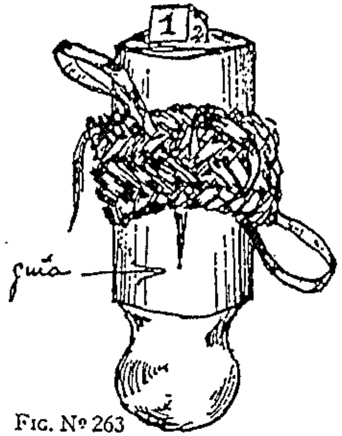
Fic. Nº 261

Fig. 261. — Sobre 3, bajo 2, sobre 4, bajo 3 (borde inferior), sobre 3, bajo 2.



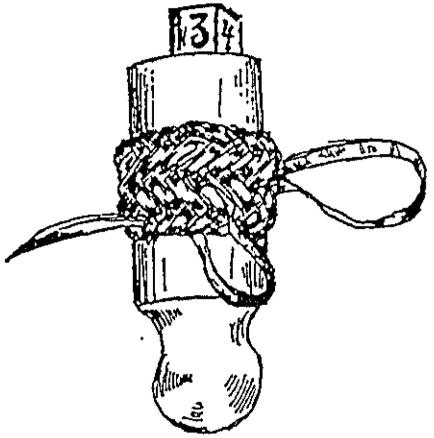
Fic. Nº 262

Fig. Nº 262. — Sobre 4, bajo 3 (borde superior), sobre 3, bajo 2.



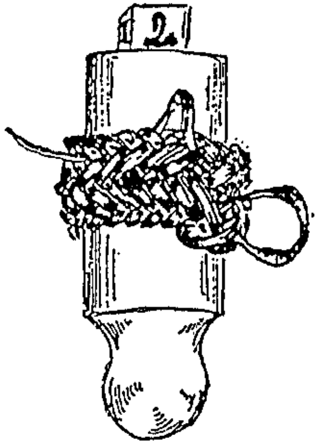
Fic. Nº 263

Fig. 263. — Sobre 4 y bajo 3 (borde inferior), sobre 3, bajo 2, sobre 4, bajo 4 (borde superior).



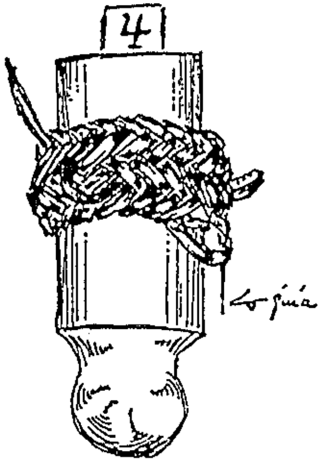
Fic. Nº 264

Fig. Nº 264. — Sobre 3, bajo 2, sobre 4, bajo 4 (borde inferior).



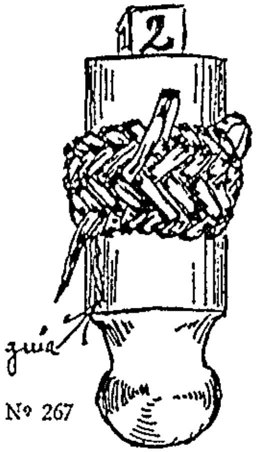
Fic. Nº 265

Fig. 265. — Sobre 3, bajo 2, sobre 4, bajo 4 (borde superior).



Fic. Nº 266

Fig. 266. — Sobre 3, bajo 3, sobre 4, bajo 4 (borde inferior), sobre 3, bajo 3, sobre 4 (borde superior).



Fic. Nº 267

Fig. 267. — Bajo 4 (borde superior), sobre 3, bajo 3, sobre 4, y al esconder la hebra que teje junto a la guía, quedará terminada la bombita.

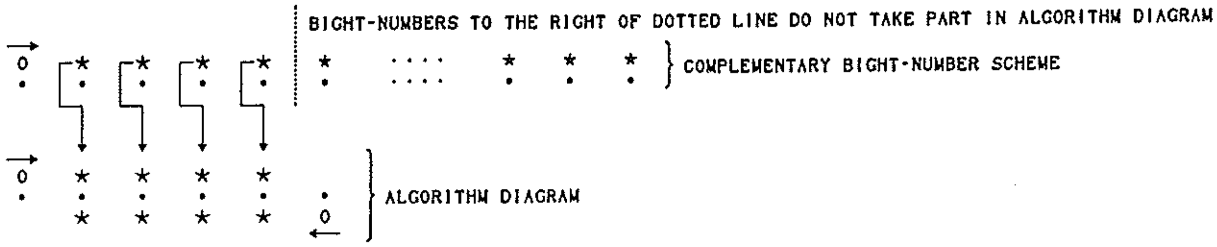

BIGHT-NUMBERS TO THE RIGHT OF DOTTED LINE DO NOT TAKE PART IN ALGORITHM DIAGRAM
COMPLEMENTARY BIGHT-NUMBER SCHEME
ALGORITHM DIAGRAM

Fig. 453 — Complementary cyclic bight-number scheme and algorithm diagram for $p = 5$ and $b > p$; the bight-numbers (i-values) are indicated by a star.

From the final three half-cycles we know that the number of bights must be greater than p, hence greater than 5, because when the number of bights is smaller than p each bight-number (i-value) is associated with at least one intersection-column and hence we cannot have more than two consecutive half-cycles (an even and its consecutive odd numbered half-cycle) with the same number of crossings.

When there are 6 bights ($b = 6$), then $i = 5$ lies to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the upper diagram in Fig. 454). Bight-number $i = 5$ is associated with the final half-cycle $2b = 12$, and $i = 4$ is associated with the half-cycles 10 (is half-cycle $2b - 2$) and 11 (is half-cycle $2b - 1$), hence each of the last three half-cycles makes the same number (4) of crossings. Thus $b = 6$ is a possibility.

When there are 7 bights ($b = 7$), then $i = 3$ and $i = 6$ lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the second diagram in Fig. 454). Bight-number $i = 3$ is associated with the half-cycles 8 (is half-cycle $2b - 6$) and 9 (is half-cycle $2b - 5$), while $i = 6$ is associated with the final half-cycle $2b = 14$. Hence each of the last three half-cycles makes the same number (4) of crossings. From our tabulation at the bottom of pg. 537 we cannot tell whether or not the half-cycles $2b - 8$ and $2b - 7$ each have the same number of crossings as the half-cycles $2b - 6$ and $2b - 5$. Thus $b = 7$ is a possibility.

When there are 8 bights ($b = 8$), then $i = 2$, $i = 5$ and $i = 7$ lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the third diagram in Fig. 454). Bight-number $i = 5$ is associated with the half-cycles 12 (is half-cycle $2b - 4$) and 13 (is half-cycle $2b - 3$), hence each of the half-cycles $2b - 6$, $2b - 5$, $2b - 4$, and $2b - 3$ have the same number of crossings. Since this does not agree with our tabulation, it follows that $b = 8$ is not a possibility.

When there are 9 bights ($b = 9$), then $i = 2$, $i = 4$, $i = 6$ and $i = 8$ lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the fourth diagram in Fig. 454). Here bight-number $i = 6$ acts the same as bight-number $i = 5$ in the case $b = 8$, hence $b = 9$ is not a possibility.

When there are 11 bights ($b = 11$), then $i = 1$, $i = 3$, $i = 5$, $i = 7$, $i = 9$ and $i = 10$ lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the fifth diagram in Fig. 454). The last 5 half-cycles have each the same number of crossings (due to the i-values 9 and 10 which lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453). Since this does not agree with our tabulation, it follows that $b = 11$ is not a possibility.

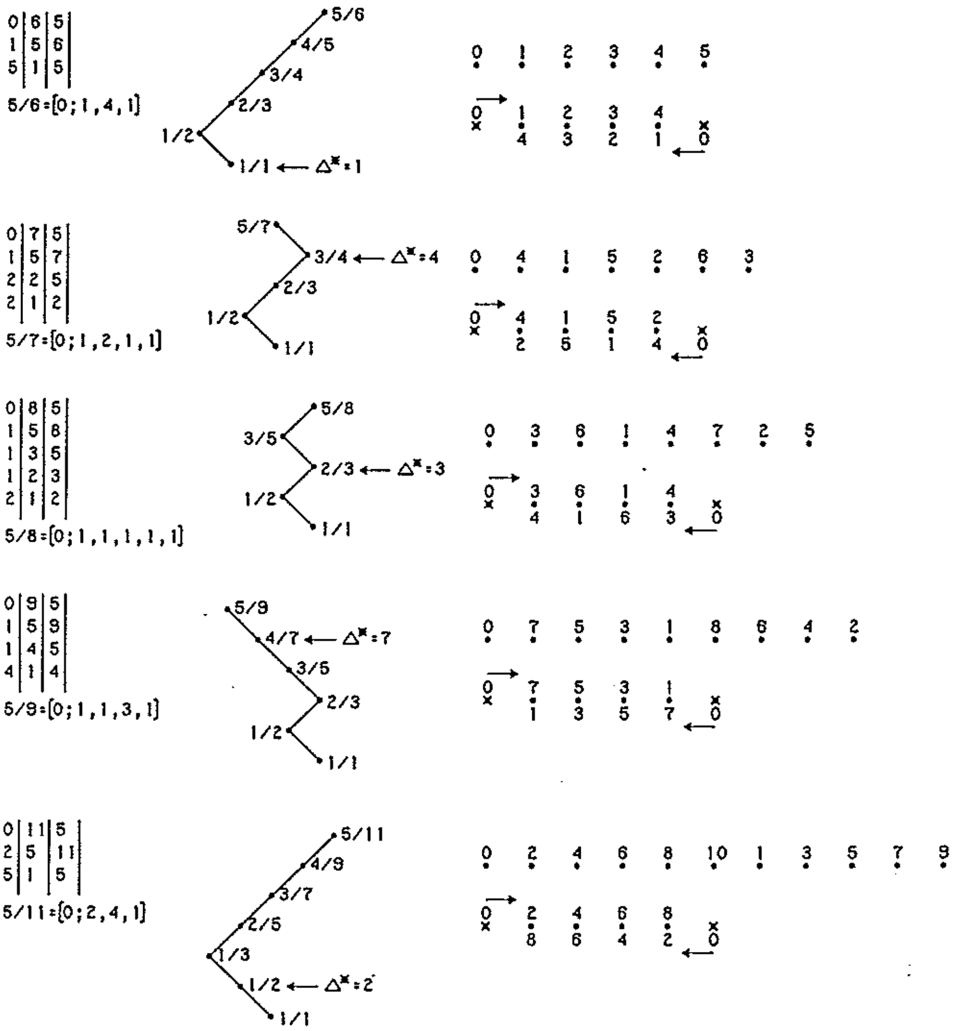


Fig. 454 — Complementary cyclic bight-number schemes and algorithm diagrams for $p/b = 5/6$ to $p/b = 5/11$.

The investigation so far not only doesn't tell us what happens when $b > 11$, but it can considerably be simplified by knowing a little more about the general outlay of a complementary cyclic bight-number scheme. Hence lets first have a look into that.

We know that p and b have to be coprime (g.c.d. $(p, b) = 1$) for a single string Regular Cylindrical Braid (a Regular Knot). Let b be greater than p, and lets find out when $|i|-p|_b|_b = p$.

$$\begin{aligned} b &> p\,,\\ |i|-p|_b|_b &= p\,,\\ |i(b-p)|_b &= p\,,\\ |-ip|_b &= p\,,\\ nb - ip &= p\,; \qquad \text{where } n = \text{a natural number}\,,\\ (i+1)p &= nb\,,\\ i+1 &= \frac{nb}{p}\,, \end{aligned}$$

$$\begin{aligned}&\text{since g.c.d.}\,(p,b)=1 \quad\rightarrow\quad n=p \text{ since } 0\le i\le b-1\,,\\ &\qquad i+1=b\,,\\ &\qquad\quad i=b-1\,,\\ &\qquad\quad i=b-1 \text{ is associated with half-cycle } 2i+2=2b\,.\end{aligned}$$

Thus the position p in a complementary cyclic bight-number scheme for $b>p$ carries the bight-number $i=b-1$.

When the last three half-cycles each have the same number of crossings, then $i=b-1$ is associated with a position to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453, hence with the position $=p$, and $i=b-2$ is associated with a position to the left of the dotted line in that complementary cyclic bight-number scheme, hence with a position $<p$.

$$\begin{aligned}&|(b-1)|-p|_b|_b=p \quad\rightarrow\quad |p|_b=p \text{ hence } b>p\,,\\ &|(b-2)|-p|_b|_b<p \quad\rightarrow\quad |2p|_b<p\,,\\ &\text{Hence :}\quad p<b<2p\,.\end{aligned}$$

When the last five half-cycles each have the same number of crossings, then $(b-1)\ge i\ge(b-2)$ is associated with a position to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453, hence with a position $\ge p$, and $i=b-3$ is associated with a position to the left of the dotted line in that complementary cyclic bight-number scheme, hence with a position $<p$.

$$\begin{aligned}&|(b-2)|-p|_b|_b>p \quad\rightarrow\quad |2p|_b>p \text{ hence } b>2p\,,\\ &|(b-3)|-p|_b|_b<p \quad\rightarrow\quad |3p|_b<p\,,\\ &\text{Hence :}\quad 2p<b<3p\,.\end{aligned}$$

When the last $(2n+1)$ half-cycles, where n is a natural number, each have the same number of crossings, then $(b-1)\ge i\ge(b-n)$ is associated with a position to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453, hence with a position $\ge p$, and $i=b-n-1$ is associated with a position to the left of the dotted line in that complementary cyclic bight-number scheme, hence with a position $<p$.

$$\begin{aligned}&|(b-n)|-p|_b|_b>p \quad\rightarrow\quad |np|_b>p \text{ hence } b>np\,,\\ &|(b-n-1)|-p|_b|_b<p \quad\rightarrow\quad |(n+1)p|_b<p\,,\\ &\text{Hence :}\quad np<b<(n+1)p\,.\end{aligned}$$

Thus from our tabulation at the bottom of pg. 537 we know now that since $n=1$, the number of bights must be greater than p, hence greater than 5, but less than $2p$, hence less than 10.

When $b>p$ the overall number of half-cycles of the string-run consist of p sets of half-cycles, where all the half-cycles of a set have each the same number of crossings. The number of half-cycles in the first set (the free-run half-cycles, the half-cycles each of which intersects zero other half-cycles) is equal to:

$$2\left\lfloor\frac{b}{p}\right\rfloor+1\,.$$

The number of half-cycles in the subsequent consecutive sets greater equal 2 and less than p, hence excepting the last set (the p^{th} set), is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor-\left\lfloor\frac{nb}{p}\right\rfloor\right),\qquad \text{where}\quad n=1\,,2\,,3\,,\cdots\,,(p-2).$$

The number of half-cycles in the last set (the p^{th} set) is equal to:

$$2\left(b-\left\lfloor\frac{(p-1)b}{p}\right\rfloor\right)-1=2\left\lfloor\frac{b}{p}\right\rfloor+1\,.$$

With these formulae it is easy to prove that the sequence of the number of half-cycles of the sequential sets is palindromic[†].

⋆⋆ *Prove the formulae given above for the half-cycle sets, and their palindromic nature.*

These formulae give us much essential information about the algorithm diagram associated with the Regular Knot p/b with $b>p$.

Applying these formulae to our case, we know from the formula for the p^{th} set of half-cycles (the last set of half-cycles) that:

$$2\left\lfloor\frac{b}{p}\right\rfloor+1=3\,,\ \text{hence}\ p<b<2p\,,\ \text{consequently}\ 5<b<10\,.$$

Hence also the first set of half-cycles, the free-run half-cycles, consists of 3 half-cycles, due to the palindromic nature of the sets.

For $b=9$ the number of half-cycles in the penultimate set, for which $n=p-2=5-2=3$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor-\left\lfloor\frac{nb}{p}\right\rfloor\right)=2\left(\left\lfloor\frac{4\times 9}{5}\right\rfloor-\left\lfloor\frac{3\times 9}{5}\right\rfloor\right)=2(7-5)=4\,.$$

This does not agree with our findings on pg. 537, hence $b=9$ is not correct.

For $b=8$ the number of half-cycles in the penultimate set, for which $n=p-2=5-2=3$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor-\left\lfloor\frac{nb}{p}\right\rfloor\right)=2\left(\left\lfloor\frac{4\times 8}{5}\right\rfloor-\left\lfloor\frac{3\times 8}{5}\right\rfloor\right)=2(6-4)=4\,.$$

This does not agree with our findings on pg. 537, hence $b=8$ is not correct.

For $b=7$ the number of half-cycles in the penultimate set, for which $n=p-2=5-2=3$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor-\left\lfloor\frac{nb}{p}\right\rfloor\right)=2\left(\left\lfloor\frac{4\times 7}{5}\right\rfloor-\left\lfloor\frac{3\times 7}{5}\right\rfloor\right)=2(5-4)=2\,.$$

This does agree with our findings on pg. 537, hence $b=7$ is a possibility.

For $b=6$ the number of half-cycles in the penultimate set, for which $n=p-2=5-2=3$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor-\left\lfloor\frac{nb}{p}\right\rfloor\right)=2\left(\left\lfloor\frac{4\times 6}{5}\right\rfloor-\left\lfloor\frac{3\times 6}{5}\right\rfloor\right)=2(4-3)=2\,.$$

This does agree with our findings on pg. 537, hence $b=6$ is a possibility.

For $b=7$ the number of half-cycles in the second to last set, for which $n=p-3=5-3=2$, is equal to:

[†] A palindromic sequence of numbers is a sequence of numbers which, when taken in reverse order, give the same sequence of numbers. For example: 3, 2, 4, 2, 3.

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor - \left\lfloor\frac{nb}{p}\right\rfloor\right) = 2\left(\left\lfloor\frac{3\times 7}{5}\right\rfloor - \left\lfloor\frac{2\times 7}{5}\right\rfloor\right) = 2(4-2) = 4\,.$$

This is a possibility as far as our observations are concerned.

For $b = 6$ the number of half-cycles in the second to last set, for which $n = p - 3 = 5 - 3 = 2$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor - \left\lfloor\frac{nb}{p}\right\rfloor\right) = 2\left(\left\lfloor\frac{3\times 6}{5}\right\rfloor - \left\lfloor\frac{2\times 6}{5}\right\rfloor\right) = 2(3-2) = 2\,.$$

This is a possibility as far as our observations are concerned.

Now it is the time to turn to the algorithm diagrams associated with $b = 6$ and $b = 7$ in order to find out which one is applicable in our case. These algorithm diagrams are depicted in Fig. 455.

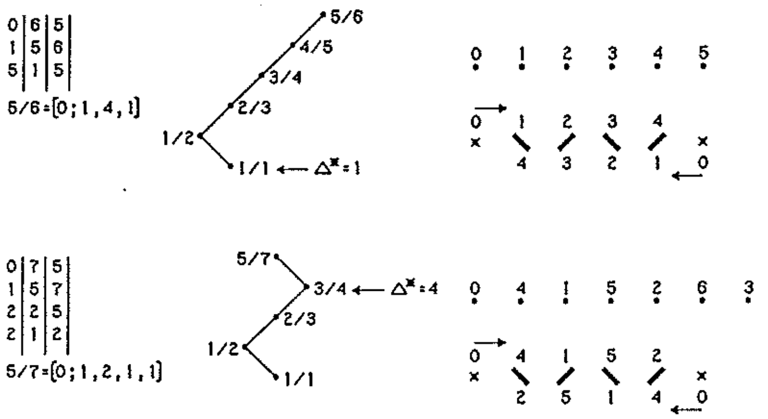


Fig. 455 — The algorithm diagrams for $p/b = 5/6$ and $p/b = 5/7$.

The half-cycle algorithms for $p/b = 5/6$ are obtained in the usual way from its algorithm diagram:

1. $L \longrightarrow R$: Free run.
2. $i = 0$ $R \longrightarrow L$: Free run.
3. $i = 0$ $L \longrightarrow R$: Free run.
4. $i = 1$ $R \longrightarrow L$: u.
5. $i = 1$ $L \longrightarrow R$: u.
6. $i = 2$ $R \longrightarrow L$: $u - o$.
7. $i = 2$ $L \longrightarrow R$: $u - o$.
8. $i = 3$ $R \longrightarrow L$: $u - o - u$.
9. $i = 3$ $L \longrightarrow R$: $u - o - u$.
10. $i = 4$ $R \longrightarrow L$: $u - o - u - o$.
11. $i = 4$ $L \longrightarrow R$: $u - o - u - o$.
12. $i = 5$ $R \longrightarrow L$: $u - o - u - o$.

Half-cycles 8 and 9 do not agree with Fig. N° 240, which indicates for each of these two half-cycles: $u - 2o$. Furthermore, half-cycles 6 and 7 do not agree with Fig. N° 241, which indicates for each of these two half-cycles: $2o$. Hence $b = 6$ is not correct, and $b = 7$ will be the correct one. We confirm this by checking out its associated half-cycle algorithms obtained from its algorithm diagram with the sketches available to us:

1. $L \longrightarrow R$: Free run.
2. $i = 0$ $R \longrightarrow L$: Free run.
3. $i = 0$ $L \longrightarrow R$: Free run.

4. $i=1 \quad R \longrightarrow L \quad : \quad o.$
5. $i=1 \quad L \longrightarrow R \quad : \quad o.$
6. $i=2 \quad R \longrightarrow L \quad : \quad 2o.$
7. $i=2 \quad L \longrightarrow R \quad : \quad 2o.$
8. $i=3 \quad R \longrightarrow L \quad : \quad 2o.$
9. $i=3 \quad L \longrightarrow R \quad : \quad 2o.$
10. $i=4 \quad R \longrightarrow L \quad : \quad u-2o.$
11. $i=4 \quad L \longrightarrow R \quad : \quad u-2o.$
12. $i=5 \quad R \longrightarrow L \quad : \quad u-o-u-o.$
13. $i=5 \quad L \longrightarrow R \quad : \quad u-o-u-o.$
14. $i=5 \quad R \longrightarrow L \quad : \quad u-o-u-o.$

This agrees with the available Fig. Nº 239, 240, 241 and 242.

Now draw the grid-diagram of this foundation knot (see upper leftmost diagram in Fig. 456). Follow in this grid-diagram the instructions given by Lopez Osornio in Figs. Nº 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 and 256 till the lower bight-point, hence stop before forming this lower bight.

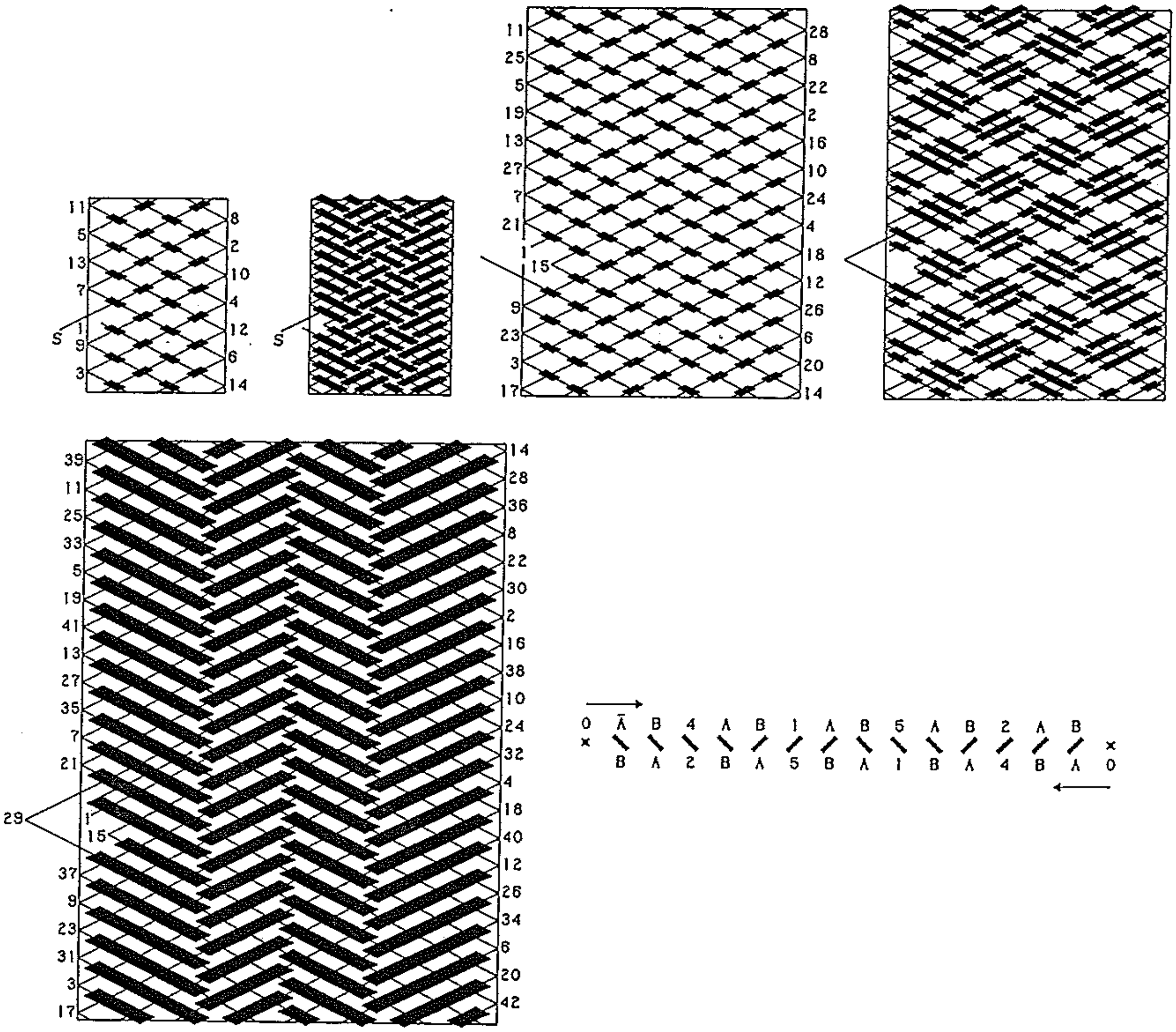


Fig. 456 — The braiding stages of the knot described by Lopez Osornio.

We thus obtain the second diagram in Fig. 456. Redraw this diagram to a larger scale; this gives the third diagram in Fig. 456. Follow in this last diagram the instructions given by Lopez Osornio in Figs. Nº 256 (from lower bight up to the left), 257, 258, 259, 260, 261, 262, 263, 264, 265, 266 and 267. We thus obtain the fourth diagram (upper rightmost) in Fig. 456. Redraw this diagram to a larger scale and finish off the Working-end as descibed by Lopez Osornio in Fig. Nº 267; this gives us the lower leftmost diagram in Fig. 456.

We note that the overall knot is a single string imitation $p/b = 15/21$ Column-coded Semi-Regular Knot. Obviously this knot has, due to its single string imitation form, an irregularity on the left bight-boundary. Although this irregularity might not be observed by novices or non-braiders, any braider of a reasonable standard would never use this or a similar knot. The more experienced braider can just as easy braid the single string Regular Knot $p/b = 15/22$ with the same Column-coding which would not have any irregularity at all. The less experienced braider can braid the single string Regular Knot $p/b = 15/19$ with the same Column-coding, but without any irregularities, much more easily (especially when the Standing-end is placed on the right-hand bight-boundary, and the knot is braided with the string-run from upper-right to lower-left and upper left to lower right).

Observe that the half-cycle algorithms for the by Lopez Osornio descibed imitation can readily be read from the algorithm diagram for the genuine $p/b = 15/21$ Semi-Regular Knot by omitting the first $\overline{A}$ for half-cycle 15 from lower-left to upper-right. Note that for the first stage (the foundation knot, upper leftmost diagram in Fig. 456), hence the half-cycles 1–14, the A and B positions in the algorithm diagram are neglected; for the second stage, resulting in the third diagram in Fig. 456, hence the half-cycles 15–28 (with 15=1 to 28=14 for the algorithm diagram), the B positions in the algorithm diagram are neglected and $\overline{A}$ is only neglected for half-cycle 15; for the third stage, resulting in the lower leftmost diagram in Fig. 456, hence the half-cycles 29–42 (with 29=1 to 42=14 for the algorithm diagram), all positions are taken into account. Hence we obtain:

1. $L \longrightarrow R$: Free run.
2. $i = 0$ $R \longrightarrow L$: Free run.
3. $i = 0$ $L \longrightarrow R$: Free run.
4. $i = 1$ $R \longrightarrow L$: o.
5. $i = 1$ $L \longrightarrow R$: o.
6. $i = 2$ $R \longrightarrow L$: $2o$.
7. $i = 2$ $L \longrightarrow R$: $2o$.
8. $i = 3$ $R \longrightarrow L$: $2o$.
9. $i = 3$ $L \longrightarrow R$: $2o$.
10. $i = 4$ $R \longrightarrow L$: $u - 2o$.
11. $i = 4$ $L \longrightarrow R$: $u - 2o$.
12. $i = 5$ $R \longrightarrow L$: $u - o - u - o$.
13. $i = 5$ $L \longrightarrow R$: $u - o - u - o$.
14. $i = 6$ $R \longrightarrow L$: $u - o - u - o$.

15. $L \longrightarrow R$: $u - o - u - o$. (neglect $\overline{A}$)
16. $i = 0$ $R \longrightarrow L$: $2u - o - u - o$.
17. $i = 0$ $L \longrightarrow R$: $2u - o - u - o$.

18. $i = 1 \quad R \longrightarrow L \quad : \quad 2u - 2o - u - o\,.$
19. $i = 1 \quad L \longrightarrow R \quad : \quad 2u - 2o - u - o\,.$
20. $i = 2 \quad R \longrightarrow L \quad : \quad 2u - 2o - u - 2o\,.$
21. $i = 2 \quad L \longrightarrow R \quad : \quad 2u - 2o - u - 2o\,.$
22. $i = 3 \quad R \longrightarrow L \quad : \quad 2u - 2o - u - 2o\,.$
23. $i = 3 \quad L \longrightarrow R \quad : \quad 2u - 2o - u - 2o\,.$
24. $i = 4 \quad R \longrightarrow L \quad : \quad 3u - 2o - u - 2o\,.$
25. $i = 4 \quad L \longrightarrow R \quad : \quad 3u - 2o - u - 2o\,.$
26. $i = 5 \quad R \longrightarrow L \quad : \quad 3u - 2o - 2u - 2o\,.$
27. $i = 5 \quad L \longrightarrow R \quad : \quad 3u - 2o - 2u - 2o\,.$
28. $i = 6 \quad R \longrightarrow L \quad : \quad 3u - 2o - 2u - 2o\,.$

29. $\quad L \longrightarrow R \quad : \quad 3u - 2o - 2u - 3o\,.$
30. $i = 0 \quad R \longrightarrow L \quad : \quad 3u - 2o - 2u - 3o\,.$
31. $i = 0 \quad L \longrightarrow R \quad : \quad 3u - 2o - 2u - 3o\,.$
32. $i = 1 \quad R \longrightarrow L \quad : \quad 3u - 3o - 2u - 3o\,.$
33. $i = 1 \quad L \longrightarrow R \quad : \quad 3u - 3o - 2u - 3o\,.$
34. $i = 2 \quad R \longrightarrow L \quad : \quad 3u - 3o - 2u - 4o\,.$
35. $i = 2 \quad L \longrightarrow R \quad : \quad 3u - 3o - 2u - 4o\,.$
36. $i = 3 \quad R \longrightarrow L \quad : \quad 3u - 3o - 2u - 4o\,.$
37. $i = 3 \quad L \longrightarrow R \quad : \quad 3u - 3o - 2u - 4o\,.$
38. $i = 4 \quad R \longrightarrow L \quad : \quad 4u - 3o - 2u - 4o\,.$
39. $i = 4 \quad L \longrightarrow R \quad : \quad 4u - 3o - 2u - 4o\,.$
40. $i = 5 \quad R \longrightarrow L \quad : \quad 4u - 3o - 3u - 4o\,.$
41. $i = 5 \quad L \longrightarrow R \quad : \quad 4u - 3o - 3u - 4o\,.$
42. $i = 6 \quad R \longrightarrow L \quad : \quad 4u - 3o - 3u - 4o\,.$

Lets first give the half-cycle algorithms for the above mentioned single string $p/b = 15/22$ (see Fig. 457) and $p/b = 15/19$ (see Fig. 458) knots.

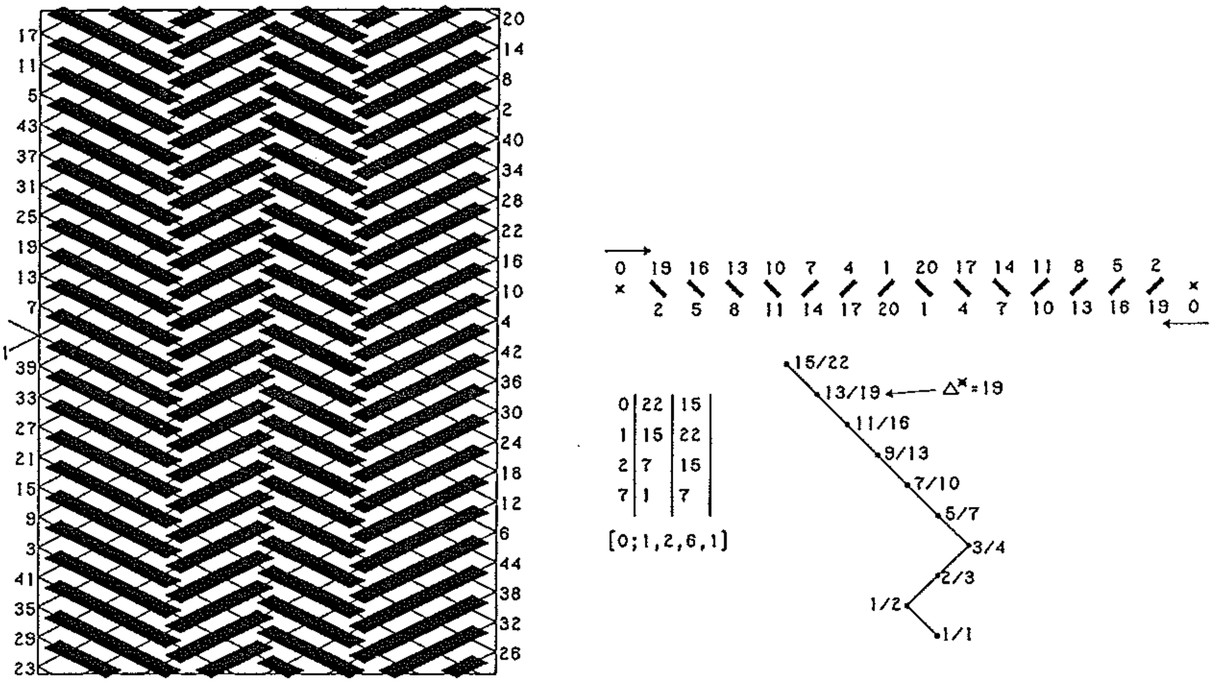


Fig. 457 — The $p/b = 15/22$ Regular Knot.

1. $L \longrightarrow R$: Free run.
2. $i = 0 \quad R \longrightarrow L$: Free run.
3. $i = 0 \quad L \longrightarrow R$: Free run.
4. $i = 1 \quad R \longrightarrow L$: o.
5. $i = 1 \quad L \longrightarrow R$: o.
6. $i = 2 \quad R \longrightarrow L$: $2o$.
7. $i = 2 \quad L \longrightarrow R$: $2o$.
8. $i = 3 \quad R \longrightarrow L$: $2o$.
9. $i = 3 \quad L \longrightarrow R$: $2o$.
10. $i = 4 \quad R \longrightarrow L$: $3o$.
11. $i = 4 \quad L \longrightarrow R$: $3o$.
12. $i = 5 \quad R \longrightarrow L$: $4o$.
13. $i = 5 \quad L \longrightarrow R$: $4o$.
14. $i = 6 \quad R \longrightarrow L$: $4o$.
15. $i = 6 \quad L \longrightarrow R$: $4o$.
16. $i = 7 \quad R \longrightarrow L$: $5o$.
17. $i = 7 \quad L \longrightarrow R$: $5o$.
18. $i = 8 \quad R \longrightarrow L$: $6o$.
19. $i = 8 \quad L \longrightarrow R$: $6o$.
20. $i = 9 \quad R \longrightarrow L$: $6o$.
21. $i = 9 \quad L \longrightarrow R$: $6o$.
22. $i = 10 \quad R \longrightarrow L$: $u - 6o$.
23. $i = 10 \quad L \longrightarrow R$: $u - 6o$.
24. $i = 11 \quad R \longrightarrow L$: $u - 7o$.
25. $i = 11 \quad L \longrightarrow R$: $u - 7o$.
26. $i = 12 \quad R \longrightarrow L$: $u - 7o$.
27. $i = 12 \quad L \longrightarrow R$: $u - 7o$.
28. $i = 13 \quad R \longrightarrow L$: $2u - 7o$.
29. $i = 13 \quad L \longrightarrow R$: $2u - 7o$.
30. $i = 14 \quad R \longrightarrow L$: $2u - 3o - u - 4o$.
31. $i = 14 \quad L \longrightarrow R$: $2u - 3o - u - 4o$.
32. $i = 15 \quad R \longrightarrow L$: $2u - 3o - u - 4o$.
33. $i = 15 \quad L \longrightarrow R$: $2u - 3o - u - 4o$.
34. $i = 16 \quad R \longrightarrow L$: $3u - 3o - u - 4o$.
35. $i = 16 \quad L \longrightarrow R$: $3u - 3o - u - 4o$.
36. $i = 17 \quad R \longrightarrow L$: $3u - 3o - 2u - 4o$.
37. $i = 17 \quad L \longrightarrow R$: $3u - 3o - 2u - 4o$.
38. $i = 18 \quad R \longrightarrow L$: $3u - 3o - 2u - 4o$.
39. $i = 18 \quad L \longrightarrow R$: $3u - 3o - 2u - 4o$.
40. $i = 19 \quad R \longrightarrow L$: $4u - 3o - 2u - 4o$.
41. $i = 19 \quad L \longrightarrow R$: $4u - 3o - 2u - 4o$.
42. $i = 20 \quad R \longrightarrow L$: $4u - 3o - 3u - 4o$.
43. $i = 20 \quad L \longrightarrow R$: $4u - 3o - 3u - 4o$.
44. $i = 21 \quad R \longrightarrow L$: $4u - 3o - 3u - 4o$.

In order to avoid initial under-movements, we can either braid the $p/b = 15/19$ knot in Fig. 458 as shown by the left-hand grid-diagram, or we can rotate this grid-diagram through an angle of 180° about an axis perpendicular to the paper so as to obtain the right-hand grid-diagram. The knot can then be braided in accordance with this right-hand grid-diagram in the normal way.

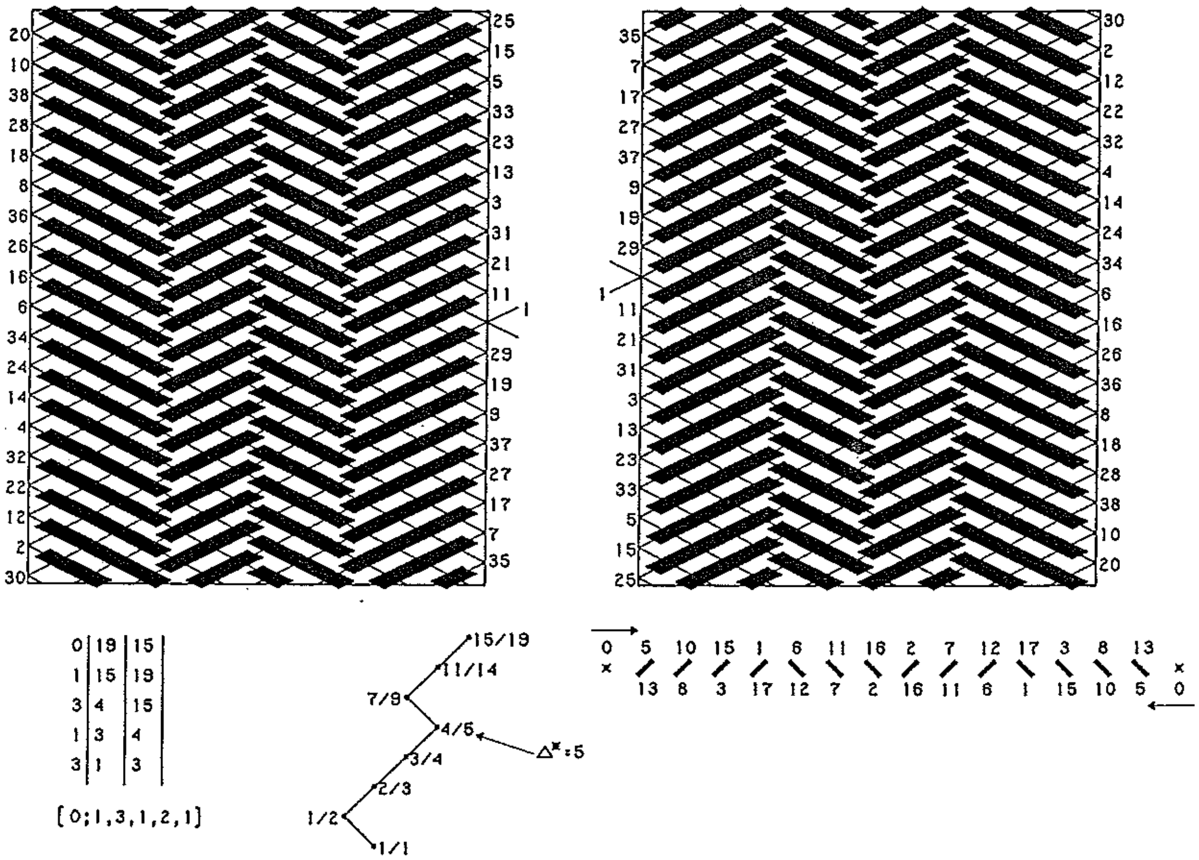


Fig. 458 — The $p/b = 15/19$ Regular Knot.

1. $L \longrightarrow R$: Free run.
2. $i = 0$ $R \longrightarrow L$: Free run.
3. $i = 0$ $L \longrightarrow R$: Free run.
4. $i = 1$ $R \longrightarrow L$: o.
5. $i = 1$ $L \longrightarrow R$: o.
6. $i = 2$ $R \longrightarrow L$: $2o$.
7. $i = 2$ $L \longrightarrow R$: $2o$.
8. $i = 3$ $R \longrightarrow L$: $2o - u$.
9. $i = 3$ $L \longrightarrow R$: $2o - u$.
10. $i = 4$ $R \longrightarrow L$: $2o - u$.
11. $i = 4$ $L \longrightarrow R$: $2o - u$.
12. $i = 5$ $R \longrightarrow L$: $3o - u$.
13. $i = 5$ $L \longrightarrow R$: $3o - u$.
14. $i = 6$ $R \longrightarrow L$: $2o - u - o - u$.
15. $i = 6$ $L \longrightarrow R$: $2o - u - o - u$.
16. $i = 7$ $R \longrightarrow L$: $2o - u - 2o - u$.
17. $i = 7$ $L \longrightarrow R$: $2o - u - 2o - u$.
18. $i = 8$ $R \longrightarrow L$: $2o - u - 2o - 2u$.
19. $i = 8$ $L \longrightarrow R$: $2o - u - 2o - 2u$.
20. $i = 9$ $R \longrightarrow L$: $2o - u - 2o - 2u$.
21. $i = 9$ $L \longrightarrow R$: $2o - u - 2o - 2u$.

22. $i = 10\ R \longrightarrow L$: $3o - u - 2o - 2u$.
23. $i = 10\ L \longrightarrow R$: $3o - u - 2o - 2u$.
24. $i = 11\ R \longrightarrow L$: $3o - 2u - 2o - 2u$.
25. $i = 11\ L \longrightarrow R$: $3o - 2u - 2o - 2u$.
26. $i = 12\ R \longrightarrow L$: $3o - 2u - 3o - 2u$.
27. $i = 12\ L \longrightarrow R$: $3o - 2u - 3o - 2u$.
28. $i = 13\ R \longrightarrow L$: $3o - 2u - 3o - 3u$.
29. $i = 13\ L \longrightarrow R$: $3o - 2u - 3o - 3u$.
30. $i = 14\ R \longrightarrow L$: $3o - 2u - 3o - 3u$.
31. $i = 14\ L \longrightarrow R$: $3o - 2u - 3o - 3u$.
32. $i = 15\ R \longrightarrow L$: $4o - 2u - 3o - 3u$.
33. $i = 15\ L \longrightarrow R$: $4o - 2u - 3o - 3u$.
34. $i = 16\ R \longrightarrow L$: $4o - 3u - 3o - 3u$.
35. $i = 16\ L \longrightarrow R$: $4o - 3u - 3o - 3u$.
36. $i = 17\ R \longrightarrow L$: $4o - 3u - 3o - 4u$.
37. $i = 17\ L \longrightarrow R$: $4o - 3u - 3o - 4u$.
38. $i = 18\ R \longrightarrow L$: $4o - 3u - 3o - 4u$.

Lets return to the knot $p/b = 15/21$ by Lopez Osornio. As we mentioned on pg. 548, this knot has no merit due to the irregularity on the left-hand bight-boundary. Suppose, however, that a single string imitation of the genuine $p/b = 15/21$ Semi-Regular Knot is required, but that no irregularity should be detectable. This can, for example, readily be achieved with the string-run depicted in Fig. 459.

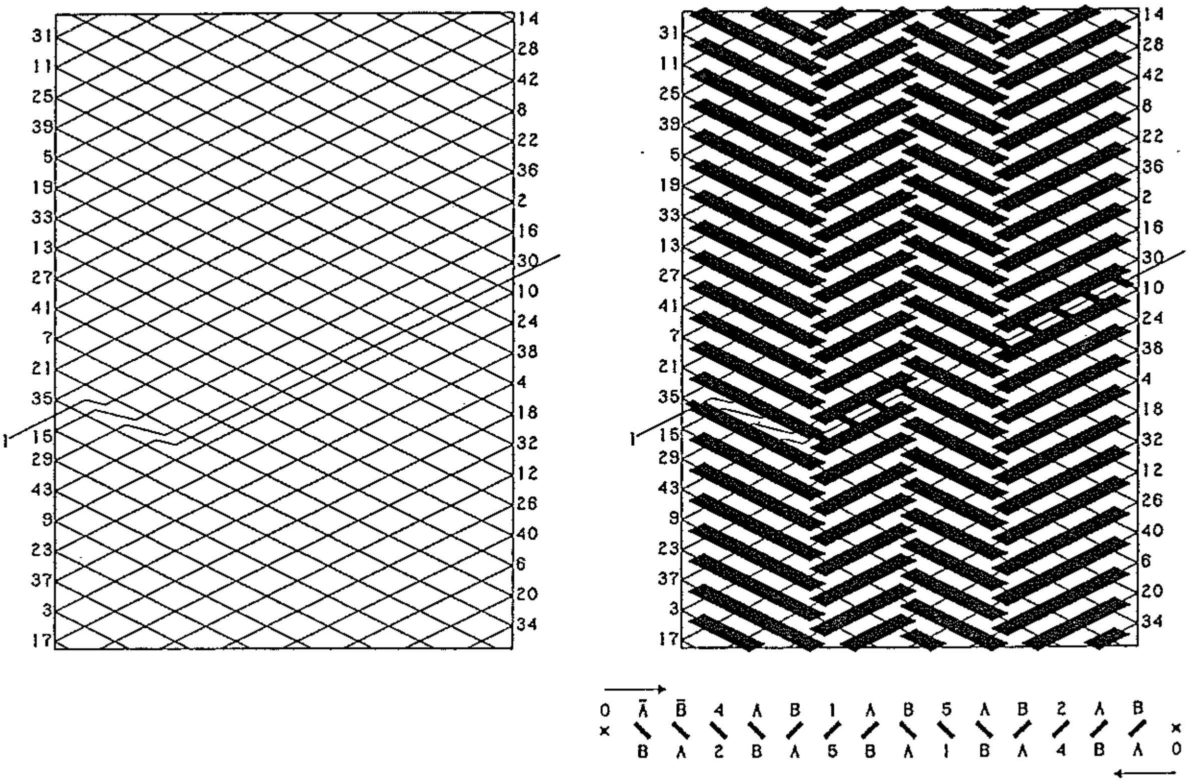


Fig. 459 — An acceptable single string imitation of the $p/b = 15/21$.

Also here can we use with a small modification the algorithm diagram of the true $p/b = 15/21$ Semi-Regular Knot. The modification required is that for the first half-cycle of the second stage, hence for half-cycle 15, $\overline{A}$ must be neglected, and that for the first half-cycle of the third stage, hence half-cycle 29, $\overline{B}$ must be neglected. Thus we obtain the half-cycle algorithms:

1. $L \longrightarrow R$: Free run.
2. $i=0$ $R \longrightarrow L$: Free run.
3. $i=0$ $L \longrightarrow R$: Free run.
4. $i=1$ $R \longrightarrow L$: o.
5. $i=1$ $L \longrightarrow R$: o.
6. $i=2$ $R \longrightarrow L$: $2o$.
7. $i=2$ $L \longrightarrow R$: $2o$.
8. $i=3$ $R \longrightarrow L$: $2o$.
9. $i=3$ $L \longrightarrow R$: $2o$.
10. $i=4$ $R \longrightarrow L$: $u-2o$.
11. $i=4$ $L \longrightarrow R$: $u-2o$.
12. $i=5$ $R \longrightarrow L$: $u-o-u-o$.
13. $i=5$ $L \longrightarrow R$: $u-o-u-o$.
14. $i=6$ $R \longrightarrow L$: $u-o-u-o$.

15. $L \longrightarrow R$: $u-o-u-o$. (neglect $\overline{A}$)
16. $i=0$ $R \longrightarrow L$: $2u-o-u-o$.
17. $i=0$ $L \longrightarrow R$: $2u-o-u-o$.
18. $i=1$ $R \longrightarrow L$: $2u-2o-u-o$.
19. $i=1$ $L \longrightarrow R$: $2u-2o-u-o$.
20. $i=2$ $R \longrightarrow L$: $2u-2o-u-2o$.
21. $i=2$ $L \longrightarrow R$: $2u-2o-u-2o$.
22. $i=3$ $R \longrightarrow L$: $2u-2o-u-2o$.
23. $i=3$ $L \longrightarrow R$: $2u-2o-u-2o$.
24. $i=4$ $R \longrightarrow L$: $3u-2o-u-2o$.
25. $i=4$ $L \longrightarrow R$: $3u-2o-u-2o$.
26. $i=5$ $R \longrightarrow L$: $3u-2o-2u-2o$.
27. $i=5$ $L \longrightarrow R$: $3u-2o-2u-2o$.
28. $i=6$ $R \longrightarrow L$: $3u-2o-2u-2o$.

29. $L \longrightarrow R$: $2u-2o-2u-3o$. (neglect $\overline{B}$)
30. $i=0$ $R \longrightarrow L$: $3u-2o-2u-3o$.
31. $i=0$ $L \longrightarrow R$: $3u-2o-2u-3o$.
32. $i=1$ $R \longrightarrow L$: $3u-3o-2u-3o$.
33. $i=1$ $L \longrightarrow R$: $3u-3o-2u-3o$.
34. $i=2$ $R \longrightarrow L$: $3u-3o-2u-4o$.
35. $i=2$ $L \longrightarrow R$: $3u-3o-2u-4o$.
36. $i=3$ $R \longrightarrow L$: $3u-3o-2u-4o$.
37. $i=3$ $L \longrightarrow R$: $3u-3o-2u-4o$.
38. $i=4$ $R \longrightarrow L$: $4u-3o-2u-4o$.
39. $i=4$ $L \longrightarrow R$: $4u-3o-2u-4o$.
40. $i=5$ $R \longrightarrow L$: $4u-3o-3u-4o$.
41. $i=5$ $L \longrightarrow R$: $4u-3o-3u-4o$.
42. $i=6$ $R \longrightarrow L$: $4u-3o-3u-4o$.
43. $L \longrightarrow R$: $14u$.

Note that although the string-run in Fig. 459 is quite different to the string-run in the lower-left diagram of Fig. 456, the difference in their respective half-cycle algorithms is restricted to their associated half-cycles 29 only. This clearly illustrates the importance of **always** depicting, with a half-cycle algorithm table, the associated grid-diagram.

Nested Cylindrical Braids

On pg. 502 (*The Braider*, Issue No. 22) we have seen that the Regular Nested Cylindrical Braids have the left sequence set $1A(A-1)(A-2)(A-3)\cdots 432$ and the right sequence set $k|k+1|_A|k+2|_A|k+3|_A\cdots A123\cdots|k-2|_A|k-1|_A$, where $1 \leq k \leq A$. Consequently for a general left and right cycle we obtained in Fig. 433 (*The Braider*, pg. 510, Issue No. 22):

$$l_{i+1} = |l_i + x - 2(l_i + r_i)|_A\,.$$
$$r_{i+1} = |r_i + x - 2(r_i + l_{i+1})|_A\,.$$

In Fig. 460 are tabulated the values for $(l_i + r_i)$.

$l_i \longrightarrow r_i$	$l_i + r_i$
$1 \longrightarrow k$	$k+1$
$A \longrightarrow k+1$	$A+k+1$
$A-1 \longrightarrow k+2$	$A+k+1$
$A-2 \longrightarrow k+3$	$A+k+1$
$\vdots$	$\vdots$
$A-(z-1) \longrightarrow k+z$	$A+k+1$
$\vdots$	$\vdots$
$k+2 \longrightarrow A-1$	$A+k+1$
$k+1 \longrightarrow A$	$A+k+1$
$k \longrightarrow 1$	$k+1$
$k-1 \longrightarrow 2$	$k+1$
$k-2 \longrightarrow 3$	$k+1$
$\vdots$	$\vdots$
$4 \longrightarrow k-3$	$k+1$
$3 \longrightarrow k-2$	$k+1$
$2 \longrightarrow k-1$	$k+1$

Fig. 460 — The values of $(l_i + r_i)$ in Regular Nested Cylindrical Braids.

Hence:

$$l_{i+1} = |l_i + x - 2(l_i + r_i)|_A = |l_i + x - 2(k+1)|_A = |l_i + \Delta|_A\,.$$
$$r_{i+1} = |r_i + x - 2(r_i + l_{i+1})|_A = |r_i - [x - 2(l_i + r_i)]|_A = |r_i - \Delta|_A\,.$$

where $\Delta = |x - 2(k+1)|_A$.

$$\Delta = |x - 2(k+1)|_A\,,$$
$$\Delta + nA = x - 2(k+1) \qquad \text{where } n = \text{an integer}\,,$$
$$2k = x - \Delta - 2 - nA\,,$$

Hence the values of k in association with A, x and Δ are:

$$A=\text{odd}\begin{cases} x=\text{odd}\begin{cases}\Delta=\text{odd} & \rightarrow\quad k=\left|\dfrac{x-\Delta-2}{2}\right|_A .\\[2ex] \Delta=\text{even} & \rightarrow\quad k=\left|\dfrac{x-\Delta-2+A}{2}\right|_A .\end{cases}\\[4ex] x=\text{even}\begin{cases}\Delta=\text{odd} & \rightarrow\quad k=\left|\dfrac{x-\Delta-2+A}{2}\right|_A .\\[2ex] \Delta=\text{even} & \rightarrow\quad k=\left|\dfrac{x-\Delta-2}{2}\right|_A .\end{cases}\end{cases}$$

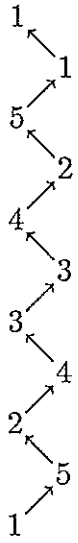
$$A=\text{even}\begin{cases} x=\text{odd}\begin{cases}\Delta=\text{odd} & \rightarrow\quad k=\left|\dfrac{x-\Delta-2}{2}\right|_A ;\ k=\left|\dfrac{x-\Delta-2+A}{2}\right|_A .\\[2ex] \Delta=\text{even} & \rightarrow\quad \text{not possible.}\end{cases}\\[4ex] x=\text{even}\begin{cases}\Delta=\text{odd} & \rightarrow\quad \text{not possible.}\\[2ex] \Delta=\text{even} & \rightarrow\quad k=\left|\dfrac{x-\Delta-2}{2}\right|_A ;\ k=\left|\dfrac{x-\Delta-2+A}{2}\right|_A .\end{cases}\end{cases}$$

Note that the above formulae for $\Delta=\text{even}$ become the formulae at the bottom of pg. 513 and top of pg. 514 when $\Delta=0$. Hence $\Delta=0$ is associated with the Standard Regular Nested Cylindrical Braids and the Semi-Standard Regular Nested Cylindrical Braids.

Example 1:

Let $A=5$; $x=23$; $\Delta=1$.

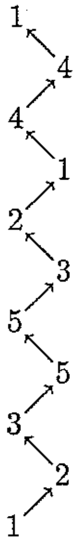
Then $k=\left|\frac{x-\Delta-2}{2}\right|_A=\left|\frac{23-1-2}{2}\right|_5=|10|_5=0=5$. Hence the first-return string-run is:



Example 2:

Let $A=5$; $x=23$; $\Delta=2$.

Then $k = \left|\frac{x-\Delta-2+A}{2}\right|_A = \left|\frac{23-2-2+5}{2}\right|_5 = |12|_5 = 2$. Hence the first-return string-run is:

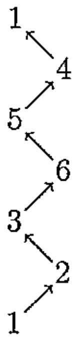


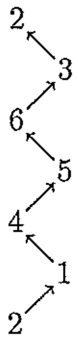
Example 3:

Let $A = 6$; $x = 20$; $\Delta = 2$.

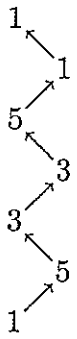
Then:

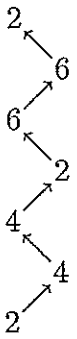
(1). $k = \left|\frac{x-\Delta-2}{2}\right|_A = \left|\frac{20-2-2}{2}\right|_6 = |8|_6 = 2$. Hence the first-return string-runs are:





(2). $k = \left|\frac{x-\Delta-2+A}{2}\right|_A = \left|\frac{20-2-2+6}{2}\right|_6 = |11|_6 = 5$. Hence the first-return string-runs are:





The Regular Nested Cylindrical Braids with $\Delta \neq 0$ can be divided into the following four families:

(i). The Perfect Regular Nested Cylindrical Braids. Here g.c.d.$(\Delta, A) = 1$, and g.c.d.$(P_c, B^*) = 1$. Hence they have one first-return string-run (thus consist of one component); this component requires only one string in its construction. It are the single string Regular Nested Cylindrical Braids.

(ii). The Semi-Perfect Regular Nested Cylindrical Braids. Here g.c.d.$(\Delta, A) = 1$, and g.c.d.$(P_c, B^*) \neq 1$. Hence they also have one first-return string-run (thus consist of one component), but this component requires more than one string in its construction.

(iii). The Compound Regular Nested Cylindrical Braids. Here g.c.d.$(\Delta, A) \neq 1$, and g.c.d.$(P_c, B^*) = 1$. Hence they have more than one first-return string-run (thus consist of more than one component), where each component requires only one string in its construction.

(iv). The Semi-Compound Regular Nested Cylindrical Braids. Here g.c.d.$(\Delta, A) \neq 1$, and at least one g.c.d.$(P_c, B^*) \neq 1$. Hence they have more than one first-return string-run (thus consist of more than one component), and at least one component requires more than one string in its construction.

For the Regular Nested Cylindrical Braids let the shortest vertical distance, in the circumferential direction of braiding (the distance along a bight-edge in the circumferential direction of braiding), from the apex of a left-hand nest of bights to the apex of a right-hand nest of bights be equal to y **row**-units, then:

$$\begin{aligned} y &= |2(A - l_1) + x + 2(A - r_1)|_{2A} \\ &= |2(A - 1) + x + 2(A - k)|_{2A} \\ &= |x - 2(1 + k)|_{2A}\,. \end{aligned}$$

Hence:

$$A = \text{odd}\begin{cases} x = \text{odd}\begin{cases} \Delta = \text{odd} & \rightarrow \quad y = |\Delta|_{2A} = \Delta\,. \\ \Delta = \text{even} & \rightarrow \quad y = |\Delta + A|_{2A} = \Delta + A\,. \end{cases} \\ x = \text{even}\begin{cases} \Delta = \text{odd} & \rightarrow \quad y = |\Delta + A|_{2A} = \Delta + A\,. \\ \Delta = \text{even} & \rightarrow \quad y = |\Delta|_{2A} = \Delta\,. \end{cases} \end{cases}$$

$$A = \text{even}\begin{cases} x = \text{odd}\begin{cases} \Delta = \text{odd} & \rightarrow \quad y = |\Delta|_{2A} = \Delta\ ;\ y = |\Delta + A|_{2A} = \Delta + A\,. \\ \Delta = \text{even} & \rightarrow \quad \text{not possible}\,. \end{cases} \\ x = \text{even}\begin{cases} \Delta = \text{odd} & \rightarrow \quad \text{not possible}\,. \\ \Delta = \text{even} & \rightarrow \quad y = |\Delta|_{2A} = \Delta\ ;\ y = |\Delta + A|_{2A} = \Delta + A\,. \end{cases} \end{cases}$$

In most practical applications we take $x \geq 2$, although x can be less than 2, hence can be negative. For $x \leq 2$ the following conditions apply:

$$\begin{aligned} 2 - A \leq x &\leq 2\,. \\ y_{min} &= 2 - x\,. \\ y_{max} &= 2A + x - 2\,. \end{aligned}$$

$\star\star$ *Prove the above conditions for* $x \leq 2$.

There are two basic complementary forms associated with a first-return string-run:

1. **The basic complementary form 1 first-return string-run is the mirror-image of a first-return string-run.** Note that the mirror-image from "left" to "right" is identical to the mirror-image from "top" to "bottom".

Example 4:

$$\begin{tikzcd}
1 & & & & 1 & & 1 & \\
& 4 \arrow[ul] & & 4 \arrow[ur] & & & & 2 \arrow[ul] \\
4 \arrow[ur] & & & & 4 \arrow[ul] & & 3 \arrow[ur] & \\
& 1 \arrow[ul] & & 1 \arrow[ur] & & & & 5 \arrow[ul] \\
2 \arrow[ur] & & & & 2 \arrow[ul] & & 5 \arrow[ur] & \\
& 3 \arrow[ul] & \longrightarrow & 3 \arrow[ur] & & = & & 3 \arrow[ul] \\
5 \arrow[ur] & & & & 5 \arrow[ul] & & 2 \arrow[ur] & \\
& 5 \arrow[ul] & & 5 \arrow[ur] & & & & 1 \arrow[ul] \\
3 \arrow[ur] & & & & 3 \arrow[ul] & & 4 \arrow[ur] & \\
& 2 \arrow[ul] & & 2 \arrow[ur] & & & & 4 \arrow[ul] \\
1 \arrow[ur] & & & & 1 \arrow[ul] & & 1 \arrow[ur] &
\end{tikzcd}$$

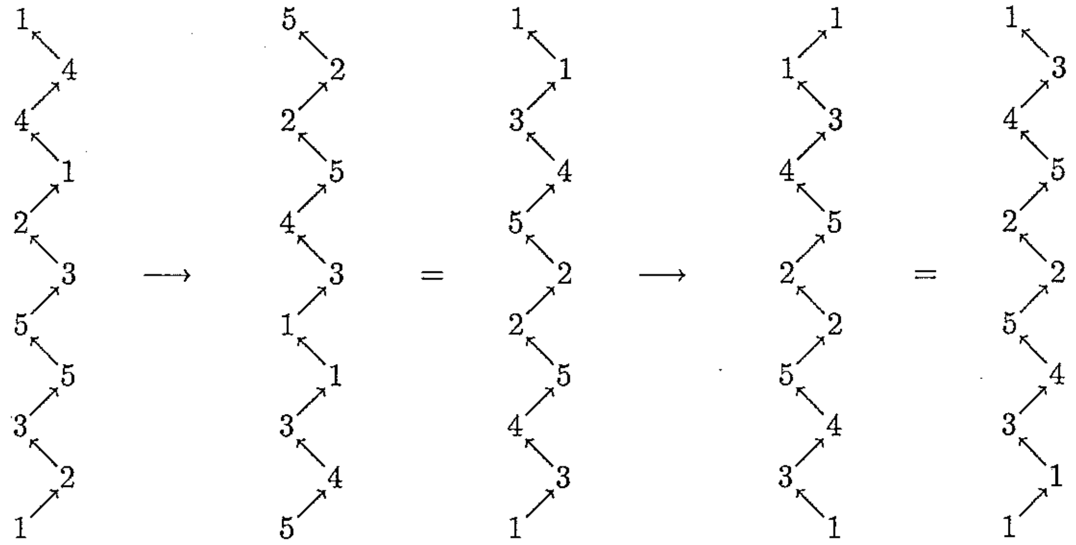
2. **The basic complementary form 2 first-return string-run involves the half-cycle complements with respect to $A_l = A$ and $A_r = A$ (the (A_l, A_r)-complement, where the complement of l_i is equal to $(A_l + 1 - l_i)$ and the complement of r_i is equal to $(A_r + 1 - r_i)$; hence in the case where $A_l = A_r = A$, the (A, A)-complement, where the complement of l_i is equal to $(A + 1 - l_i)$ and the complement of r_i is equal to $(A + 1 - r_i)$).**

Example 5:

$$\begin{tikzcd}
1 & & & 5 & & & 1 & \\
& 4 \arrow[ul] & & & 2 \arrow[ul] & & & 1 \arrow[ul] \\
4 \arrow[ur] & & & 2 \arrow[ur] & & & 3 \arrow[ur] & \\
& 1 \arrow[ul] & & & 5 \arrow[ul] & & & 4 \arrow[ul] \\
2 \arrow[ur] & & & 4 \arrow[ur] & & & 5 \arrow[ur] & \\
& 3 \arrow[ul] & \longrightarrow & & 3 \arrow[ul] & = & & 2 \arrow[ul] \\
5 \arrow[ur] & & & 1 \arrow[ur] & & & 2 \arrow[ur] & \\
& 5 \arrow[ul] & & & 1 \arrow[ul] & & & 5 \arrow[ul] \\
3 \arrow[ur] & & & 3 \arrow[ur] & & & 4 \arrow[ur] & \\
& 2 \arrow[ul] & & & 4 \arrow[ul] & & & 3 \arrow[ul] \\
1 \arrow[ur] & & & 5 \arrow[ur] & & & 1 \arrow[ur] &
\end{tikzcd}$$

The combination of these two basic complementary first-return string-run forms results in a compound-complementary first-return string-run which is the mirror-imaged (A_l, A_r)-complement of the given first-return string-run. Hence in the case where $A_l = A_r = A$, the combination of these two basic complementary first-return string-run forms results in a compound-complementary first-return string-run which is the mirror-imaged (A, A)-complement of the given first-return string-run.

Example 6:



At the left-hand side in Fig. 461 are depicted some of the half-cycles of the string-run of a Regular Nested Cylindrical Braid with nesting-number A, while at the right-hand side are depicted their mirror-images.

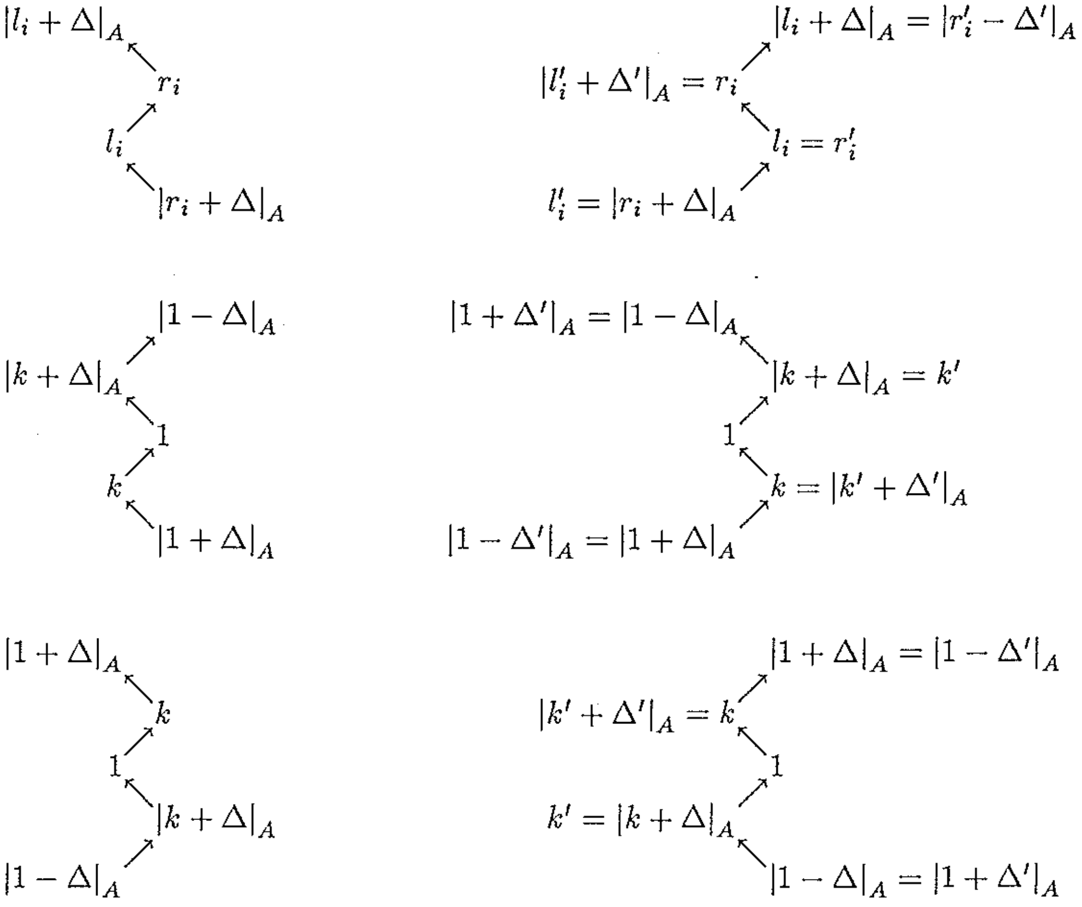


Fig. 461 — Some half-cycles of the string-run of a Regular Nested Cylindrical Braid and their mirror-image.

Let x, y and Δ be associated with the string-run of a Regular Nested Cylindrical

Braid with nesting-number A, and let x', y' and Δ' be associated with its mirror-imaged string-run. Then:

$$\begin{aligned}
y &= |x-2(k+1)|_{2A}\,.\\
\Delta &= |x-2(k+1)|_A = |y|_A\,.\\
2\Delta &= 2|x-2(k+1)|_A = |2\{x-2(k+1)\}|_{2A}\,.\\
k' &= |k+\Delta|_A\,.\\
\Delta' &= |-\Delta|_A\,.
\end{aligned}$$

$$\begin{aligned}
y' &= |x'-2(k'+1)|_{2A}\\
&= |x'-2|k+\Delta|_A-2|_{2A}\\
&= |x'-|2(k+\Delta)|_{2A}-2|_{2A}\\
&= |x'-2k-2\Delta-2|_{2A}\\
&= |x'-2(k+1)-2\Delta|_{2A}\\
&= |x'-2(k+1)-2x+4(k+1)|_{2A}\\
&= |x'-2x+2(k+1)|_{2A}\\
\text{for}\quad x'=x \rightarrow\quad y' &= |-\{x-2(k+1)\}|_{2A}\\
&= |-|x-2(k+1)|_{2A}|_{2A}\\
&= |-y|_{2A}\,.
\end{aligned}$$

$$\text{Thus :}\qquad y' = |-y|_{2A}\ ;\ \Delta' = |-\Delta|_A\ ;\ k' = |k+\Delta|_A\,.$$

The relationship $y' = |-y|_{2A}$ follows, with $\Delta' = |-\Delta|_A$, also immediately from the relationships shown on pg. 557 between A, x, Δ and y.

At the left-hand side in Fig. 462 are depicted some of the half-cycles of the string-run of a Regular Nested Cylindrical Braid with nesting-number A, while at the right-hand side are depicted their mirror-imaged (A,A)-complements.

Let x, y and Δ be associated with the string-run of a Regular Nested Cylindrical Braid with nesting-number A, and let x'', y'' and Δ'' be associated with its mirror-imaged (A,A)-complementary string-run. Then:

$$\begin{aligned}
y &= |x-2(k+1)|_{2A}\,.\\
\Delta &= |x-2(k+1)|_A = |y|_A\,.\\
k'' &= |-k-\Delta|_A\,.\\
\Delta'' &= \Delta\,.
\end{aligned}$$

$$\begin{aligned}
\Delta'' &= |x''-2(k''+1)|_A\\
&= |x''+2k+2\Delta-2|_A\\
&= |x''+2k+\Delta+x-2k-2-2|_A\\
&= |x''+x-4+\Delta|_A\,.\\
nA &= x''+x-4\,.
\end{aligned}$$

$$\frac{x''+x}{2} = 2+\frac{nA}{2}\,,\qquad \text{where } n = \text{whole number.}$$

$$\begin{tikzcd}
|l_i+\Delta|_A & \\
 & r_i \arrow[ul] \\
l_i \arrow[ur] & \\
 & |r_i+\Delta|_A \arrow[ul]
\end{tikzcd}
\qquad
\begin{tikzcd}
 & |1-l_i-\Delta|_A = |r_i''-\Delta''|_A \\
|l_i''+\Delta''|_A = |1-r_i|_A \arrow[ur] & \\
 & |1-l_i|_A = r_i'' \arrow[ul] \\
l_i'' = |1-r_i-\Delta|_A \arrow[ur] &
\end{tikzcd}$$

$$\begin{tikzcd}
 & |1-\Delta|_A \\
|k+\Delta|_A \arrow[ur] & \\
 & 1 \arrow[ul] \\
k \arrow[ur] & \\
 & |1+\Delta|_A \arrow[ul]
\end{tikzcd}
\qquad
\begin{tikzcd}
|A+\Delta''|_A = |A+\Delta|_A & \\
 & |1-k-\Delta|_A = |k''+1|_A \arrow[ul] \\
A \arrow[ur] & \\
 & |1-k|_A = |k''+1+\Delta''|_A \arrow[ul] \\
|A-\Delta''|_A = |A-\Delta|_A \arrow[ur] &
\end{tikzcd}$$

$$\begin{tikzcd}
 & |k+1-\Delta|_A \\
|A+\Delta|_A \arrow[ur] & \\
 & |k+1|_A \arrow[ul] \\
A \arrow[ur] & \\
 & |k+1+\Delta|_A \arrow[ul] \\
|A-\Delta|_A \arrow[ur] &
\end{tikzcd}
\qquad
\begin{tikzcd}
|k''+2\Delta''|_A = |-k+\Delta|_A & \\
 & |1-\Delta|_A = |1-\Delta''|_A \arrow[ul] \\
|k''+\Delta''|_A = |-k|_A \arrow[ur] & \\
 & 1 \arrow[ul] \\
k'' = |-k-\Delta|_A \arrow[ur] & \\
 & |1+\Delta|_A = |1+\Delta''|_A \arrow[ul]
\end{tikzcd}$$

$$\begin{tikzcd}
 & |A-\Delta|_A \\
|k+1+\Delta|_A \arrow[ur] & \\
 & A \arrow[ul] \\
|k+1|_A \arrow[ur] & \\
 & |A+\Delta|_A \arrow[ul]
\end{tikzcd}
\qquad
\begin{tikzcd}
|1+\Delta''|_A = |1+\Delta|_A & \\
 & |-k-\Delta|_A = k'' \arrow[ul] \\
1 \arrow[ur] & \\
 & |-k|_A = |k''+\Delta''|_A \arrow[ul] \\
|1-\Delta''|_A = |1-\Delta|_A \arrow[ur] &
\end{tikzcd}$$

$$\begin{tikzcd}
|1+\Delta|_A & \\
 & k \arrow[ul] \\
1 \arrow[ur] & \\
 & |k+\Delta|_A \arrow[ul] \\
|1-\Delta|_A \arrow[ur] &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & |A-\Delta|_A = |A-\Delta''|_A \\
|k''+1+\Delta''|_A = |1-k|_A \arrow[ur] & \\
 & A \arrow[ul] \\
|k''+1|_A = |1-k-\Delta|_A \arrow[ur] & \\
 & |A+\Delta|_A = |A+\Delta''|_A \arrow[ul]
\end{tikzcd}$$

Fig. 462 — Some half-cycles of the string-run of a Regular Nested Cylindrical Braid and their mirror-imaged (A, A)-complements.

$$\begin{aligned} y'' &= |x'' - 2(k''+1)|_{2A} \\ &= |x'' + 2k + 2\Delta - 2|_{2A} \\ &= |x'' + 2k + 2x - 4k - 4 - 2|_{2A} \\ &= |x'' + x + y - 4|_{2A} \end{aligned}$$

$$\text{for} \quad y'' = y \rightarrow \quad \frac{x''+x}{2} = 2 + \frac{2mA}{2}, \qquad \text{where } m = \text{whole number.}$$

$$\text{Hence :} \qquad n = 2m$$

The table in Fig. 463 tabulates the Regular Nested Cylindrical Braids for A, x and y. From the foregoing it follows that:

The first-return string-runs belonging to cell F_1 and the cell F_2 are the mirror-image of each other. The first-return string-runs belonging to cell F_3 and the cell F_4 are the mirror-image of each other.

The first-return string-runs belonging to cell F_1 and the cell F_3 are the mirror-imaged (A,A)-complements of each other. The first-return string-runs belonging to cell F_2 and the cell F_4 are the mirror-imaged (A,A)-complements of each other.

The first-return string-runs belonging to cell F_1 and the cell F_4 are the (A,A)-complements of each other. The first-return string-runs belonging to cell F_2 and the cell F_3 are the (A,A)-complements of each other.

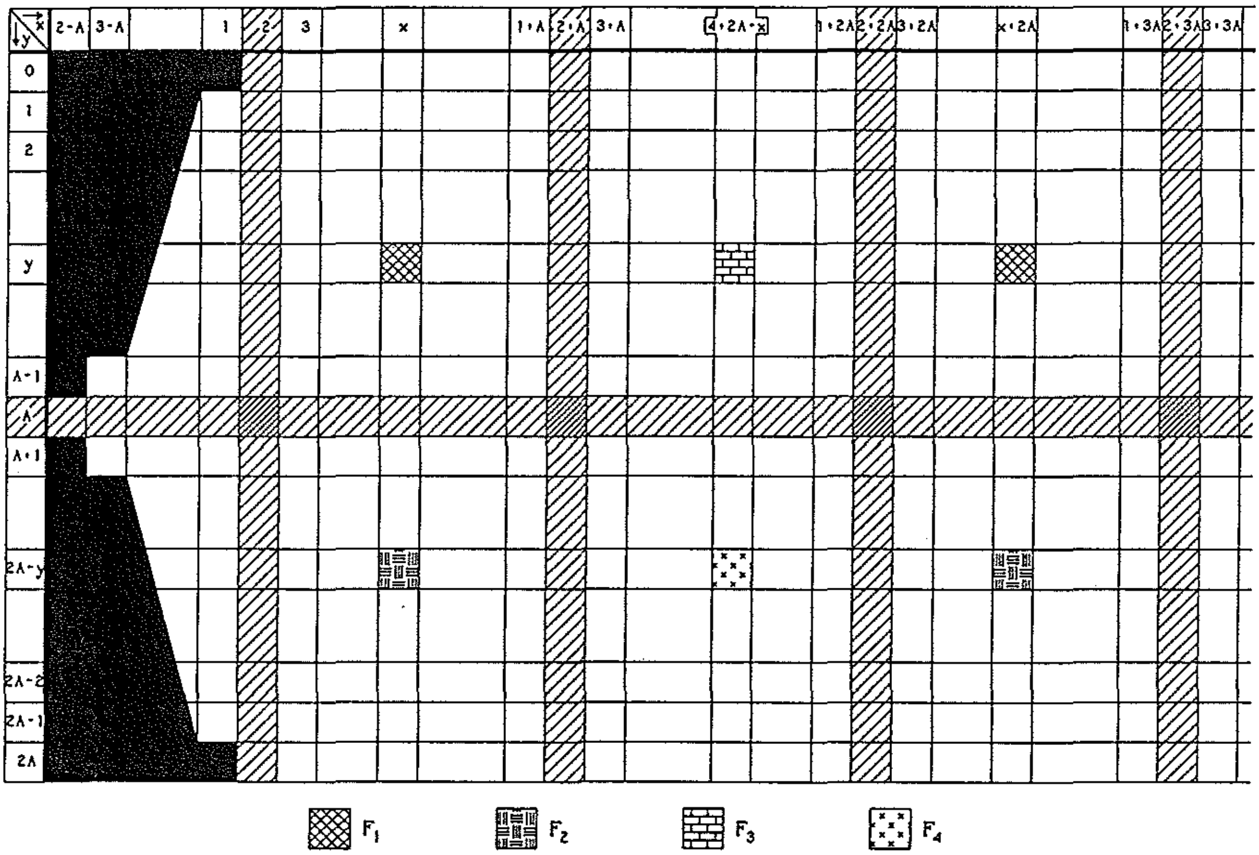


Fig. 463 — The A, x, y table.

THE BRAIDER

APPENDIX 2000

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A quarterly publication
for
the braiding artisan

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Knot nomenclature

In *The Braider*, Issue No. 21, pp. 465–466, we introduced a braid name change in order to alleviate any possible misinterpretations. Most readers will know that any particular issue of *The Braider* has been written a few years before its publication date, and that this necessitated the introduction of a yearly Appendix. Issue No. 21, for example, was written in March 1997. To some braiders we had indicated that with the first Issue of *The Braider* in the year 2000 (hence Issue No. 21) we would introduce the name change discussed in that Issue. In some ensuing correspondence it became clear that Tom Hall had misinterpreted the use of the term **over–under** or **under–over** coding. It is therefore possible that also some other braiders might need to be reminded to read carefully in which context the term **over–under** or **under–over** coding is being used. Hence we feel that, hopefully, the following examples might facilitate in the alleviation of any misinterpretations.
(1). The terms **over–under coding** or **under–over coding** and **over–under coded Braid** or **under–over coded Braid** mean that the braid has an over–under coding or under–over coding, but no further specifics about the braidform are necessarily implied, hence the term may, for example, apply to any form of **flat braid**, **round braid**, **torus braid**, **cylindrical braid**, etc.
(2). The term **over–under coded Cylindrical Braid** or **under–over coded Cylindrical Braid** means that the braid is a Cylindrical Braid with an over–under or under–over coding throughout, but no further specifics about the cylindrical braidform are necessarily implied.
(3). The term **over–under coded Regular Cylindrical Braid** or **under–over coded Regular Cylindrical Braid** means that the braid is a Regular Cylindrical Braid with an over–under or under–over coding throughout, but no further specifics about the regular cylindrical braidform are necessarily implied.
(4). The term **over–under coded Knot** or **under–over coded Knot** means that the braid is made from one string and has an over–under or under–over coding throughout, but no further specifics about the knotform are necessarily implied.
(5). The term **over–under coded Cylindrical Knot** or **under–over coded Cylindrical Knot** means that the braid is a Cylindrical Knot (hence a single-string Cylindrical Braid) with an over–under or under–over coding throughout, but no further specifics about the cylindrical knotform are necessarily implied.
(6). The term **over–under coded Regular Knot** or **under–over coded Regular Knot** means that the braid is a Regular Knot (hence a single-string Regular Cylindrical Braid) with an over–under or under–over coding throughout, but no further specifics about the regular knotform are necessarily implied.

It is of course up to each individual whether or not he/she will follow the discussed name change. No doubt many will not, but that is irrelevant to us. When we feel that a name change might alleviate some possible confusion, or might avoid a possible misconception, we will certainly introduce it. Hence suggestions in that field are greatly appreciated, and will of course be acknowledged.

In the Dutch knotting periodical *Het Knoopeknauwertje*, Issue No. 12, we find on the pages 8 to 10 an article by Tom Hall entitled *Sobre-gecodeerde Knopen* (*Sobre-coded Knots*). We have already discussed in *The Braider — Appendix 1999*, pp. *iii-v*, that the term **Casa** as defined by Tom Hall has no place in knot nomenclature. In

Het Knoopeknauwertje No. 12 he talks about the **Casa Knot** and the **Sobre-Casa Knot**. In that article he restricts the **Casa Knot** to an over–under coded Regular Knot in which every half-cycle from lower-left to upper-right starts with an **under** crossing, while every half-cycle from lower-left to upper-right in the **Sobre-Casa Knot** starts with an **over** crossing. We see here again the introduction of another senseless term as far as knot nomenclature is concerned:
(1.) If there is a **sobre**-something knot, then there must be a **bajo**-something knot.[†] Not only should the sobre-something knot and the bajo-something knot be structurally different from each other, but they should also structurally differ from the knot without the sobre and bajo terms.
(2.) If the **Sobre-Casa Knot** has an **odd** number of parts and we rotate this knot through an angle of 180°, then it becomes a **Casa Knot** (a **Bajo-Casa Knot**). Hence, since the knot does not physically change (*it has only been rotated, but* **not** *changed in string-run or coding*), a **Sobre-Casa Knot** with an odd number of parts is identical to a **Casa Knot** and a **Bajo-Casa Knot** with the identical number of parts and the identical number of bights. An Example is shown in Fig. 1 where A depicts a section of a 5-part Casa Knot or Bajo-Casa Knot and B depicts this section rotated through an angle of 180° (the stars in the respective algorithm diagrams represent the i-values). However, if we rotate a **Sobre-Casa Knot** with an **even** number of parts through an angle of 180°, then it **does not** become a **Casa Knot** (a **Bajo-Casa Knot**).

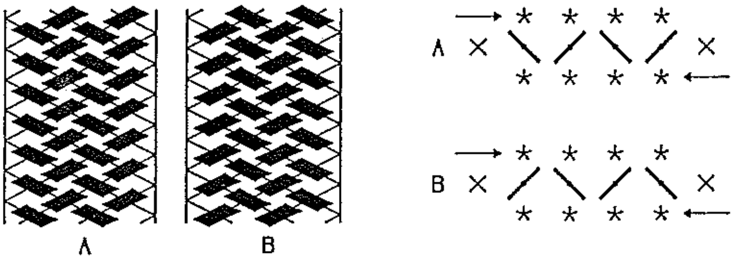

Fig. 1 — A 5-part Casa Knot and its identical Sobre-Casa Knot.

It will thus be obvious that the term **sobre** (as well as the term **bajo**) has, in the context used, no place in knot nomenclature.

Those who were able to read the article (the article concerned was in Dutch) and have read it will thus see that it is ridiculous to talk of a **Gaucho Knot** and a **Sobre-Gaucho Knot** since a Gaucho-coding always requires the same number of **over-crossings** as **under-crossings** for every half-cycle from lower-left to upper-right, and hence an n-pass **Sobre-Gaucho Knot** is identical to an n-pass **Gaucho Knot** with identical n, the same number of parts and the same number of bights.

The essential coding of Pineapple Knots[††] clearly indicates that for Standard Pineapple Knots the coding of the interwoven knots is not determined; furthermore, their components have an **odd** number of parts[‡]. For the Standard (Herringbone) Pineapple Knots the interwoven knots are Casa Knots, hence are identical to Sobre-Casa Knots since they have an **odd** number of parts.

Although an A-pass Standard Herringbone Pineapple knot can be braided in A!

[†] *sobre* is the Spanish word for *over*, while *bajo* is the Spanish word for *under*.

[††] See *Book 4/1, BRAIDING — Standard Herringbone Pineapple Knots*, pp. 16-17; see *The Braider*, Issue No. 44, pg. 1041.

[‡] See *The Braider*, Issue No. 23, pg. 526.

different ways[†], there are thus another $A!$ ways in which to braid it.

The Standard Herringbone Knots are Regular Cylindrical Braids which consist of interwoven under–over coded Regular Knots, all of which have the same number of parts and the same number of bights.

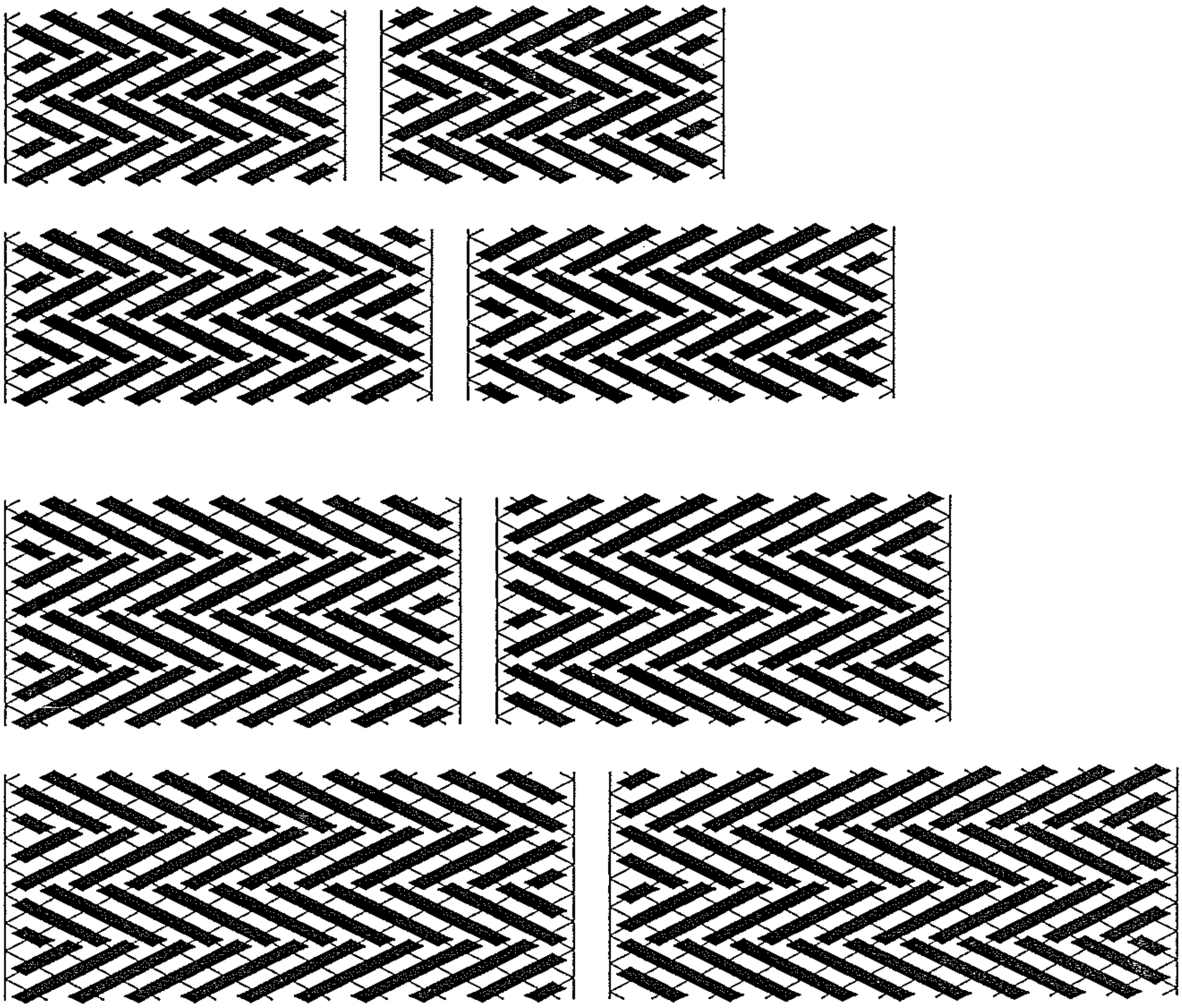

Fig. 2 — Sections of Standard Herringbone Knots.

Although an n-pass Standard Herringbone Knot can be braided in $n!$ different ways, there are another $n!$ different ways to braid it when n is **even**. When n is **odd** there are only another $n!$ different ways in which to braid a Standard Herringbone Knot when the interwoven under–over coded Regular Knots have all an **odd** number of parts.[‡] Fig. 3 shows an Example of a 2-pass Standard Herringbone Knot in which the two under–over coded Regular Knot components have each 5-parts. When section A is rotated through an angle of 180° we obtain section B. Hence 2-pass Standard Herringbone Knots with identical bight numbers and with respectively section A and section B are identical; each can be braided in $n! = 2! = 2$ different ways (see the two times two sets of two algorithm diagrams for interbraiding the two components).

[†] See *The Braider*, Issue No. 23, pp.523-525 & 527.

[‡] Fig. 2 shows, on the left-hand side, sections of four Standard Herringbone Knots: the upper two with $n = 3$, respectively with $p_{component} = 4$ and with $p_{component} = 5$; the lower two with $n = 4$, respectively with $p_{component} = 4$ and with $p_{component} = 5$. When we rotate these sections through 180° we obtain the sections on their right. This rotation does not physically change the knot, but when braiding its half-cycles from lower-left to upper-right and lower-right to upper-left, it offers another $n!$ different ways to braid it when the half-cycles in the rotated section differ from the half-cycles in the

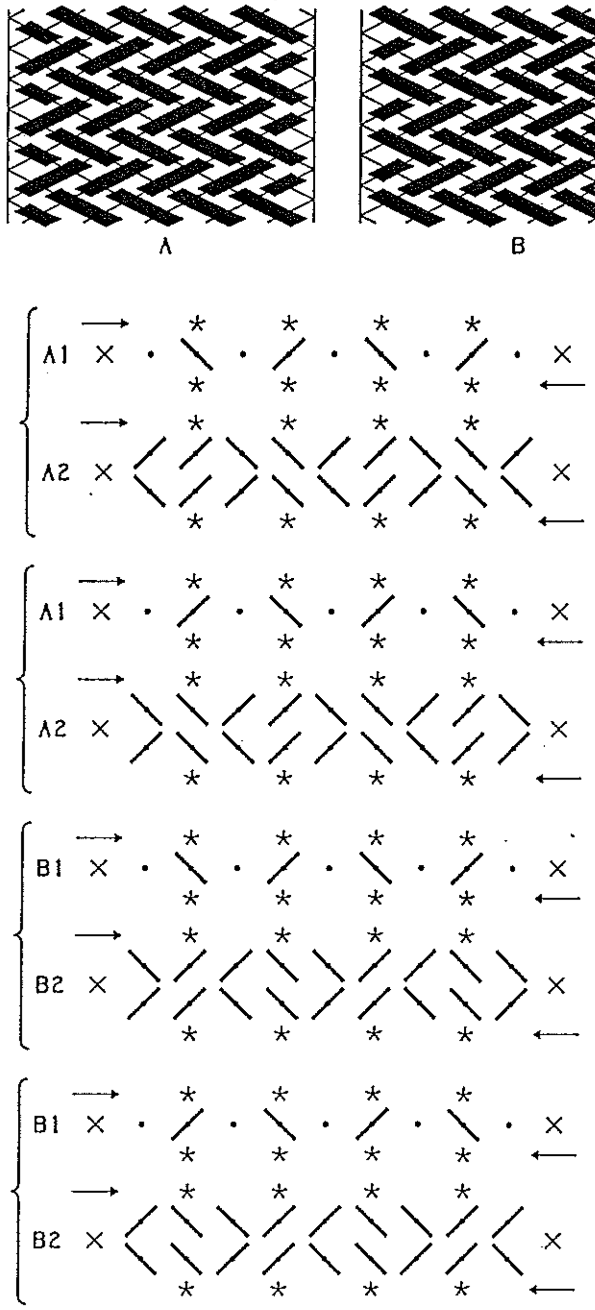

Fig. 3 — The 2-pass 10-parts Standard Herringbone Knot.

Fig. 4 shows an Example of a 3-pass Standard Herringbone Knot in which the two under–over coded Regular Knot components have each 5-parts. When section A is

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not rotated section. We see that only the half-cycles in the rotated section with $n = 3$ and $p_{component} = 4$ are identical to the half-cycles in its associated not rotated section, hence when n is **odd** with $p_{component}$ **even**, rotation does not offer another $n!$ different ways in which to braid such a Standard Herringbone Knot.

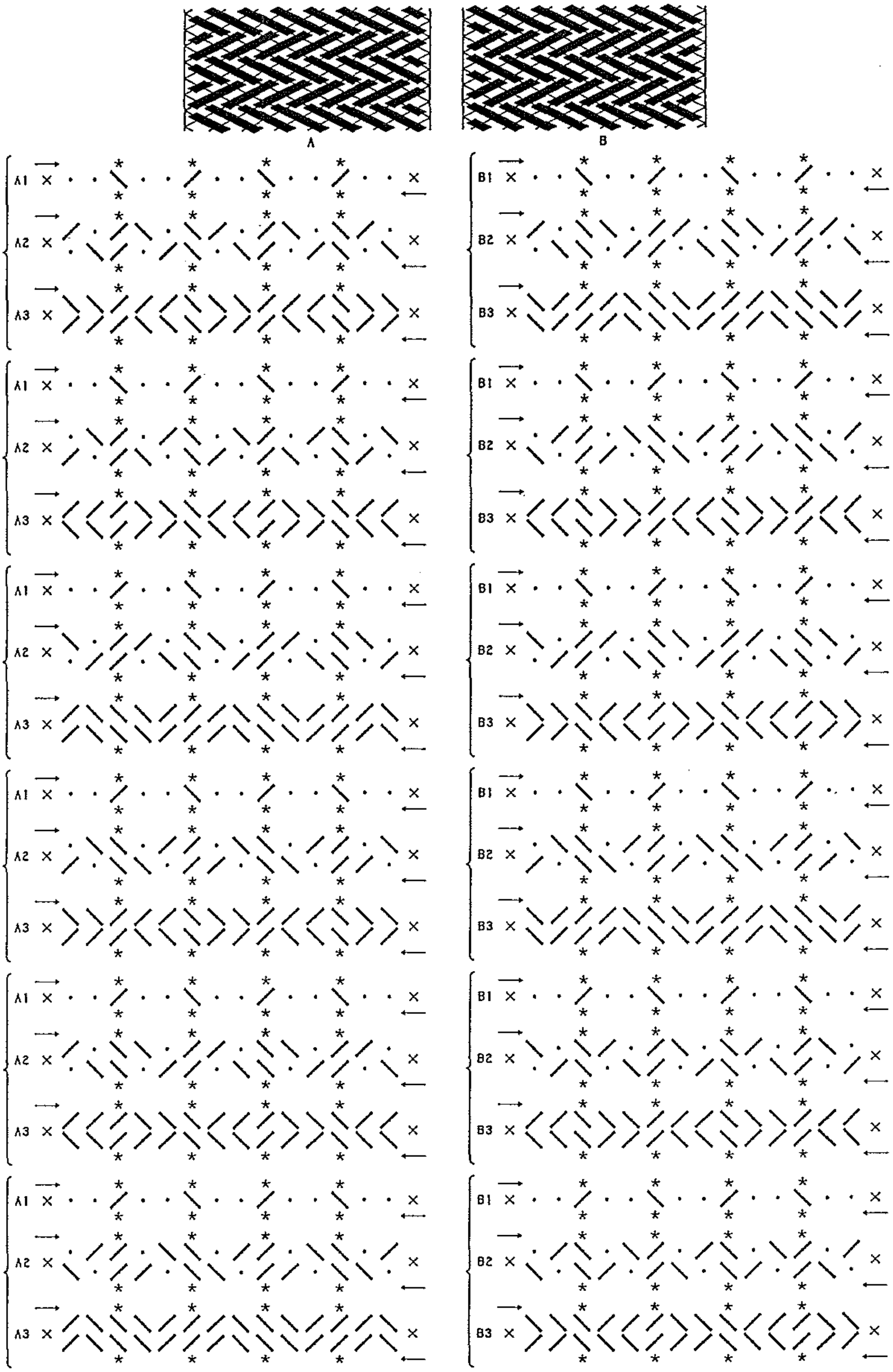

Fig. 4 — The 3-pass 15-parts Standard Herringbone Knot.

rotated through an angle of 180° we obtain section B. Hence 3-pass Standard Herringbone Knots with identical bight numbers and with respectively section A and section B are identical; each can be braided in $n! = 3! = 6$ different ways (see the two times six sets of three algorithm diagrams for interbraiding the three components).

One naturally would ask how such an introduction of ambiguous terms, in fact senseless terms in the context of their use, could occur. It is a typical example of mixing up **construction procedures** with the **naming** (nomenclature) of knots and braids.† Obviously it results from a desire to identify the knot or braid in a clear-cut manner to those who cannot draw and/or read its grid-diagram. We cannot stress enough that a half-cycle braiding algorithm table or any other textual braiding instruction of a knot or braid without its grid-diagram is at least cumbersome if not useless in the visualisation of the knot or braid concerned.

In the Australia periodical *The Australian Whipmaker*, Issue No. 50, pp. 972-973, and in the Dutch periodical *Het Knoopeknauwertje*, Issue No. 15, pp. 26-28, at the end of an article by Tom Hall it is stated that the term **Standard** in the name of a knot means that the knot is tied from interwoven **Casa knots**.

With reference to the earlier discussed article by Tom Hall, the *Casa Knot* in this article is defined as a Regular Knot with an over-one — under-one weaving pattern made with **one strand**‡. Hence in this article by Tom Hall, the **Casa Knot** is not only the **Casa Knot** in the earlier discussed article by Tom Hall (the **Bajo-Casa Knot**), but is also the **Sobre-Casa Knot** in that article. There is thus an obvious inconsistency here, especially since nothing about redefining has been mentioned!

We like to point out that we introduced the term **Standard** in the early nineteen eighties as a direct or indirect reference to a commonly used knot or braidform. The term **Standard** has thus nothing to do with any form of interbraiding. It is purely coincidental that **Standard Herringbone Pineapple Knots** and **Standard Herringbone Knots** are interbraids of under–over coded Regular Knots (Casa Knots). An A-pass **Standard Herringbone Grant Knot** for example can consist of one single string when A is odd.††

Every braider and knot-tyer should appreciate that Knot nomenclature will never be able to give a clear impression of a knot or braid in every case, not even with a comprehensive textual description. Grid-diagrams are essential not only for this task, but also for construction procedures associated with a braid or knot. For Nested Knots we still see half-cycle braiding algorithm tables with half-cycle braiding algorithms containing MIBS, MOBS or SOBS instead of the bight-boundaries between which a half-cycle runs. Those MIBS, MOBS and SOBS tables are in most cases suplied without a grid-diagram and hence for that reason alone are not worth the paper they are printed on; they no doubt will only supply for most mobs a plentiful source for sobs.

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† Such a mix-up is relatively common amongst braiders and knot-tyers. Another example of such a mix-up will be dealt with in *The Braider*, Issue No. 38, pp. 881-902.

‡ See also *The Braider*, Appendix 1999, pp. *iii-v*.

†† See *The Braider*, Issue No. 46, pg. 1084. The Grant knots are discussed in *The Braider*, Issue No. 45, pp. 1066-1076 and Issue No. 46, pp. 1077-1090.

The Double Twin Scallop Barcus Knots

We received a letter from Jim Barcus containing two single-string knots and their associated half-cycle braiding algorithm tables with MIBS and SOBS. Jim asked us if we could draw up the grid-diagrams for these two knots. A few days later we received a letter from Bill Budd with a draft copy of *The Elko County Braiders Association News Letter* in which the MIBS and SOBS half-cycle braiding algorithm table of one of these knots was published. This draft copy also contained a scanned picture of the knot braided from two different coloured strings.

We threw the MIBS and SOBS braiding algorithm tables, without really looking at it of course, in the rubbish-bin. Hence we don't know, nor were we interested to know, whether or not the MIBS and SOBS tables belonged to the single-string or the two-string variety. Since we received single-string knots, we drew their grid-diagrams and associated algorithm diagrams, and from these algorithm diagrams we read the associated half-cycle braiding algorithms.

The Double Twin Scallop Barcus Knots are beautiful knots, it are imitations of **Standard Regular Nested Knots**, hence imitations of **Regular Nested Cylindrical Braids** with the string-run

$(222\cdots/x{=}2k{+}nA{+}2/222\cdots)\{(1)(A)(A{-}1)\cdots(4)(3)(2)/(k)(k{+}1)(k{+}2)\cdots(A)(1)(2)\cdots(k{-}3)(k{-}2)(k{-}1)\}AB^*$,
where $n=-2,-1,0,1,2,3\cdots$, $y=A$ and $1\le k\le A$.
In our case $A=4$, $k=A=4$, $n=-1$ hence $x=6$, and $y=4$.

The nonimitation type would consist of A (hence in our case 4) interwoven Regular Knots, hence requires A essential strings. The imitation has one essential string. Normally an imitation has some irregularity, which in many cases can be hidden by means of a suitable modification in its string-run, while when not such a suitable modification takes place the irregularity becomes quite visible. The charm of the two knots does not only lie in their pretty appearance, but lies mainly in the fact that without modifying their string-run significantly, their actual appearance cannot be distinguished from that of the nonimitation type. This is, of course, due to their coding. An additional charm is that we can braid these knots with one string as if they were nonimitation types, excepting the initial half-cycle of the third set of half-cycles which requires a change. This means that we can employ the algorithm diagrams of the nonimitation types. We have shown the algorithm diagrams for the nonimitation types and it will be seen that half-cycle $1''$ (which is half-cycle 29 in the Double Twin Scallop Barcus Knot 1 (see Fig. 5), and is half-cycle 21 in the Double Twin Scallop Barcus Knot 2 (see Fig. 6)) needs a change only. We have indicated that in the third algorithm diagram for each knot. The half-cycle braiding algorithm are now read off from these algorithm diagrams. Normally we like the ingoing and outgoing half-cycle at the ingoing respectively outgoing end to be **under** as many opposing half-cycles as possible. Consequently we use in practice the grid-diagrams shown at the right instead of those on the left which are the grid-diagrams associated with the knots we received. This means that the half-cycle braiding algorithm of the very last half-cycle must be modified at its exit end. It will no doubt be obvious why we would braid these knots as indicated in the grid-diagrams shown, hence in the position as shown, since half-cycle 1 will then automatically be laid down in its final correct position.

If we modify the Double Twin Scallop Barcus Knots 1 and 2 so that one string is used for the first two sets of braiding sequences and another different colour string for the last two sets of braiding sequences, then the resulting knot has a lopsided colour-pattern and hence will rarely be of any practical value. However, a similar type of knot with a good balanced colour-pattern can readily be created.

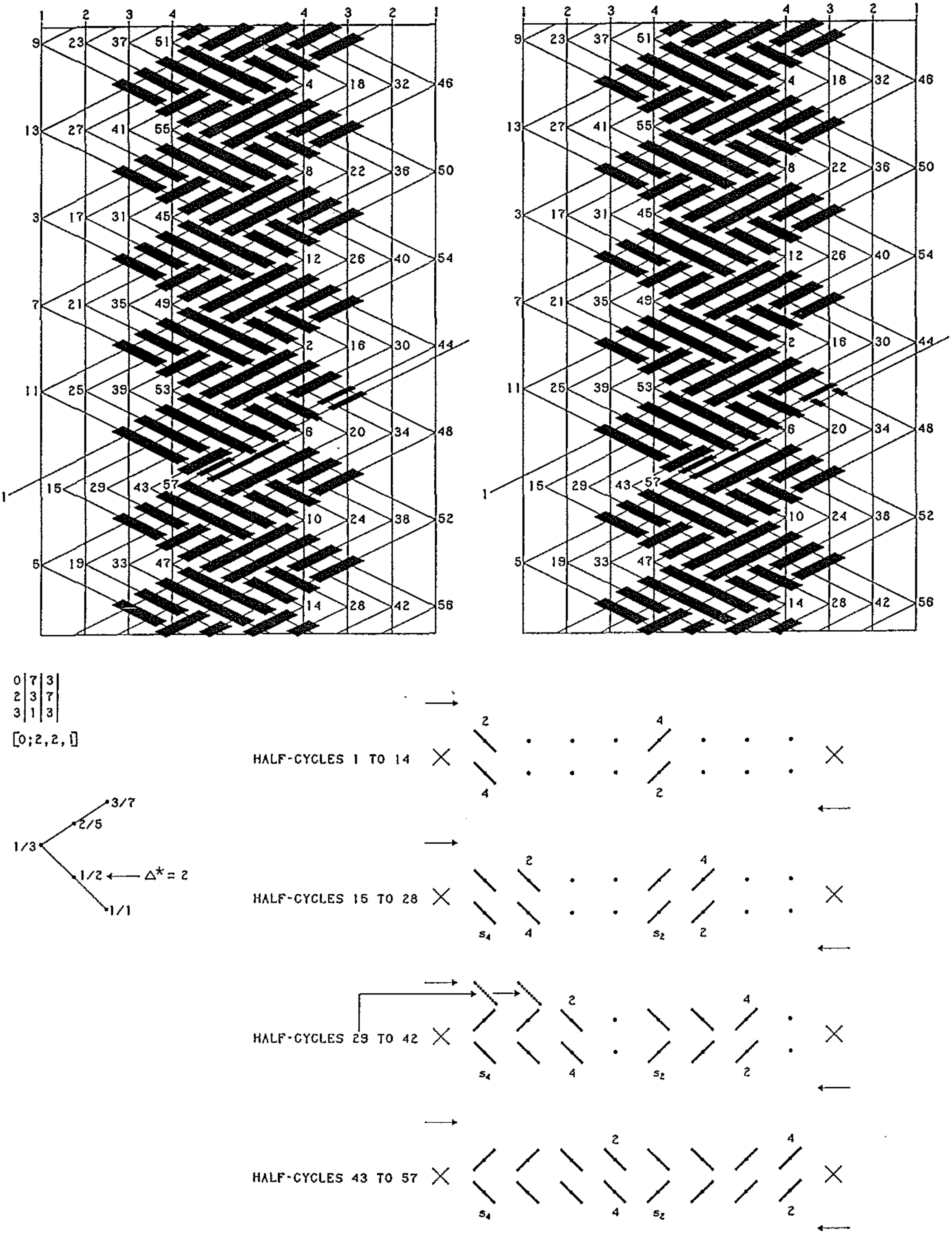

Fig. 5 — The Double Twin Scallop Barcus Knot 1.

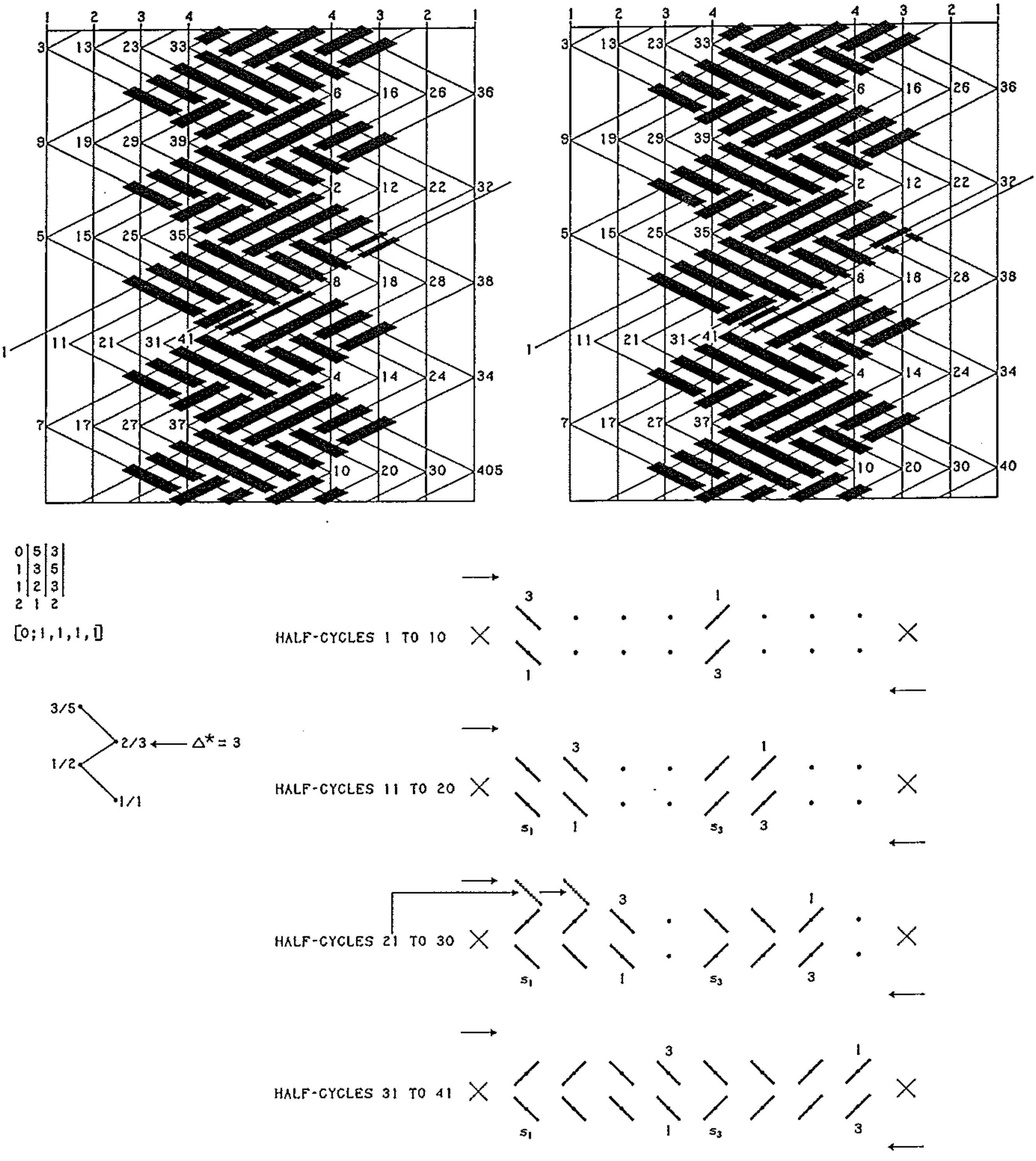

Fig. 6 — The Double Twin Scallop Barcus Knot 2.

The half-cycle braiding algorithms associated with Fig. 5 :

half-cycle 1 : $L_1 \longrightarrow R_4$: Free run.
half-cycle 2 : $i = 0$; $R_4 \longrightarrow L_1$: Free run.
half-cycle 3 : $i = 0$; $L_1 \longrightarrow R_4$: Free run.
half-cycle 4 : $i \leq 1$; $R_4 \longrightarrow L_1$: Free run.
half-cycle 5 : $i \leq 1$; $L_1 \longrightarrow R_4$: Free run.
half-cycle 6 : $i \leq 2$; $R_4 \longrightarrow L_1$: $(s)u$.

half-cycle 7 : $i \leq 2$; $L_1 \longrightarrow R_4$: u .
half-cycle 8 : $i \leq 3$; $R_4 \longrightarrow L_1$: u .
half-cycle 9 : $i \leq 3$; $L_1 \longrightarrow R_4$: u .
half-cycle 10 : $i \leq 4$; $R_4 \longrightarrow L_1$: $u - (s)o$.
half-cycle 11 : $i \leq 4$; $L_1 \longrightarrow R_4$: $u - o$.
half-cycle 12 : $i \leq 5$; $R_4 \longrightarrow L_1$: $u - o$.
half-cycle 13 : $i \leq 5$; $L_1 \longrightarrow R_4$: $u - o$.
half-cycle 14 : $i \leq 6$; $R_4 \longrightarrow L_1$: $u - o$.

At this stage the $p/b = 3/7$ *under-over* coded Regular Knot has been obtained. In practice it is obtained from the $p/b = 3/1$ *under-over* coded Regular Knot (see *The Braider*, Appendix 1996).

half-cycle 15 = 1′ : $L_2{-} \longrightarrow R_3$: $u - o$.
half-cycle 16 = 2′ : $i = 0$; $R_3 \longrightarrow L_2$: $u - o$.
half-cycle 17 = 3′ : $i = 0$; $L_2 \longrightarrow R_3$: $u - o$.
half-cycle 18 = 4′ : $i \leq 1$; $R_3 \longrightarrow L_2$: $u - o$.
half-cycle 19 = 5′ : $i \leq 1$; $L_2 \longrightarrow R_3$: $u - o$.
half-cycle 20 = 6′ : $i \leq 2$; $R_3 \longrightarrow L_2$: $(1, s)2u - o$.
half-cycle 21 = 7′ : $i \leq 2$; $L_2 \longrightarrow R_3$: $2u - o$.
half-cycle 22 = 8′ : $i \leq 3$; $R_3 \longrightarrow L_2$: $2u - o$.
half-cycle 23 = 9′ : $i \leq 3$; $L_2 \longrightarrow R_3$: $2u - o$.
half-cycle 24 = 10′ : $i \leq 4$; $R_3 \longrightarrow L_2$: $2u - (1, s)2o$.
half-cycle 25 = 11′ : $i \leq 4$; $L_2 \longrightarrow R_3$: $2u - 2o$.
half-cycle 26 = 12′ : $i \leq 5$; $R_3 \longrightarrow L_2$: $2u - 2o$.
half-cycle 27 = 13′ : $i \leq 5$; $L_2 \longrightarrow R_3$: $2u - 2o$.
half-cycle 28 = 14′ : $i \leq 6$; $R_3 \longrightarrow L_2$: $2u - 2o$.

half-cycle 29 = 1″ : $L_3{-} \longrightarrow R_2$: $4u$.
half-cycle 30 = 2″ : $i = 0$; $R_2 \longrightarrow L_3$: $2u - 2o$.
half-cycle 31 = 3″ : $i = 0$; $L_3 \longrightarrow R_2$: $2o - 2u$.
half-cycle 32 = 4″ : $i \leq 1$; $R_2 \longrightarrow L_3$: $2u - 2o$.
half-cycle 33 = 5″ : $i \leq 1$; $L_3 \longrightarrow R_2$: $2o - 2u$.
half-cycle 34 = 6″ : $i \leq 2$; $R_2 \longrightarrow L_3$: $(2, s)3u - 2o$.
half-cycle 35 = 7″ : $i \leq 2$; $L_3 \longrightarrow R_2$: $2o - 3u$.
half-cycle 36 = 8″ : $i \leq 3$; $R_2 \longrightarrow L_3$: $3u - 2o$.
half-cycle 37 = 9″ : $i \leq 3$; $L_3 \longrightarrow R_2$: $2o - 3u$.
half-cycle 38 = 10″ : $i \leq 4$; $R_2 \longrightarrow L_3$: $3u - (2, s)3o$.
half-cycle 39 = 11″ : $i \leq 4$; $L_3 \longrightarrow R_2$: $2o - 3u - o$.
half-cycle 40 = 12″ : $i \leq 5$; $R_2 \longrightarrow L_3$: $3u - 3o$.
half-cycle 41 = 13″ : $i \leq 5$; $L_3 \longrightarrow R_2$: $2o - 3u - o$.
half-cycle 42 = 14″ : $i \leq 6$; $R_2 \longrightarrow L_3$: $3u - 3o$.

half-cycle 43 = 1‴ : $L_4{-} \longrightarrow R_1$: $2o - 3u - o$.
half-cycle 44 = 2‴ : $i = 0$; $R_1 \longrightarrow L_4$: $3u - 3o$.
half-cycle 45 = 3‴ : $i = 0$; $L_4 \longrightarrow R_1$: $2o - 3u - o$.
half-cycle 46 = 4‴ : $i \leq 1$; $R_1 \longrightarrow L_4$: $3u - 3o$.
half-cycle 47 = 5‴ : $i \leq 1$; $L_4 \longrightarrow R_1$: $2o - 3u - o$.
half-cycle 48 = 6‴ : $i \leq 2$; $R_1 \longrightarrow L_4$: $(3, s)4u - 3o$.
half-cycle 49 = 7‴ : $i \leq 2$; $L_4 \longrightarrow R_1$: $2o - 4u - o$.
half-cycle 50 = 8‴ : $i \leq 3$; $R_1 \longrightarrow L_4$: $4u - 3o$.

half-cycle 51 = $9'''$: $i \leq 3$; $L_4 \longrightarrow R_1$: $2o - 4u - o$.
half-cycle 52 = $10'''$: $i \leq 4$; $R_1 \longrightarrow L_4$: $4u - (3, s)4o$.
half-cycle 53 = $11'''$: $i \leq 4$; $L_4 \longrightarrow R_1$: $2o - 4u - 2o$.
half-cycle 54 = $12'''$: $i \leq 5$; $R_1 \longrightarrow L_4$: $4u - 4o$.
half-cycle 55 = $13'''$: $i \leq 5$; $L_4 \longrightarrow R_1$: $2o - 4u - 2o$.
half-cycle 56 = $14'''$: $i \leq 6$; $R_1 \longrightarrow L_4$: $4u - 4o$.
half-cycle 57 = $15'''$: $i \leq 6$; $L_{4+} \longrightarrow R$: $2o - 4u - 2o$.
or
half-cycle 57 = $15'''$: $i \leq 6$; $L_{4+} \longrightarrow R$: $2o - 6u$.

The half-cycle braiding algorithms associated with Fig. 6:

half-cycle 1 : $L_1 \longrightarrow R_4$: Free run.
half-cycle 2 : $i = 0$; $R_4 \longrightarrow L_1$: Free run.
half-cycle 3 : $i = 0$; $L_1 \longrightarrow R_4$: Free run.
half-cycle 4 : $i \leq 1$; $R_4 \longrightarrow L_1$: $(s)o$.
half-cycle 5 : $i \leq 1$; $L_1 \longrightarrow R_4$: o.
half-cycle 6 : $i \leq 2$; $R_4 \longrightarrow L_1$: o.
half-cycle 7 : $i \leq 2$; $L_1 \longrightarrow R_4$: o.
half-cycle 8 : $i \leq 3$; $R_4 \longrightarrow L_1$: $(s)u - o$.
half-cycle 9 : $i \leq 3$; $L_1 \longrightarrow R_4$: $u - o$.
half-cycle 10 : $i \leq 4$; $R_4 \longrightarrow L_1$: $u - o$.

At this stage the $p/b = 3/5$ *under-over* coded Regular Knot has been obtained. In practice it is obtained from the $p/b = 3/2$ *under-over* coded Regular Knot (see *The Braider*, Appendix 1996).

half-cycle 11 = $1'$: $L_{2-} \longrightarrow R_3$: $u - o$.
half-cycle 12 = $2'$: $i = 0$; $R_3 \longrightarrow L_2$: $u - o$.
half-cycle 13 = $3'$: $i = 0$; $L_2 \longrightarrow R_3$: $u - o$.
half-cycle 14 = $4'$: $i \leq 1$; $R_3 \longrightarrow L_2$: $u - (1, s)2o$.
half-cycle 15 = $5'$: $i \leq 1$; $L_2 \longrightarrow R_3$: $u - 2o$.
half-cycle 16 = $6'$: $i \leq 2$; $R_3 \longrightarrow L_2$: $u - 2o$.
half-cycle 17 = $7'$: $i \leq 2$; $L_2 \longrightarrow R_3$: $u - 2o$.
half-cycle 18 = $8'$: $i \leq 3$; $R_3 \longrightarrow L_2$: $(1, s)2u - 2o$.
half-cycle 19 = $9'$: $i \leq 3$; $L_2 \longrightarrow R_3$: $2u - 2o$.
half-cycle 20 = $10'$: $i \leq 4$; $R_3 \longrightarrow L_2$: $2u - 2o$.

half-cycle 21 = $1''$: $L_{3-} \longrightarrow R_2$: $4u$.
half-cycle 22 = $2''$: $i = 0$; $R_2 \longrightarrow L_3$: $2u - 2o$.
half-cycle 23 = $3''$: $i = 0$; $L_3 \longrightarrow R_2$: $2o - 2u$.
half-cycle 24 = $4''$: $i \leq 1$; $R_2 \longrightarrow L_3$: $2u - (2, s)3o$.
half-cycle 25 = $5''$: $i \leq 1$; $L_3 \longrightarrow R_2$: $2o - 2u - o$.
half-cycle 26 = $6''$: $i \leq 2$; $R_2 \longrightarrow L_3$: $2u - 3o$.
half-cycle 27 = $7''$: $i \leq 2$; $L_3 \longrightarrow R_2$: $2o - 2u - o$.
half-cycle 28 = $8''$: $i \leq 3$; $R_2 \longrightarrow L_3$: $(2, s)3u - 3o$.
half-cycle 29 = $9''$: $i \leq 3$; $L_3 \longrightarrow R_2$: $2o - 3u - o$.
half-cycle 30 = $10''$: $i \leq 4$; $R_2 \longrightarrow L_3$: $3u - 3o$.

half-cycle 31 = $1'''$: $L_{4-} \longrightarrow R_1$: $2o - 3u - o$.
half-cycle 32 = $2'''$: $i = 0$; $R_1 \longrightarrow L_4$: $3u - 3o$.
half-cycle 33 = $3'''$: $i = 0$; $L_4 \longrightarrow R_1$: $2o - 3u - o$.
half-cycle 34 = $4'''$: $i \leq 1$; $R_1 \longrightarrow L_4$: $3u - (3, s)4o$.

half-cycle $35 = 5'''$: $i \le 1$; $L_4 \longrightarrow R_1$: $2o - 3u - 2o$.
half-cycle $36 = 6'''$: $i \le 2$; $R_1 \longrightarrow L_4$: $3u - 4o$.
half-cycle $37 = 7'''$: $i \le 2$; $L_4 \longrightarrow R_1$: $2o - 3u - 2o$.
half-cycle $38 = 8'''$: $i \le 3$; $R_1 \longrightarrow L_4$: $(3,s)4u - 4o$.
half-cycle $39 = 9'''$: $i \le 3$; $L_4 \longrightarrow R_1$: $2o - 4u - 2o$.
half-cycle $40 = 10'''$: $i \le 4$; $R_1 \longrightarrow L_4$: $4u - 4o$.
half-cycle $41 = 11'''$: $i \le 4$; $L_{4}+ \longrightarrow R$: $2o - 4u - 2o$.
or
half-cycle $41 = 11'''$: $i \le 4$; $L_{4+} \longrightarrow R$: $2o - 6u$.

The grid-diagrams of the knots we received should immediately direct one to the grid-diagram of a similar 2-string knot with a properly balanced colour-pattern.

In the single-string knots we received, we saw that $A = 4$, $k = A = 4$, $n = -1$ hence $x = 6$, and $y = 4$. If we now take $A = 4$, $k = A - 2 = 2$, $n = 1$ hence $x = 10$, and $y = 4$, and braid the half-cycles between the bight-boundaries $1 \longleftrightarrow 2$ and $2 \longleftrightarrow 1$ with one coloured string and the half-cycles between the bight-boundaries $3 \longleftrightarrow 4$ and $4 \longleftrightarrow 3$ with the other coloured string, then for example a knot similar to the Double Twin Scallop Barcus Knot 1, but with a balanced colour-pattern, is created. In fact, this knot is nicer than the Double Twin Scallop Barcus Knot 1. Fig. 7 shows this knot with its associated algorithm diagrams. From these algorithm diagrams we read the following half-cycle braiding algorithms:

half-cycle 1 : $L_1 \longrightarrow R_2$: Free run.
half-cycle 2 : $i = 0$; $R_2 \longrightarrow L_1$: Free run.
half-cycle 3 : $i = 0$; $L_1 \longrightarrow R_2$: Free run.
half-cycle 4 : $i \le 1$; $R_2 \longrightarrow L_1$: $(s)o$.
half-cycle 5 : $i \le 1$; $L_1 \longrightarrow R_2$: o.
half-cycle 6 : $i \le 2$; $R_2 \longrightarrow L_1$: $(1,s)2o$.
half-cycle 7 : $i \le 2$; $L_1 \longrightarrow R_2$: $2o$.
half-cycle 8 : $i \le 3$; $R_2 \longrightarrow L_1$: $2o$.
half-cycle 9 : $i \le 3$; $L_1 \longrightarrow R_2$: $2o$.
half-cycle 10 : $i \le 4$; $R_2 \longrightarrow L_1$: $(s)u - 2o$.
half-cycle 11 : $i \le 4$; $L_1 \longrightarrow R_2$: $u - 2o$.
half-cycle 12 : $i \le 5$; $R_2 \longrightarrow L_1$: $u - o - (s)u - o$.
half-cycle 13 : $i \le 5$; $L_1 \longrightarrow R_2$: $u - o - u - o$.
half-cycle 14 : $i \le 6$; $R_2 \longrightarrow L_1$: $u - o - u - o$.

At this stage the $p/b = 5/7$ *under-over* coded Regular Knot has been obtained.

half-cycle $15 = 1^\star$: $L_{2}- \longrightarrow R_1$: $u - o - u - o$.
half-cycle $16 = 2^\star$: $i = 0$; $R_1 \longrightarrow L_2$: $u - o - u - o$.
half-cycle $17 = 3^\star$: $i = 0$; $L_2 \longrightarrow R_1$: $u - o - u - o$.
half-cycle $18 = 4^\star$: $i \le 1$; $R_1 \longrightarrow L_2$: $u - (1,s)2o - u - o$.
half-cycle $19 = 5^\star$: $i \le 1$; $L_2 \longrightarrow R_1$: $u - 2o - u - o$.
half-cycle $20 = 6^\star$: $i \le 2$; $R_1 \longrightarrow L_2$: $u - 2o - u - (1,s)2o$.
half-cycle $21 = 7^\star$: $i \le 2$; $L_2 \longrightarrow R_1$: $u - 2o - u - 2o$.
half-cycle $22 = 8^\star$: $i \le 3$; $R_1 \longrightarrow L_2$: $u - 2o - u - 2o$.
half-cycle $23 = 9^\star$: $i \le 3$; $L_2 \longrightarrow R_1$: $u - 2o - u - 2o$.
half-cycle $24 = 10^\star$: $i \le 4$; $R_1 \longrightarrow L_2$: $(1,s)2u - 2o - u - 2o$.
half-cycle $25 = 11^\star$: $i \le 4$; $L_2 \longrightarrow R_1$: $2u - 2o - u - 2o$.
half-cycle $26 = 12^\star$: $i \le 5$; $R_1 \longrightarrow L_2$: $2u - 2o - (1,s)2u - 2o$.
half-cycle $27 = 13^\star$: $i \le 5$; $L_2 \longrightarrow R_1$: $2u - 2o - 2u - 2o$.

half-cycle $28 = 14^\star$: $i \leq 6$; $R_1 \longrightarrow L_2$: $2u - 2o - 2u - 2o$.
half-cycle $29 = 15^\star$: $i \leq 6$; $L_{2+} \longrightarrow R$: $8u$.

half-cycle $1'$: $L_3 \longrightarrow R_4$: $4u - 2o$.
half-cycle $2'$: $i = 0$; $R_4 \longrightarrow L_3$: $2o - 2u - 2o$.
half-cycle $3'$: $i = 0$; $L_3 \longrightarrow R_4$: $2o - 2u - 2o$.
half-cycle $4'$: $i \leq 1$; $R_4 \longrightarrow L_3$: $3o - 2u - 2o$.
half-cycle $5'$: $i \leq 1$; $L_3 \longrightarrow R_4$: $2o - 2u - 2o$.
half-cycle $6'$: $i \leq 2$; $R_4 \longrightarrow L_3$: $2o - (s,2)3u - 2o$.
half-cycle $7'$: $i \leq 2$; $L_3 \longrightarrow R_4$: $2o - 3u - 2o$.
half-cycle $8'$: $i \leq 3$; $R_4 \longrightarrow L_3$: $2o - 3u - 3o$.
half-cycle $9'$: $i \leq 3$; $L_3 \longrightarrow R_4$: $2o - 3u - 2o$.
half-cycle $10'$: $i \leq 4$; $R_4 \longrightarrow L_3$: $2o - 3u - (s,2)3o$.
half-cycle $11'$: $i \leq 4$; $L_3 \longrightarrow R_4$: $2o - 3u - 3o$.
half-cycle $12'$: $i \leq 5$; $R_4 \longrightarrow L_3$: $2o - 3u - 3o$.
half-cycle $13'$: $i \leq 5$; $L_3 \longrightarrow R_4$: $2o - 3u - 3o$.
half-cycle $14'$: $i \leq 6$; $R_4 \longrightarrow L_3$: $3o - 3u - 3o$.

half-cycle $15' = 1'^\star$: $L_{4-} \longrightarrow R_3$: $2o - 3u - 3o$.
half-cycle $16' = 2'^\star$: $i = 0$; $R_3 \longrightarrow L_4$: $2o - 3u - 3o$.
half-cycle $17' = 3'^\star$: $i = 0$; $L_4 \longrightarrow R_3$: $2o - 3u - 3o$.
half-cycle $18' = 4'^\star$: $i \leq 1$; $R_3 \longrightarrow L_4$: $2o - u - o - 2u - 3o$.
half-cycle $19' = 5'^\star$: $i \leq 1$; $L_4 \longrightarrow R_3$: $2o - 3u - 3o$.
half-cycle $20' = 6'^\star$: $i \leq 2$; $R_3 \longrightarrow L_4$: $2o - (1,s,2)4u - 3o$.
half-cycle $21' = 7'^\star$: $i \leq 2$; $L_4 \longrightarrow R_3$: $2o - 4u - 3o$.
half-cycle $22' = 8'^\star$: $i \leq 3$; $R_3 \longrightarrow L_4$: $2o - 4u - 4o$.
half-cycle $23' = 9'^\star$: $i \leq 3$; $L_4 \longrightarrow R_3$: $2o - 4u - 3o$.
half-cycle $24' = 10'^\star$: $i \leq 4$; $R_3 \longrightarrow L_4$: $2o - 4u - (1,s,2)4o$.
half-cycle $25' = 11'^\star$: $i \leq 4$; $L_4 \longrightarrow R_3$: $2o - 4u - 4o$.
half-cycle $26' = 12'^\star$: $i \leq 5$; $R_3 \longrightarrow L_4$: $2o - 4u - 4o$.
half-cycle $27' = 13'^\star$: $i \leq 5$; $L_4 \longrightarrow R_3$: $2o - 4u - 4o$.
half-cycle $28' = 14'^\star$: $i \leq 6$; $R_3 \longrightarrow L_4$: $3o - 4u - 4o$.
half-cycle $29' = 15'^\star$: $i \leq 6$; $L_{4+} \longrightarrow R$: $2o - 10u$.

Once again, we see here the importance of being able to draw grid-diagrams. Not only to enable someone else to see immediately what a knot or braid is going to look like, but it will normally also direct the person who created a knot or braid by trial and error immediately to other somewhat similar and often better knots or braids. Hence we cannot stress strongly enough that a braider should get a pad of isometric graph-paper and exercise in drawing and reading grid-diagrams.† It should at all times be remembered that a grid-diagram should be easy to read, hence should be uniform in style, not contain confusing layouts and unnecessary line-elements.

Anyone who experiences difficulties in drawing the grid-diagram of a knot or braid he or she created by trial and error is, of course, most welcome to send the knot to us and we will send him or her its associated grid-diagram.

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† Surely, a pad of graph-paper with a pencil and rubber should be in reach of everybody. For supply sources refer to *The Braider*, Issue No. 1, pg. 3 ; *The Braider*, Appendix 1995, pg. *i*; *The Braider*, Issue No. 5, pg. 106.

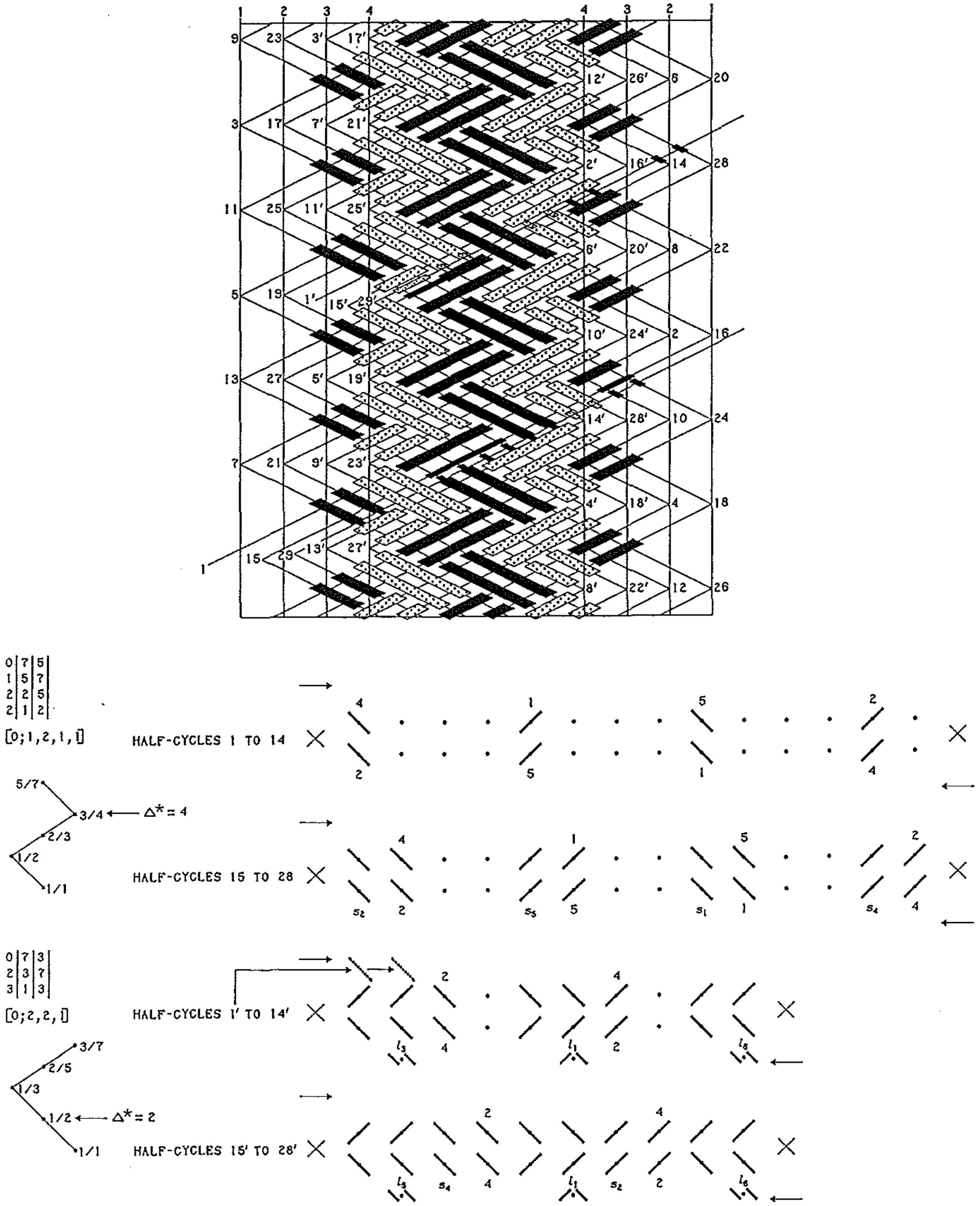

Fig. 7 — A Double Twin Scallop knot with a balanced colour-pattern.

Could it have fallen on Deaf Ears?

In *The Braider*, Issue No. 20, pg. 464, we asked if there was someone who could solve whether the name **Rosendahl** by Geoffrey Budworth or the name **Rosenthal** by Brion Toss was correct. Unfortunately we haven't received an answer to that question as yet. We know that some of our readers are a member of the IGKT and we would have thought that at least one of them would have found out from the IGKT's upper echelon, or from a historical addict in its ranks, the correct answer (we have no contact with the IGKT). We hope that the person who does find out the answer to our question will let us know.

A little Contemplation would not have gone amiss.

We came across a most ridiculous article entitled *Het Xaviera Knoopje (The Xaviera Knot)* in the Dutch knotting periodical *Het Knoopeknauwertje*, Issue No. 24, pp. 10-13. In this article the author (Pieter van de Griend) mentions that a Gaucho coding consists of an even number of **lanes**, each with a set number of crossings. He does not define what a **lane** is, but we can deduce from the article that a **lane** in an α-pass Gaucho Knot contains α consecutive columns all of which have the same coding. Hence an α-pass Gaucho Knot with n lanes has $(n\alpha + 1)$ parts.[†]

The author poses the question whether or not it is possible to increase the number of crossings in the two central **lanes** of a 4-lane 2-pass Gaucho Knot with an interbraid (the Gaucho Knot itself has thus $4 \times 2 + 1 = 9$ parts, and the interbraided knot is supposedly a Regular Knot).

He erroneously states, and erroneously tries to prove in a foggy academic way, that this is not possible.

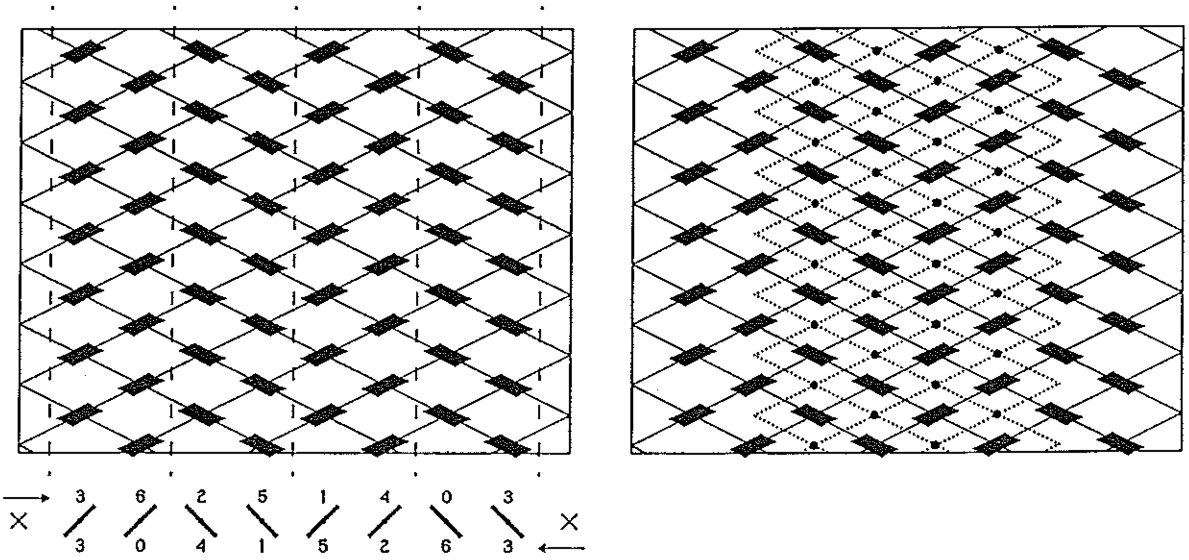

Fig. 8 — A 4-lane 2-pass Gaucho Knot provided with a Regular Knot interbraid.

[†] In *The Braider*, Issue No. 11, pg. 234, we indicated the number of parts in an α-pass Gaucho Knot by $(2m\alpha + 1)$, hence n lanes in an α-pass Gaucho Knot contain $2m\alpha$ intersection columns.

The grid-diagram at the left in Fig. 8 depicts a 4-lane 2-pass Gaucho Knot. If we provide this Gaucho Knot with a Regular Knot interbraid, then the boundaries of the lanes which fall within this interbraid contain crossings as shown in the grid-diagram at the right in Fig. 8. Hence the question whether or not it is possible to increase the number of crossings in **lanes** becomes a senseless question, because not only is such a lane going to contain more intersection columns, but also its boundaries become intersection columns. **Consequently there is no specific lane to which such a lane-boundary intersection column belongs.** It will thus be obvious that the **lane-concept** in Gaucho Knots becomes a senseless concept as soon as we provide the Gaucho Knot with a Regular Knot interbraid. From the grid-diagram at the right in Fig. 8 it will be observed that the intersection columns in the interbraided Regular Knot itself (the columns with the dots) coincide with intersection columns in the Gaucho Knot itself. **Hence we can in any Gaucho Knot centrally interbraid a Regular Cylindrical Braid with any odd number of parts.**

In Fig. 9 the Gaucho Knot in Fig. 8 has been centrally interbraided with a 5-parts Regular Knot. The 5-parts Regular Knot has in the upper grid-diagram a 2-pass Spanish Ring Knot coding and in the lower grid-diagram it has an over–under coding.

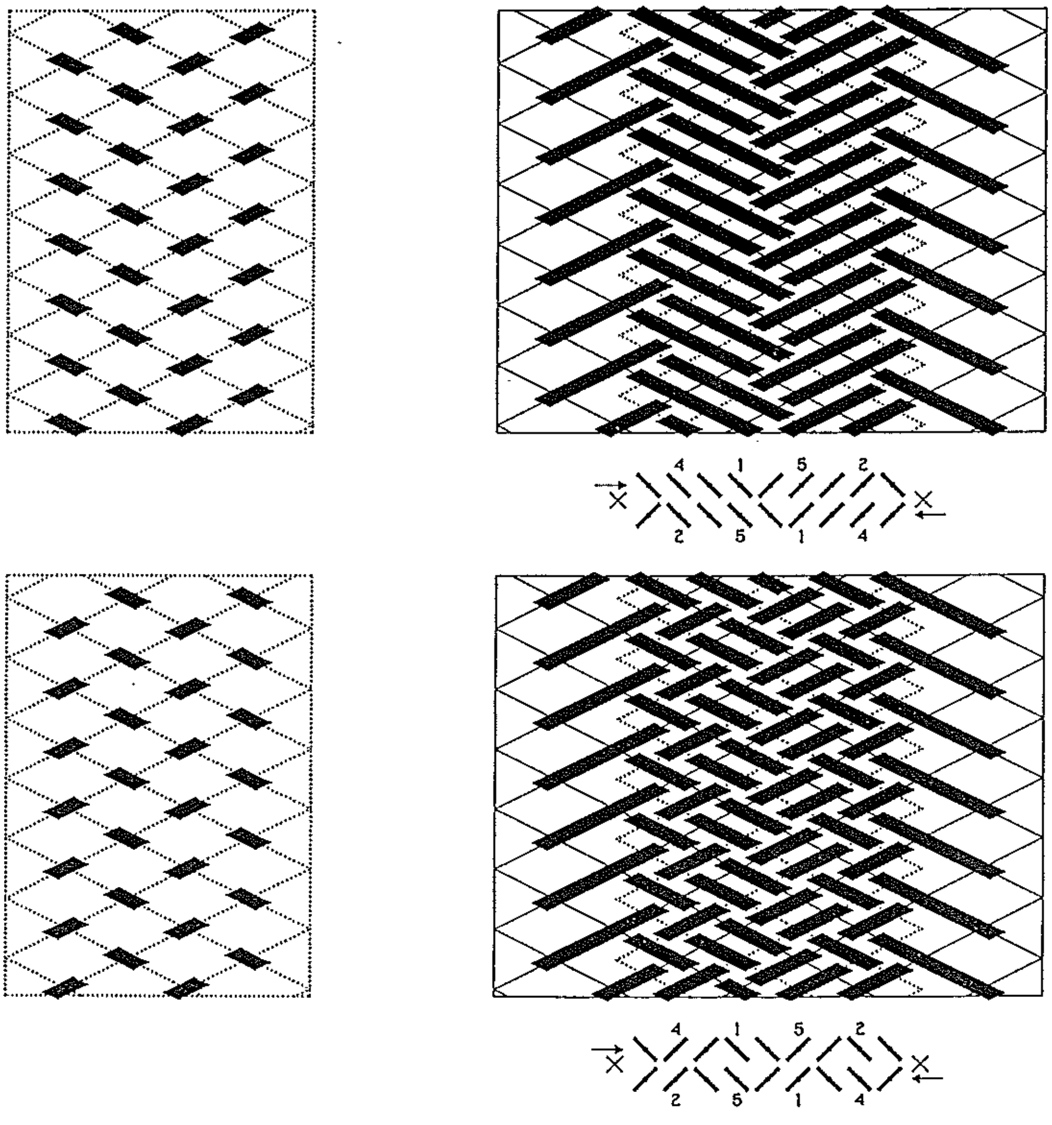

Fig. 9 — The Gaucho knot in Fig. 8 centrally interbraided with a 5-parts Regular Knot.

The half-cycle braiding algorithms for the Gaucho Knot are read from its associated algorithm diagram (see Fig. 8) and are as follows:

half-cycle 1 : $L \longrightarrow R$: Free run.
half-cycle 2 : $i = 0$; $R \longrightarrow L$: $(s)u$.
half-cycle 3 : $i = 0$; $L \longrightarrow R$: u .
half-cycle 4 : $i \leq 1$; $R \longrightarrow L$: $(s)o - u$.
half-cycle 5 : $i \leq 1$; $L \longrightarrow R$: $o - u$.
half-cycle 6 : $i \leq 2$; $R \longrightarrow L$: $(s)u - o - u$.
half-cycle 7 : $i \leq 2$; $L \longrightarrow R$: $u - o - u$.
half-cycle 8 : $i \leq 3$; $R \longrightarrow L$: $(s)o - u - o - (1,s)2u$.
half-cycle 9 : $i \leq 3$; $L \longrightarrow R$: $o - u - o - 2u$.
half-cycle 10 : $i \leq 4$; $R \longrightarrow L$: $o - u - (1,s)2o - 2u$.
half-cycle 11 : $i \leq 4$; $L \longrightarrow R$: $o - u - 2o - 2u$.
half-cycle 12 : $i \leq 5$; $R \longrightarrow L$: $o - (1,s)2u - 2o - 2u$.
half-cycle 13 : $i \leq 5$; $L \longrightarrow R$: $o - 2u - 2o - 2u$.
half-cycle 14 : $i \leq 6$; $R \longrightarrow L$: $(1,s)2o - 2u - 2o - 2u$.

The half-cycle braiding algorithms for the interbraid in the upper right-hand grid-diagram of Fig. 9 are read from its associated algorithm diagram (see Fig. 9) as follows:

half-cycle 1′ : $L_2 \longrightarrow R_2$: $2u - 2o - u$.
half-cycle 2′ : $i = 0$; $R_2 \longrightarrow L_2$: $2u - 2o - u$.
half-cycle 3′ : $i = 0$; $L_2 \longrightarrow R_2$: $2u - 2o - u$.
half-cycle 4′ : $i \leq 1$; $R_2 \longrightarrow L_2$: $(2,s)3u - 2o - u$.
half-cycle 5′ : $i \leq 1$; $L_2 \longrightarrow R_2$: $3u - 3o - u$.
half-cycle 6′ : $i \leq 2$; $R_2 \longrightarrow L_2$: $3u - (2,s)3o - u$.
half-cycle 7′ : $i \leq 2$; $L_2 \longrightarrow R_2$: $3u - 3o - u$.
half-cycle 8′ : $i \leq 3$; $R_2 \longrightarrow L_2$: $3u - 3o - u$.
half-cycle 9′ : $i \leq 3$; $L_2 \longrightarrow R_2$: $3u - 3o - u$.
half-cycle 10′ : $i \leq 4$; $R_2 \longrightarrow L_2$: $(1,s,2)4u - 3o - u$.
half-cycle 11′ : $i \leq 4$; $L_2 \longrightarrow R_2$: $4u - 3o - u$.
half-cycle 12′ : $i \leq 5$; $R_2 \longrightarrow L_2$: $4u - (1,s,2)4o - u$.
half-cycle 13′ : $i \leq 5$; $L_2 \longrightarrow R_2$: $4u - 4o - u$.
half-cycle 14′ : $i \leq 6$; $R_2 \longrightarrow L_2$: $4u - 4o - u$.

The half-cycle braiding algorithms for the interbraid in the lower right-hand grid-diagram of Fig. 9 are read from its associated algorithm diagram (see Fig. 9) as follows:

half-cycle 1′ : $L_2 \longrightarrow R_2$: $u - o - u - o - u$.
half-cycle 2′ : $i = 0$; $R_2 \longrightarrow L_2$: $u - o - u - o - u$.
half-cycle 3′ : $i = 0$; $L_2 \longrightarrow R_2$: $u - o - u - o - u$.
half-cycle 4′ : $i \leq 1$; $R_2 \longrightarrow L_2$: $u - o - (s,1)2u - o - u$.
half-cycle 5′ : $i \leq 1$; $L_2 \longrightarrow R_2$: $u - o - 2u - o - u$.
half-cycle 6′ : $i \leq 2$; $R_2 \longrightarrow L_2$: $u - o - 2u - o - (s,1)2u$.
half-cycle 7′ : $i \leq 2$; $L_2 \longrightarrow R_2$: $u - o - 2u - o - 2u$.
half-cycle 8′ : $i \leq 3$; $R_2 \longrightarrow L_2$: $u - o - 2u - o - 2u$.
half-cycle 9′ : $i \leq 3$; $L_2 \longrightarrow R_2$: $u - o - 2u - o - 2u$.
half-cycle 10′ : $i \leq 4$; $R_2 \longrightarrow L_2$: $u - (s,1)2o - 2u - o - 2u$.
half-cycle 11′ : $i \leq 4$; $L_2 \longrightarrow R_2$: $u - 2o - 2u - o - 2u$.
half-cycle 12′ : $i \leq 5$; $R_2 \longrightarrow L_2$: $u - 2o - 2u - (s,1)2o - 2u$.
half-cycle 13′ : $i \leq 5$; $L_2 \longrightarrow R_2$: $u - 2o - 2u - 2o - 2u$.
half-cycle 14′ : $i \leq 6$; $R_2 \longrightarrow L_2$: $u - 2o - 2u - 2o - 2u$.

The Xaviera Knot in the Dutch knotting periodical *Het Knoopeknauwertje* is in practical terms a monstrosity and hence of no practical use at all. It is Pieter van de Griend's solution to interbraiding the two central lanes of a 4-lane 2-pass Gaucho Knot, which according to his absurd theory necessitates a modification of one of these lanes. The grid-diagrams associated with this modification and its resulting interbraided contraption by the name of Xaviera Knot are shown in Fig. 10.

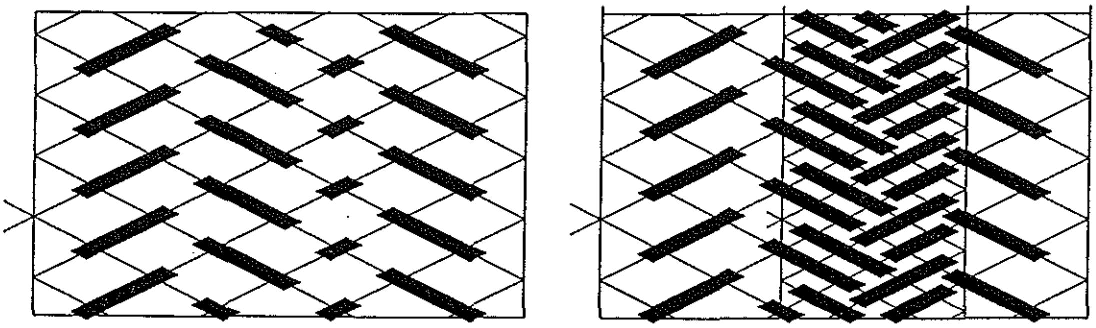

Fig. 10 — The Xaviera Knot.

ERRATA—*The Braider*

The list below lists the typing errors which were noted in at least some copies of the following Issues of *The Braider*.

No. 25

pg. 563, line 19 — read as: greater than or equal to 2 and less than p

No. 26

pg. 593, line 6 — delete parenthesis after half-cycle

No. 27

pg. 631, line 18 — **Herringbone**
pg. 633, line 33 — **Herringbone**
pg. 634, line 3 — **Herringbone**

No. 28

pg. 640, line 2 — **algorithms**
pg. 647, lines 11 & 20 below Fig. 517 — **finished**
pg. 648, lines 1, 10, 19, 28, 37 & 46 — **finished**
pg. 649, lines 8 & 17 — **finished**
pg. 650, line 4 — read as: greater than 1 and less than or equal to 17
pg. 650, lines 5 & 6 — read as: greater than or equal to 3 and less than or equal to 17
pg. 650, line 4 below Fig. 519 — read as: greater than 1 and less than or equal to 23
pg. 650, line 6 below Fig. 519 — read as: greater than or equal to 3 and less than or equal to 23
pg. 651, line 3 — read as: greater than or equal to 2 and less than or equal to 8
pg. 651, line 4 below Fig. 521 — read as: greater than or equal to 2 and less than or equal to 12

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Solutions to the Questions in Issue No. 24

Question on pg. 545.

Consider the development of the string-run in the case where $p < b$. By laying down the string-run, we lay down the consecutive half-cycles. Let the first half-cycle (the Standing-End half-cycle) run from lower-left to upper-right. Only lower-right to upper-left half-cycles may be able to intersect the Standing-End half-cycle, but since $p < b$ and a cycle encompasses p bights, not all lower-right to upper-left half-cycles will intersect the Standing-End half-cycle, furthermore, each of those which does intersect the Standing-End half-cycle will intersect it only once. Since $p < b$, the number of **free-run** half-cycles will thus be equal to:

$$2\left\lfloor \frac{b}{p} \right\rfloor + 1\,.$$

A half-cycle which intersects the Standing-End half-cycle and its consecutive half-cycles to, but not including, the next half-cycle which intersects the Standing-End half-cycle, form a set of half-cycles each of which intersects the same number of half-cycles (strands). The number of intersected strands per half-cycle increases by one for the next such set of half-cycles.

As we have already seen above, the number of half-cycles in the first set (the free-run half-cycles, the half-cycles which intersect zero strings) is equal to:

$$2\left\lfloor \frac{b}{p} \right\rfloor + 1\,.$$

The number of half-cycles in the subsequent consecutive sets greater equal 2 and less than p, hence excepting the last set (the p^{th} set), is thus equal to:

$$2\left(\left\lfloor \frac{(n+1)b}{p} \right\rfloor - \left\lfloor \frac{nb}{p} \right\rfloor\right), \qquad \text{where} \quad n = 1\,,\, 2\,,\, 3\,,\, \cdots\,,\, (p-2)\,.$$

The number of half-cycles in the last set (the p^{th} set) is equal to:

$$2\left(b - \left\lfloor \frac{(p-1)b}{p} \right\rfloor\right) - 1 = 2\left\lfloor \frac{b}{p} \right\rfloor + 1\,.$$

With these formulae we can readily prove that the sequence of the number of half-cycles in the sequential sets is palindromic:

As the above formulae show, the first set and the last set contain each the same number of half-cycles.

The set j, where $j = 2\,,\, 3\,,\, 4\,,\, \cdots\,,\, (p-1)$ contains:

$$2\left(\left\lfloor \frac{jb}{p} \right\rfloor - \left\lfloor \frac{(j-1)b}{p} \right\rfloor\right) \text{ half-cycles.}$$

Then the set $p - (j-1)$ contains:

$$2\left(\left\lfloor \frac{\{p-(j-1)\}b}{p} \right\rfloor - \left\lfloor \frac{(p-j)b}{p} \right\rfloor\right) = 2\left(\left\lfloor \frac{jb}{p} \right\rfloor - \left\lfloor \frac{(j-1)b}{p} \right\rfloor\right) \text{ half-cycles.}$$

Hence the sets j and $p - (j-1)$ contain each the same number of half-cycles.

Thus the sequence of the number of half-cycles in the sequential sets is palindromic.

Question on pg. 557.

The proof is shown in the Figs. 464, 465 & 466, where the left-hand bight-boundaries and bights are drawn in solid lines, while the right-hand bight-boundaries and bights are drawn in dotted lines.

Note that in Fig. 464 we must have the conditions $y_{max} \geq A$, and $y_{min} \leq A$. Consequently $x \geq 2 - A$; and since $x < 0$ it follows that $A > 2$.

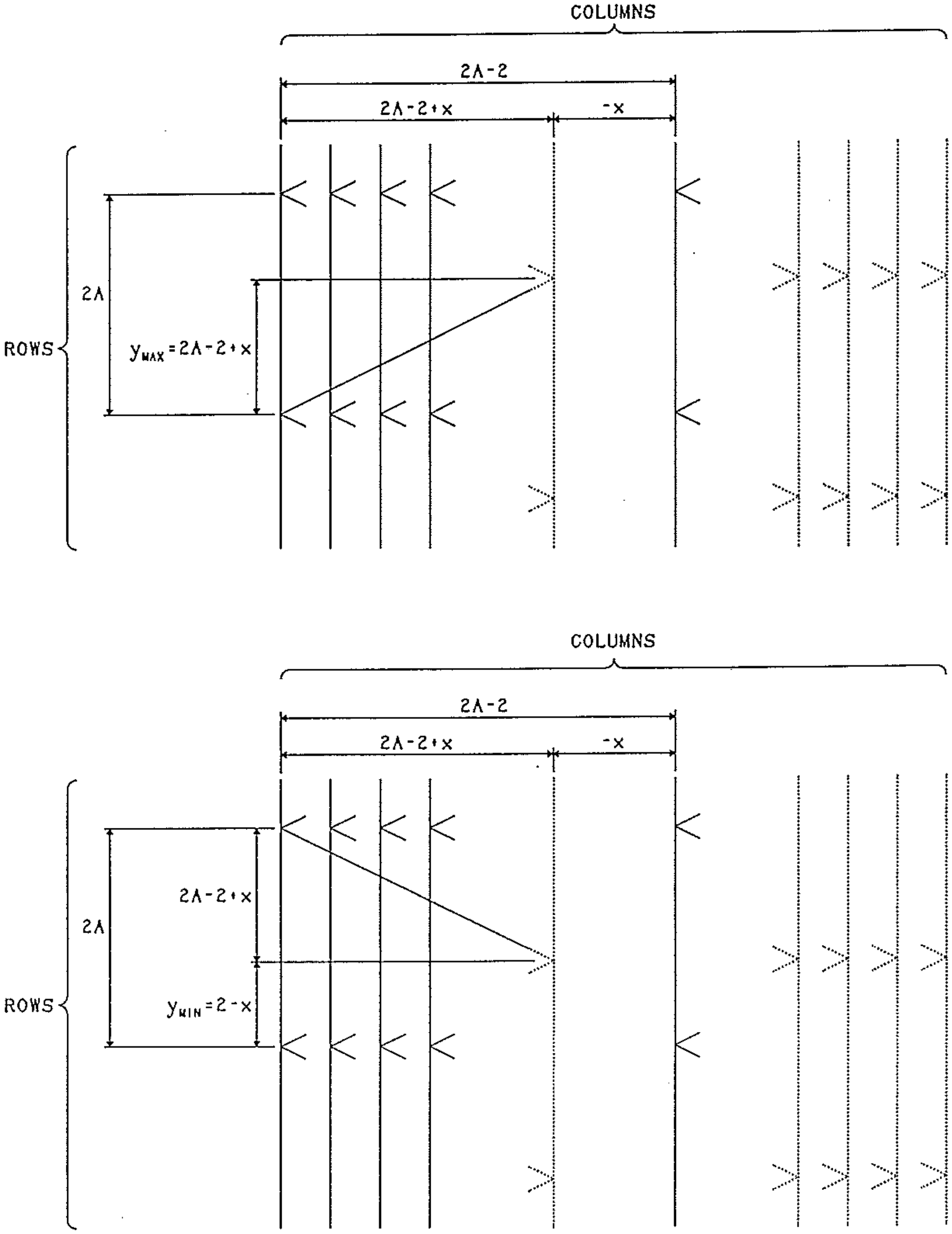


Fig. 464 — y_{max} and y_{min} for $2 - A \leq x < 0$.

Note that in Fig. 465 we must have the conditions $y_{max} \geq A$, and $y_{min} \leq A$. Consequently $A \geq 2$.

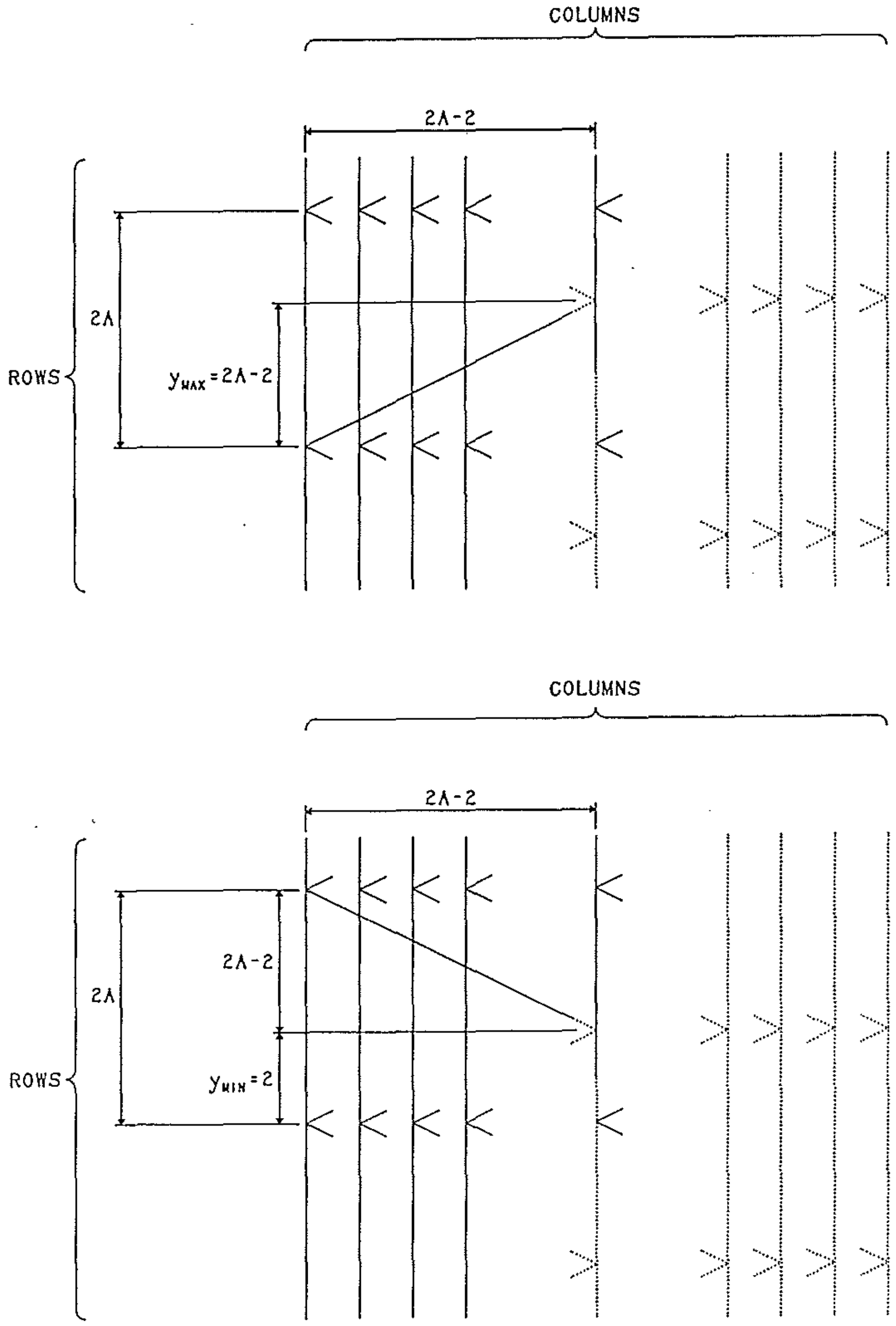


Fig. 465 — y_{max} and y_{min} for $x = 0$.

Note that in Fig. 466 for $x = 2$ we obtain $y_{max} = 2A$ and $y_{min} = 0$. This are also the maximum, respectively minimum values possible for $x > 2$. For $0 < x < 2$ we obtain $2A - 2 < y_{max} < 2A$, and $2 > y_{min} > 0$. This fulfils for $A \geq 2$ the necessary condition $y_{max} \geq A$, and $y_{min} \leq A$.

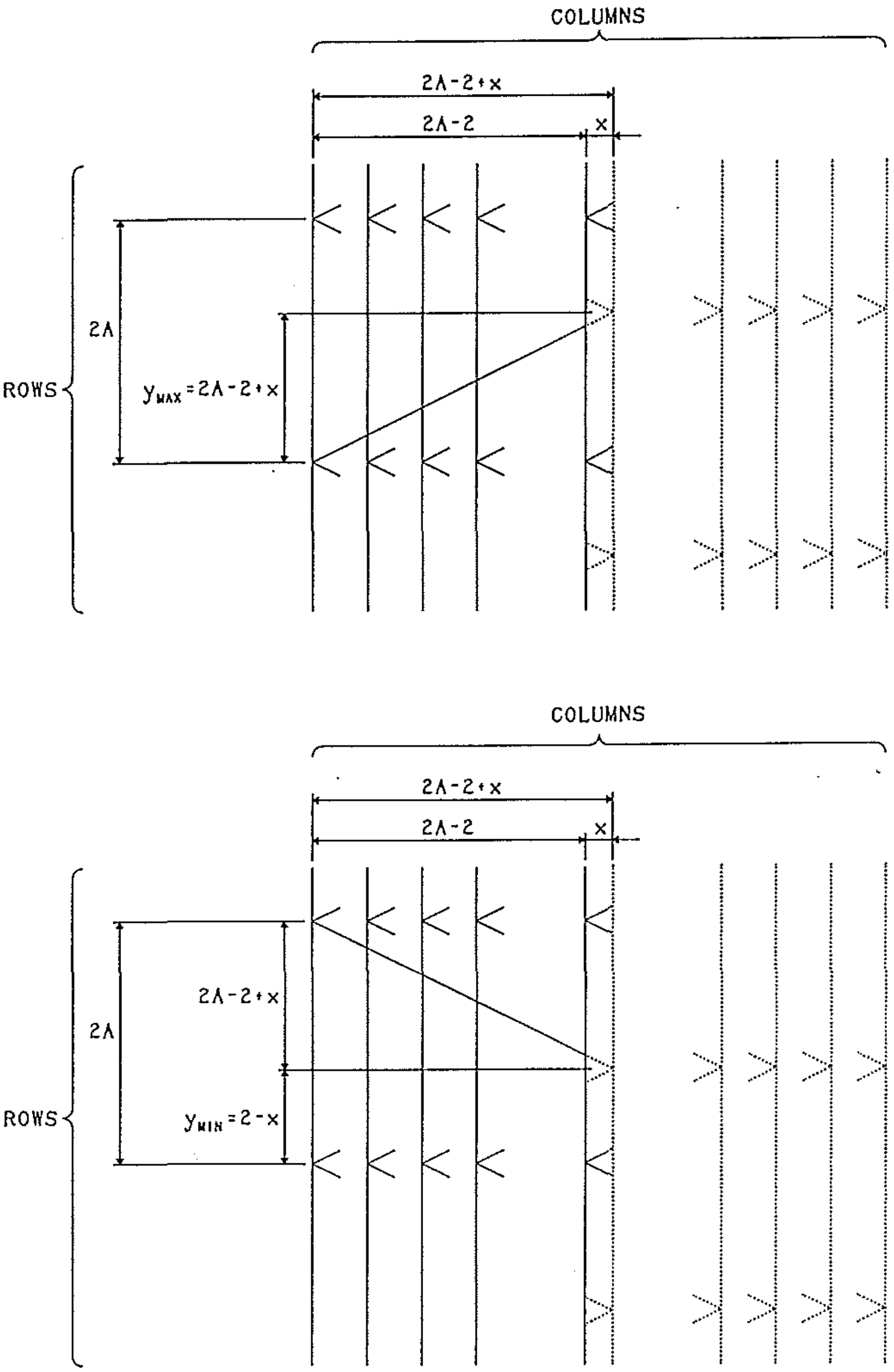


Fig. 466 — y_{max} and y_{min} for $0 < x \leq 2$.

Nested Cylindrical Braids

From the relationships $\Delta = |x - 2(k+1)|_A$ and $y = |x - 2(k+1)|_{2A}$ we obtain:

$$\Delta = |y|_A \,.$$

Furthermore, from the relationship $y = |x - 2(k+1)|_{2A}$ we obtain:

$$k = \left| \frac{x - y - 2}{2} \right|_A ,$$

hence x and y must both have the same parity (both **odd** or both **even**). Consequently, there are two types of (A, x, y)-tables, one for A = odd and one for A = even. An example of each type is shown in respectively Fig. 467 and Fig. 468. Note the two different kinds of hatched x-lines in the tables for $A = \text{odd}$.

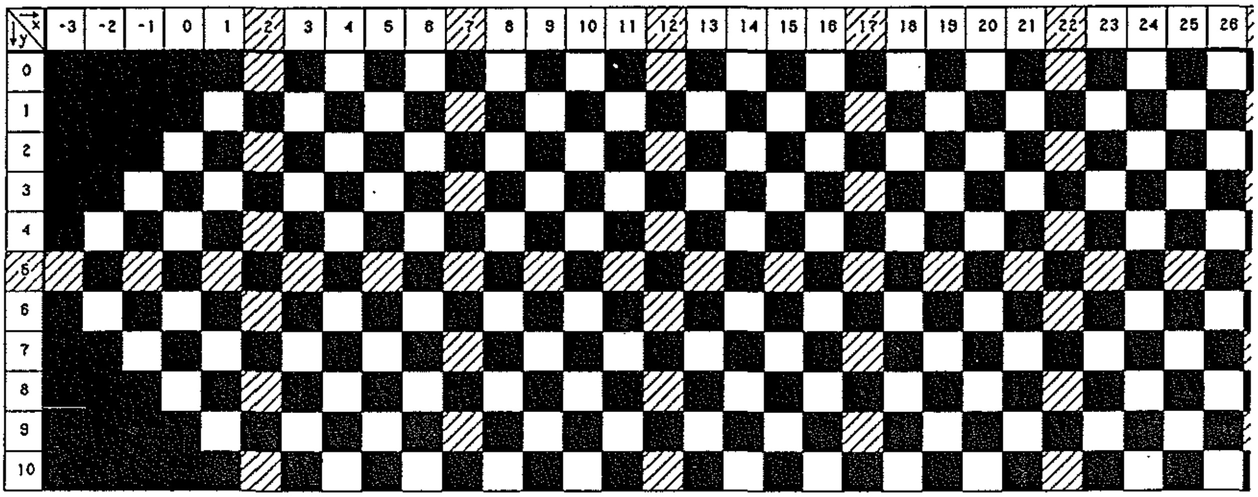


Fig. 467 — The $(A = 5, x, y)$-table.

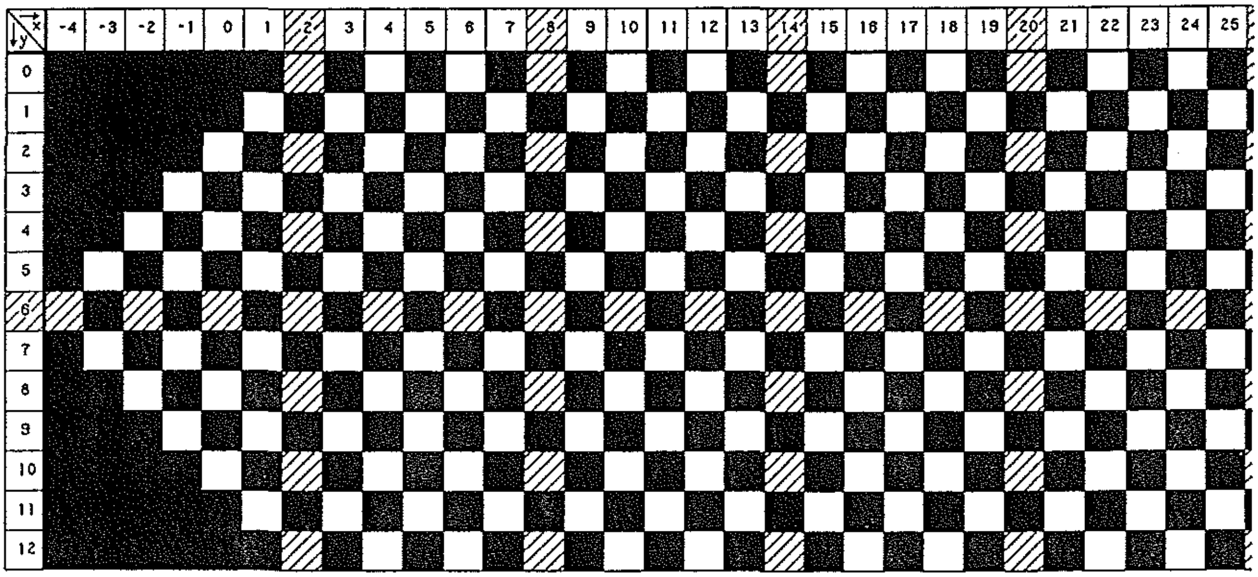
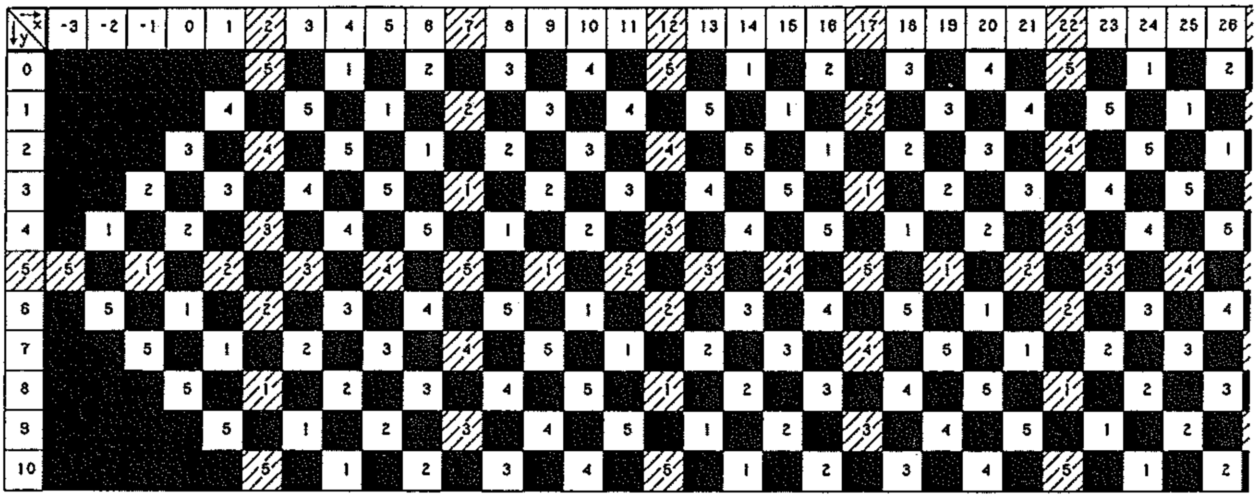


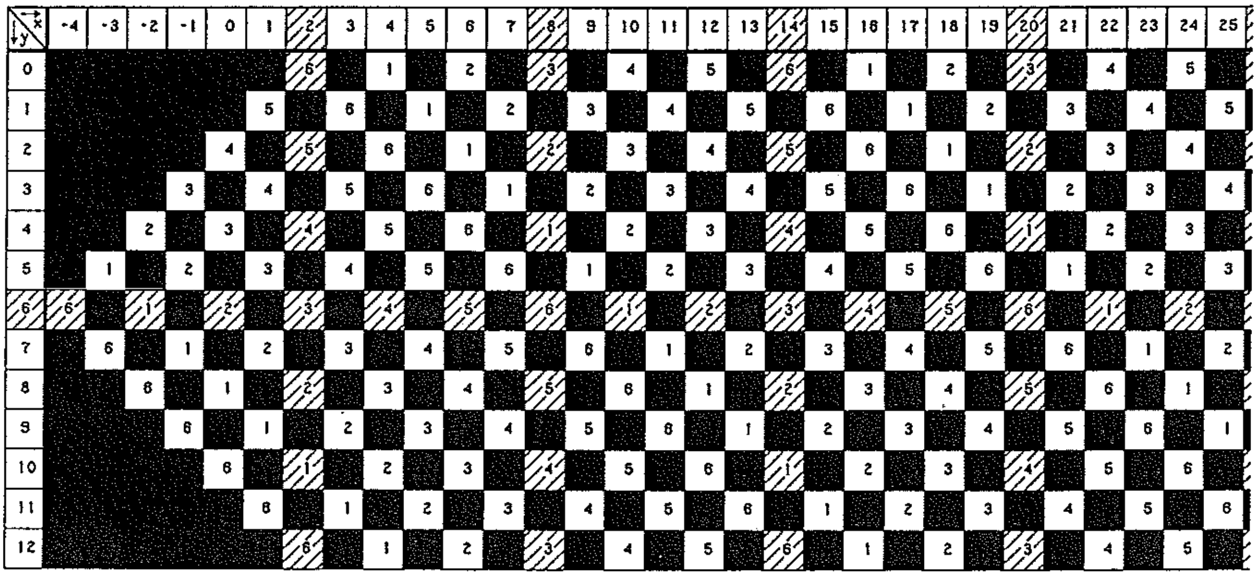
Fig. 468 — The $(A = 6, x, y)$-table.

In each non-black cell of an (A, x, y)-table we enter the associated k-value. This is shown in Fig. 469 for the table in Fig. 467, and in Fig. 470 for the table in Fig. 468.



y \ x	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
0						5		1		2		3		4		5		1		2		3		4		5		1		2
1					4		5		1		2		3		4		5		1		2		3		4		5		1	
2				3		4		5		1		2		3		4		5		1		2		3		4		5		1
3			2		3		4		5		1		2		3		4		5		1		2		3		4		5	
4		1		2		3		4		5		1		2		3		4		5		1		2		3		4		5
5	5		1		2		3		4		5		1		2		3		4		5		1		2		3		4	
6		5		1		2		3		4		5		1		2		3		4		5		1		2		3		4
7			5		1		2		3		4		5		1		2		3		4		5		1		2		3	
8				5		1		2		3		4		5		1		2		3		4		5		1		2		3
9					5		1		2		3		4		5		1		2		3		4		5		1		2	
10						5		1		2		3		4		5		1		2		3		4		5		1		2

Fig. 469 — The $(A = 5, x, y)$-table with k-values.



y \ x	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0							6		1		2		3		4		5		6		1		2		3		4		5	
1						5		6		1		2		3		4		5		6		1		2		3		4		5
2					4		5		6		1		2		3		4		5		6		1		2		3		4	
3				3		4		5		6		1		2		3		4		5		6		1		2		3		4
4			2		3		4		5		6		1		2		3		4		5		6		1		2		3	
5		1		2		3		4		5		6		1		2		3		4		5		6		1		2		3
6	6		1		2		3		4		5		6		1		2		3		4		5		6		1		2	
7		6		1		2		3		4		5		6		1		2		3		4		5		6		1		2
8			6		1		2		3		4		5		6		1		2		3		4		5		6		1	
9				6		1		2		3		4		5		6		1		2		3		4		5		6		1
10					6		1		2		3		4		5		6		1		2		3		4		5		6	
11						6		1		2		3		4		5		6		1		2		3		4		5		6
12							6		1		2		3		4		5		6		1		2		3		4		5	

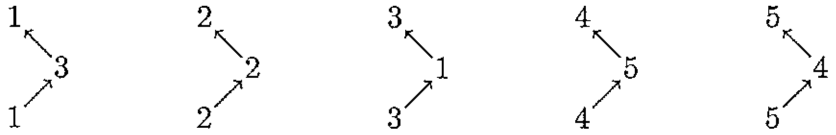
Fig. 470 — The $(A = 6, x, y)$-table with k-values.

From the relationships we derived and discussed in the previous issue, it follows that for a Regular Nested Cylindrical Braid associated with a cell in the row $y = A$, its set of first-return string-runs is the set of their mirror images. The same applies for the set of first-return string-runs of a Regular Nested Cylindrical Braid associated with a cell in the row $y = 0 = 2A$.

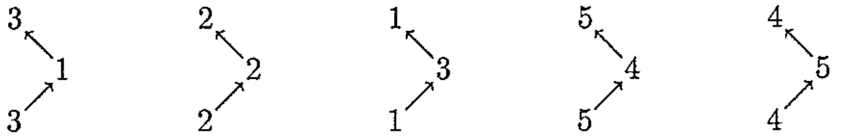
Example 1 : $A = 5$; $y = 5$; $x = 13$.

Hence $k = 3$. The value of k can be calculated with the formula $k = \left|\dfrac{x - y - 2}{2}\right|_A$, or may be obtained from the table in Fig. 469.

Since $y = 5$, it follows that $\Delta = |y|_A = 0$. Hence the set of first-return string-runs is :



The mirror image set is thus:



which is identical to the previous set of first-return string-runs.

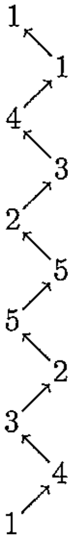
Note that $\Delta = 0$ is associated with the Standard and Semi-Standard Regular Nested Cylindrical Braids (see *The Braider*, Issues No. 23 & No. 24, pp. 513–515, pp. 519–531, pg. 555; and **Braiding**, book 4/1, Standard Herringbone Pineapple Knots).

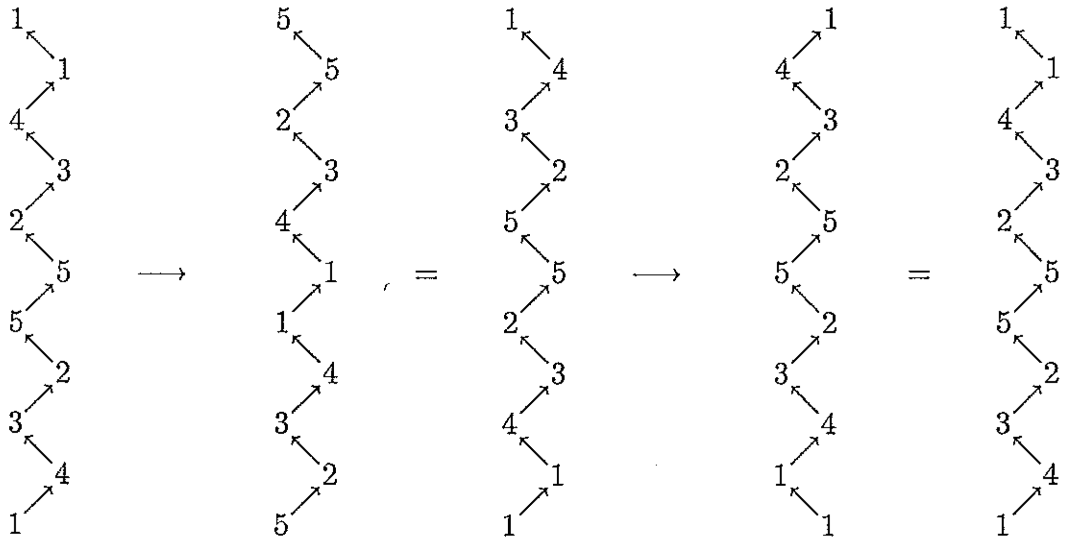
The set of first-return string-runs of a Regular Nested Cylindrical Braid associated with a cell in the column $x = 2 + nA$ is also the set of their mirror imaged (A, A)-complements.

Example 2: $A = 5$; $x = 17$; $y = 7$.

Hence $k = 4$. Again, k can be calculated with the formula $k = \left| \frac{x - y - 2}{2} \right|_A$, or may be obtained from the table in Fig. 469.

Since $y = 7$, it follows that $\Delta = |y|_A = 2$. The number of Components is equal to g.c.d. (Δ, A), and hence the set of first-return string-runs consists of only one first-return string-run (g.c.d. $(\Delta, A) = 1$):



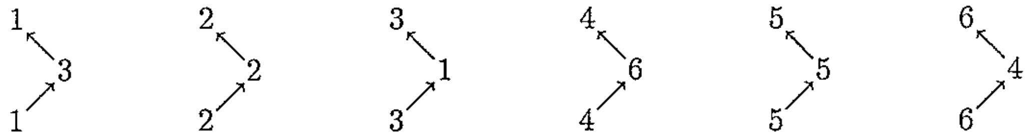


The set of first-return string-runs of a Regular Nested Cylindrical Braid associated with a cell in the column $x = 2 + nA$ and row $y = A$ or $y = 0 = 2A$ is also the set of their mirror image, the set of their (A, A)-complements, as well as the set of their mirror imaged (A, A)-complements.

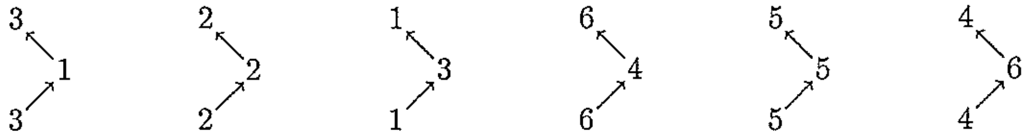
Example 3: $A = 6$; $x = 14$; $y = 6$.

Hence $k = 3$. The value of k can be calculated with the formula $k = \left|\dfrac{x - y - 2}{2}\right|_A$, or may be obtained from the table in Fig. 470.

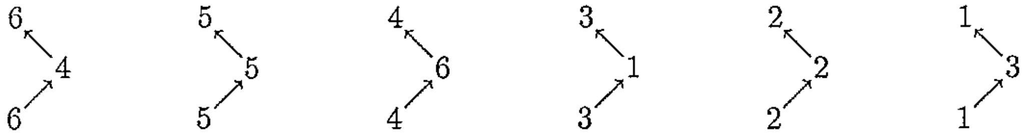
Since $y = 6$, it follows that $\Delta = |y|_A = 0$. Hence the set of first-return string-runs is:



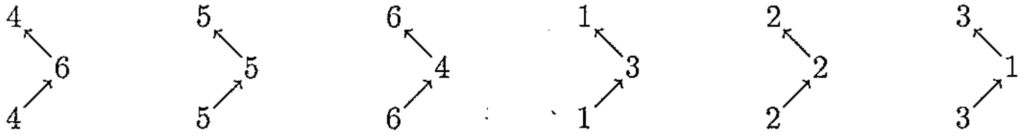
The mirror image set is thus:



The (A, A)-complemental set is thus:



The mirror imaged (A, A)-complemental set is thus:



Note that all these four sets of first-return string-runs are identical.

Since $l_{i+1} = |l_i + \Delta|_A$ and $r_{i+1} = |r_i - \Delta|_A$, it follows that Δ is an **invariant** for the Regular Nested Cylindrical Braid concerned. Consequently α (the number of bights in a first-return string-run, hence the number of bights in a **component**) is also an **invariant** for the Regular Nested Cylindrical Braid concerned.

Let $\gamma = \text{g.c.d.}\,(\Delta, A) =$ number of first-return string-runs $=$ number of components. Then $\alpha = \dfrac{A}{\gamma}$. It will thus be evident that, in the sequence of the γ first-return string-runs of a Regular Nested Cylindrical Braid, we can take for the first lower-left to upper-right half-cycle the respective half-cycles:

$$1 \longrightarrow k\ ;\ 2 \longrightarrow |k-1|_A\ ;\ 3 \longrightarrow |k-2|_A\ ;\ \cdots\cdots\ ;\ \gamma \longrightarrow |k+1-\gamma|_A\,.$$

This sequence of γ first-return string-runs, and hence their associated components, forms two sets of components. **It suffices to consider $0 \le y \le A$ due to the mirror-image complementary relationship. Hence $y' = y$ with $\Delta' = |y'|_A = |y|_A = \Delta$ for $0 \le y < A$, and $y' = 2A - y$ with $\Delta' = |y'|_A = |-\Delta|_A$ for $A \le y \le 2A$.**

The first set contains

$$\left|\frac{x + \Delta' - 2}{2}\right|_\gamma \text{ components,}$$

with each component having a number of parts equal to

$$P_c = \frac{2A-\Delta'}{\gamma} + 2\left\lceil \frac{x+\Delta'-2}{2\gamma} \right\rceil = \frac{x+2A-2-2\left|\frac{x+\Delta'-2}{2}\right|_{\gamma}}{\gamma} + 2\,.$$

The second set contains

$$\gamma - \left|\frac{x+\Delta'-2}{2}\right|_{\gamma} \text{ components,}$$

with each component having a number of parts equal to

$$P_c = \frac{2A-\Delta'}{\gamma} + 2\left\lfloor \frac{x+\Delta'-2}{2\gamma} \right\rfloor = \frac{x+2A-2-2\left|\frac{x+\Delta'-2}{2}\right|_{\gamma}}{\gamma}\,.$$

In these formulae $\Delta' = |y'|_A$ for $y' \neq A$; $\gamma = A$ when $\Delta' = 0$; $\Delta' = A$ when $y' = A$.

Example 4 : $A = 8$; $y = 12$; $x = 56$.

$y' = 2A - y = 16 - 12 = 4$ (mirror-immage complementary relationship), hence $\Delta' = |y'|_A = |4|_8 = 4$. Furthermore $\gamma = \text{g.c.d.}(\Delta, A) = \text{g.c.d.}(4,8) = 4$; $\alpha = \dfrac{A}{\gamma} = \dfrac{8}{4} = 2$; $k = \left|\dfrac{x-y-2}{2}\right|_A = \left|\dfrac{56-12-2}{2}\right|_8 = |21|_8 = 5$ with $\Delta = |y|_A = |12|_8 = 4$.

$$\#\text{ of components in } 1^{\text{st}} \text{ set} = \left|\frac{x+\Delta'-2}{2}\right|_{\gamma} = \left|\frac{56+4-2}{2}\right|_4 = |29|_4 = 1\,.$$

$$P_c = \frac{2A-\Delta'}{\gamma} + 2\left\lceil \frac{x+\Delta'-2}{2\gamma} \right\rceil = \frac{16-4}{4} + 2\left\lceil \frac{56+4-2}{8} \right\rceil = 3 + 16 = 19\,.$$

$$P_c = \frac{x+2A-2-2\left|\frac{x+\Delta'-2}{2}\right|_{\gamma}}{\gamma} + 2 = \frac{56+16-2-2\left|\frac{56+4-2}{2}\right|_4}{4} + 2 = 17 + 2 = 19\,.$$

$$\#\text{ of components in } 2^{\text{st}} \text{ set} = \gamma - \left|\frac{x+\Delta'-2}{2}\right|_{\gamma} = 4 - \left|\frac{56+4-2}{2}\right|_4 = 4 - 1 = 3\,.$$

$$P_c = \frac{2A-\Delta'}{\gamma} + 2\left\lfloor \frac{x+\Delta'-2}{2\gamma} \right\rfloor = \frac{16-4}{4} + 2\left\lfloor \frac{56+4-2}{8} \right\rfloor = 3 + 14 = 17\,.$$

$$P_c = \frac{x+2A-2-2\left|\frac{x+\Delta'-2}{2}\right|_{\gamma}}{\gamma} = \frac{56+16-2-2\left|\frac{56+4-2}{2}\right|_4}{4} = 17\,.$$

Thus the first set of components consists of one component with $P_c = 19$ parts, and the second set of components consists of three components with $P_c = 17$ parts each.

Note that a useful check may be made with the formula:
$[(\#\text{ of components in } 1^{\text{st}} \text{ set}) \times (P_c \text{ in } 1^{\text{st}} \text{ set})] + [(\#\text{ of components in } 2^{\text{nd}} \text{ set}) \times (P_c \text{ in } 2^{\text{nd}} \text{ set})] = x + 2A - 2$.
Hence in our case: $1 \times 19 + 3 \times 17 = 19 + 51 = 70 = x + 2A - 2 = 56 + 16 - 2 = 70$.

Example 5 : $A = 8$; $y = 8$; $x = 56$.

Hence $\Delta' = A = 8$ $(y' = A)$; $\gamma = \text{g.c.d.}(\Delta, A) = \text{g.c.d.}(8,8) = 8$; $\alpha = \dfrac{A}{\gamma} = \dfrac{8}{8} = 1$; $k = \left|\dfrac{x-y-2}{2}\right|_A = \left|\dfrac{56-8-2}{2}\right|_8 = |23|_8 = 7$ with $\Delta = |y|_A$.

$$\#\text{ of components in } 1^{\text{st}} \text{ set} = \left|\frac{x+\Delta'-2}{2}\right|_{\gamma} = \left|\frac{56+8-2}{2}\right|_8 = |31|_8 = 7\,.$$

$$P_c = \frac{2A-\Delta'}{\gamma} + 2\left\lceil\frac{x+\Delta'-2}{2\gamma}\right\rceil = \frac{16-8}{8} + 2\left\lceil\frac{56+8-2}{16}\right\rceil = 1+8 = 9\,.$$

$$P_c = \frac{x+2A-2-2\left|\frac{x+\Delta'-2}{2}\right|_\gamma}{\gamma} + 2 = \frac{56+16-2-2\left|\frac{56+8-2}{2}\right|_8}{8} + 2 = 7+2 = 9\,.$$

$$\#\text{ of components in } 2^{\text{nd}} \text{ set} = \gamma - \left|\frac{x+\Delta'-2}{2}\right|_\gamma = 8 - \left|\frac{56+8-2}{2}\right|_8 = 8-7 = 1\,.$$

$$P_c = \frac{2A-\Delta'}{\gamma} + 2\left\lfloor\frac{x+\Delta'-2}{2\gamma}\right\rfloor = \frac{16-8}{8} + 2\left\lfloor\frac{56+8-2}{16}\right\rfloor = 1+6 = 7\,.$$

$$P_c = \frac{x+2A-2-2\left|\frac{x+\Delta'-2}{2}\right|_\gamma}{\gamma} = \frac{56+16-2-2\left|\frac{56+8-2}{2}\right|_8}{8} = 7\,.$$

Hence the first set of components consists of seven components, each of which has $P_c = 9$ parts, and the second set of components consists of one component with $P_c = 7$ parts.

Check: $7 \times 9 + 1 \times 7 = 63 + 7 = 70 = x + 2A - 2 = 56 + 16 - 2 = 70\,.$

We can of course also readily tabulate in the cells of the (A, x, y)-table the number of components in the first set with the number of parts for each component in that set and the number of components in the second set with the number of parts for each component in that set. Since the first cell in each row has always an empty first set, we can readily calculate for these cells their P_c-values by means of the formula:

$$P_c = \frac{P_{total}}{\gamma}\,.$$

An example for $A = 12$ is shown in Fig. 471. Since the row $y = A$ is the row of mirror-image, it is the row of symmetry for these cell-entries.

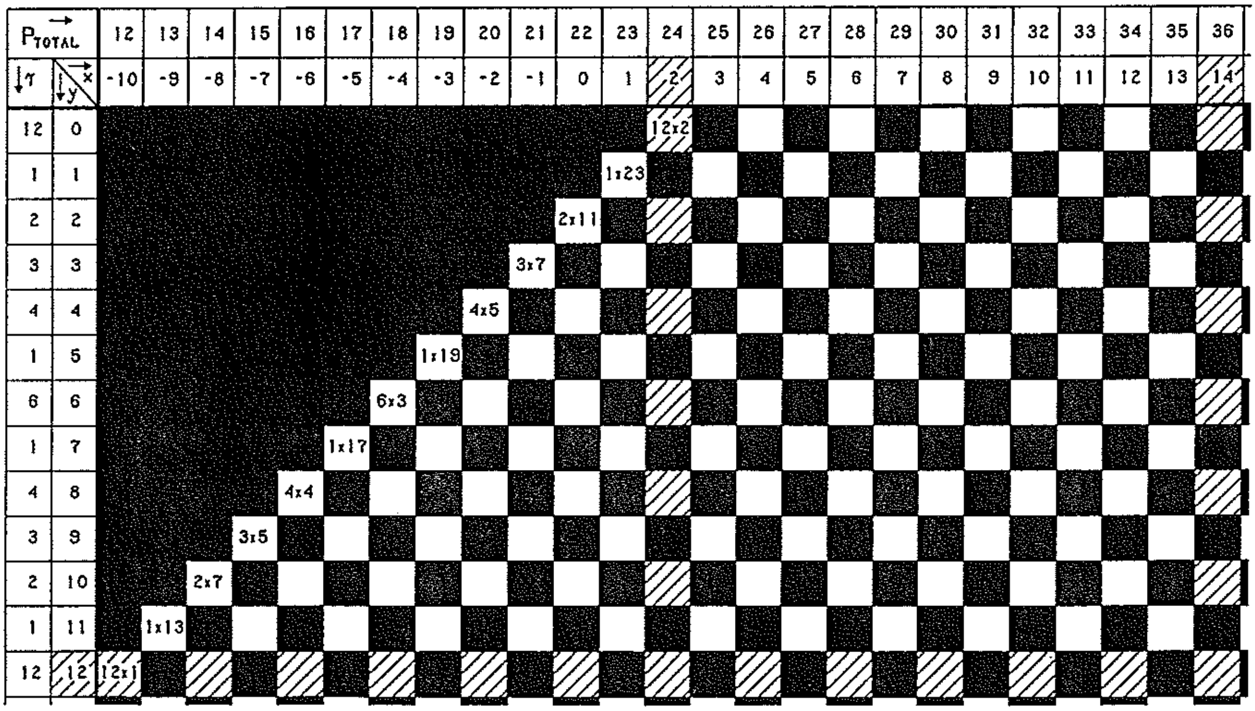


Fig. 471 — The entry in each first row-cell for $A = 12$.

The further consecutive cell-entries for each row can now be entered; a few examples are shown in Fig. 472.

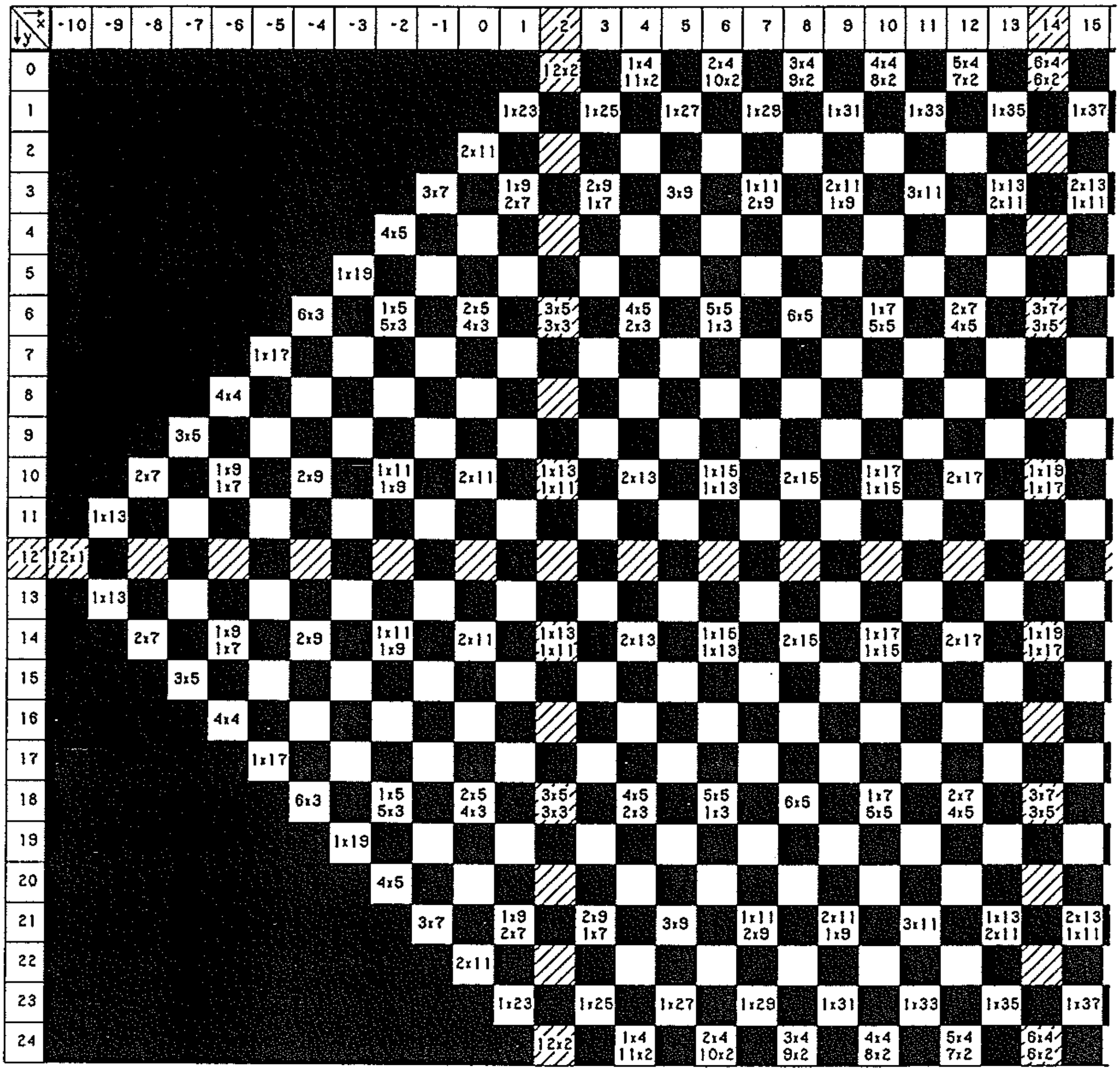


Fig. 472 — A few example rows showing the further cell-entries for $A = 12$.

★★ *Complete the cell-entries in the above table (Fig. 472). What do you observe, and what can you deduce for other A-values?*

When we know for a set of (A, x, y)-values its associated component-set and the number of components, we can readily determine for a B^*-value, the total number of essential strings required.

Example 6: $A = 8$; $x = 12$; $y = 4$; $B^* = 5$.

Hence the associated component-set is $P_c = 7$ and $Pc = 5$ with the respective number of components 3 and 1 (thus 3×7 and 1×5). Then g.c.d.$(P_c, B^*) =$ g.c.d.$(7, 5) = 1$ and g.c.d.$(P_c, B^*) =$ g.c.d.$(5, 5) = 5$. Consequently the number of essential strings is $3 \times 1 + 1 \times 5 = 8$.

Note that an (A, x, y)-table with cell-entries for the essential number of strings associated with a specific B^*-value has little or no value since an (A, x, y)-table with cell-entries for $\{(\#\text{ of components}) \times P_c\}$ is applicable to all B^*-values and the calculation $\{\text{g.c.d.}(P_c, B^*)\}$ is a simple one.

Transitions from two Round Braids to one Round Braid, and vice versa

In applications where such transitions are called for (for example in the formation of loops or eyes) it is important that the transition presents a pleasing appearance, hence does not show disturbing irregularities. The reader be reminded that some aspects of round braids were discussed in *The Braider*, Issue No. 7, pp. 146–153, and Issue No. 10, pp. 216–222.

Transitions between two 4-string round braids and one 8-string round braid.

BRAIDING TYPE											
8 STRING								4 STRING			
→				←				→		←	
U	O	U	O	O	U	O	U	U	O	O	U
1	1	1	1	1	1	1	1	1	1	1	1

We have here four good transition combinations; they are respectively depicted in Figs. 473, 474, 475 and 476.

Fig. 473:

The 4-string round braid has at one end the strings A, B, C, D and at the other end the strings 1, 2, 3, 4.

Form the crossings between the strings A, C, 2, 4.

Bring 1 from the right around the back to the left, then along the front from left to right under B, over D, under 2, over 4.

Bring B from the left around the back to the right, then along the front from right to left under 3, over A, under C, over 1.

Bring 3 from the right around the back to the left, then along the front from left to right under D, over 2, under 4, over B.

Bring D from the left around the back to the right, then along the front from right to left under A, over C, under 1, over 3.

Bring A from the right around the back to the left, then along the front from left to right under 2, over 4, under B, over D.

Bring 2 from the left around the back to the right, then along the front from right to left under C, over 1, under 3, over A.

Bring C from the right around the back to the left, then along the front from left to right under 4, over B, under D, over 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 1, over 3, under A, over C.

And so on.

The 8-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8.

Bring 1 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6, over 8.

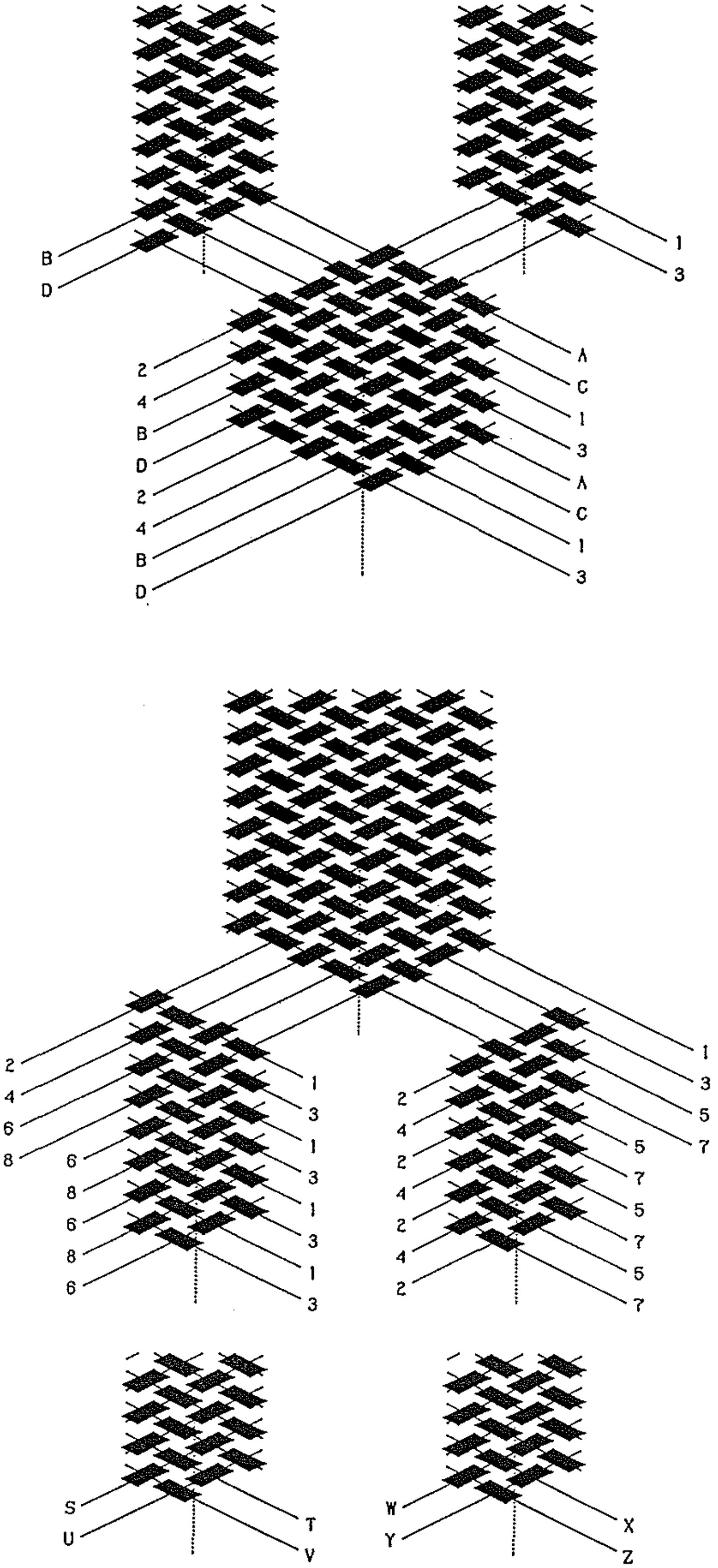


Fig. 473 — Transitions between two 4-string round braids and one 8-string round braid.

Bring 2 from the left around the back to the right, then along the front from right to left under 3, over 5, under 7.

Bring 3 from the right around the back to the left, then along the front from left to right under 4, over 6, under 8.

Bring 4 from the left around the back to the right, then along the front from right to left under 5, over 7.

Braid the left-hand round braid of 4-strings:

Bring 1 from the right around the back to the left, then along the front from left to right under 6, over 8.

Bring 6 from the left around the back to the right, then along the front from right to left under 3, over 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 8, over 6.

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3.

And so on.

Braid the right-hand round braid of 4-strings:

Bring 5 from the right around the back to the left, then along the front from left to right under 2, over 4.

Bring 2 from the left around the back to the right, then along the front from right to left under 7, over 5.

Bring 7 from the right around the back to the left, then along the front from left to right under 4, over 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 5, over 7.

And so on.

Fig. 474:

The 4-string round braid has at one end the strings *A*, *B*, *C*, *D* and at the other end the strings 1, 2, 3, 4.

Form the crossings between the strings *B*, *D*, 1, 3.

Bring *A* from the left around the back to the right, then along the front from right to left under 2, over 4, under *B*, over *D*.

Bring 2 from the right around the back to the left, then along the front from left to right under *C*, over 1, under 3, over *A*.

Bring *C* from the left around the back to the right, then along the front from right to left under 4, over *B*, under *D*, over 2.

Bring 4 from the right around the back to the left, then along the front from left to right under 1, over 3, under *A*, over *C*.

Bring 1 from the left around the back to the right, then along the front from right to left under *B*, over *D*, under 2, over 4.

Bring *B* from the right around the back to the left, then along the front from left to right under 3, over *A*, under *C*, over 1.

Bring 3 from the left around the back to the right, then along the front from right to left under *D*, over 2, under 4, over *B*.

Bring *D* from the right around the back to the left, then along the front from left to right under *A*, over *C*, under 1, over 3.

And so on.

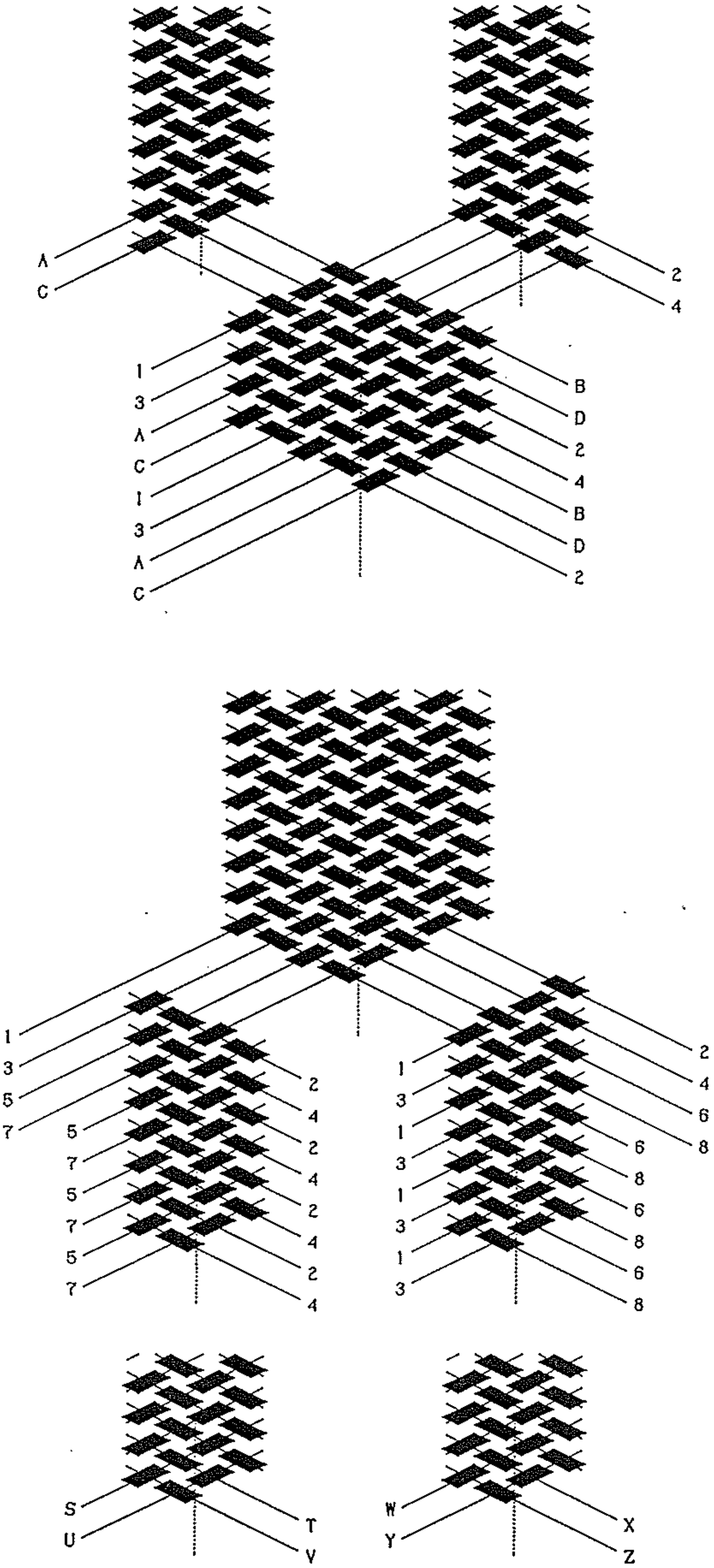


Fig. 474 — Transitions between two 4-string round braids and one 8-string round braid.

The 8-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8.

Bring 1 from the left around the back to the right, then along the front from right to left under 2, over 4, under 6, over 8.

Bring 2 from the right around the back to the left, then along the front from left to right under 3, over 5, under 7.

Bring 3 from the left around the back to the right, then along the front from right to left under 4, over 6, under 8.

Bring 4 from the right around the back to the left, then along the front from left to right under 5, over 7.

Braid the left-hand round braid of 4-strings:

Bring 5 from the left around the back to the right, then along the front from right to left under 2, over 4.

Bring 2 from the right around the back to the left, then along the front from left to right under 7, over 5.

Bring 7 from the left around the back to the right, then along the front from right to left under 4, over 2.

Bring 4 from the right around the back to the left, then along the front from left to right under 5, over 7.

And so on.

Braid the right-hand round braid of 4-strings:

Bring 1 from the left around the back to the right, then along the front from right to left under 6, over 8.

Bring 6 from the right around the back to the left, then along the front from left to right under 3, over 1.

Bring 3 from the left around the back to the right, then along the front from right to left under 8, over 6.

Bring 8 from the right around the back to the left, then along the front from left to right under 1, over 3.

And so on.

Fig. 475:

The transition from the two 4-string round braids to the one 8-string round braid is in accordance with Fig. 473, and the transition from the one 8-string round braid to the two 4-string round braids is in accordance with Fig. 474.

Fig. 476:

The transition from the two 4-string round braids to the one 8-string round braid is in accordance with Fig. 474, and the transition from the one 8-string round braid to the two 4-string round braids is in accordance with Fig. 473.

BRAIDING TYPE							
8 STRING				4 STRING			
→		←		→		←	
U	O	O	U	U	O	O	U
2	2	2	2	1	1	1	1

The transition combinations in Figs. 477 & 478 have a 3-over set of crossings. Especially in colour-work they are less preferable than the transition combinations in Figs. 479 & 480 which have a 3-under set of crossings.

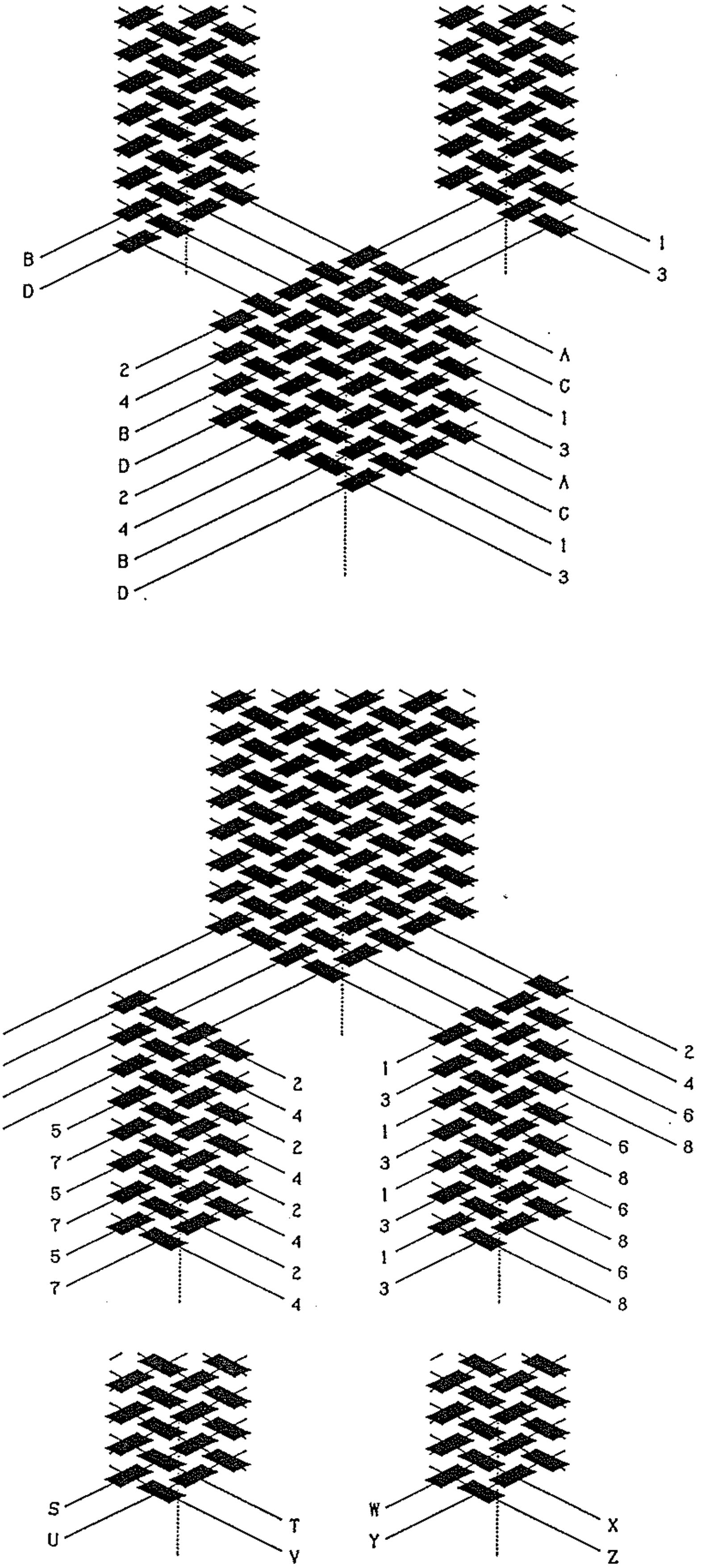


Fig. 475 — Transitions between two 4-string round braids and one 8-string round braid.

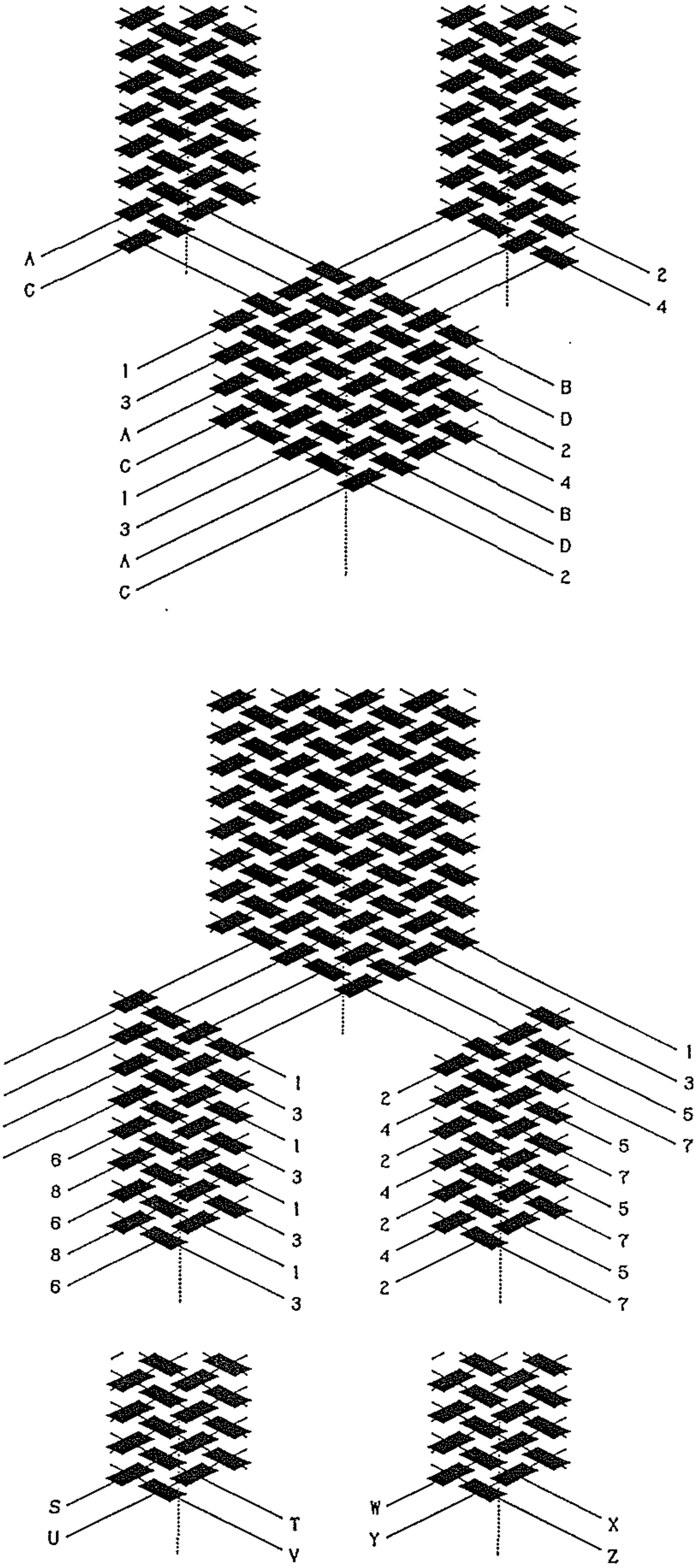


Fig. 476 — Transitions between two 4-string round braids and one 8-string round braid.

Fig. 477:

The 4-string round braid has at one end the strings A, B, C, D and at the other end the strings 1, 2, 3, 4.

Form the crossings between the strings B, D, 1, 3.

Bring 2 from the right around the back to the left, then along the front from left to right under A, over C, over 1, under 3.

Bring 4 from the right around the back to the left, then along the front from left to right under A, under C, over 1, over 3.

Bring A from the left around the back to the right, then along the front from right to left under B, under D, over 2, over 4.

Bring B from the right around the back to the left, then along the front from left to right under C, under 1, over 3, over A.

Bring C from the left around the back to the right, then along the front from right to left under D, under 2, over 4, over B.

Bring D from the right around the back to the left, then along the front from left to right under 1, under 3, over A, over C.

Bring 1 from the left around the back to the right, then along the front from right to left under 2, under 4, over B, over D.

Bring 2 from the right around the back to the left, then along the front from left to right under 3, under A, over C, over 1.

And so on.

The 8-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8.

Bring 1 from the right around the back to the left, then along the front from left to right under 2, under 4, over 6, under 8.

Bring 2 from the left around the back to the right, then along the front from right to left under 3, under 5, over 7.

Bring 3 from the right around the back to the left, then along the front from left to right under 4, under 6, over 8.

Bring 4 from the left around the back to the right, then along the front from right to left over 5, under 7.

Braid the left-hand round braid of 4-strings:

Bring 6 from the left around the back to the right, then along the front from right to left under 1, over 3.

Bring 1 from the right around the back to the left, then along the front from left to right under 8, over 6.

Bring 8 from the left around the back to the right, then along the front from right to left under 3, over 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 6, over 8.

And so on.

Braid the right-hand round braid of 4-strings:

Bring 2 from the left around the back to the right, then along the front from right to left under 5, over 7.

Bring 5 from the right around the back to the left, then along the front from left to right under 4, over 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 7, over 5.

Bring 7 from the right around the back to the left, then along the front from left to right under 2, over 4.

And so on.

Fig. 478:

The 4-string round braid has at one end the strings A, B, C, D and at the other end the strings 1, 2, 3, 4.

Form the crossings between the strings A, C, 2, 4.

Bring B from the left around the back to the right, then along the front from right to left under 1, over 3, over A, under C.

Bring D from the left around the back to the right, then along the front from right to left under 1, under 3, over A, over C.

Bring 1 from the right around the back to the left, then along the front from left to right under 2, under 4, over B, over D.

Bring 2 from the left around the back to the right, then along the front from right to left under 3, under A, over C, over 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 4, under B, over D, over 2.

Bring 4 from the left around the back to the right, then along the front from right to left under A, under C, over 1, over 3.

Bring A from the right around the back to the left, then along the front from left to right under B, under D, over 2, over 4.

Bring B from the left around the back to the right, then along the front from right to left under C, under 1, over 3, over A.

And so on.

The 8-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8.

Bring 1 from the left around the back to the right, then along the front from right to left under 2, under 4, over 6, under 8.

Bring 2 from the right around the back to the left, then along the front from left to right under 3, under 5, over 7.

Bring 3 from the left around the back to the right, then along the front from right to left under 4, under 6, over 8.

Bring 4 from the right around the back to the left, then along the front from left to right over 5, under 7.

Braid the left-hand round braid of 4-strings:

Bring 2 from the right around the back to the left, then along the front from left to right under 5, over 7.

Bring 5 from the left around the back to the right, then along the front from right to left under 4, over 2.

Bring 4 from the right around the back to the left, then along the front from left to right under 7, over 5.

Bring 7 from the left around the back to the right, then along the front from right to left under 2, over 4.

And so on.

Braid the right-hand round braid of 4-strings:

Bring 6 from the right around the back to the left, then along the front from left to right under 1, over 3.

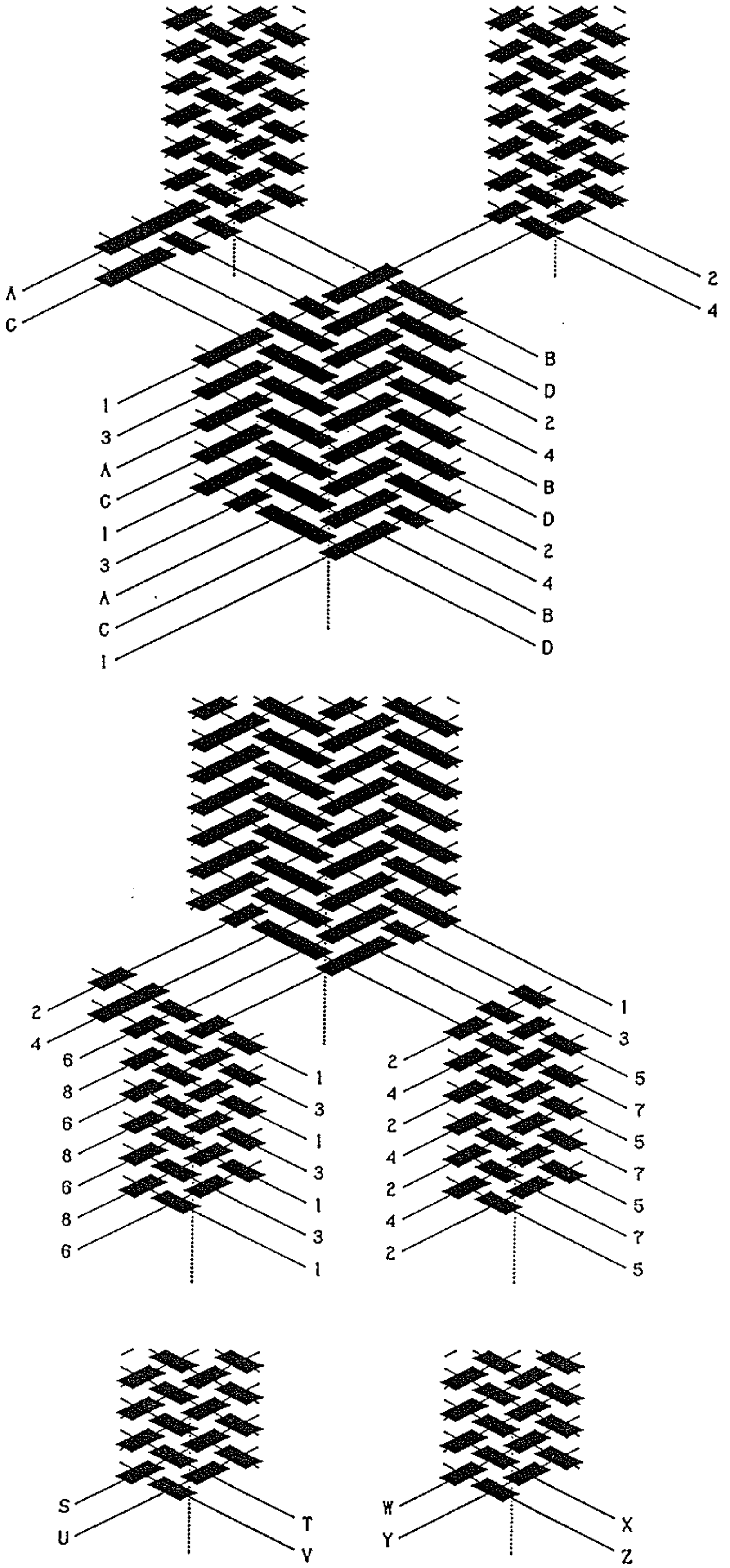


Fig. 477 — Transitions between two 4-string round braids and one 8-string round braid.

Bring 1 from the left around the back to the right, then along the front from right to left under 8, over 6.

Bring 8 from the right around the back to the left, then along the front from left to right under 3, over 1.

Bring 3 from the left around the back to the right, then along the front from right to left under 6, over 8.

And so on.

Fig. 479:

The 4-string round braid has at one end the strings A, B, C, D and at the other end the strings 1, 2, 3, 4.

Form the crossings between the strings A, C, 2, 4.

Bring B from the left around the back to the right, then along the front from right to left under 1, over 3, over A, under C.

Bring 1 from the right around the back to the left, then along the front from left to right under D, under 2, over 4, over B.

Bring D from the left around the back to the right, then along the front from right to left under 3, under A, over C, over 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 2, under 4, over B, over D.

Bring 2 from the left around the back to the right, then along the front from right to left under A, under C, over 1, over 3.

Bring A from the right around the back to the left, then along the front from left to right under 4, under B, over D, over 2.

Bring 4 from the left around the back to the right, then along the front from right to left under C, under 1, over 3, over A.

Bring C from the right around the back to the left, then along the front from left to right under B, under D, over 2, over 4.

And so on.

The 8-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8.

Bring 1 from the left around the back to the right, then along the front from right to left under 2, under 4, over 6, over 8.

Bring 2 from the right around the back to the left, then along the front from left to right under 3, under 5, over 7, over 1.

Bring 3 from the left around the back to the right, then along the front from right to left under 4, under 6, under 8, over 2.

Bring 4 from the right around the back to the left, then along the front from left to right under 5, over 7.

Bring 6 from the right around the back to the left, then along the front from left to right over 5, under 7.

Braid the left-hand round braid of 4-strings:

Bring 4 from the right around the back to the left, then along the front from left to right under 5, over 7.

Bring 5 from the left around the back to the right, then along the front from right to left under 6, over 4.

Bring 6 from the right around the back to the left, then along the front from left to right under 7, over 5.

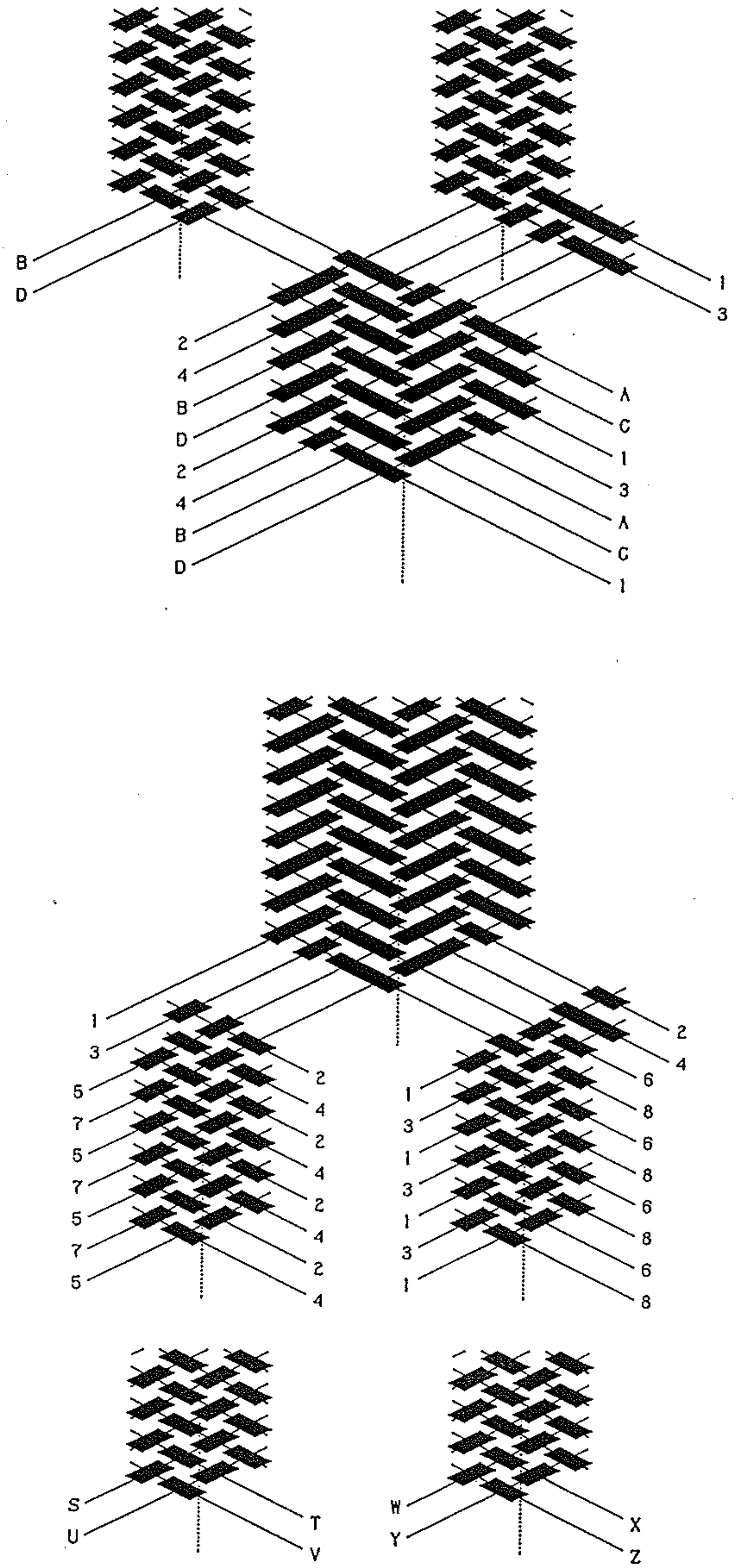


Fig. 478 — Transitions between two 4-string round braids and one 8-string round braid.

Bring 7 from the left around the back to the right, then along the front from right to left under 4, over 6.

And so on.

Braid the right-hand round braid of 4-strings:

Bring 8 from the right around the back to the left, then along the front from left to right under 1, over 3.

Bring 1 from the left around the back to the right, then along the front from right to left under 2, over 8.

Bring 2 from the right around the back to the left, then along the front from left to right under 3, over 1.

Bring 3 from the left around the back to the right, then along the front from right to left under 8, over 2.

And so on.

Fig. 480:

The 4-string round braid has at one end the strings *A*, *B*, *C*, *D* and at the other end the strings 1, 2, 3, 4.

Form the crossings between the strings *B*, *D*, 1, 3.

Bring 2 from the right around the back to the left, then along the front from left to right under *A*, under *C*, over 1, over 3.

Bring *A* from the left around the back to the right, then along the front from right to left under 4, under *B*, over *D*, over 2.

Bring 4 from the right around the back to the left, then along the front from left to right under *C*, under 1, over 3, over *A*.

Bring *C* from the left around the back to the right, then along the front from right to left under *B*, under *D*, over 2, over 4.

Bring *B* from the right around the back to the left, then along the front from left to right under 1, under 3, over *A*, over *C*.

Bring 1 from the left around the back to the right, then along the front from right to left under *D*, under 2, over 4, over *B*.

Bring *D* from the right around the back to the left, then along the front from left to right under 3, under *A*, over *C*, over 1.

Bring 3 from the left around the back to the right, then along the front from right to left under 2, under 4, over *B*, over *D*.

And so on.

The 8-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8.

Bring 1 from the right around the back to the left, then along the front from left to right under 2, under 4, over 6, over 8.

Bring 2 from the left around the back to the right, then along the front from right to left under 3, under 5, over 7, over 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 4, under 6, under 8, over 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 5, over 7.

Bring 6 from the left around the back to the right, then along the front from right to left over 5, under 7.

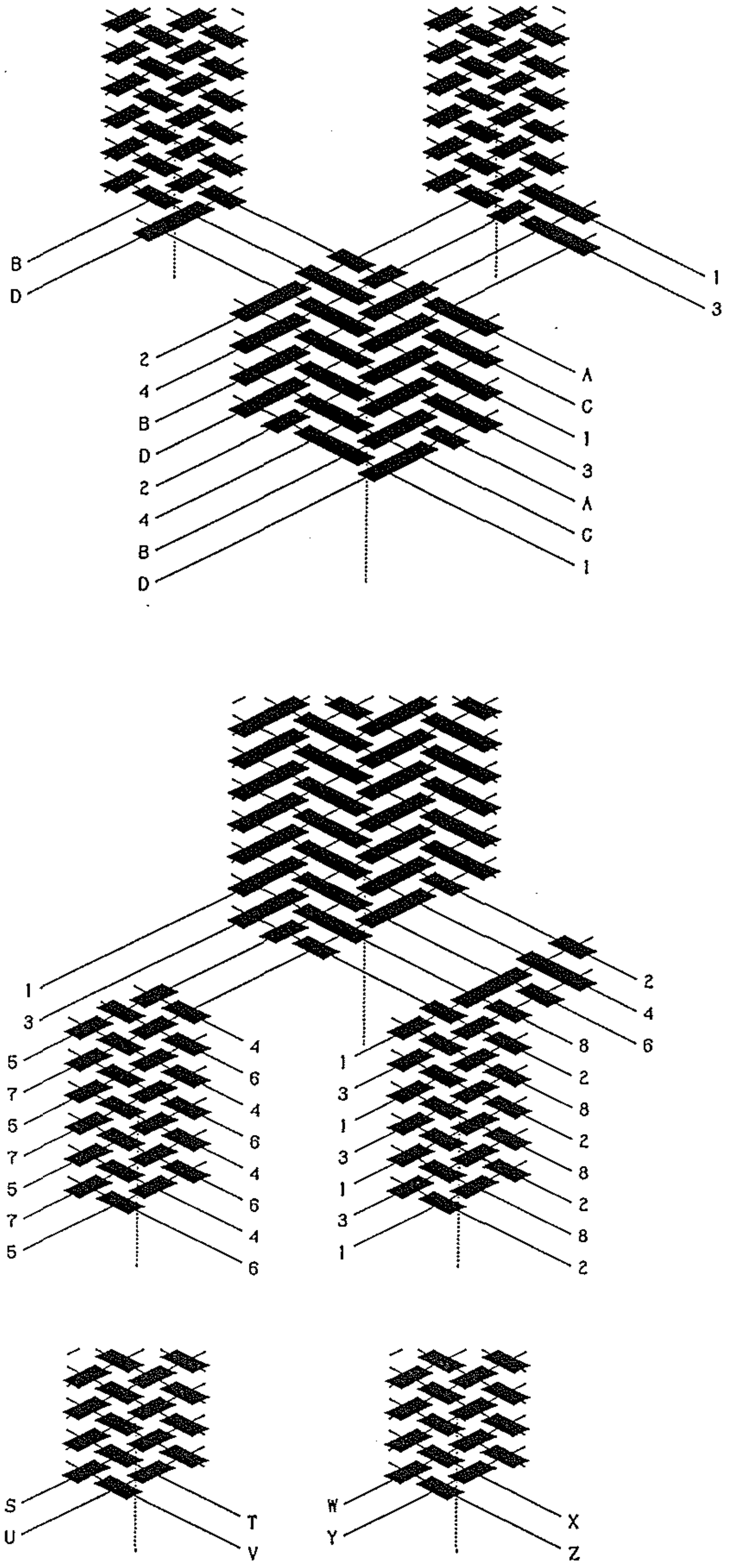


Fig. 479 — Transitions between two 4-string round braids and one 8-string round braid.

Braid the left-hand round braid of 4-strings:

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3.

Bring 1 from the right around the back to the left, then along the front from left to right under 2, over 8.

Bring 2 from the left around the back to the right, then along the front from right to left under 3, over 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 8, over 2.

And so on.

Braid the right-hand round braid of 4-strings:

Bring 4 from the left around the back to the right, then along the front from right to left under 5, over 7.

Bring 5 from the right around the back to the left, then along the front from left to right under 6, over 4.

Bring 6 from the left around the back to the right, then along the front from right to left under 7, over 5.

Bring 7 from the right around the back to the left, then along the front from left to right under 4, over 6.

And so on.

Reviews

the Century Guide to KNOTS ISBN 0 7126 0089 2 255 pages 197x136mm., hard cover, sewn and glued spine.

Authors: Mario Bigon and Guido Regazzoni.

Publisher: Century Publishing Co. Ltd., Portland House, 12/13 Greek Street, London W1V 5LE, England.

The book contains a brief description of the various rope materials and gives their recommended usage. It gives clear illustrations of how to coil a rope and how it should be hung up. The book gives excellent illustations and descriptions of four stopper knots (Overhand knot, Multiple overhand knot, Figure-eight knot, Heaving line knot), nine hitches (Half hitch, Clove hitch, Bill hitch, Cow hitch, Cat's paw, Fisherman's bend, Rolling hitch, Highwayman's hitch, Constrictor knot), eight loops (Bowline, Portuguese bowline, Spanish bowline, Jury mast knot, True-lover's knot, Three-part crown, Angler's loop, Artillery loop), four nooses (Noose, Running bowline, Hangman's knot, Tarbuck knot), three loop knots (Loop knot, Sheepshank, Knotted sheepshank), two tackles (Simple and complex tackle, Poldo tackle), nine bends (Sheet bend, Thief knot, Reef knot, Hunter's bend, Surgeon's knot, Japanese bend, Carrick bend, Water knot, Grapevine knot), twenty knots for fishermen (Seven knots for eye hooks, Five knots for flatted hooks, Barrel knot, Dropper knot, Double overhand bend, Stopper knot, Dropper loop, Loop on the bight, Two swivel hitches), twenty three decorative knots (Four-strand crown sennit, Six-strand crown sennit, Woven sennit, Turk's head, Monkey's fist, Two multi-strand lanyard knots, Two-strand diamond knot, Chinese button knot, Chain sennit, Ocean plat, Round mat, Square mat, Flat sennit, Double braid, Four types of four-strand sennits, Five-strand sennit, Six-strand sennit, Crown sennit,

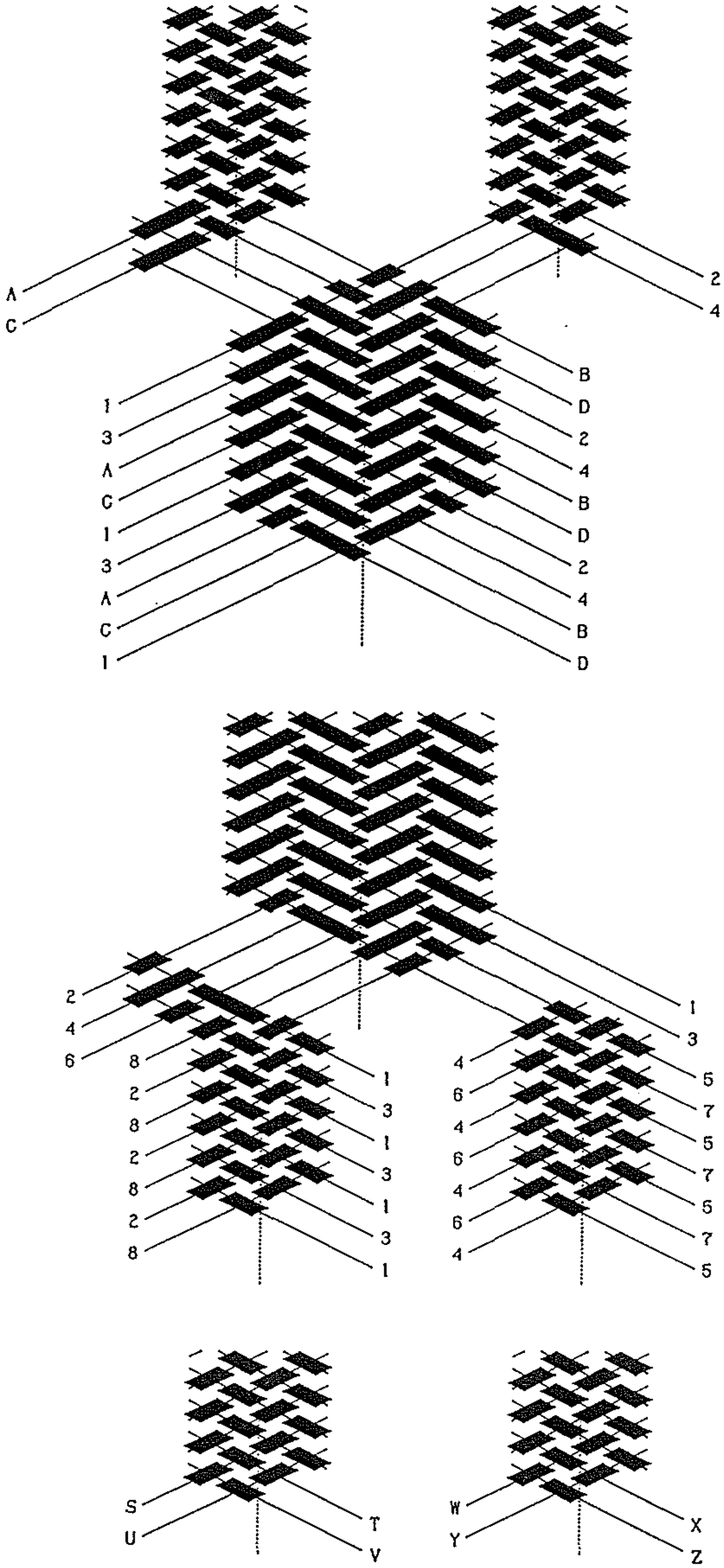


Fig. 480 — Transitions between two 4-string round braids and one 8-string round braid.

Figure-eight chain), five knot applications (Ladder, Nets, Scaffolds, Mountaineering guide knot, Safety harness).

All illustrations consist of very clear colour photographs. A good glossary is provided, as well as a listing for the uses of the knots discussed. It is one of the very best books on the knots dealt with.

A fresh approach to Knotting and Ropework ISBN 0 9592036 3 *X*
272 pages 212x135mm., soft cover, sewn and glued spine.
Author: Charles Warner.
Publisher: Charles Warner, Glenellen, Hume Highway, Yanderra, NSW 2574, Australia.

This is a rather peculiar book on knots and ropework. We find on the title-page the statement "*Knots arranged according to their structure*", and in the preface the statement "*Now, there is a logic in the construction of knots, and I have tried to follow that logic in the writing of this book*". The author has, in the whole, not only failed to arrange according to their structure several of the discussed knots, but has in general also failed to follow 'a logic' in their construction. The many illustrations are of a shoddy nature (obviously the author has little or no artistic ability, and hence should have engaged someone who has), the layout of the book is furthermore very cramped and hence most unattractive. One might also wonder what the chapter (No. 23) on "*Surveying or Practical Geometry*" has to do with knotting and ropework. The book contains virtually no information that cannot be found in very much better books (technically as well as in their layout) on knotting and ropework; nothing can be seen at a glance.

Knots ISBN 1-85348-900-*X* 80 pages 201x151mm., hard cover, sewn spine.
Author: Peter Owen.
Publisher: New Burlington Books, 6 Blundell Street, London N7 9BH, England.

This is another delightful booklet on knots. It briefly discusses rope construction and gives for the various rope materials their recommended use. All illustrations are exceptionably well done, and for each knot its use can be seen at a glance. A very useful feature is that alternative knot names are given immediately below the heading of each knot. There are described six stopper knots (Overhand knot, Overhand loop, Multiple overhand knot, Heaving line knot, Figure-of-eight knot, Figure-of-eight chain), fourteen hitches (Highwayman's hitch, Half hitch, Transom knot, Constrictor knot, Cow hitch, Timber hitch, Clove hitch, Fishermen's bend, Cat's paw, Bill hitch, Rolling hitch, Round turn and two half hitches, Prussic knot, Italian hitch), five loops (Figure-of-eight loop, Threaded figure-of-eight, bowline, Spanish bowline, Angler's loop), ten bends (Reef knot, Capsized reef knot, Thief knot, Surgeon's knot, Fisherman's knot, Double fisherman's knot, Hunter's bend, Sheet bend, Figure-of-eight bend, Carrick bend), three running knots (Running bowline, Hangman's knot, Noose), two shortenings (Sheepshank, Loop knot), nine fishing knots (Blood knot, Blood loop dropper knot, Half-tucked blood knot, Turle knot, Water knot, Grinner knot, Double grinner knot, Double loop knot, Needle knot).

A good glossary is provided. It is one of the very best booklets on the knots dealt with.

THE BRAIDER

No.26 **MAY 2001.**

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A quarterly publication
for
the braiding artisan

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Hamilton,
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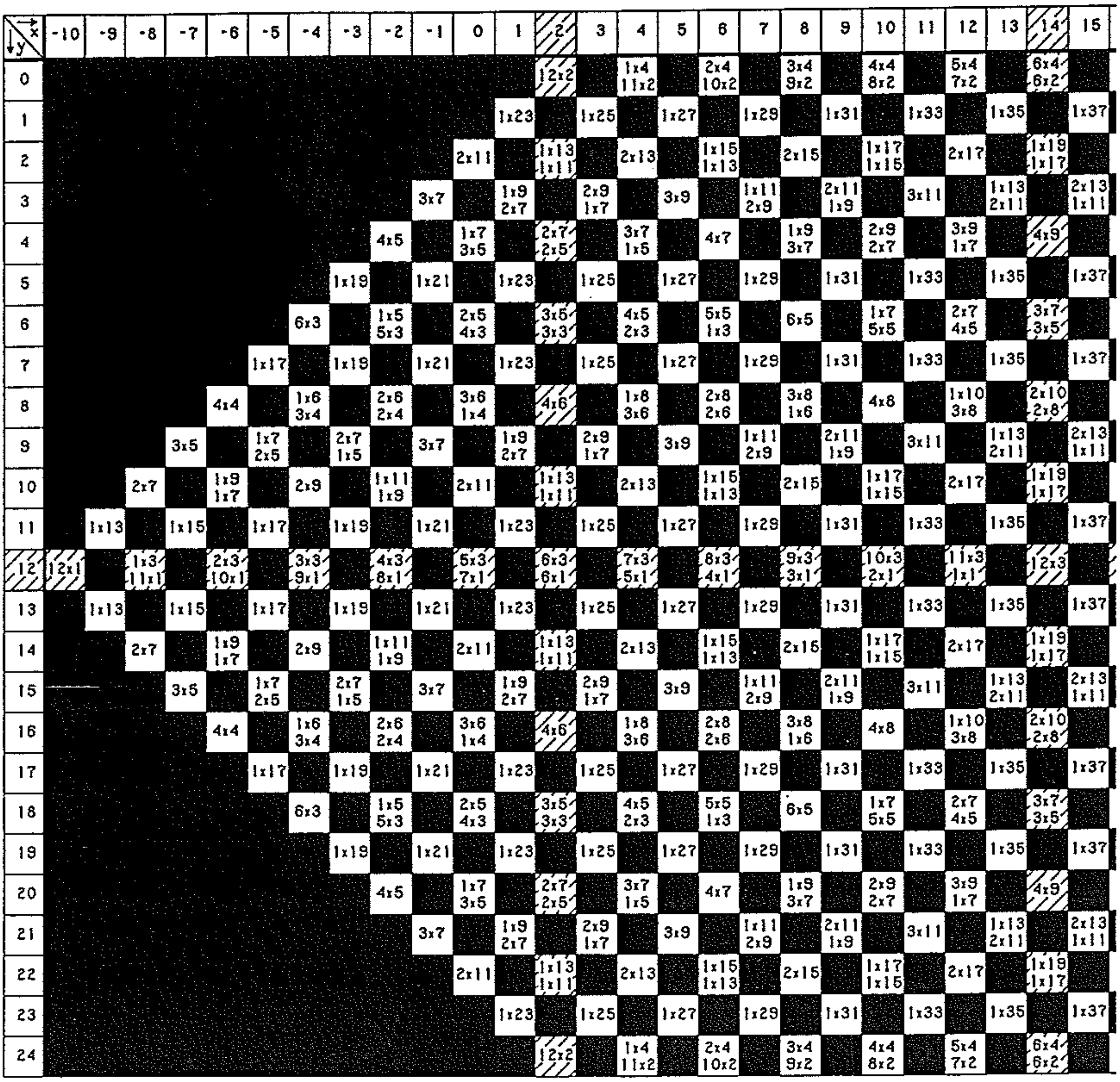
Solution to the Question in Issue No. 25

Question on pg. 573.

The completed cell-entries in the (A, x, y)-table for $A = 12$ are shown in Fig. 481.



↓y x→	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0													12x2		1x4 11x2		2x4 10x2		3x4 9x2		4x4 8x2		5x4 7x2		6x4 6x2	
1												1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
2											2x11		1x13 1x11		2x13		1x15 1x13		2x15		1x17 1x15		2x17		1x19 1x17	
3										3x7		1x9 2x7		2x9 1x7		3x9		1x11 2x9		2x11 1x9		3x11		1x13 2x11		2x13 1x11
4									4x5		1x7 3x5		2x7 2x5		3x7 1x5		4x7		1x9 3x7		2x9 2x7		3x9 1x7		4x9	
5								1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
6							6x3		1x5 5x3		2x5 4x3		3x5 3x3		4x5 2x3		5x5 1x3		6x5		1x7 5x5		2x7 4x5		3x7 3x5	
7						1x17		1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
8					4x4		1x6 3x4		2x6 2x4		3x6 1x4		4x6		1x8 3x6		2x8 2x6		3x8 1x6		4x8		1x10 3x8		2x10 2x8	
9				3x5		1x7 2x5		2x7 1x5		3x7		1x9 2x7		2x9 1x7		3x9		1x11 2x9		2x11 1x9		3x11		1x13 2x11		2x13 1x11
10			2x7		1x9 1x7		2x9		1x11 1x9		2x11		1x13 1x11		2x13		1x15 1x13		2x15		1x17 1x15		2x17		1x19 1x17	
11		1x13		1x15		1x17		1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
12	12x1		1x3 11x1		2x3 10x1		3x3 9x1		4x3 8x1		5x3 7x1		6x3 6x1		7x3 5x1		8x3 4x1		9x3 3x1		10x3 2x1		11x3 1x1		12x3	
13		1x13		1x15		1x17		1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
14			2x7		1x9 1x7		2x9		1x11 1x9		2x11		1x13 1x11		2x13		1x15 1x13		2x16		1x17 1x15		2x17		1x19 1x17	
15				3x5		1x7 2x5		2x7 1x5		3x7		1x9 2x7		2x9 1x7		3x9		1x11 2x9		2x11 1x9		3x11		1x13 2x11		2x13 1x11
16					4x4		1x6 3x4		2x6 2x4		3x6 1x4		4x6		1x8 3x6		2x8 2x6		3x8 1x6		4x8		1x10 3x8		2x10 2x8	
17						1x17		1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
18							6x3		1x5 5x3		2x5 4x3		3x5 3x3		4x5 2x3		5x5 1x3		6x5		1x7 5x5		2x7 4x5		3x7 3x5	
19								1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
20									4x5		1x7 3x5		2x7 2x5		3x7 1x5		4x7		1x9 3x7		2x9 2x7		3x9 1x7		4x9	
21										3x7		1x9 2x7		2x9 1x7		3x9		1x11 2x9		2x11 1x9		3x11		1x13 2x11		2x13 1x11
22											2x11		1x13 1x11		2x13		1x15 1x13		2x15		1x17 1x15		2x17		1x19 1x17	
23												1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
24													12x2		1x4 11x2		2x4 10x2		3x4 9x2		4x4 8x2		5x4 7x2		6x4 6x2	

Fig. 481 — The (A, x, y)-table for $A = 12$.

As mentioned on pg. 570 of *The Braider*, Issue No. 25, we only have to determine the cell-entries for $0 \leq y \leq A$ since they provide the cell-entries for $A < y \leq 2A$ by means of the mirror-image complementary relationship. Apart from the determination of the cell-entries for $y = A$, it is, however, not necessary to calculate the cell-entries for each y-value in the range $0 < y < A$.

Say we have determined the cell-entries for $y = y_2$, where $0 < y_2 < A$. Then for the same x, the cell-entry for $0 < y_1 = y_2 - 2n\{\text{g.c.d.}(y_2, A)\}$, where n is a natural number, is identical to the cell-entry for y_2 if and only if $\text{g.c.d.}(y_1, A) = \text{g.c.d.}(y_2, A)$. Thus in the case $A = 12$ we first determine the cell-entries for $y = A = 12$. Then we determine the cell-entries for $y_2 = 11$, and from these cell-entries we obtain directly the cell-entries for $y_1 = 7, 5, 1$ (hence $n = 2, 3, 5$ respectively). Next we determine the cell-entries for $y_2 = 10$, and from these cell-entries we obtain directly the cell-entries for $y_1 = 2$ (hence $n = 2$). Next we determine the cell-entries for $y_2 = 9$, and from these cell-entries we obtain directly the cell-entries for $y_1 = 3$ (hence $n = 1$). Next we determine the cell-entries for $y_2 = 8$; from these cell-entries we cannot obtain directly the cell-entries for a y-value which is less than 8. Next we determine the cell-entries for $y_2 = 6$; from these cell-entries we cannot obtain directly the cell-entries for a y-value which is less than 6. Next we determine the cell-entries for $y_2 = 4$; from these cell-entries we cannot obtain directly the cell-entries for a y-value which is less than 4. Next we determine the cell-entries for $y = 0$. Then we use the mirror-image complementary relationship for the cell-entries $A < y \leq 2A$, hence for $12 < y \leq 24$.

Nested Cylindrical Braids

In order to obtain an overview of the actual positions of the bights on the bight-boundaries, we attach to the bight-boundary number of a bight the **nest-index number** of the nest to which the bight belongs.

Let the Standing End half-cycle, running from lower-left to upper-right, start at the left-hand bight-boundary l_1 and end at the right-hand bight-boundary r_1. The nest, to which its left-hand bight-point belongs, receives the nest-index number $I_L = I_{L_1} = 0$, and the nest, to which its right-hand bight-point belongs, receives the nest-index number $I_R = I_{R_1} = 0$. The left-hand bight-point of the Standing End half-cycle is indicated by $I_L/l_1 = I_{L_1}/l_1 = 0/l_1$, and the right-hand bight-point of the Standing End half-cycle is indicated by $r_1/I_R = r_1/I_{R_1} = r_1/0$.

The next half-cycle (the second half-cycle), running from lower-right to upper-left, starts at the right-hand bight-point $r_1/0$ and ends at the left-hand bight-boundary l_2. The nest, to which its bight-point on the left-hand bight-boundary l_2 belongs, receives the nest-index number $I_L = I_{L_2} = |I_{L_1} + 4A + x - (l_1 + l_2 + 2r_1)|_B$. The left-hand bight-point of the second half-cycle is indicated by $I_L/l_2 = I_{L_2}/l_2$.

The next half-cycle (the third half-cycle), running from lower-left to upper-right, starts at the left-hand bight-point I_{L_2}/l_2 and ends at the right-hand bight-boundary r_2. The nest, to which its bight-point on the right-hand bight-boundary r_2 belongs, receives the nest-index number $I_R = I_{R_2} = |I_{R_1} + 4A + x - (r_1 + r_2 + 2l_2)|_B$. The right-hand bight-point of the third half-cycle is indicated by r_2/I_{R_2}.

The next half-cycle (the fourth half-cycle), running from lower-right to upper-left, starts at the right-hand bight-point r_2/I_{R_2} and ends at the left-hand bight-boundary l_3. The nest, to which its bight-point on the left-hand bight-boundary l_3 belongs, receives the nest-index number $I_L = I_{L_3} = |I_{L_2} + 4A + x - (l_2 + l_3 + 2r_2)|_B$. The left-hand bight-point of the fourth half-cycle is indicated by $I_L/l_3 = I_{L_3}/l_3$.

The next half-cycle (the fifth half-cycle), running from lower-left to upper-right, starts at the left-hand bight-point I_{L_3}/l_3 and ends at the right-hand bight-boundary r_3. The nest, to which its bight-point on the right-hand bight-boundary r_3 belongs, receives the nest-index number $I_R = I_{R_3} = |I_{R_2} + 4A + x - (r_2 + r_3 + 2l_3)|_B$. The right-hand bight-point of the fifth half-cycle is indicated by r_3/I_{R_3}.

And so on.

In general:

The $(2n)^{th}$ half-cycle, where $n = 1, 2, 3, \cdots$, running from lower-right to upper-left, starts at the right-hand bight-point r_n/I_{R_n} and ends at the left-hand bight-boundary l_{n+1}. The nest, to which its bight-point on the left-hand bight-boundary l_{n+1} belongs, receives the nest-index number $I_L = I_{L_{n+1}} = |I_{L_n} + 4A + x - (l_n + l_{n+1} + 2r_n)|_B$. The left-hand bight-point of the $(2n)^{th}$ half-cycle, where $n = 1, 2, 3, \cdots$, is indicated by $I_L/l_{n+1} = I_{L_{n+1}}/l_{n+1}$.

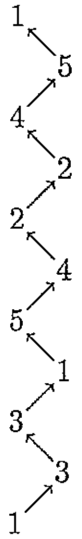
The $(2n+1)^{th}$ half-cycle), running from lower-left to upper-right, starts at the left-hand bight-point $I_{L_{n+1}}/l_{n+1}$ and ends at the right-hand bight-boundary r_{n+1}. The nest, to which its bight-point on the right-hand bight-boundary r_{n+1} belongs, receives the nest-index number $I_R = I_{R_{n+1}} = |I_{R_n} + 4A + x - (r_n + r_{n+1} + 2l_{n+1})|_B$. The right-hand bight-point of the $(2n+1)^{th}$ half-cycle is indicated by $r_{n+1}/I_{R_{n+1}}$.

Example 1 : (2222/10/2222){15432/34512}20

Hence:

$$\begin{aligned} A &= 5. \\ l_1 &= 1. \\ k = r_1 &= 3. \\ y &= |x - 2(k+1)|_{2A} = |10 - 2(3+1)|_{10} = 2. \\ \Delta &= |x - 2(k+1)|_A = |y|_A = |2|_5 = 2. \end{aligned}$$

The first-return string-run is:



$$\begin{array}{lll} l_1 = 1 \longrightarrow 3 = r_1 & \to & l_2 = |l_1 + \Delta|_5 = |1+2|_5 = 3. \\ l_2 = 3 \longleftarrow 3 = r_1 & \to & r_2 = |r_1 - \Delta|_5 = |3-2|_5 = 1. \\ l_2 = 3 \longrightarrow 1 = r_2 & \to & l_3 = |l_2 + \Delta|_5 = |3+2|_5 = 5. \\ l_3 = 5 \longleftarrow 1 = r_2 & \to & r_3 = |r_2 - \Delta|_5 = |1-2|_5 = 4. \\ l_3 = 5 \longrightarrow 4 = r_3 & \to & l_4 = |l_3 + \Delta|_5 = |5+2|_5 = 2. \\ l_4 = 2 \longleftarrow 4 = r_3 & \to & r_4 = |r_3 - \Delta|_5 = |4-2|_5 = 2. \\ l_4 = 2 \longrightarrow 2 = r_4 & \to & l_5 = |l_4 + \Delta|_5 = |2+2|_5 = 4. \\ l_5 = 4 \longleftarrow 2 = r_4 & \to & r_5 = |r_4 - \Delta|_5 = |2-2|_5 = 5. \\ l_5 = 4 \longrightarrow 5 = r_5 & \to & l_6 = |l_5 + \Delta|_5 = |4+2|_5 = 1 = l_1. \end{array}$$

$$P_c = 4\cdot\alpha + \frac{\alpha\cdot x - 2\sum(l_i + r_i)}{A} = 4\times 5 + \frac{5\times10-2\{(1+3+5+2+4)+(3+1+4+2+5)\}}{5} = 18.$$

$\text{g.c.d.}\,(P_c\,,\,B^*) = \text{g.c.d.}\,(18\,,\,4) = 2.$

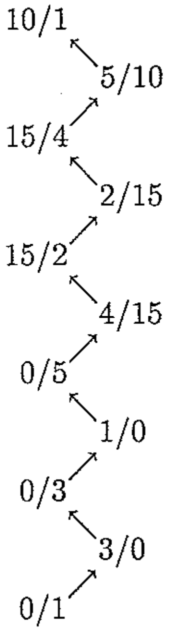
Hence:

$$P_{total} = \sum P_{component} = 18.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = \sum \text{g.c.d.}\,(P_c\,,\,B^*) = 2.$$

Let's now attach nest-index numbers to the bight-boundary numbers of the first-return string-run. The end-points of the first half-cycle receive both the nest-index number $I = 0$.



$$\begin{aligned}
I_L = I_1 &= 0 .\\
I_R = I_{1'} &= 0 .\\
I_L = I_2 &= |I_1 + 4A + x - (l_1 + l_2 + 2r_1)|_B =\\
&\quad |0 + 4 \times 5 + 10 - (1 + 3 + 2 \times 3)|_{20} = 0 .\\
I_R = I_{2'} &= |I_{1'} + 4A + x - (r_1 + r_2 + 2l_2)|_B =\\
&\quad |0 + 4 \times 5 + 10 - (3 + 1 + 2 \times 3)|_{20} = 0 .\\
I_L = I_3 &= |I_2 + 4A + x - (l_2 + l_3 + 2r_2)|_B =\\
&\quad |0 + 4 \times 5 + 10 - (3 + 5 + 2 \times 1)|_{20} = 0 .\\
I_R = I_{3'} &= |I_{2'} + 4A + x - (r_2 + r_3 + 2l_3)|_B =\\
&\quad |0 + 4 \times 5 + 10 - (1 + 4 + 2 \times 5)|_{20} = 15 .\\
I_L = I_4 &= |I_3 + 4A + x - (l_3 + l_4 + 2r_3)|_B =\\
&\quad |0 + 4 \times 5 + 10 - (5 + 2 + 2 \times 4)|_{20} = 15 .\\
I_R = I_{4'} &= |I_{3'} + 4A + x - (r_3 + r_4 + 2l_4)|_B =\\
&\quad |15 + 4 \times 5 + 10 - (4 + 2 + 2 \times 2)|_{20} = 15 .\\
I_L = I_5 &= |I_4 + 4A + x - (l_4 + l_5 + 2r_4)|_B =\\
&\quad |15 + 4 \times 5 + 10 - (2 + 4 + 2 \times 2)|_{20} = 15 .\\
I_R = I_{5'} &= |I_{4'} + 4A + x - (r_4 + r_5 + 2l_5)|_B =\\
&\quad |15 + 4 \times 5 + 10 - (2 + 5 + 2 \times 4)|_{20} = 10 .\\
I_L = I_6 &= |I_5 + 4A + x - (l_5 + l_6 + 2r_5)|_B =\\
&\quad |15 + 4 \times 5 + 10 - (4 + 1 + 2 \times 5)|_{20} = 10 .
\end{aligned}$$

For each of the two sub-components, this first-return string-run with its attached nest-index numbers provides us with the necessary data for writing down the consecutive half-cycles with their associated bight-boundary numbers and nest-index numbers at the ends.

Since there are four nests on the left bight-edge, and since the bight-edge has a total length of twenty bights, it follows that the nest-index numbers are 0, 5, 10, 15. When the Standing End of the first sub-component has the left-hand nest-index number 0, then the consecutive cycles of its string-run will be associated with the left-hand nest-index numbers: $\underline{0}$; 0; 0; 15; 15; $\underline{10}$; $10 = |10 + 0|_B$; $10 = |10 + 0|_B$; $5 = |10 + 15|_B$; $5 = |10 + 15|_B$; $\underline{0} = |10 + 10|_B$.

A first-return string-run runs between two consecutive underlined nest-index numbers. Hence the string-run of the first sub-component consists of two first-return string-runs. The first one starts at bight-boundary 1 and nest-index number 0, the second one starts at bight-boundary 1 and nest-index number 10.

Hence the second sub-component can either start at bight-boundary 1 and nest-index number 5, or at bight-boundary 1 and nest-index number 15. We shall take the first option. Note that we don't have to start the second, or for that matter the first sub-component, with the half-cycle $1 \longrightarrow 3$, but could start with any of the lower-left to upper-right half-cycles of the first-return string-run.

With the chosen start for the second sub-component at bight-boundary 1 and nest-index number 5, the consecutive cycles of its string-run will be associated with the left-hand nest-index numbers: $\underline{5}$; $5 = |5 + 0|_B$; $5 = |5 + 0|_B$; $0 = |5 + 15|_B$; $0 = |5 + 15|_B$; $\underline{15} = |5 + 10|_B$; $15 = |15 + 0|_B$; $15 = |15 + 0|_B$; $10 = |15 + 15|_B$; $10 = |15 + 15|_B$; $\underline{5} = |15 + 10|_B$.

Thus the string-runs of the two sub-components are:

$$\begin{tikzcd}
\underline{0/1} & \\
 & 5/5 \arrow[ul] \\
5/4 \arrow[ur] & \\
 & 2/5 \arrow[ul] \\
5/2 \arrow[ur] & \\
 & 4/5 \arrow[ul] \\
10/5 \arrow[ur] & \\
 & 1/10 \arrow[ul] \\
10/3 \arrow[ur] & \\
 & 3/10 \arrow[ul] \\
\underline{10/1} \arrow[ur] & \\
 & 5/10 \arrow[ul] \\
15/4 \arrow[ur] & \\
 & 2/15 \arrow[ul] \\
15/2 \arrow[ur] & \\
 & 4/15 \arrow[ul] \\
0/5 \arrow[ur] & \\
 & 1/0 \arrow[ul] \\
0/3 \arrow[ur] & \\
 & 3/0 \arrow[ul] \\
\underline{0/1} \arrow[ur] &
\end{tikzcd}$$

$$\begin{tikzcd}
\underline{5/1} & \\
 & 5/10 \arrow[ul] \\
10/4 \arrow[ur] & \\
 & 2/10 \arrow[ul] \\
10/2 \arrow[ur] & \\
 & 4/10 \arrow[ul] \\
15/5 \arrow[ur] & \\
 & 1/15 \arrow[ul] \\
15/3 \arrow[ur] & \\
 & 3/15 \arrow[ul] \\
\underline{15/1} \arrow[ur] & \\
 & 5/0 \arrow[ul] \\
0/4 \arrow[ur] & \\
 & 2/0 \arrow[ul] \\
0/2 \arrow[ur] & \\
 & 4/0 \arrow[ul] \\
5/5 \arrow[ur] & \\
 & 1/5 \arrow[ul] \\
5/3 \arrow[ur] & \\
 & 3/5 \arrow[ul] \\
\underline{5/1} \arrow[ur] &
\end{tikzcd}$$

We can now readily draw the relative bight-positions of the two sub-components (see Fig. 482), which gives us some general overview of the braid.

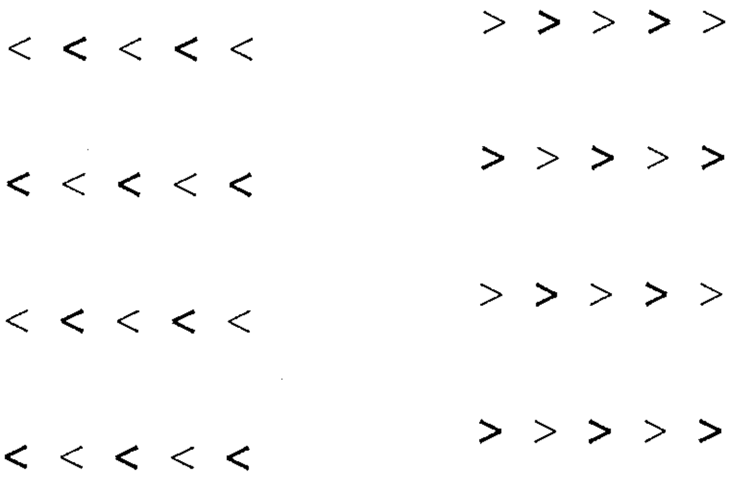


Fig. 482 — The relative bight-positions of (2222/10/2222){15432/34512}20.

The complete string-run diagram of (2222/10/2222){15432/34512}20 is shown in Fig. 483.

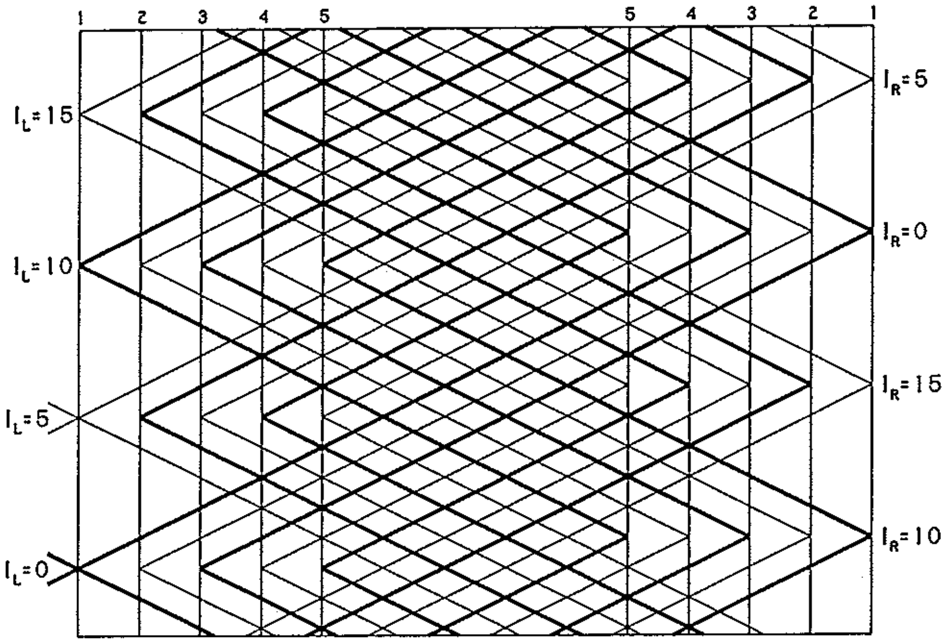


Fig. 483 — The string-run of (2222/10/2222){15432/34512}20.

The above example was a simple one in that it had only one component, hence there was only one first-return string-run and hence there was only one first half-cycle type, with the consequence that the first half-cycle of any sub-component had the same length. If we have, however, more than one component, then there are more than one first-return string-run and hence more than one first half-cycle type, with the consequence that the sub-components of different components may have their first half-cycles of a different length.

In general, let the first component have a first-return string-run in which the first half-cycle, running from lower-left to upper-right, starts at the left-hand bight-boundary $l_1 = 1$ and ends at the right-hand bight-boundary $r_1 = k$, where $1 \leq k \leq A$. Let furthermore the left-hand nest-index number and the right-hand nest-index number of the first half-cycle of the first sub-component of this first component be respectively equal to $I_{L_1} = 0$ and $I_{R_1} = 0$.

Let the second component have a first-return string-run in which the first half-cycle, running from lower-left to upper-right, starts at the left-hand bight-boundary l'_1 and ends at the right-hand bight-boundary r'_1. Let furthermore the left-hand nest-index number of the first half-cycle of the first sub-component of this second component be equal to I'_{L_1}. Then the right-hand nest-index number of the first half-cycle of the first sub-component of this second component is equal to $I'_{R_1} = \left|I'_{L_1} + (l_1 + r_1) - (l'_1 + r'_1)\right|_B = \left|I'_{L_1} + (k+1) - (l'_1 + r'_1)\right|_B$. In *The Braider*, Issue No. 23, on pg. 521 we have seen that for Regular Nested Cylindrical Braids $(l_i + r_i)$ is either $(k+1)$ or $(A+k+1)$ for $1 \leq k < A$, and $(l_i + r_i) = (A+1)$ for $k = A$. Hence when $(l'_1 + r'_1) = (k+1)$, then $I'_{R_1} = I'_{L_1}$, and when $(l'_1 + r'_1) = (A+k+1)$ (hence only possible when $1 \leq k < A$), then $I'_{R_1} = \left|I'_{L_1} - A\right|_B$.

Let the third component have a first-return string-run in which the first half-cycle, running from lower-left to upper-right, starts at the left-hand bight-boundary l''_1 and ends at the right-hand bight-boundary r''_1. Let furthermore the left-hand nest-index number of the first half-cycle of the first sub-component of this third component be equal to I''_{L_1}. Then the right-hand nest-index number of the first half-cycle of the first sub-component of this third component is equal to $I''_{R_1} = \left|I''_{L_1} + (l_1 + r_1) - (l''_1 + r''_1)\right|_B = \left|I''_{L_1} + (k+1) - (l''_1 + r''_1)\right|_B$. For Regular Nested Cylindrical Braids $(l_i + r_i)$ is either $(k+1)$ or $(A+k+1)$ for $1 \le k < A$, and $(l_i + r_i) = (A+1)$ for $k = A$. Hence when $(l''_1 + r''_1) = (k+1)$, then $I''_{R_1} = I''_{L_1}$, and when $(l''_1 + r''_1) = (A+k+1)$ (hence only possible when $1 \le k < A$), then $I''_{R_1} = \left|I''_{L_1} - A\right|_B$.

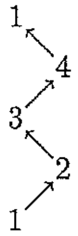
And so on.

Example 2: (222/12/222){1432/2341}16

Hence:

$$\begin{aligned} A &= 4. \\ l_1 &= 1. \\ k = r_1 &= 2. \\ y &= |x - 2(k+1)|_{2A} = |12 - 2(2+1)|_8 = 6. \\ \Delta &= |x - 2(k+1)|_A = |y|_A = |6|_4 = 2. \\ \text{g.c.d.}\,(\Delta, A) &= \text{g.c.d.}\,(2, 4) = 2. \end{aligned}$$

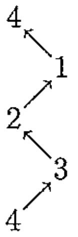
The first-return string-runs are:



$$\begin{array}{lcl} l_1 = 1 \longrightarrow 2 = r_1 & \to & l_2 = |l_1 + \Delta|_4 = |1+2|_4 = 3. \\ l_2 = 3 \longleftarrow 2 = r_1 & \to & r_2 = |r_1 - \Delta|_4 = |2-2|_4 = 4. \\ l_2 = 3 \longrightarrow 4 = r_2 & \to & l_3 = |l_2 + \Delta|_4 = |3+2|_4 = 1. \end{array}$$

$$P_{c_1} = 4\cdot\alpha + \frac{\alpha\cdot x - 2\sum(l_i + r_i)}{A} = 4\times 2 + \tfrac{2\times12-2\{(1+3)+(2+4)\}}{4} = 9.$$

g.c.d. $(P_{c_1}, B^*) = $ g.c.d. $(9, 4) = 1$.



$$\begin{array}{lcl} l'_1 = 4 \longrightarrow 3 = r'_1 & \to & l'_2 = |l'_1 + \Delta|_4 = |4+2|_4 = 2. \\ l'_2 = 2 \longleftarrow 3 = r'_1 & \to & r'_2 = |r'_1 - \Delta|_4 = |3-2|_4 = 1. \\ l'_2 = 2 \longrightarrow 1 = r'_2 & \to & l'_3 = |l'_2 + \Delta|_4 = |2+2|_4 = 4. \end{array}$$

$$P_{c_2} = 4\cdot\alpha + \frac{\alpha\cdot x - 2\sum(l_i + r_i)}{A} = 4\times 2 + \tfrac{2\times12-2\{(4+2)+(3+1)\}}{4} = 9.$$

g.c.d. $(P_{c_2}, B^*) = $ g.c.d. $(9, 4) = 1$.

Hence:

$$P_{total} = \sum P_{component} = P_{c_1} + P_{c_2} = 18 = 2A + x - 2.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 2.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = \sum \text{g.c.d.}\,(P_c, B^*) = 2.$$

By attaching nest-index numbers to the bight-boundary numbers of the first-return string-runs we obtain:

$$\begin{tikzcd}
4/1 & \\
& 4/0 \arrow[ul] \\
4/3 \arrow[ur] & \\
& 2/0 \arrow[ul] \\
0/1 \arrow[ur] &
\end{tikzcd}$$

$$\begin{aligned}
I_L = I_1 &= 0\,. \\
I_R = I_{1'} &= 0\,. \\
I_L = I_2 &= |I_1 + 4A + x - (l_1 + l_2 + 2r_1)|_B = \\
&\quad |0 + 4 \times 4 + 12 - (1 + 3 + 2 \times 2)|_{16} = 4\,. \\
I_R = I_{2'} &= |I_{1'} + 4A + x - (r_1 + r_2 + 2l_2)|_B = \\
&\quad |0 + 4 \times 4 + 12 - (2 + 4 + 2 \times 3)|_{16} = 0\,. \\
I_L = I_3 &= |I_2 + 4A + x - (l_2 + l_3 + 2r_2)|_B = \\
&\quad |4 + 4 \times 4 + 12 - (3 + 1 + 2 \times 4)|_{16} = 4\,.
\end{aligned}$$

$$\begin{tikzcd}
12/4 & \\
& 1/8 \arrow[ul] \\
8/2 \arrow[ur] & \\
& 3/4 \arrow[ul] \\
8/4 \arrow[ur] &
\end{tikzcd}$$

$$\begin{aligned}
I_L = I'_1 &= 8\,. \\
I_R = I'_{1'} &= |8 - A|_B = |8 - 4|_{16} = 4 \text{ since } l'_1 + r'_1 = A + k + 1\,. \\
I_L = I'_2 &= |I'_1 + 4A + x - (l'_1 + l'_2 + 2r'_1)|_B = \\
&\quad |8 + 4 \times 4 + 12 - (4 + 2 + 2 \times 3)|_{16} = 8\,. \\
I_R = I'_{2'} &= |I'_{1'} + 4A + x - (r'_1 + r'_2 + 2l'_2)|_B = \\
&\quad |4 + 4 \times 4 + 12 - (3 + 1 + 2 \times 2)|_{16} = 8\,. \\
I_L = I'_3 &= |I'_2 + 4A + x - (l'_2 + l'_3 + 2r'_2)|_B = \\
&\quad |8 + 4 \times 4 + 12 - (2 + 4 + 2 \times 1)|_{16} = 12\,.
\end{aligned}$$

Thus the string-runs of the two components are:

$$\begin{tikzcd}
\underline{0}/\underline{1} & \\
& 4/12 \arrow[ul] \\
0/3 \arrow[ur] & \\
& 2/12 \arrow[ul] \\
\underline{12}/\underline{1} \arrow[ur] & \\
& 4/8 \arrow[ul] \\
12/3 \arrow[ur] & \\
& 2/8 \arrow[ul] \\
\underline{8}/\underline{1} \arrow[ur] & \\
& 4/4 \arrow[ul] \\
8/3 \arrow[ur] & \\
& 2/4 \arrow[ul] \\
\underline{4}/\underline{1} \arrow[ur] & \\
& 4/0 \arrow[ul] \\
4/3 \arrow[ur] & \\
& 2/0 \arrow[ul] \\
\underline{0}/\underline{1} \arrow[ur] &
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\underline{8}/\underline{4} & \\
& 1/4 \arrow[ul] \\
4/2 \arrow[ur] & \\
& 3/0 \arrow[ul] \\
\underline{4}/\underline{4} \arrow[ur] & \\
& 1/0 \arrow[ul] \\
0/2 \arrow[ur] & \\
& 3/12 \arrow[ul] \\
\underline{0}/\underline{4} \arrow[ur] & \\
& 1/12 \arrow[ul] \\
12/2 \arrow[ur] & \\
& 3/8 \arrow[ul] \\
\underline{12}/\underline{4} \arrow[ur] & \\
& 1/8 \arrow[ul] \\
8/2 \arrow[ur] & \\
& 3/4 \arrow[ul] \\
\underline{8}/\underline{4} \arrow[ur] &
\end{tikzcd}$$

We can now draw the relative bight-positions of the two components (see Fig. 484), which gives us some general overview of the braid.

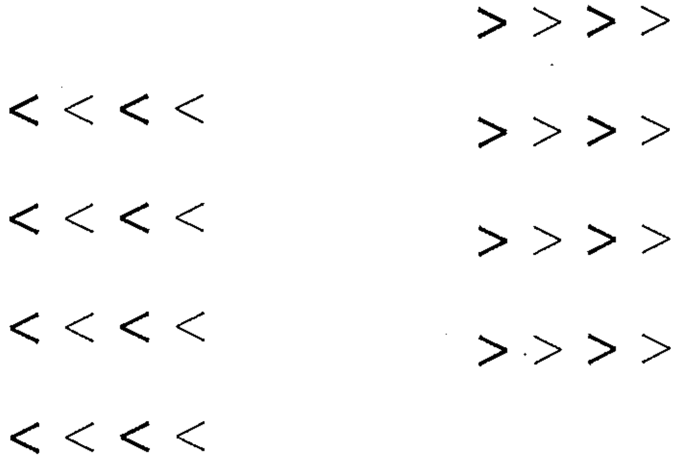


Fig. 484 — The relative bight-positions of (222/12/222){1432/2341}16.

The complete string-run diagram of (222/12/222){1432/2341}16 is depicted in Fig. 485.

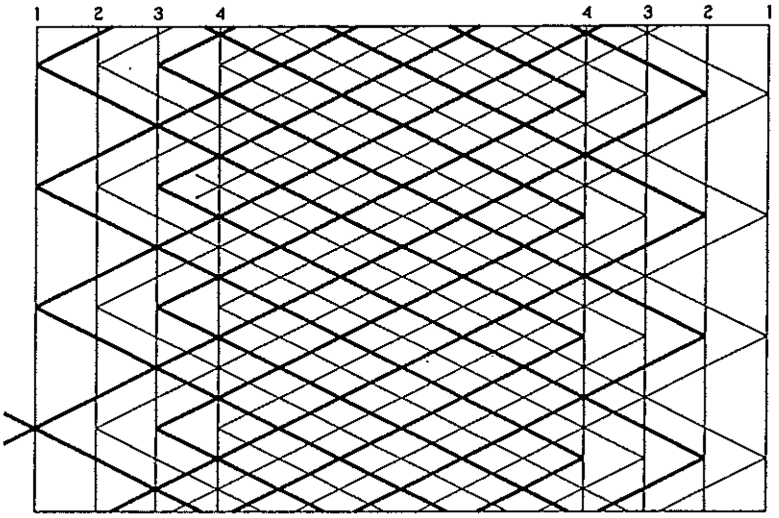


Fig. 485 — The string-run of (222/12/222){1432/2341}16.

THE BRAIDER'S NOTEBOOK

We have mentioned in the past that, if one wants to further his or her braiding skills, it is important to become thoroughly familiar with grid-diagrams. Only then will the braider really be able to design new braid-forms and hence will immediately see new possibilities resulting from a simple new knot which may have been discovered by experimentation.

From Eugene Ulrich, an eldery American knot-tyer, braider and horseman from Faith in South Dakota, we received in 1997 a letter in which he described an apparently new knot which he discovered by experimentation. In his letter he wrote:

The enclosed diagram [see Fig. 486] *is of, I think, a new knot. I can't describe the knot using your system, so will do it in the following way: When disregarding the dotted* *lines, we have a two strand Matthew Walker knot. With the dotted lines it becomes a doubled two strand Matthew Walker knot. Start at the smaller arrow points and follow each dotted line around the second time on top of itself to the end of the bigger arrow point. When working the knot down and into position, it will usually be necessary to push the strands coming from the base of the knot up and on top of the other strands and all the way around. Do this when the strands are still quite loose. This knot is at its best when tied with three or four strands and in color.*

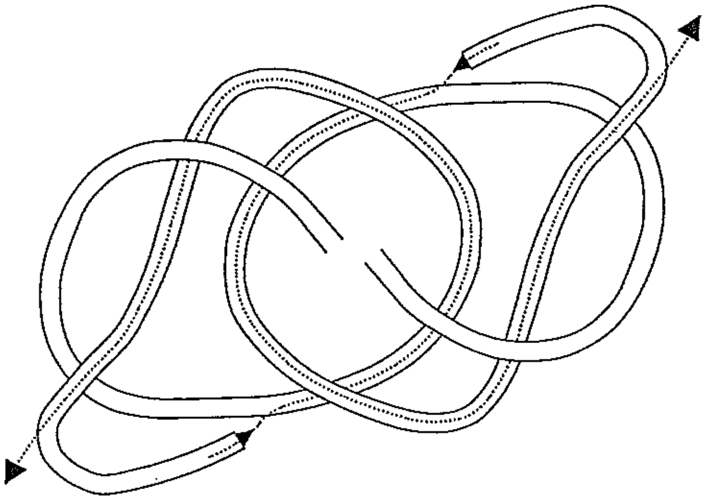


Fig. 486 — The doubled two strand Matthew Walker knot.

The grid-diagram of this doubled two strand Matthew Walker knot, which has a left-hand helix, is shown at the right in Fig. 487. The left-hand grid-diagram in Fig. 487 depicts the type with a right-hand helix. Observe that the nesting-number $A = 2$.

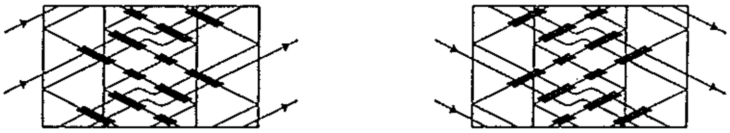


Fig. 487 — The two types of doubled two strand Matthew Walker knot ($A = 2$).

The three-strand and four-strand versions are depicted in Fig. 488.

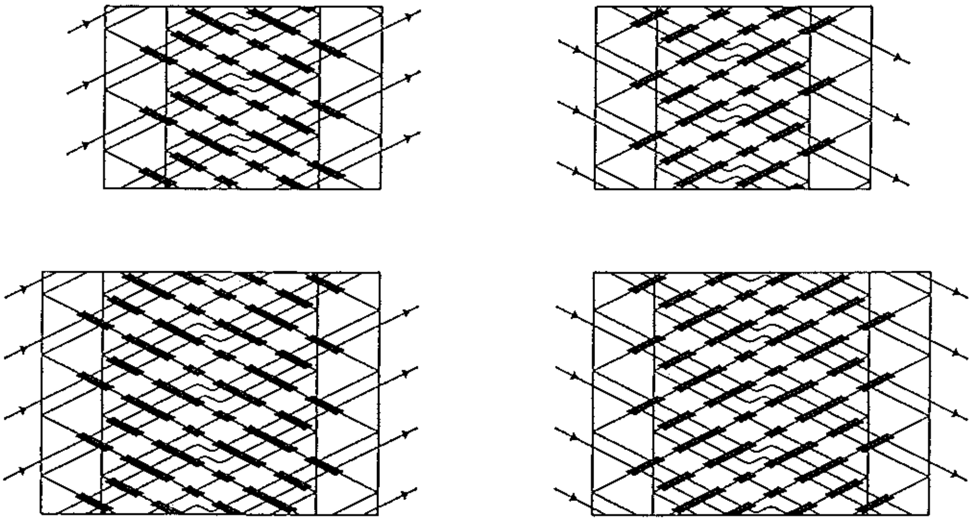


Fig. 488 — The three-strand and four-strand versions for $A = 2$.

Note that these knots with a right-hand helix are based on the string-run of the Regular Nested Cylindrical Braids $(2/2n - 1/2)\{12/12\}2n$, and that those with a left-hand helix are based on the string-run of the Regular Nested Cylindrical Braids $(2/2n - 1/2)\{12/21\}2n$, where n is the number of strands.

It should now have become evident that there must be similar knots with a nesting-number $A = 3$ (and with nesting-numbers greater than 3, although those are in general of little or no use in practical braidwork).

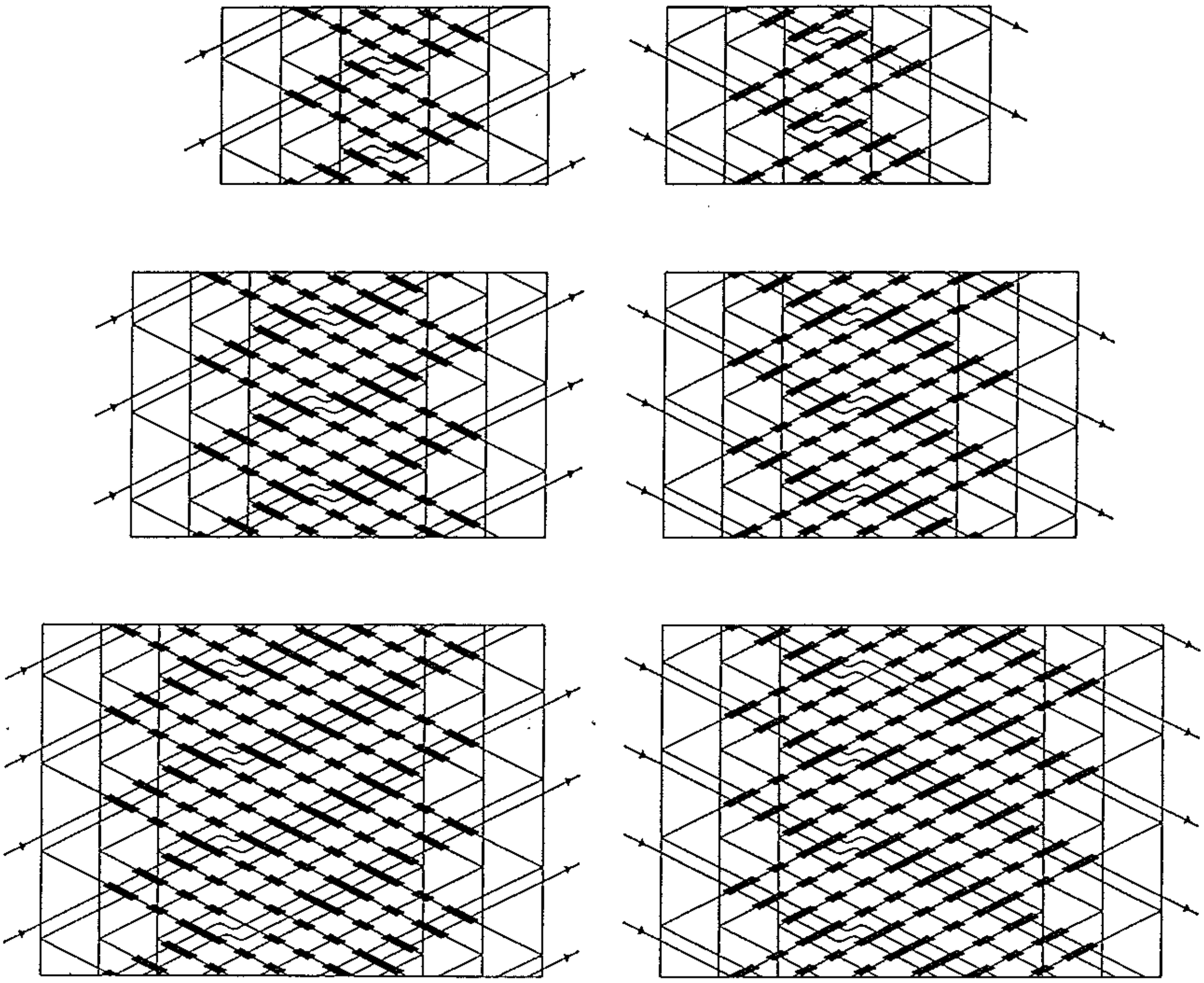


Fig. 489 — The two-, three- and four-strand versions for $A = 3$.

↓n A→	2	3	4	5	6	7	8	9	10	11	12
2	3	3	3	3	3	3	3	3	3	3	3
3	5	6	7	8	9	10	11	12	13	14	15
4	7	9	11	13	15	17	19	21	23	25	27
5	9	12	15	18	21	24	27	30	33	36	39
6	11	15	19	23	27	31	35	39	43	47	51
7	13	18	23	28	33	38	43	48	53	58	63
8	15	21	27	33	39	45	51	57	63	69	75

Fig. 490 — The relationships between A, n and x.

In Fig. 490 are tabulated some (A, n, x)-values. Thus for $A = 2$ we have $x = 2n - 1$; for $A = 3$ we have $x = 3n - 3$; or in general for A we have $x = A(n-2) + 3$.

Hence in general, for a nesting-number A and a number of strands equal to n, this type of knot with a right-hand helix is based on the string-run of the Regular Nested Cylindrical Braid $(222\cdots/A(n-2)+3/222\cdots)\{1A(A-1)(A-2)\cdots 2/1234\cdots A\}An$, and with a left-hand helix is based on the string-run of the Regular Nested Cylindrical Braid $(222\cdots/A(n-2)+3/222\cdots)\{1A(A-1)(A-2)\cdots 2/A123\cdots(A-1)\}An$.

We can of course design further colour-patterns. An example is shown by the grid-diagrams in Fig. 491. This type, with a number of strands equal to n, is based on the string-run of the Regular Nested Cylindrical Braid $(22/3n-2/22)\{132/231\}3n$ for a right-hand helix, and is based on the string-run of the Regular Nested Cylindrical Braid $(22/3n-2/22)\{132/312\}3n$ for a left-hand helix. The two-tone colour-pattern in the uppermost and lowermost grid-diagrams of Fig. 491 is especially suited for $n =$ even.

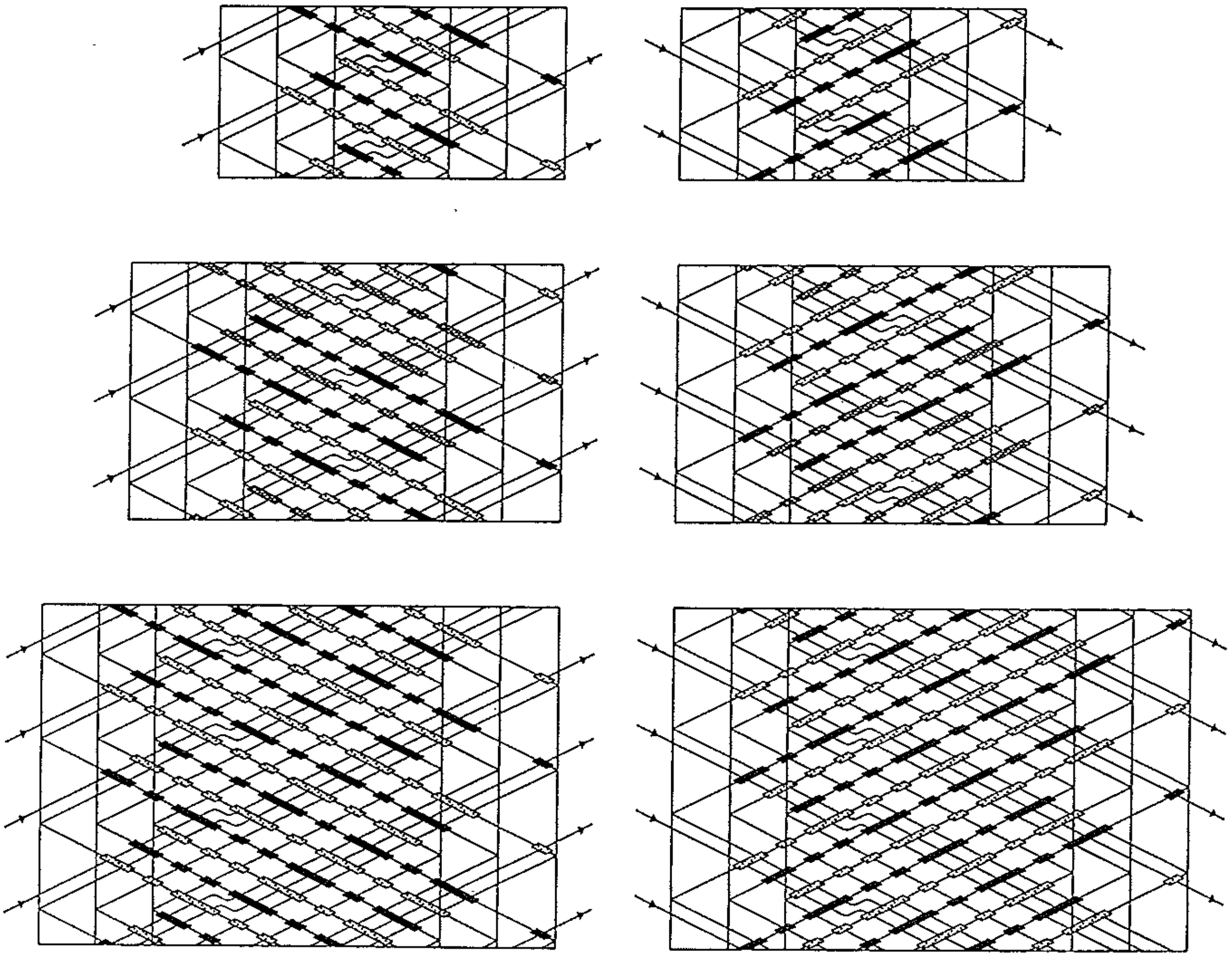


Fig. 491 — A colour-pattern example for $A = 3$.

We hope to have shown here once again the importance of grid-diagrams in facilitating the design of new knots. Such design may be an extension to some experimental discovery as in the case discussed, or it may be a direct design without an experimental bases. Another very important aspect of grid-diagrams is that they readily enable us to generalise these designed braids into families.

Transitions from two Round Braids to one Round Braid, and vice versa

Transitions between two 6-string round braids and one 12-string round braid.

BRAIDING TYPE																	
12 STRING												6 STRING					
→						←						→			←		
U	O	U	O	U	O	O	U	O	U	O	U	U	O	U	U	O	U
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Similar to the transition between two 4-string round braids and one 8-string round braid as discussed in *The Braider*, Issue No. 25, pp. 574–578, we have here four good transition combinations. We have only depicted the type which is similar to the one on pg. 575 (see Fig. 492).

Fig. 492:

The 6-string round braid has at one end the strings *A*, *B*, *C*, *D*, *E*, *F* and at the other end the strings 1, 2, 3, 4, 5, 6.

Form the crossings between the strings *A*, *C*, *E*, 2, 4, 6.

Bring 1 from the right around the back to the left, then along the front from left to right under *B*, over *D*, under *F*, over 2, under 4, over 6.

Bring *B* from the left around the back to the right, then along the front from right to left under 3, over 5, under *A*, over *C*, under *E*, over 1.

Bring 3 from the right around the back to the left, then along the front from left to right under *D*, over *F*, under 2, over 4, under 6, over *B*.

Bring *D* from the left around the back to the right, then along the front from right to left under 5, over *A*, under *C*, over *E*, under 1, over 3.

Bring 5 from the right around the back to the left, then along the front from left to right under *F*, over 2, under 4, over 6, under *B*, over *D*.

Bring *F* from the left around the back to the right, then along the front from right to left under *A*, over *C*, under *E*, over 1, under 3, over 5.

Bring *A* from the right around the back to the left, then along the front from left to right under 2, over 4, under 6, over *B*, under *D*, over *F*.

Bring 2 from the left around the back to the right, then along the front from right to left under *C*, over *E*, under 1, over 3, under 5, over *A*.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6, over 8, under 10, over 12.

Bring 2 from the left around the back to the right, then along the front from right to left under 3, over 5, under 7, over 9, under 11.

Bring 3 from the right around the back to the left, then along the front from left to right under 4, over 6, under 8, over 10, under 12.

Bring 4 from the left around the back to the right, then along the front from right to left under 5, over 7, under 9, over 11.

Bring 5 from the right around the back to the left, then along the front from left to right under 6, over 8, under 10, over 12.

Bring 6 from the left around the back to the right, then along the front from right to left under 7, over 9, under 11.

Braid the left-hand round braid of 6-strings:

Bring 1 from the right around the back to the left, then along the front from left to right under 8, over 10, under 12.

Bring 8 from the left around the back to the right, then along the front from right to left under 3, over 5, under 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 10, over 12, under 8.

Bring 10 from the left around the back to the right, then along the front from right to left under 5, over 1, under 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 12, over 8, under 10.

Bring 12 from the left around the back to the right, then along the front from right to left under 1, over 3, under 5.

Bring 1 from the right around the back to the left, then along the front from left to right under 8, over 10, under 12.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 7 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6.

Bring 2 from the left around the back to the right, then along the front from right to left under 9, over 11, under 7.

Bring 9 from the right around the back to the left, then along the front from left to right under 4, over 6, under 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 11, over 7, under 9.

Bring 11 from the right around the back to the left, then along the front from left to right under 6, over 2, under 4.

Bring 6 from the left around the back to the right, then along the front from right to left under 7, over 9, under 11.

And so on.

BRAIDING TYPE									
12 STRING						6 STRING			
→			←			→		←	
U	O	U	U	O	U	U	O	O	U
2	2	2	2	2	2	1	2	2	1

Fig. 493:

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6.

Form the crossings between the strings B, D, F, 2, 4, 6.

Bring 1 from the right around the back to the left, then along the front from left to right under A and C, over E and 2, under 4 and 6.

Bring A from the left around the back to the right, then along the front from right to left under 3 and 5, over B and D, under F and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under C and E, over 2 and 4, under 6 and A.

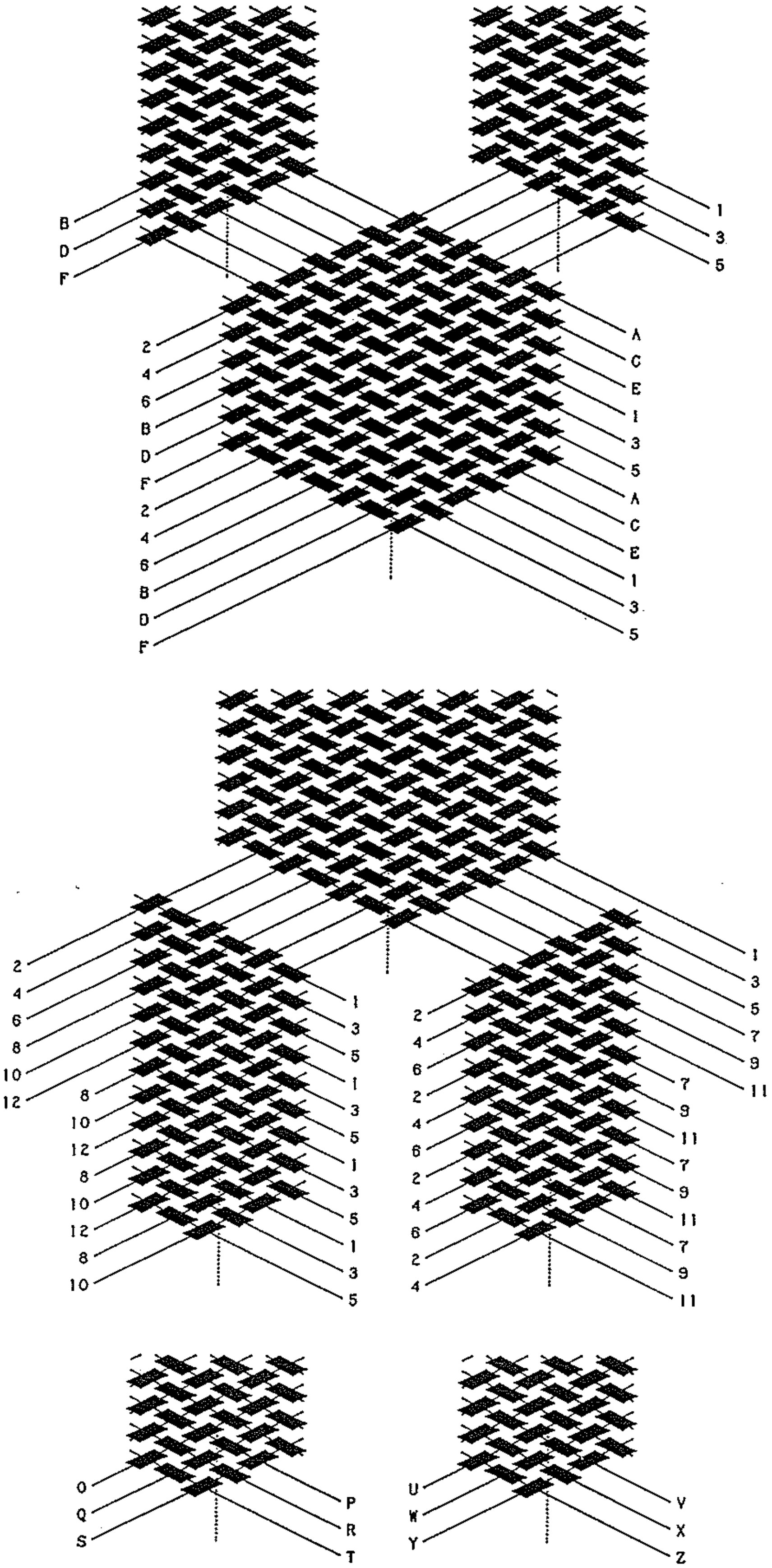


Fig. 492 — Transition between two 6-string round braids and one 12-string round braid.

Bring C from the left around the back to the right, then along the front from right to left under 5 and B, over D and F, under 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under E and 2, over 4 and 6, under A and C.

Bring E from the left around the back to the right, then along the front from right to left under B and D, over F and 1, under 3 and 5.

Bring B from the right around the back to the left, then along the front from left to right under 2 and 4, over 6 and A, under C and E.

Bring 2 from the left around the back to the right, then along the front from right to left under D and F, over 1 and 3, under 5 and B.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6 and 8, under 10 and 12.

Bring 2 from the left around the back to the right, then along the front from right to left under 3 and 5, over 7 and 9, under 11.

Bring 3 from the right around the back to the left, then along the front from left to right under 4 and 6, over 8 and 10, under 12.

Bring 4 from the left around the back to the right, then along the front from right to left under 5 and 7, over 9 and 11.

Bring 5 from the right around the back to the left, then along the front from left to right under 6 and 8, over 10 and 12.

Bring 6 from the left around the back to the right, then along the front from right to left under 7 and 9, over 11.

Braid the left-hand round braid of 6-strings:

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3 and 5.

Bring 1 from the right around the back to the left, then along the front from left to right under 10, over 12 and 8.

Bring 10 from the left around the back to the right, then along the front from right to left under 3, over 5 and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 12, over 8 and 10.

Bring 12 from the left around the back to the right, then along the front from right to left under 5, over 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 8, over 10 and 12.

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3 and 5.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 7 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6.

Bring 2 from the left around the back to the right, then along the front from right to left under 9 and 11, over 7.

Bring 9 from the right around the back to the left, then along the front from left to right under 4, over 6, under 2.

Bring 4 from the left around the back to the right, then along the front from right

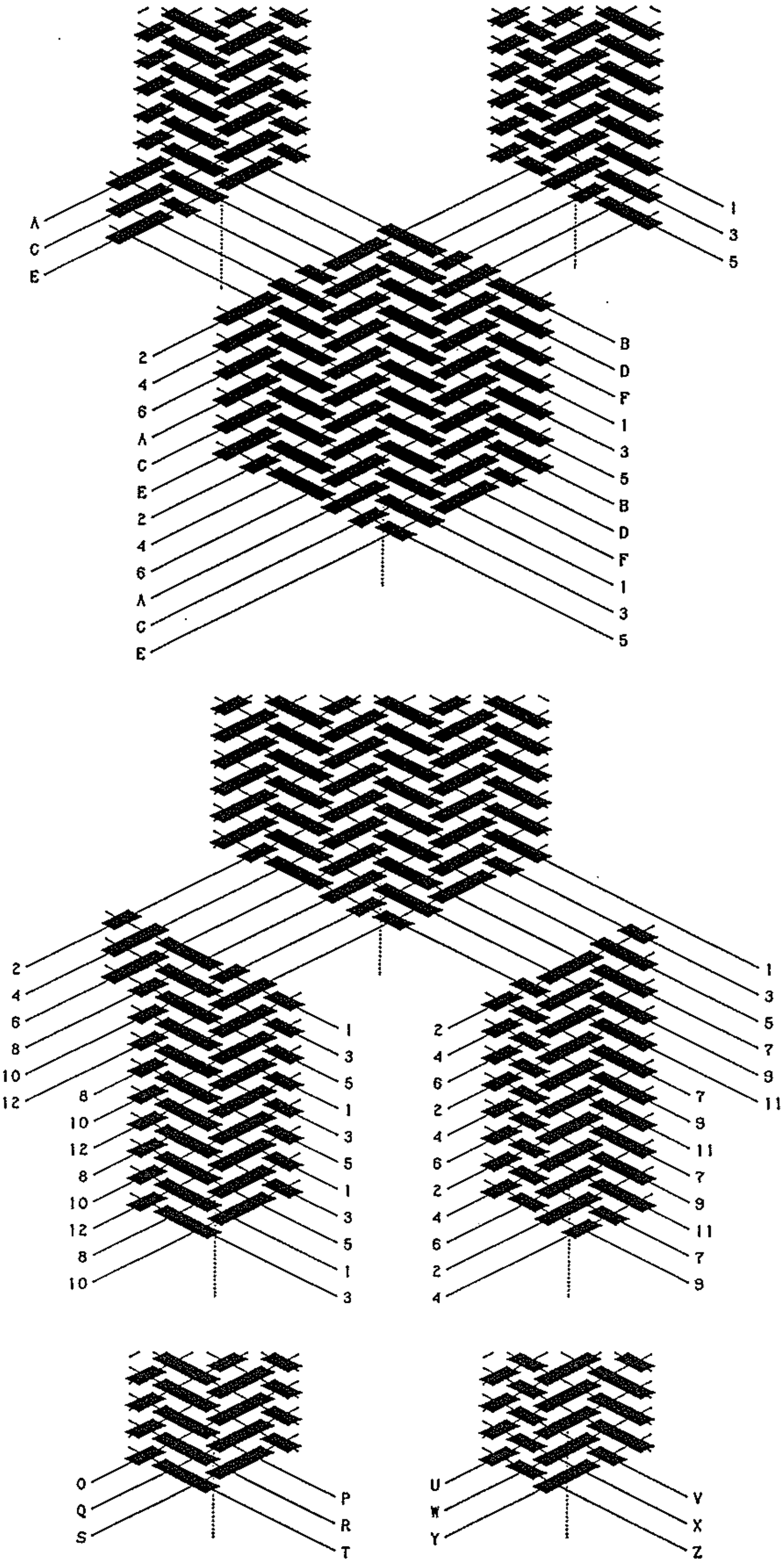


Fig. 493 — Transition between two 6-string round braids and one 12-string round braid.

to left under 11 and 7, over 9.

Bring 11 from the right around the back to the left, then along the front from left to right under 6, over 2, under 4.

Bring 6 from the left around the back to the right, then along the front from right to left under 7 and 9, over 11.

And so on.

BRAIDING TYPE									
12 STRING						6 STRING			
→			←			→		←	
U	O	U	U	O	U	U	O	O	U
2	2	2	2	2	2	2	1	2	1

Fig. 494:

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6. Bring 2 from the left around the back to the right, then along the front from right to left under 1 and 3, over 5 as indicated by arrow.

Form the crossings between the strings B, D, F, 4, 6, 2.

Bring 1 from the right around the back to the left, then along the front from left to right under A and C, over E and 4, under 6 and 2.

Bring A from the left around the back to the right, then along the front from right to left under 3 and 5, over B and D, under F and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under C and E, over 4 and 6, under 2 and A.

Bring C from the left around the back to the right, then along the front from right to left under 5 and B, over D and F, under 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under E and 4, over 6 and 2, under A and C.

Bring E from the left around the back to the right, then along the front from right to left under B and D, over F and 1, under 3 and 5.

Bring B from the right around the back to the left, then along the front from left to right under 4 and 6, over 2 and A, under C and E.

Bring 4 from the left around the back to the right, then along the front from right to left under D and F, over 1 and 3, under 5 and B.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6 and 8, under 10 and 12.

Bring 2 from the left around the back to the right, then along the front from right to left under 3 and 5, over 7 and 9, under 11.

Bring 3 from the right around the back to the left, then along the front from left to right under 4 and 6, over 8 and 10, under 12.

Bring 4 from the left around the back to the right, then along the front from right to left under 5 and 7, over 9 and 11.

Bring 5 from the right around the back to the left, then along the front from left to right under 6 and 8, over 10 and 12.

Bring 6 from the left around the back to the right, then along the front from right to left under 7 and 9, over 11.

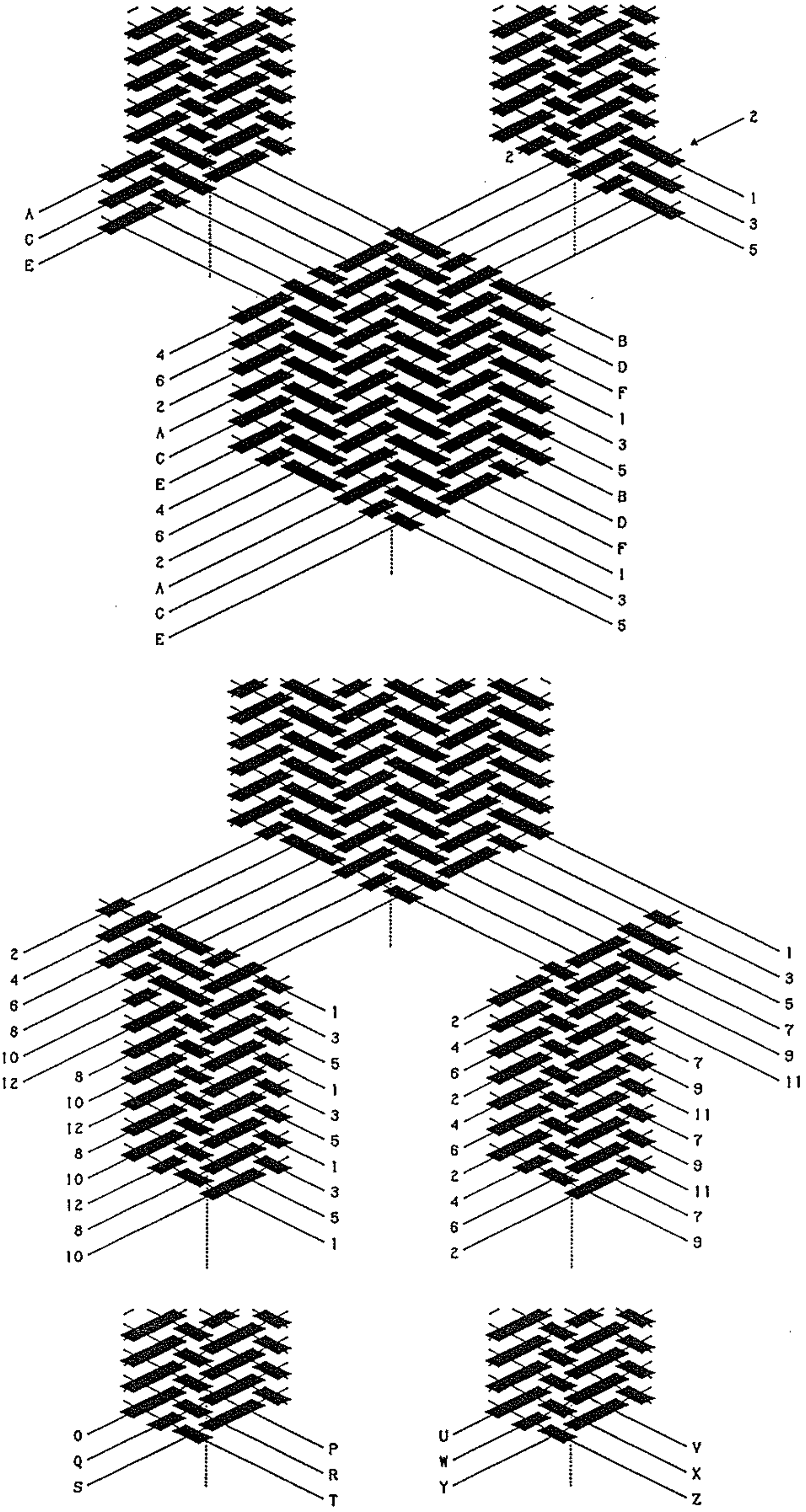


Fig. 494 — Transition between two 6-string round braids and one 12-string round braid.

Braid the left-hand round braid of 6-strings:

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3 and 5.

Bring 1 from the right around the back to the left, then along the front from left to right under 10 and 12, over 8.

Bring 10 from the left around the back to the right, then along the front from right to left under 3, over 5 and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 12 and 8, over 10.

Bring 12 from the left around the back to the right, then along the front from right to left under 5, over 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 8 and 10, over 12.

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3 and 5.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 7 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6.

Bring 9 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6.

Bring 2 from the left around the back to the right, then along the front from right to left under 11, over 7 and 9.

Bring 11 from the right around the back to the left, then along the front from left to right under 4 and 6, over 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 7, over 9 and 11.

Bring 7 from the right around the back to the left, then along the front from left to right under 6 and 2, over 4.

And so on.

Fig. 495:

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6.

Form the crossings between the strings B, D, F, 2, 4, 6.

Bring A from the left around the back to the right, then along the front from right to left under 1 and 3, over 5 and B, under D and F.

Bring 1 from the right around the back to the left, then along the front from left to right under C and E, over 2 and 4, under 6 and A.

Bring C from the left around the back to the right, then along the front from right to left under 3 and 5, over B and D, under F and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under E and 2, over 4 and 6, under A and C.

Bring E from the left around the back to the right, then along the front from right to left under 5 and B, over D and F, under 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6 and A, under C and E.

Bring 2 from the left around the back to the right, then along the front from right

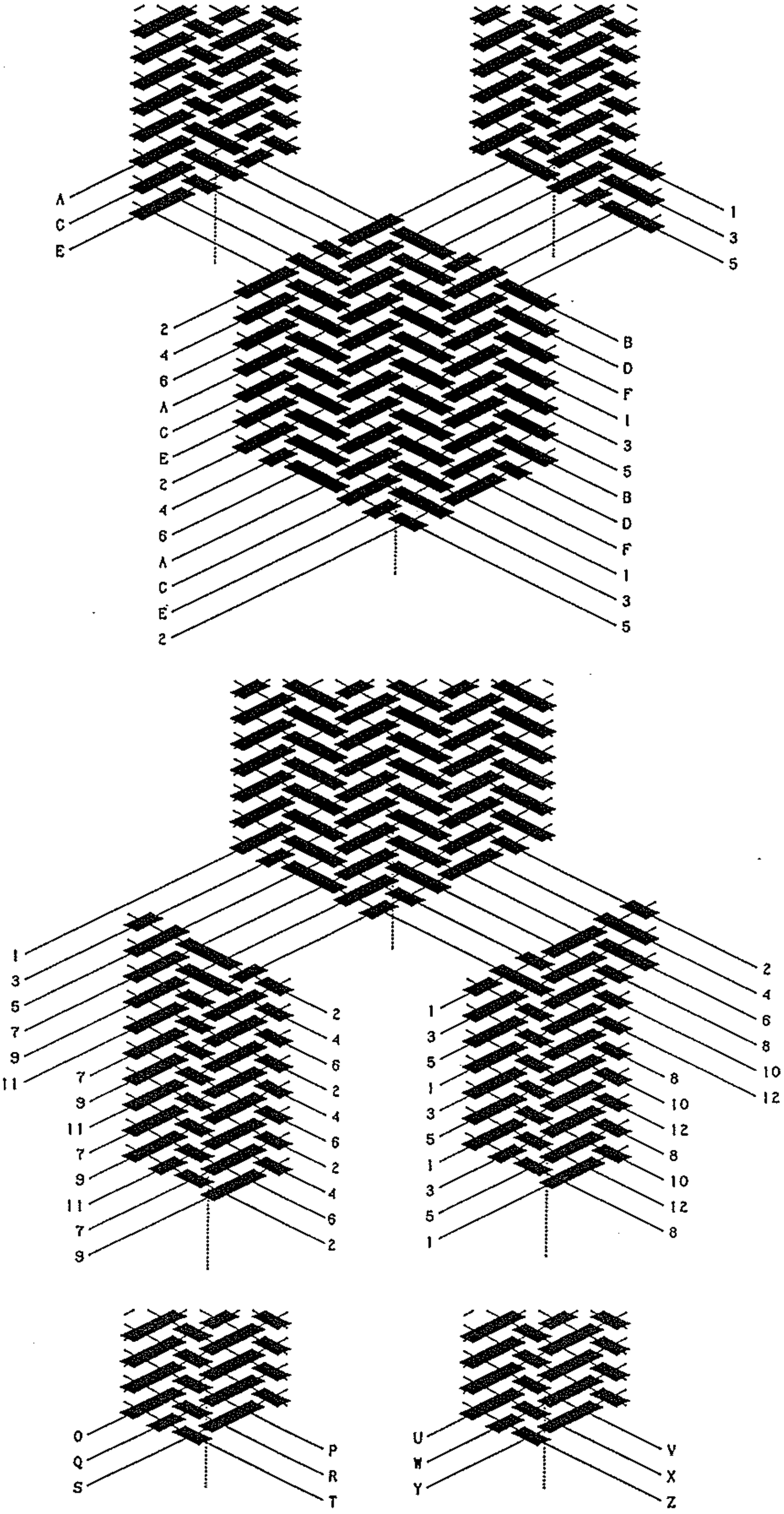


Fig. 495 — Transition between two 6-string round braids and one 12-string round braid.

to left under B and D, over F and 1, under 3 and 5.

Bring B from the right around the back to the left, then along the front from left to right under 4 and 6, over A and C, under E and 2.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the left around the back to the right, then along the front from right to left under 2 and 4, over 6 and 8, under 10 and 12.

Bring 2 from the right around the back to the left, then along the front from left to right under 3 and 5, over 7 and 9, under 11.

Bring 3 from the left around the back to the right, then along the front from right to left under 4 and 6, over 8 and 10, under 12.

Bring 4 from the right around the back to the left, then along the front from left to right under 5 and 7, over 9 and 11.

Bring 5 from the left around the back to the right, then along the front from right to left under 6 and 8, over 10 and 12.

Bring 6 from the right around the back to the left, then along the front from left to right under 7 and 9, over 11.

Braid the left-hand round braid of 6-strings:

Bring 7 from the left around the back to the right, then along the front from right to left under 2, over 4 and 6.

Bring 2 from the right around the back to the left, then along the front from left to right under 9 and 11, over 7.

Bring 9 from the left around the back to the right, then along the front from right to left under 4, over 6 and 2.

Bring 4 from the right around the back to the left, then along the front from left to right under 11 and 7, over 9.

Bring 11 from the left around the back to the right, then along the front from right to left under 6, over 2 and 4.

Bring 6 from the right around the back to the left, then along the front from left to right under 7 and 9, over 11.

Bring 7 from the left around the back to the right, then along the front from right to left under 2, over 4 and 6.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 8 from the right around the back to the left, then along the front from left to right under 1 and 3, over 5.

Bring 1 from the left around the back to the right, then along the front from right to left under 10, over 12 and 8.

Bring 10 from the right around the back to the left, then along the front from left to right under 3 and 5, over 1.

Bring 3 from the left around the back to the right, then along the front from right to left under 12, over 8 and 10.

Bring 12 from the right around the back to the left, then along the front from left to right under 5 and 1, over 3.

Bring 5 from the left around the back to the right, then along the front from right to left under 8, over 10 and 12.

And so on.

THE BRAIDER

No.27 **AUGUST 2001.**

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Nested Cylindrical Braids 623

A quarterly publication
for
the braiding artisan

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Transitions from two Round Braids to one Round Braid, and vice versa

We continue with the transitions between two 6-string round braids and one 12-string round braid.

BRAIDING TYPE											
12 STRING								6 STRING			
→				←				→		←	
U	O	U	O	O	U	O	U	U	O	O	U
2	1	2	1	2	1	2	1	2	1	2	1

Fig. 496 :

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6.

Form the crossings between the strings A, C, E, 2, 4, 6.

Bring 1 from the right around the back to the left, then along the front from left to right under B and D, over F, under 2 and 4, over 6.

Bring B from the left around the back to the right, then along the front from right to left under 3, over 5 and A, under C, over E and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under D and F, over 2, under 4 and 6, over B.

Bring D from the left around the back to the right, then along the front from right to left under 5, over A and C, under E, over 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under F and 2, over 4, under 6 and B, over D.

Bring F from the left around the back to the right, then along the front from right to left under A, over C and E, under 1, over 3 and 5.

Bring A from the right around the back to the left, then along the front from left to right under 2 and 4, over 6, under B and D, over F.

Bring 2 from the left around the back to the right, then along the front from right to left under C, over E and 1, under 3, over 5 and A.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6, under 8 and 10, over 12.

Bring 2 from the left around the back to the right, then along the front from right to left under 3, over 5 and 7, under 9, over 11.

Bring 3 from the right around the back to the left, then along the front from left to right under 4 and 6, over 8, under 10 and 12.

Bring 4 from the left around the back to the right, then along the front from right to left under 5, over 7 and 9, under 11.

Bring 5 from the right around the back to the left, then along the front from left to right under 6 and 8, over 10, under 12.

Bring 6 from the left around the back to the right, then along the front from right to left under 7, over 9 and 11.

Braid the left-hand round braid of 6-strings:

Bring 1 from the right around the back to the left, then along the front from left to right under 8 and 10, over 12.

Bring 8 from the left around the back to the right, then along the front from right to left under 3, over 5 and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 10 and 12, over 8.

Bring 10 from the left around the back to the right, then along the front from right to left under 5, over 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 12 and 8, over 10.

Bring 12 from the left around the back to the right, then along the front from right to left under 1, over 3 and 5.

Bring 1 from the right around the back to the left, then along the front from left to right under 8 and 10, over 12.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 7 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6.

Bring 2 from the left around the back to the right, then along the front from right to left under 9, over 11 and 7.

Bring 9 from the right around the back to the left, then along the front from left to right under 4 and 6, over 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 11, over 7 and 9.

Bring 11 from the right around the back to the left, then along the front from left to right under 6 and 2, over 4.

Bring 6 from the left around the back to the right, then along the front from right to left under 7, over 9 and 11.

And so on.

BRAIDING TYPE													
12 STRING								6 STRING					
→				←				→			←		
U	O	U	O	O	U	O	U	U	O	U	U	O	U
2	1	1	2	2	1	1	2	1	1	1	1	1	1

Fig. 497:

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6.

Form the crossings between the strings B, D, F, 1, 3, 5.

Bring 2 from the right around the back to the left, then along the front from left to right under A and C, over E, under 1, over 3 and 5.

Bring A from the left around the back to the right, then along the front from right to left under 4 and 6, over B, under D, over F and 2.

Bring 4 from the right around the back to the left, then along the front from left to right under C and E, over 1, under 3, over 5 and A.

Bring C from the left around the back to the right, then along the front from right to left under 6 and B, over D, under F, over 2 and 4.

Bring 6 from the right around the back to the left, then along the front from left to right under E and 1, over 3, under 5, over A and C.

Bring E from the left around the back to the right, then along the front from right

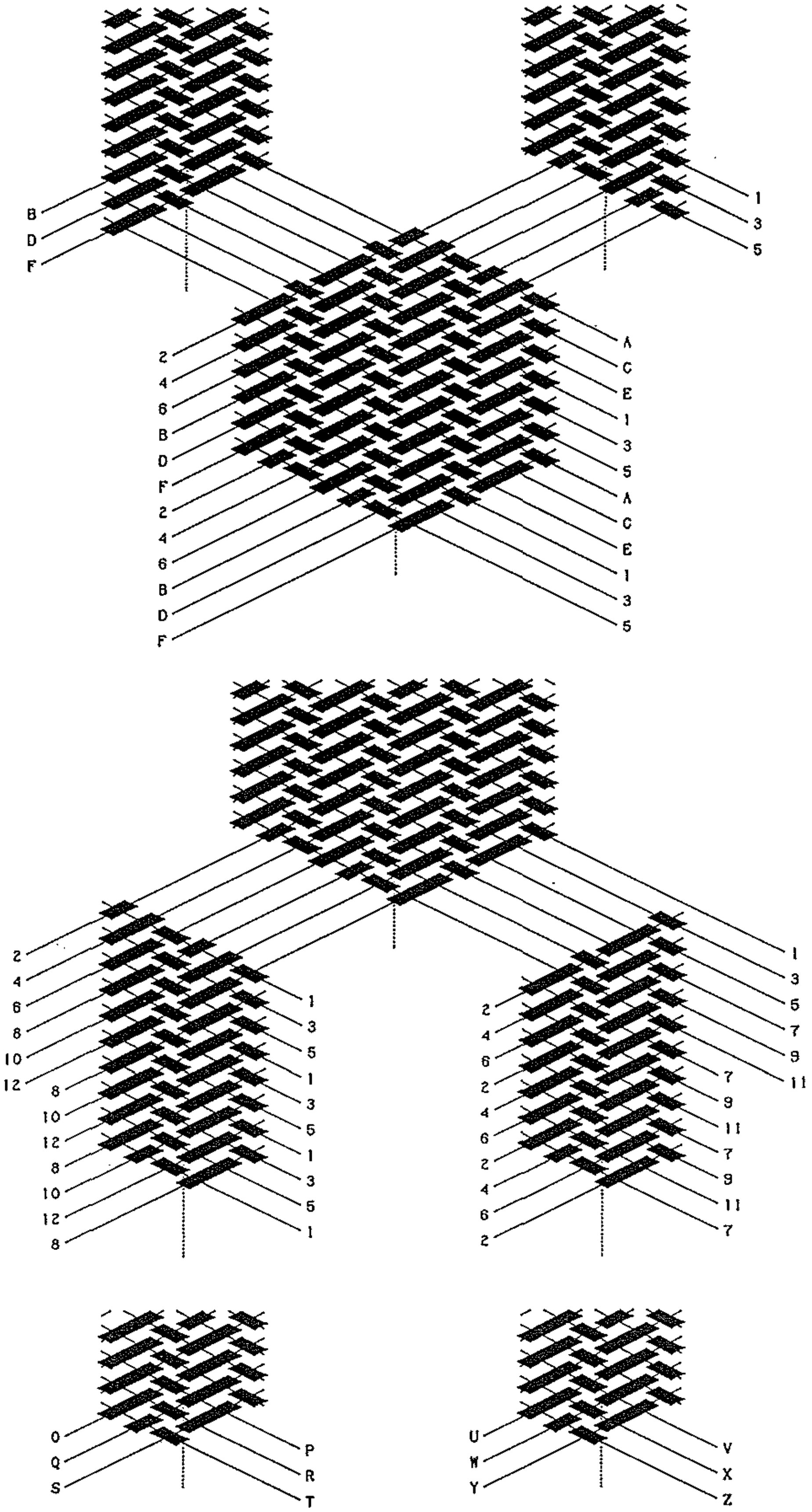

Fig. 496 — Transition between two 6-string round braids and one 12-string round braid.

to left under B and D, over F, under 2, over 4 and 6.

Bring B from the right around the back to the left, then along the front from left to right under 1 and 3, over 5, under A, over C and E.

Bring 1 from the left around the back to the right, then along the front from right to left under D and F, over 2, under 4, over 6 and B.

Bring D from the right around the back to the left, then along the front from left to right under 3 and 5, over A, under C, over E and 1.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6, under 8, over 10, under 12.

Bring 2 from the left around the back to the right, then along the front from right to left under 3 and 5, over 7, under 9, over 11.

Bring 3 from the right around the back to the left, then along the front from left to right under 4 and 6, over 8, under 10, over 12.

Bring 4 from the left around the back to the right, then along the front from right to left under 5 and 7, over 9, under 11.

Bring 5 from the right around the back to the left, then along the front from left to right under 6 and 8, over 10, under 12.

Bring 6 from the left around the back to the right, then along the front from right to left over 7, under 9, over 11.

Braid the left-hand round braid of 6-strings:

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3, under 5.

Bring 1 from the right around the back to the left, then along the front from left to right under 10, over 12, under 8.

Bring 10 from the left around the back to the right, then along the front from right to left under 3, over 5, under 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 12, over 8, under 10.

Bring 12 from the left around the back to the right, then along the front from right to left under 5, over 1, under 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 8, over 10, under 12.

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3, under 5.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 2 from the left around the back to the right, then along the front from right to left under 7, over 9, under 11.

Bring 7 from the right around the back to the left, then along the front from left to right under 4, over 6, under 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 9, over 11, under 7.

Bring 9 from the right around the back to the left, then along the front from left to right under 6, over 2, under 4.

Bring 6 from the left around the back to the right, then along the front from right to left under 11, over 7, under 9.

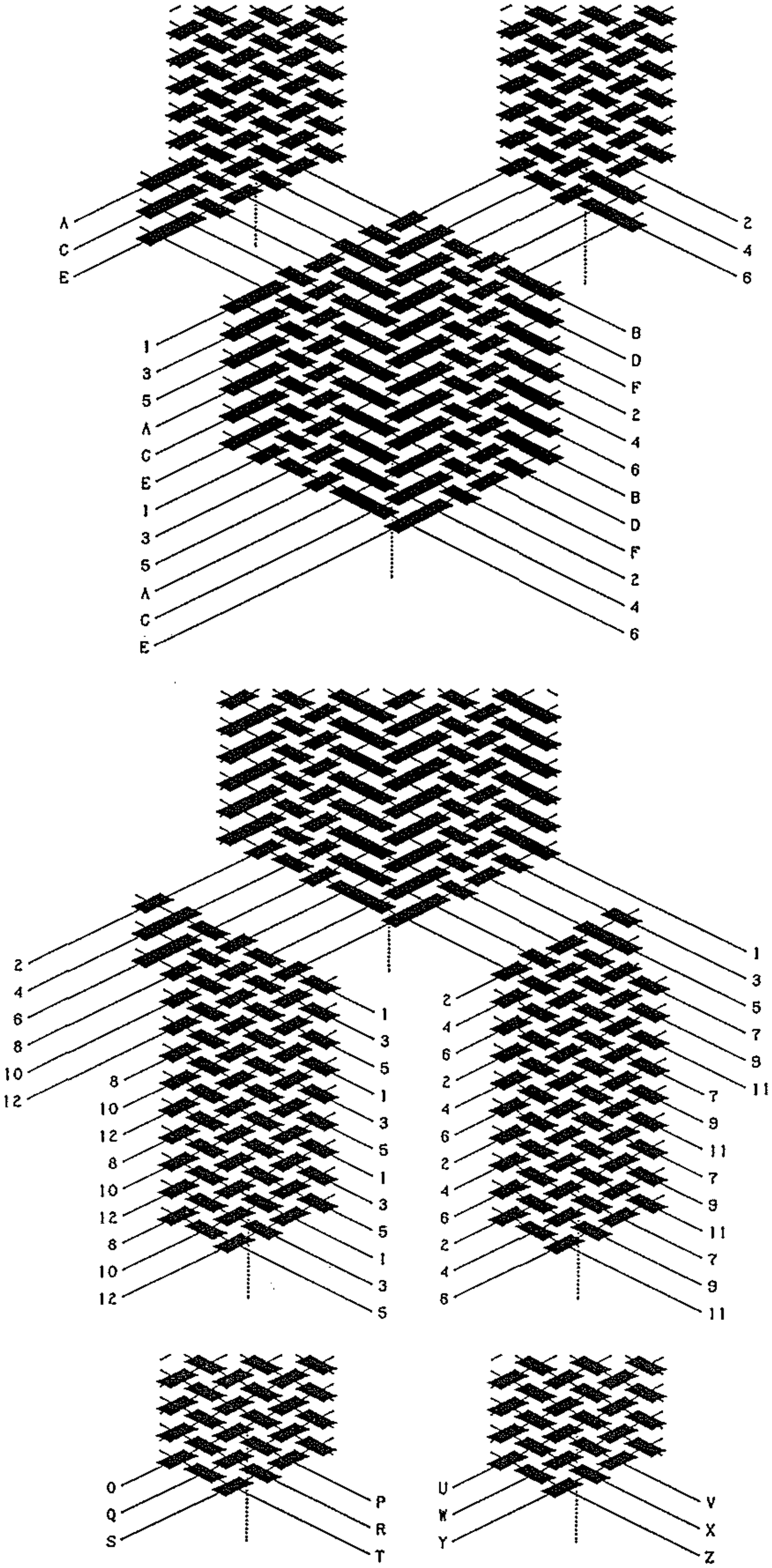

Fig. 497 — Transition between two 6-string round braids and one 12-string round braid.

Bring 11 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6.

Bring 2 from the left around the back to the right, then along the front from right to left under 7, over 9, under 11.

And so on.

A transition between two 6-string round braids and one 8-string round braid with a 4-string round braid core.

BRAIDING TYPE																	
8 STRING MANTLE								4 STRING CORE				6 STRING					
→				←				→		←		→			←		
U	O	U	O	O	U	O	U	U	O	O	U	U	O	U	U	O	U
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Figs. 498, 499 and 500:

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6.

The strings A, B, 1, 2 go over into the 4-string round braid core, the preparation of which is depicted in the upper part of Fig. 498; bring A around the back of the left-hand 6-string round braid from left to right, and bring 2 around the back of the right-hand 6-string round braid from right to left. Note that in the 4-string round braid core the flesh-side of the strings is outermost. The braiding of this core continues as follows:

Bring 2 from the left around the back to the right, then along the front from right to left under 1, over A.

Bring 1 from the right around the back to the left, then along the front from left to right under B, over 2.

Bring B from the left around the back to the right, then along the front from right to left under A, over 1.

Bring A from the right around the back to the left, then along the front from left to right under 2, over B.

And so on.

The strings C, D, E, F, 3, 4, 5, 6 go over into the 8-string round braid mantle.

Form the crossings between the strings D, F, 3, 5 as depicted in the lower part of Fig. 498.

Next bring 4 from the right around the back to the left, then along the front from left to right under C, over E, under 3, over 5.

Bring C from the left around the back to the right, then along the front from right to left under 6, over D, under F, over 4.

Bring 6 from the right around the back to the left, then along the front from left to right under E, over 3, under 5, over C.

Bring E from the left around the back to the right, then along the front from right to left under D, over F, under 4, over 6.

Bring D from the right around the back to the left, then along the front from left to right under 3, over 5, under C, over E.

Bring 3 from the left around the back to the right, then along the front from right to left under F, over 4, under 6, over D.

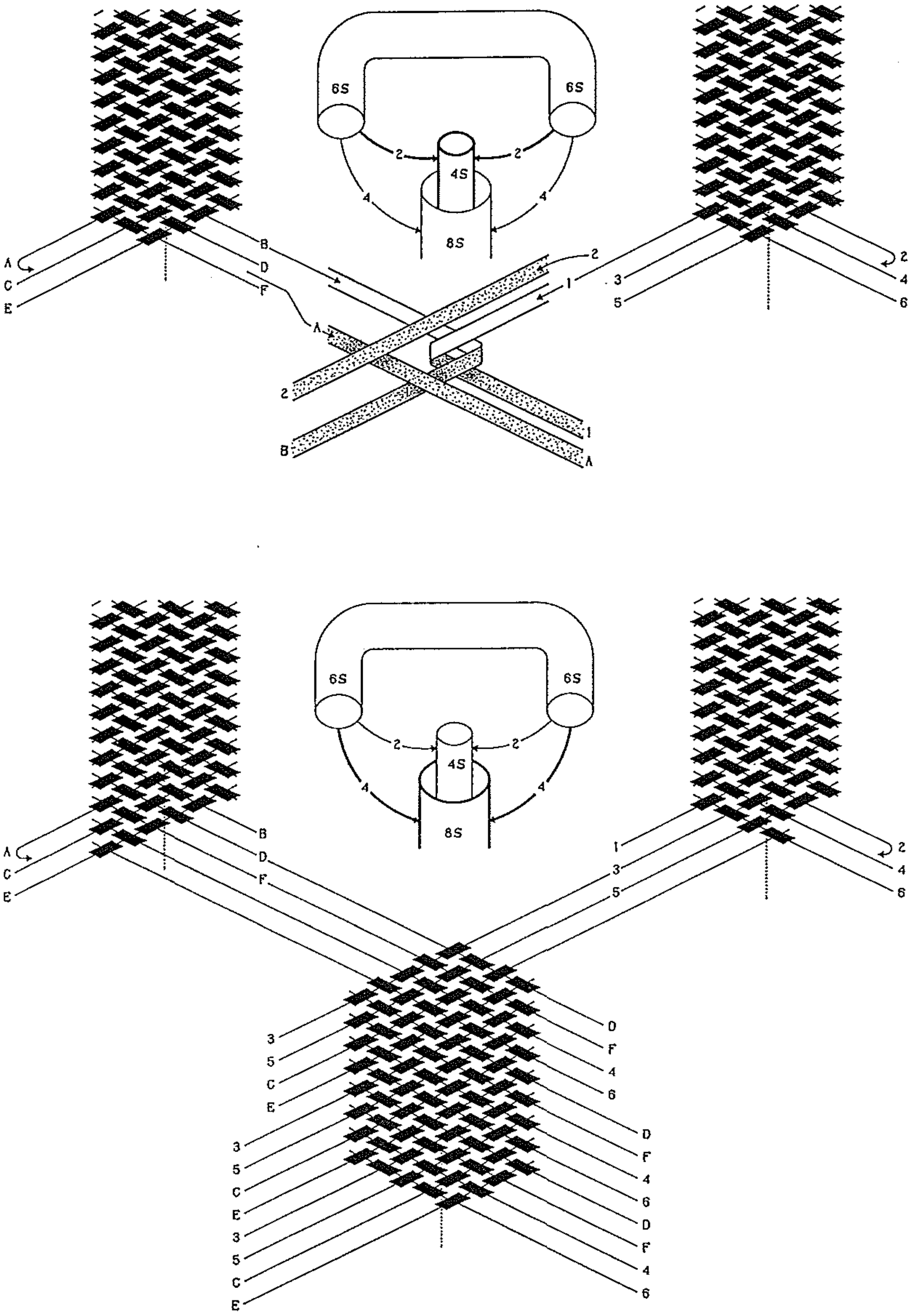

Fig. 498 — Transition from two 6-string round braids to 8-string round braid with core.

Bring F from the right around the back to the left, then along the front from left to right under 5, over C, under E, over 3.

Bring 5 from the left around the back to the right, then along the front from right to left under 4, over 6, under D, over F.

Bring 4 from the right around the back to the left, then along the front from left to right under C, over E, under 3, over 5.

Bring C from the left around the back to the right, then along the front from right to left under 6, over D, under F, over 4.

Bring 6 from the right around the back to the left, then along the front from left to right under E, over 3, under 5, over C.

Bring E from the left around the back to the right, then along the front from right to left under D, over F, under 4, over 6.

And so on.

The 4-string round braid core has the strings 1, 2, 3, 4 (see Fig. 499 upper part, and Fig. 500), the 8-string round braid mantle has the strings A, B, C, D, E, F, G, H (see Fig. 499 lower part, and Fig. 500).

The preparation of the 4-string round braid core (Fig. 499 upper part and Fig. 500):

Bring 1 around the back from right to left with the hair-side up on left. Place 1 between 2 and 4.

Bring 2 around the back from left to right with the hair-side up on right. Place 2 below 3.

The preparation of the 8-string round braid mantle (Fig. 499 lower part and Fig. 500):

Bring A from the right around the back to the left, then along the front from left to right under B, over D, under F, over H.

Bring B from the left around the back to the right, then along the front from right to left under C, over E, under G.

Bring C from the right around the back to the left, then along the front from left to right under D, over F, under H.

Bring D from the left around the back to the right, then along the front from right to left under E, over G.

Braiding the two 6-string round braids (see Fig. 500):

Braid the left-hand round braid of 6-strings:

Bring 1 over 4, then along the front from right to left over A, under C.

Bring 4 around the back from the right to the left, then along the front from left to right under F, over H, under 1.

Bring F from the left around the back to the right, then along the front from right to left under A, over C, under 4.

Bring A from the right around the back to the left, then along the front from left to right under H, over 1, under F.

Bring H from the left around the back to the right, then along the front from right to left under C, over 4, under A.

Bring C from the right around the back to the left, then along the front from left to right under 1, over F, under H.

Bring 1 from the left around the back to the right, then along the front from right to left under 4, over A, under C.

Bring 4 from the right around the back to the left, then along the front from left to right under F, over H, under 1.

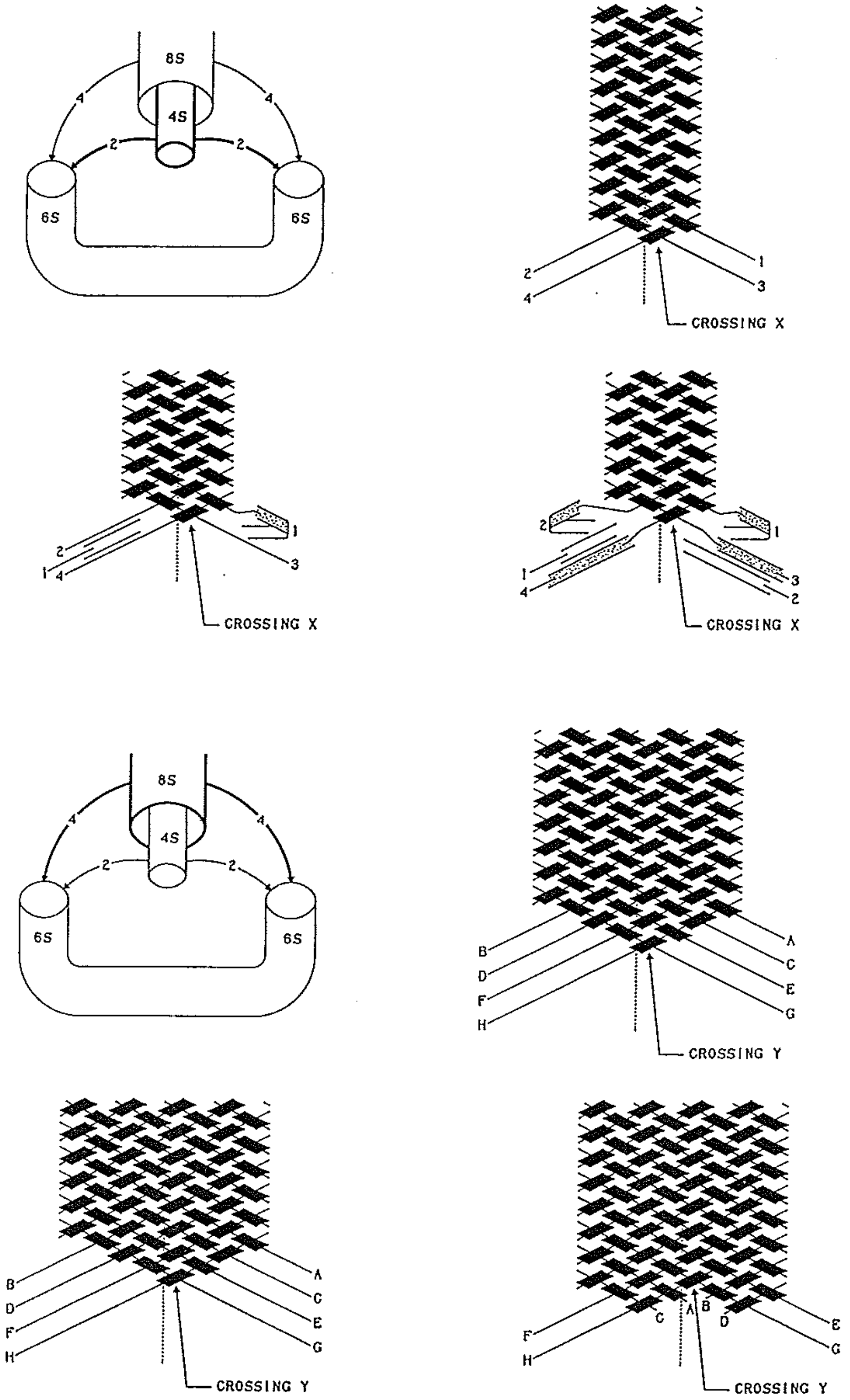

Fig. 499 — Transition from 8-string round braid with core to two 6-string round braids.

Before starting the preparation of the 8-string round braid mantle, continue braiding this mantle until crossing "Y" of the mantle is at the same level as crossing "X" of the core.

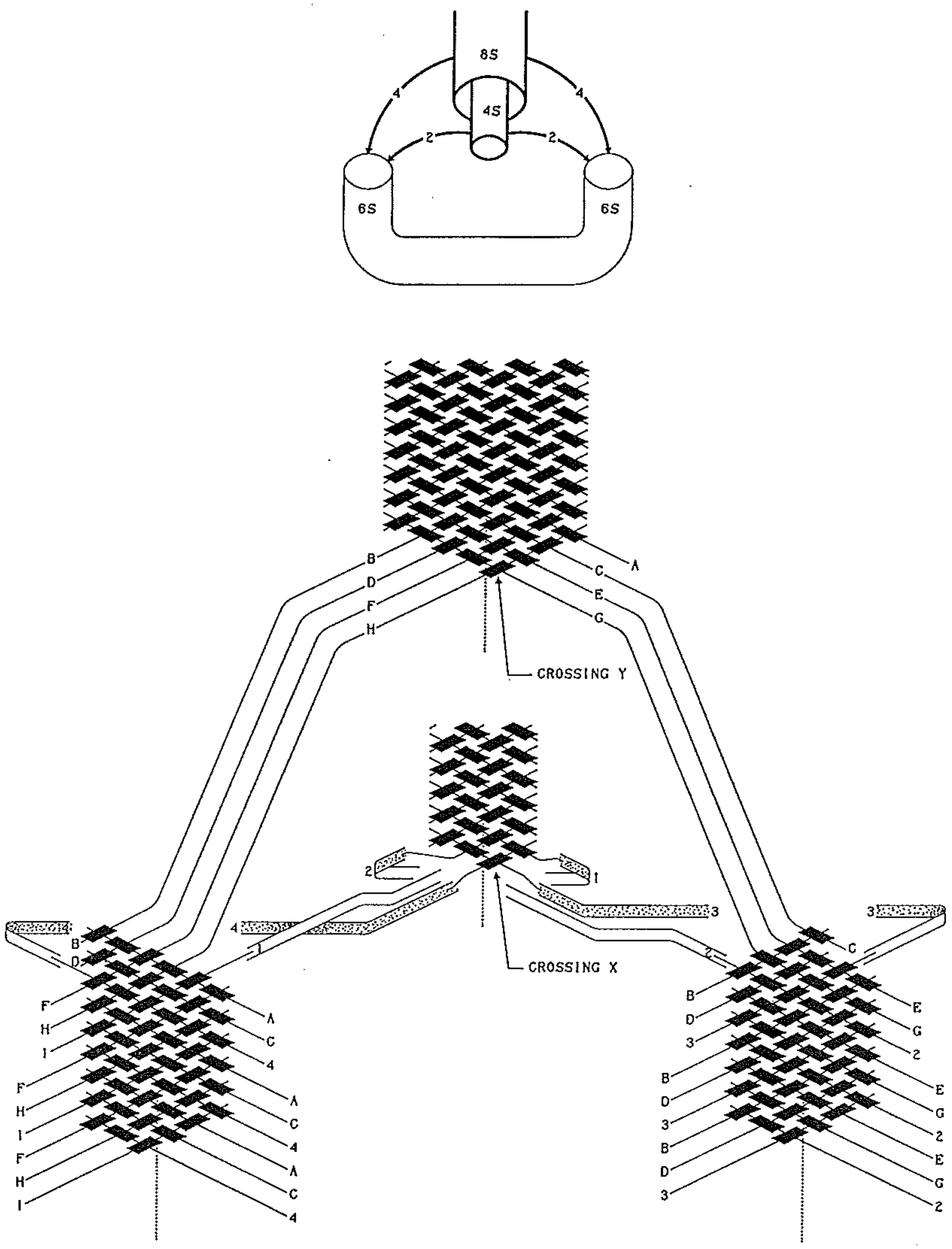

Fig. 500 — Transition from 8-string round braid with core to two 6-string round braids.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 3 around the back from the left to the right, then along the front from right to left over E, under G.

Bring 2 along the front from left to right under B, over D, under 3.

Bring B from the left around the back to the right, then along the front from right to left under E, over G, under 2.

Bring E from the right around the back to the left, then along the front from left to right under D, over 3, under B.

Bring D from the left around the back to the right, then along the front from right to left under G, over 2, under E.

Bring G from the right around the back to the left, then along the front from left to right under 3, over B, under D.

Bring 3 from the left around the back to the right, then along the front from right to left under 2, over E, under G.

Bring 2 from the right around the back to the left, then along the front from left to right under B, over D, under 3.

And so on.

Nested Cylindrical Braids

We have seen that the minimum x-value in the string-run of **Regular Nested Cylindrical Braids** is equal to $2 - A$ (see *The Braider*, Issues No. 24 and 25), and that for this x-value $y_{min} = y_{max} = A$. Since there are for this case no string-crossings, the 'braid' will fall apart, hence for practical applications the value of x must always be greater than $2 - A$. The minimum x-value for practical applications, however, depends on the superimposed coding.

In *The Braider*, Issue No. 23, we have already discussed some aspects of the **Standard** and **Semi-Standard Herringbone Pineapple Knots**, and in this Issue we will have a look at their colour-patterns when strings of a different colour are being used in their construction.

For the Standard and Semi-Standard Herringbone Pineapple Knots the minimum x-value for practical applications is $A + 2$. For their string-run $\gamma = A$, $y = A$ and $\Delta = 0$.

Since $\gamma = A$, their string-run consists of A components.

g.c.d. $(P_c, B^*) = 1$ for every component of a **Standard Herringbone Pineapple Knot**, hence the total number of essential strings in a **Standard Herringbone Pineapple Knot** is equal to $\gamma = A$. The coding-pattern of each component consists of one or more circumferential chains of B^* "<", in which all "<" are of the same colour. When a component consists of more than one circumferential chain, then its consecutive parallel circumferential chains are A chains apart.

There is in a **Semi-Standard Herrringbone Pineapple Knot** at least one component which consists of two or more sub-components (at least for one component g.c.d. $(P_c, B^*) \neq 1$). The coding-pattern of each component consists of one or more circumferential chains of B^* "<", in which all "<" are of the same colour only for components with g.c.d.$(P_c\,,\,B^*) = 1$. When a component consists of more than one circumferential chain, then its consecutive parallel circumferential chains are A chains apart.

The coding-pattern of the **Standard** and **Semi-Standard Herringbone Pineapple Knots** thus consists of B^* circumferential sets of V regularly horizontally stacked "<", where $V = \dfrac{A+x-2}{2} = \dfrac{P-A}{2}$.

For practical applications of the Standard and Semi-Standard Herringbone Pineapple Knots we thus obtain the following formulae:

$$\begin{aligned}
V &= \frac{A+x-2}{2} && \longrightarrow \quad x = 2V+2-A \quad \rightarrow \quad k = |V|_A\,. \\
V &= \frac{P-A}{2} && \longrightarrow \quad P = A+2V = 2A+x-2\,. \\
x_{min} &= A+2 && \longrightarrow \quad V_{min} = A\,. \\
P_{min} &= 3A\,. \\
A_{max} &= V\,. \\
P_{max} &= 3V\,.
\end{aligned}$$

Since $y = A$ it follows that $y' = 2A - y = A$ and hence $\Delta' = A$.[†] Thus the first set of components (the components in which $l_i + r_i = k+1$, where $1 \le k < A$) contains

$$\left|\frac{x+\Delta'-2}{2}\right|_\gamma = |V|_A \text{ components,}$$

with each component having a number of parts equal to

$$P_c = \frac{2V - 2|V|_A}{A} + 3\,.$$

And the second set of components (the components in which $l_i + r_i = k+1+A$, where $1 \le k < A$) contains

$$\gamma - \left|\frac{x+\Delta'-2}{2}\right|_\gamma = \gamma - |V|_A \text{ components,}$$

with each component having a number of parts equal to

$$P_c = 1 + 2\left\lfloor\frac{V}{A}\right\rfloor = \frac{2V - 2|V|_A}{A} + 1\,.$$

When $k = A$ (hence $V = nA$, where $n = 1\,,2\,,3\,,\cdots$), the first set of components is empty and the second set of components contains

$$\gamma = A \text{ components,}$$

with each component having a number of parts equal to

$$P_c = 1 + 2\left\lfloor\frac{V}{A}\right\rfloor = \frac{2V - 2|V|_A}{A} + 1\,.$$

The colour-pattern of a **Standard Herringbone Pineapple Knot** is thus represented by the colour-pattern of the V regularly horizontally stacked "<".

After we have designed for a **Standard Herringbone Pineapple Knot** a suitable colour-pattern for the in our application required V regularly horizontally stacked "<", we can then determine the valid A-values for such a colour-pattern.

[†] See *The Braider*, Issue No. 25, pp. 570 - 571.

Say the regularly horizontally stacked "<" have the following pattern of two colours:

< < < < **<** **<** < < **<** < < **<** **<** < < < **<**

Since $V = 17$, it follows that $2 \leq A \leq 17$. The examination of each A-value in association with the given colour-pattern of the seventeen horizontally regularly stacked "<" is presented in Fig. 501. We see that:
$A = 2$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 3$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 4$ is not possible: although the colour-repetition of the first "<" agrees with the required colour-pattern, the colour-repetition of the second "<" does not agree with the required colour-pattern.
$A = 5$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 6$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 7$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 8$ is not possible: although the colour-repetition of the first, the second and the third "<" agrees with the required colour-pattern, the colour-repetition of the fourth "<" does not agree with the required colour-pattern.
$A = 9$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 10$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 11$ is not possible: although the colour-repetition of the first "<" agrees with the required colour-pattern, the colour-repetition of the second "<" does not agree with the required colour-pattern.
$A = 12$ is possible: the colour-repetition of the first twelve successive "<" does agree with the required colour-pattern.
$A = 13$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 14$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 15$ is not possible: already the colour-repetition of the first "<" does not agree with the required colour-pattern.
$A = 16$ is possible: the colour-repetition of the first sixteen successive "<" does agree with the required colour-pattern.
$A = 17$ is possible: the colour-repetition of the seventeen successive "<" does agree with the required colour-pattern.

Hence the only valid A-values for our required two colour colour-pattern are 12, 16 and 17.

For $A = 12$, the rightmost "<" which is associated with the first "<" is the fifth "<" from the right. Hence $k = 5$. Thus the first half-cycles of the components are:
$1 \nearrow^{5}, 2 \nearrow^{4}, 3 \nearrow^{3}, 4 \nearrow^{2}, 5 \nearrow^{1}, 6 \nearrow^{12}, 7 \nearrow^{11}, 8 \nearrow^{10}, 9 \nearrow^{9}, 10 \nearrow^{8}, 11 \nearrow^{7}, 12 \nearrow^{6}.$
The P_c-values of these respective components are: $5, 5, 5, 5, 5, 3, 3, 3, 3, 3, 3, 3$.

Hence B^* should not be divisible by 3 and/or 5.
Note also that k may be calculated with the formula $|V|_A = |17|_{12} = 5$.

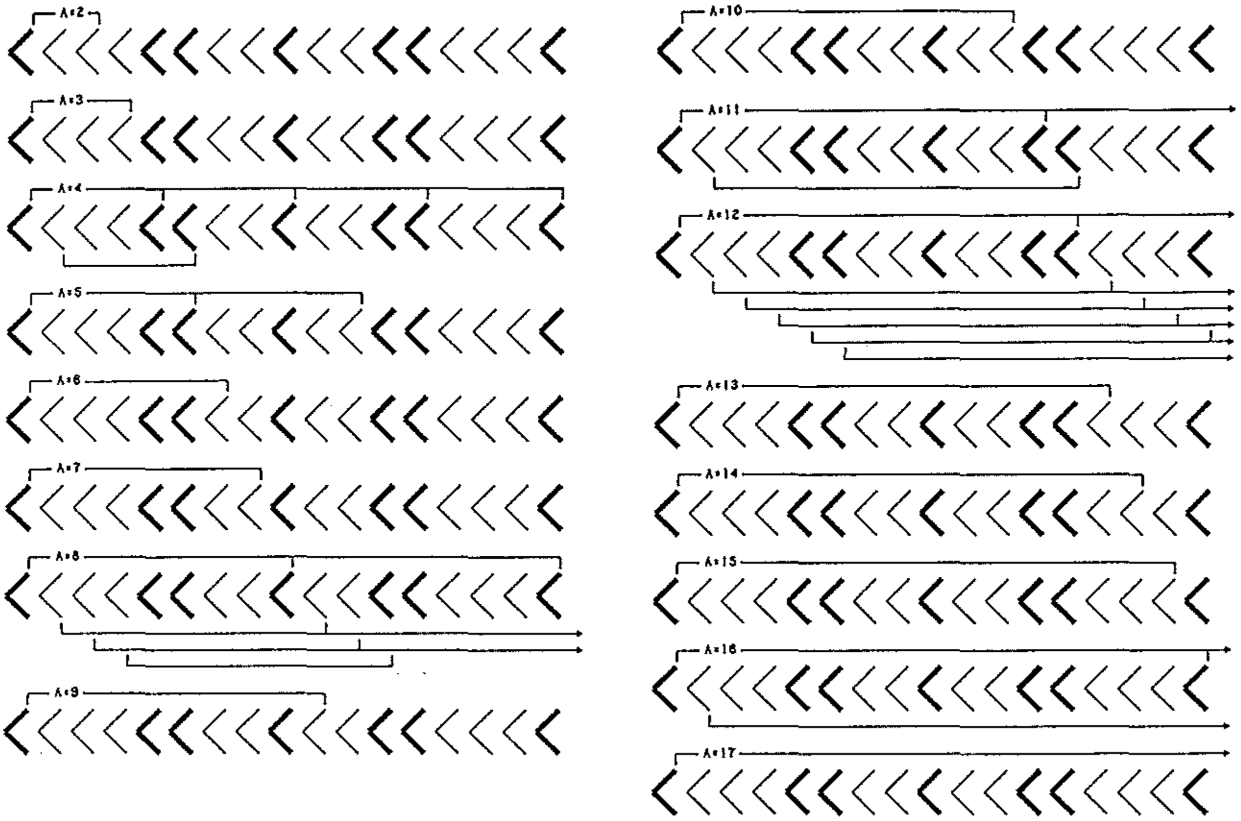

Fig. 501 — The examination of the A-values in the interval $2 \le A \le V$.

For $A = 16$, the rightmost "<" which is associated with the first "<" is the first "<" from the right. Hence $k = 1$. Thus the first half-cycles of the components are:
$1\nearrow^{1},\ 2\nearrow^{16},\ 3\nearrow^{15},\ 4\nearrow^{14},\ 5\nearrow^{13},\ 6\nearrow^{12},\ 7\nearrow^{11},\ 8\nearrow^{10},\ 9\nearrow^{9},\ 10\nearrow^{8},$
$11\nearrow^{7},\ 12\nearrow^{6},\ 13\nearrow^{5},\ 14\nearrow^{4},\ 15\nearrow^{3},\ 16\nearrow^{2}.$
The P_c-values of these respective components are: $5, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3$.
Hence B^* should not be divisible by 3 and/or 5.
Note also that k may be calculated with the formula $|V|_A = |17|_{16} = 1$.

For $A = V$, the value of k is equal to A. Hence $k = 17$. Thus the first half-cycles of the components are:
$1\nearrow^{17},\ 2\nearrow^{16},\ 3\nearrow^{15},\ 4\nearrow^{14},\ 5\nearrow^{13},\ 6\nearrow^{12},\ 7\nearrow^{11},\ 8\nearrow^{10},\ 9\nearrow^{9},\ 10\nearrow^{8},$
$11\nearrow^{7},\ 12\nearrow^{6},\ 13\nearrow^{5},\ 14\nearrow^{4},\ 15\nearrow^{3},\ 16\nearrow^{2},\ 17\nearrow^{1}.$
Each component has a P_c-value of 3.
Hence B^* should not be divisible by 3.
Note also that k may be calculated with the formula $|V|_A = |17|_{17} = A$.

Example 1:

The pattern of two colours of the $V = 12$ regularly horizontally stacked "<" is :

< < < **< <** < < **< <** < < **<**

Since $V = 12$, it follows that $2 \leq A \leq 12$. The examination of each A-value in association with the given colour pattern of the twelve horizontally regularly stacked "<" shows that only $A = 4$, $A = 8$, $A = 11$ and $A = 12$ are possible.

$k = 4$ for $A = 4$. Furthermore $x = 22$ and $P = 28$. Thus the first half-cycles of the components are:

${}_1\nearrow^4, {}_2\nearrow^3, {}_3\nearrow^2, {}_4\nearrow^1.$

Each component has a P_c-value of 7.

Hence B^* should not be divisible by 7.

$k = 4$ for $A = 8$. Furthermore $x = 18$ and $P = 32$. Thus the first half-cycles of the components are:

${}_1\nearrow^4, {}_2\nearrow^3, {}_3\nearrow^2, {}_4\nearrow^1, {}_5\nearrow^8, {}_6\nearrow^7, {}_7\nearrow^6, {}_8\nearrow^5.$

The P_c-values of these respective components are: $5, 5, 5, 5, 3, 3, 3, 3$.

Hence B^* should not be divisible by 3 and/or 5.

$k = 1$ for $A = 11$. Furthermore $x = 15$ and $P = 35$. Thus the first half-cycles of the components are:

${}_1\nearrow^1, {}_2\nearrow^{11}, {}_3\nearrow^{10}, {}_4\nearrow^9, {}_5\nearrow^8, {}_6\nearrow^7, {}_7\nearrow^6, {}_8\nearrow^5, {}_9\nearrow^4, {}_{10}\nearrow^3, {}_{11}\nearrow^2.$

The P_c-values of these respective components are: $5, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3$.

Hence B^* should not be divisible by 3 and/or 5.

$k = A = 12$ for $A = V = 12$. Furthermore $x = 14$ and $P = 36$. Thus the first half-cycles of the components are:

${}_1\nearrow^{12}, {}_2\nearrow^{11}, {}_3\nearrow^{10}, {}_4\nearrow^9, {}_5\nearrow^8, {}_6\nearrow^7, {}_7\nearrow^6, {}_8\nearrow^5, {}_9\nearrow^4, {}_{10}\nearrow^3,$
${}_{11}\nearrow^2, {}_{12}\nearrow^1.$

Each component has a P_c-value of 3.

Hence B^* should not be divisible by 3.

Example 2:

The pattern of two colours of the $V = 8$ regularly horizontally stacked "<" is :

< < < **<** **<** < < **<**

Since $V = 8$, it follows that $2 \leq A \leq 8$. The examination of each A-value in association with the given colour pattern of the eight horizontally regularly stacked "<" shows that only $A = 4$, $A = 7$, and $A = 8$ are possible.

$k = 4$ for $A = 4$. Furthermore $x = 14$ and $P = 20$. Thus the first half-cycles of the components are:

${}_1\nearrow^4, {}_2\nearrow^3, {}_3\nearrow^2, {}_4\nearrow^1.$

Each component has a P_c-value of 5.

Hence B^* should not be divisible by 5.

$k = 1$ for $A = 7$. Furthermore $x = 11$ and $P = 23$. Thus the first half-cycles of the components are:

${}_1\nearrow^1, {}_2\nearrow^7, {}_3\nearrow^6, {}_4\nearrow^5, {}_5\nearrow^4, {}_6\nearrow^3, {}_7\nearrow^2.$

The P_c-values of these respective components are: $5, 3, 3, 3, 3, 3, 3$.

Hence B^* should not be divisible by 3 and/or 5.

$k = A = 8$ for $A = V = 8$. Furthermore $x = 10$ and $P = 24$. Thus the first half-cycles of the components are:

$$1 \nearrow^{8},\ 2 \nearrow^{7},\ 3 \nearrow^{6},\ 4 \nearrow^{5},\ 5 \nearrow^{4},\ 6 \nearrow^{3},\ 7 \nearrow^{2},\ 8 \nearrow^{1}.$$

Each component has a P_c-value of 3.

Hence B^* should not be divisible by 3.

Example 3:

The pattern of two colours of the $V = 7$ regularly horizontally stacked "<" is :

$$\boldsymbol{<}\,\boldsymbol{<}\,\boldsymbol{<}\,<\,\boldsymbol{<}\,\boldsymbol{<}\,\boldsymbol{<}$$

Since $V = 7$, it follows that $2 \leq A \leq 7$. The examination of each A-value in association with the given colour pattern of the seven horizontally regularly stacked "<" shows that $A = 4$, $A = 5$, $A = 6$ and $A = 7$ are possible.

$k = 3$ for $A = 4$. Furthermore $x = 12$ and $P = 18$. Thus the first half-cycles of the components are:

$$1 \nearrow^{3},\ 2 \nearrow^{2},\ 3 \nearrow^{1},\ 4 \nearrow^{4}.$$

The P_c-values of these respective components are: $5, 5, 5, 3$.

Hence B^* should not be divisible by 3 and/or 5.

$k = 2$ for $A = 5$. Furthermore $x = 11$ and $P = 19$. Thus the first half-cycles of the components are:

$$1 \nearrow^{2},\ 2 \nearrow^{1},\ 3 \nearrow^{5},\ 4 \nearrow^{4},\ 5 \nearrow^{3}.$$

The P_c-values of these respective components are: $5, 5, 3, 3, 3$.

Hence B^* should not be divisible by 3 and/or 5.

$k = 1$ for $A = 6$. Furthermore $x = 10$ and $P = 20$. Thus the first half-cycles of the components are:

$$1 \nearrow^{1},\ 2 \nearrow^{6},\ 3 \nearrow^{5},\ 4 \nearrow^{4},\ 5 \nearrow^{3},\ 6 \nearrow^{2}.$$

The P_c-values of these respective components are: $5, 3, 3, 3, 3, 3$.

Hence B^* should not be divisible by 3 and/or 5.

$k = A = 7$ for $A = V = 7$. Furthermore $x = 9$ and $P = 21$. Thus the first half-cycles of the components are:

$$1 \nearrow^{7},\ 2 \nearrow^{6},\ 3 \nearrow^{5},\ 4 \nearrow^{4},\ 5 \nearrow^{3},\ 6 \nearrow^{2},\ 7 \nearrow^{1}.$$

Each component has a P_c-value of 3.

Hence B^* should not be divisible by 3.

Example 4:

The pattern of two colours of the $V = 11$ regularly horizontally stacked "<" is :

$$\boldsymbol{<}\,<\,\boldsymbol{<}\,\boldsymbol{<}\,<\,\boldsymbol{<}\,<\,\boldsymbol{<}\,\boldsymbol{<}\,<\,\boldsymbol{<}$$

Since $V = 11$, it follows that $2 \leq A \leq 11$. The examination of each A-value in association with the given colour pattern of the eleven horizontally regularly stacked "<" shows that $A = 5$, $A = 8$, $A = 10$ and $A = 11$ are possible.

$k = 1$ for $A = 5$. Furthermore $x = 19$ and $P = 27$. Thus the first half-cycles of the components are:

${}_1\nearrow^{1},\ {}_2\nearrow^{5},\ {}_3\nearrow^{4},\ {}_4\nearrow^{3},\ {}_5\nearrow^{2}.$

The P_c-values of these respective components are: $7, 5, 5, 5, 5$.

Hence B^* should not be divisible by 5 and/or 7.

$k = 3$ for $A = 8$. Furthermore $x = 16$ and $P = 30$. Thus the first half-cycles of the components are:

${}_1\nearrow^{3},\ {}_2\nearrow^{2},\ {}_3\nearrow^{1},\ {}_4\nearrow^{8},\ {}_5\nearrow^{7},\ {}_6\nearrow^{6},\ {}_7\nearrow^{5},\ {}_8\nearrow^{4}.$

The P_c-values of these respective components are: $5, 5, 5, 3, 3, 3, 3, 3$.

Hence B^* should not be divisible by 3 and/or 5.

$k = 1$ for $A = 10$. Furthermore $x = 14$ and $P = 32$. Thus the first half-cycles of the components are:

${}_1\nearrow^{1},\ {}_2\nearrow^{10},\ {}_3\nearrow^{9},\ {}_4\nearrow^{8},\ {}_5\nearrow^{7},\ {}_6\nearrow^{6},\ {}_7\nearrow^{5},\ {}_8\nearrow^{4},\ {}_9\nearrow^{3},\ {}_{10}\nearrow^{2}.$

The P_c-values of these respective components are: $5, 3, 3, 3, 3, 3, 3, 3, 3, 3$.

Hence B^* should not be divisible by 3 and/or 5.

$k = A = 11$ for $A = V = 11$. Furthermore $x = 13$ and $P = 33$. Thus the first half-cycles of the components are:

${}_1\nearrow^{11},\ {}_2\nearrow^{10},\ {}_3\nearrow^{9},\ {}_4\nearrow^{8},\ {}_5\nearrow^{7},\ {}_6\nearrow^{6},\ {}_7\nearrow^{5},\ {}_8\nearrow^{4},\ {}_9\nearrow^{3},\ {}_{10}\nearrow^{2},\ {}_{11}\nearrow^{1}.$

Each component has a P_c-value of 3.

Hence B^* should not be divisible by 3.

In a **Semi-Standard Herringbone Pineapple Knot** there is at least one component for which g.c.d.$(P_c, B^*) \neq 1$, and any such a component requires more than one essential string. Unless all the essential strings of a component have the same colour, the colour of the circumferential chain(s) of "<" belonging to such a component is not uniform. Hence for the **Semi-Standard Herringbone Pineapple Knots** we can create further colour-patterns.

Example 5:

First we design a colour-pattern for the components and do as if each component requires one essential string only:

Lets take the following two-tone colour-pattern for the regularly horizontally stacked "<" of such components:

$$< < < \boldsymbol{<} \boldsymbol{<} < \ < <$$

Since $V = 8$, it follows that $2 \leq A \leq 8$. The examination of each A-value in association with the given colour pattern of the eight horizontally regularly stacked "<" shows that $A = 5$, $A = 6$, $A = 7$ and $A = 8$ are possible.

Lets take $A = 5$, then $k = |V|_A = |8|_5 = 3$, $x = 13$, and $P = 21$. The first half-cycles of the components are:

${}_1\nearrow^{3},\ {}_2\nearrow^{2},\ {}_3\nearrow^{1},\ {}_4\nearrow^{5},\ {}_5\nearrow^{4}.$

The P_c-values of these respective components are: $5, 5, 5, 3, 3$.

The components with the first half-cycles ${}_4\nearrow^{5}$ and ${}_5\nearrow^{4}$ are the components which produce the two central "$\boldsymbol{<}$". If we take $B^* = 6$, then for each of these two components g.c.d.$(P_c, B^*) =$ g.c.d.$(3, 6) = 3$, and hence each consists of three subcomponents, thus each requires three essential strings.

Note that g.c.d.$(P_c\,, B^*) =$ g.c.d.$(5\,, 6) = 1$ for the components with the first half-cycles ${}_1\nearrow^3$, ${}_2\nearrow^2$ and ${}_3\nearrow^1$, which produce the left three and the right three "<". Hence these components each require one essential string.

We can thus create the colour-pattern depicted in Fig. 502 for our **Semi-Standard Herringbone Pineapple Knot.**

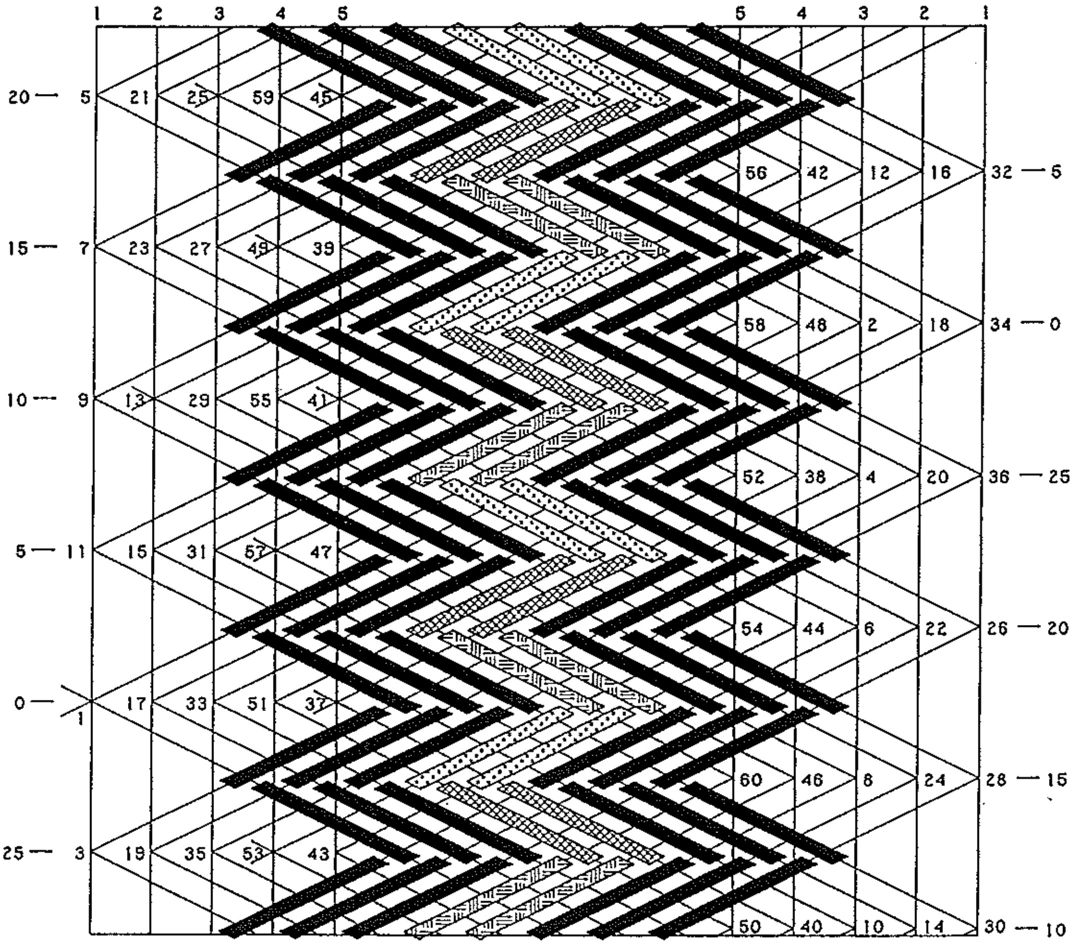

Fig. 502 — The Semi-Standard Herringbone Pineapple Knot with $A = 5\,, V = 8\,, B^* = 6\,.$

The herringbone coding-form depicted in Fig. 502 is that of the general form for nesting-number A depicted in the lower diagram of Fig. 446, pag. 525 (see *The Braider*, Issue No. 23). In this coding-form, a lower-left to upper-right half-cycle running from l_i to r_i has the coding-sequence:

$$(l_i - 1)u - Ao - Au - \cdots - Au - Ao - (r_i)u\,,$$

in which each set of its P_c sets of crossings, except the first set $(l_i{-}1)u$, has one crossing belonging to the string-run of the interbraided knot between the bight-boundaries l_i and r_i.

A lower-right to upper-left half-cycle running from r_i to l_i has the coding-sequence:

$$(r_i - 1)u - Ao - Au - \cdots - Au - Ao - (l_i)u\,.$$

in which each set of its P_c sets of crossings, except the first set $(r_i{-}1)u$, has one crossing belonging to the string-run of the interbraided knot between the bight-boundaries l_i and r_i.

Hence the reference half-cycle sequences, consequently the first lower-left to upper-right half-cycle sequence, are from lower-left l_i to upper-right r_i:

$$(l_i - 1)u - (A-1)o - (A-1)u - \cdots - (A-1)u - (A-1)o - (r_i - 1)u\,.$$

from lower-right r_i to upper-left l_i:

$$(r_i - 1)u - (A-1)o - (A-1)u - \cdots - (A-1)u - (A-1)o - (l_i - 1)u\,.$$

Thus the general form of an algorithm diagram for an imterbraided over–under coded Regular or Semi-Regular Knot between the bight-boundaries l_i and r_i is as follows:

$$\begin{array}{ccccccccccc}
\longrightarrow & l_i-1 & A-1 & A-1 & A-1 & \ldots & A-1 & A-1 & r_i-1 & & \\
 & 0 & \star & \star & \star & \ldots & \star & \star & \star & & \\
 & \backslash & / & \backslash & / & \ldots & \backslash & / & \backslash & / & \\
 & & \star & \star & \star & \ldots & \star & \star & \star & 0 & \\
 & & l_i-1 & A-1 & A-1 & \ldots & A-1 & A-1 & A-1 & r_i-1 & \longleftarrow
\end{array}$$

The positions of the stars are occupied by the i-values of the complementary bight-number scheme associated with the knot to be interbraided. The value above an upper star, or below a lower star, increases by 1 when its associated i-value is applicable to the half-cycle concerned. The first entry for the lower-left to upper-right half-cycles, $(l_i - 1)u$, remains the same for all of them. Similarly for all the lower-right to upper-left half-cycles, the first entry $(r_i - 1)u$ remains the same.

The Semi-Standard Herrinbone Pineapple Knot in our Example is an interbraid of three 5-parts/6-bights over–under coded Regular Knots and two 3-parts/6-bights over–under coded Semi-Regular Knots.

The general algorithm diagram for the three interbraided $P_c/B^* = 5/6$ Regular Knots with $\Delta^* = 1$ is:

$$\begin{array}{cccccccc}
\longrightarrow & l_i-1 & A-1 & A-1 & A-1 & r_i-1 & & \\
 & 0 & 1 & 2 & 3 & 4 & & \\
 & \backslash & / & \backslash & / & \backslash & / & \\
 & & 4 & 3 & 2 & 1 & 0 & \\
 & & l_i-1 & A-1 & A-1 & A-1 & r_i-1 & \longleftarrow
\end{array}$$

The general algorithm diagram for the two interbraided $P_c/B^* = 3/6$ Semi-Regular Knots[†] with $\Delta^* = 1$ is:

$$\begin{array}{cccccc}
\longrightarrow & l_i-1 & A-1 & r_i-1 & & \\
 & 0 & X & Y & & \\
 & \backslash & / & \backslash & / & \\
 & & Y & X & 0 & \\
 & & l_i-1 & A-1 & r_i-1 & \longleftarrow
\end{array}$$

According to the grid-diagram in Fig. 502 we first braid in the $A = 5$ Semi-Standard Herringbone Pineapple Knot the over–under coded $P_c/B^* = 5/6$ Regular Knot between its left bight-boundary 1 and its right bight-boundary 3. This is the first knot we braid, hence it is the foundation knot, thus $A = 1$ with $l_i = 1$ and $r_i = 1$. Consequently we obtain the following half-cycle algorithms from its associated general algorithm diagram:

half-cycle 1		:	$L \longrightarrow R$	Free Run.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	Free Run.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	Free Run.
half-cycle 4	$i \leq 1$	:	$L \longleftarrow R$	o.

[†] See *The Braider*, Issue No. 8, pp. 173-176.

half-cycle 5	$i \leq 1$	:	$L \longrightarrow R$	o.
half-cycle 6	$i \leq 2$	:	$L \longleftarrow R$	$o - u$.
half-cycle 7	$i \leq 2$	:	$L \longrightarrow R$	$o - u$.
half-cycle 8	$i \leq 3$	:	$L \longleftarrow R$	$o - u - o$.
half-cycle 9	$i \leq 3$	:	$L \longrightarrow R$	$o - u - o$.
half-cycle 10	$i \leq 4$	:	$L \longleftarrow R$	$o - u - o - u$.
half-cycle 11	$i \leq 4$	:	$L \longrightarrow R$	$o - u - o - u$.
half-cycle 12	$i \leq 5$	:	$L \longleftarrow R$	$o - u - o - u$.

Next we braid in the $A = 5$ Semi-Standard Herringbone Pineapple Knot the over–under coded $P_c/B^* = 5/6$ Regular Knot between its left bight-boundary 2 and its right bight-boundary 2. For this knot $P_c/B^* = 5/6$, with $A = 2$ and $l_i = 2$; $r_i = 1$. Hence from its associated general algorithm diagram we obtain the following half-cycle algorithms:

half-cycle 13		:	$L \longrightarrow R$	$u - o - u - o$.
half-cycle 14	$i = 0$	:	$L \longleftarrow R$	$o - u - o - u$.
half-cycle 15	$i = 0$	:	$L \longrightarrow R$	$u - o - u - o$.
half-cycle 16	$i \leq 1$	:	$L \longleftarrow R$	$2o - u - o - u$.
half-cycle 17	$i \leq 1$	:	$L \longrightarrow R$	$u - 2o - u - o$.
half-cycle 18	$i \leq 2$	:	$L \longleftarrow R$	$2o - 2u - o - u$.
half-cycle 19	$i \leq 2$	:	$L \longrightarrow R$	$u - 2o - 2u - o$.
half-cycle 20	$i \leq 3$	:	$L \longleftarrow R$	$2o - 2u - 2o - u$.
half-cycle 21	$i \leq 3$	:	$L \longrightarrow R$	$u - 2o - 2u - 2o$.
half-cycle 22	$i \leq 4$	:	$L \longleftarrow R$	$2o - 2u - 2o - 2u$.
half-cycle 23	$i \leq 4$	:	$L \longrightarrow R$	$u - 2o - 2u - 2o - u$.
half-cycle 24	$i \leq 5$	:	$L \longleftarrow R$	$2o - 2u - 2o - 2u$.

Next we braid in the $A = 5$ Semi-Standard Herringbone Pineapple Knot the over–under coded $P_c/B^* = 5/6$ Regular Knot between its left bight-boundary 3 and its right bight-boundary 1. For this knot $P_c/B^* = 5/6$, with $A = 3$ and $l_i = 3$; $r_i = 1$. Hence we obtain the following half-cycle algorithms from its associated general algorithm diagram:

half-cycle 25		:	$L \longrightarrow R$	$2u - 2o - 2u - 2o$.
half-cycle 26	$i = 0$	:	$L \longleftarrow R$	$2o - 2u - 2o - 2u$.
half-cycle 27	$i = 0$	:	$L \longrightarrow R$	$2u - 2o - 2u - 2o$.
half-cycle 28	$i \leq 1$	:	$L \longleftarrow R$	$3o - 2u - 2o - 2u$.
half-cycle 29	$i \leq 1$	:	$L \longrightarrow R$	$2u - 3o - 2u - 2o$.
half-cycle 30	$i \leq 2$	:	$L \longleftarrow R$	$3o - 3u - 2o - 2u$.
half-cycle 31	$i \leq 2$	:	$L \longrightarrow R$	$2u - 3o - 3u - 2o$.
half-cycle 32	$i \leq 3$	:	$L \longleftarrow R$	$3o - 3u - 3o - 2u$.
half-cycle 33	$i \leq 3$	:	$L \longrightarrow R$	$2u - 3o - 3u - 3o$.
half-cycle 34	$i \leq 4$	:	$L \longleftarrow R$	$3o - 3u - 3o - 3u$.
half-cycle 35	$i \leq 4$	:	$L \longrightarrow R$	$2u - 3o - 3u - 3o - u$.
half-cycle 36	$i \leq 5$	:	$L \longleftarrow R$	$3o - 3u - 3o - 3u$.

Next we braid in the $A = 5$ Semi-Standard Herringbone Pineapple Knot the over–under coded $P_c/B^* = 3/6$ Semi-Regular Knot between its left bight-boundary 5 and its right bight-boundary 4. For this knot $P_c/B^* = 3/6$, with $A = 4$ and $l_i = 4$; $r_i = 4$. Hence from its associated general algorithm diagram we obtain the following half-cycle algorithms:

half-cycle 37 : $L \longrightarrow R$ $3u - 3o - 3u$.
half-cycle 38 $i = 0$: $L \longleftarrow R$ $3u - 3o - 3u$.
half-cycle 39 $i = 0$: $L \longrightarrow R$ $3u - 3o - 3u$.
half-cycle 40 $i \leq 1$: $L \longleftarrow R$ $3u - 3o - 3u$.
half-cycle 41 $i = X$: $L \longrightarrow R$ $3u - 4o - 3u$.
half-cycle 42 $i = 0\,; i = X$: $L \longleftarrow R$ $3u - 4o - 3u$.
half-cycle 43 $i = 0\,; i = X$: $L \longrightarrow R$ $3u - 4o - 3u$.
half-cycle 44 $i \leq 1\,; i = X$: $L \longleftarrow R$ $3u - 4o - 3u$.
half-cycle 45 $i = X, Y$: $L \longrightarrow R$ $3u - 4o - 4u$.
half-cycle 46 $i = 0\,; i = X, Y$: $L \longleftarrow R$ $3u - 4o - 4u$.
half-cycle 47 $i = 0\,; i = X, Y$: $L \longrightarrow R$ $3u - 4o - 4u$.
half-cycle 48 $i \leq 1\,; i = X, Y$: $L \longleftarrow R$ $3u - 4o - 4u$.

Finally we braid in the $A = 5$ Semi-Standard Herringbone Pineapple Knot the over–under coded $P_c/B^* = 3/6$ Semi-Regular Knot between its left bight-boundary 4 and its right bight-boundary 5. For this knot $P_c/B^* = 3/6$, with $A = 5$ and $l_i = 4; r_i = 5$. Hence from its associated general algorithm diagram we obtain the following half-cycle algorithms:

half-cycle 49 : $L \longrightarrow R$ $3u - 4o - 4u$.
half-cycle 50 $i = 0$: $L \longleftarrow R$ $4u - 4o - 3u$.
half-cycle 51 $i = 0$: $L \longrightarrow R$ $3u - 4o - 4u$.
half-cycle 52 $i \leq 1$: $L \longleftarrow R$ $4u - 4o - 3u$.
half-cycle 53 $i = X$: $L \longrightarrow R$ $3u - 5o - 4u$.
half-cycle 54 $i = 0\,; i = X$: $L \longleftarrow R$ $4u - 5o - 3u$.
half-cycle 55 $i = 0\,; i = X$: $L \longrightarrow R$ $3u - 5o - 4u$.
half-cycle 56 $i \leq 1\,; i = X$: $L \longleftarrow R$ $4u - 5o - 3u$.
half-cycle 57 $i = X, Y$: $L \longrightarrow R$ $3u - 5o - 5u$.
half-cycle 58 $i = 0\,; i = X, Y$: $L \longleftarrow R$ $4u - 5o - 4u$.
half-cycle 59 $i = 0\,; i = X, Y$: $L \longrightarrow R$ $3u - 5o - 5u$.
half-cycle 60 $i \leq 1\,; i = X, Y$: $L \longleftarrow R$ $4u - 5o - 4u$.

The 5-pass Standard Herringbone Pineapple Knot has now been completed.

Although the general algorithm diagram for an interbraided over–under coded Regular or Semi-Regular Knot between the bight-boundaries l_i and r_i in the coding-layout of Fig. 503 for a Standard or Semi-Standard Herrinbone Pineapple Knot is

$$\begin{array}{ccccccccccc}
\longrightarrow & l_i - 1 & A-1 & A-1 & A-1 & \ldots & A-1 & A-1 & r_i - 1 & & \\
 & 0 & \star & \star & \star & \ldots & \star & \star & \star & & \\
 & \backslash & / & \backslash & / & \ldots & \backslash & / & \backslash & / & \\
 & & \star & \star & \star & \ldots & \star & \star & \star & 0 & \\
 & & l_i - 1 & A-1 & A-1 & \ldots & A-1 & A-1 & A-1 & r_i - 1 & \longleftarrow
\end{array}$$

where the positions of the stars are being occupied by the i-values of the complementary bight-number scheme associated with the knot to be interbraided, and where the reference value above an upper star, or below a lower star, increases by 1 when its associated i-value is applicable to the half-cycle concerned, excepting the **first** entry $(l_i - 1)u$ which remains the same for each of the lower-left to upper-right half-cycles, and the **first** entry $(r_i - 1)u$ which remains the same for each of the lower-right to upper-left half-cycles, while the general algorithm diagram for an interbraided over–under coded Regular or Semi-Regular Knot between the bight-boundaries l_i and r_i in the coding-layout of Fig. 504 for a Standard or Semi-Standard Herrinbone Pineapple Knot is[†]

$$\begin{array}{ccccccccc}
\longrightarrow & l_i-1 & A-1 & A-1 & \ldots & A-1 & A-1 & A-1 & r_i-1 \\
0 & \star & \star & \star & \ldots & \star & \star & \star & \\
/ & \backslash & / & \backslash & \ldots & / & \backslash & / & \backslash \\
 & \star & \star & \star & \ldots & \star & \star & \star & 0 \\
l_i-1 & A-1 & A-1 & A-1 & \ldots & A-1 & A-1 & r_i-1 & \longleftarrow
\end{array}$$

where the positions of the stars are being occupied by the i-values of the complementary bight-number scheme associated with the knot to be interbraided, and where the reference value above an upper star, or below a lower star, increases by 1 when its associated i-value is applicable to the half-cycle concerned, excepting the **last** entry $(r_i-1)u$ which remains the same for each of the lower-left to upper-right half-cycles, and the **last** entry $(l_i-1)u$ which remains the same for each of the lower-right to upper-left half-cycles, the final Standard or Semi-Standard Herringbone Pineapple knots for a given A, x and B^* are identical.

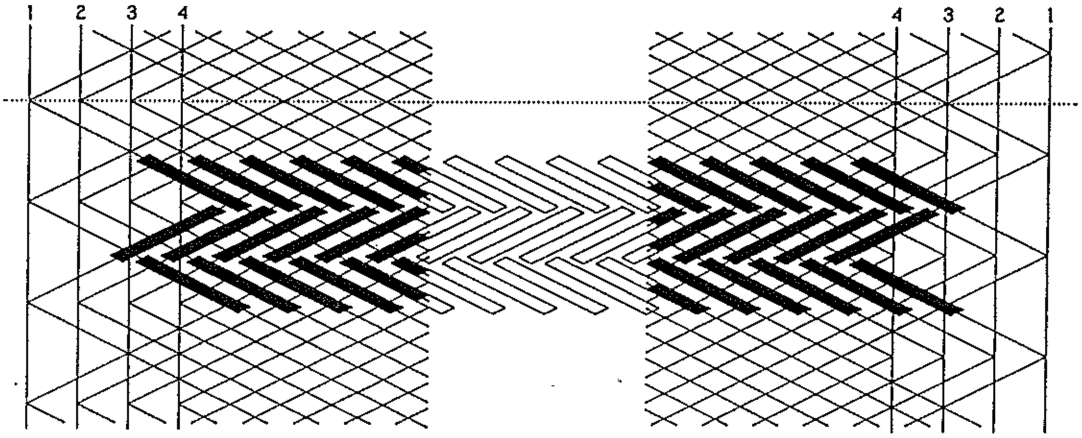

Fig. 503 — The Standard and Semi-Standard Herringbone Pineapple Knots.

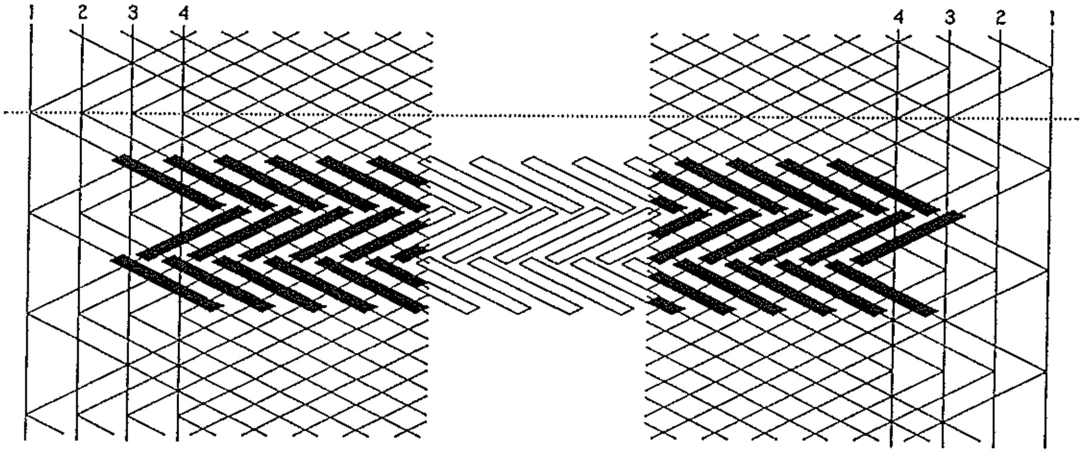

Fig. 504 — The Standard and Semi-Standard Herringbone Pineapple Knots.

[†] See *The Braider*, Issue No. 23, pp. 525-526.

THE BRAIDER

No.28 **NOVEMBER 2001.**

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A quarterly publication
for
the braiding artisan

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THE BRAIDER'S NOTEBOOK

In some previous issues of *The Braider* we have touched on some aspects relating to "discoveries" in the braiding arena.[†] Even today, as in the past, a braid is generally treated as a unique entity by most braiders, and consequently its associated wider picture remains invisible to them. It is, however, just this wider picture which is so important for a full understanding of the possibilities offered by a particular braid. A "newly discovered" braid is nearly always a simple braid, and before one can proceed to more involved directly or indirectly associated braids, a grid-diagram of the "newly discovered" braid is in such a process essential and hence is a must. Many braiders and especially the editors of braiding publications don't seem to comprehend the importance of grid-diagrams as is testified by the fancy and not so fancy, but in technical terms useless illustrations in their published material. Lets have a look at one of the many of such examples that may be found in the Dutch publication *Het Knoopeknauwertje*[‡].

In *Het Knoopeknauwertje*, Issue No. 6, pp. 4-5, we find a description of **the CK knot**. This is a 'flat' knot, developed by Cornelis Kooiman of Rotterdam in the Netherlands, which has to serve the purpose of 'closing' the open end of, for example, a Regular Nested Cylindrical Braid which closes insufficiently at the bight-edge desired to do so. It should be pointed out that for a purely closure purpose, no separate knot or disc of any form should be used, but instead the nesting-number (A) of the Regular Nested Cylindrical Braid at the bight-edge concerned should be large enough to achieve with four nests ($B^* = 4$) at this bight-edge a complete closure. It should also have been pointed out that the CK knot is only suitable for round braiding material, and not for normal flat material. However, such 'flat' closure-knots may be of some value when they either form an integral part with the knot which doesn't close, or are extended to a further decorative knot on the outside of it. A series of drawings by the editor, of the usual 2-dimensional lay out, accompany the CK knot in *Het Knoopeknauwertje*, and depict a way in which it may be constructed, but the all important grid-diagram of the knot has not been shown. Consequently little if anything can be learned from this knot, which is a great shame since the grid-diagram of this elementary CK knot clearly points to several other not only somewhat more decorative 'flat' closure-knots, but also to 'flat' closure-knots of a different diameter.

Below, in Fig. 505, we have shown the grid-diagram of the CK knot described in *Het Knoopeknauwertje*. The front and back appearances of the knot are shown in Fig. 506.

The knot can of course readily be constructed with the aid of its grid-diagram. Hence it would have been more fruitfull when the series of drawings accompanying this knot in *Het Knoopeknauwertje* had been replaced by its grid-diagram.

It will readily be seen that various modifications to this grid-diagram lead to other

---

[†] See for example *The Braider*, Issue No. 8, pg. 162, and Issue No. 11, pg. 245.

[‡] We have selected here the publication *Het Knoopeknauwertje* since its name intends to mean *The unraveller of knot-mysteries*, but does unfortunately nothing in that respect. The reader should, however, be aware that the contents of an article in *Het Knoopeknauwertje* does not necessarily represent the true contents nor the gist of the article submitted by its author, as besides the illustrations by the editor which apparently replace those by the author, the editor states that he reserves the right to abbreviate a submitted article, but does not take any responsibility for its contents!!!

'flat' closure-knots for round-string braiding material, one of which is depicted in Fig. 507. The front and back appearances of this knot are shown in Fig. 508.

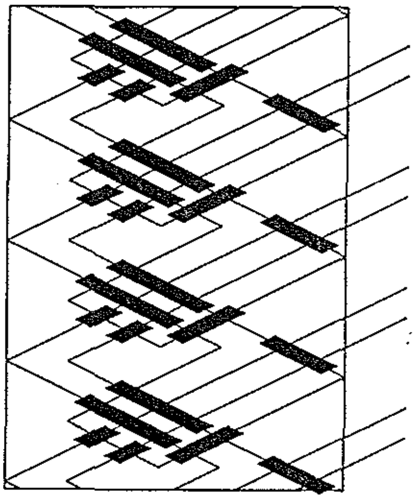
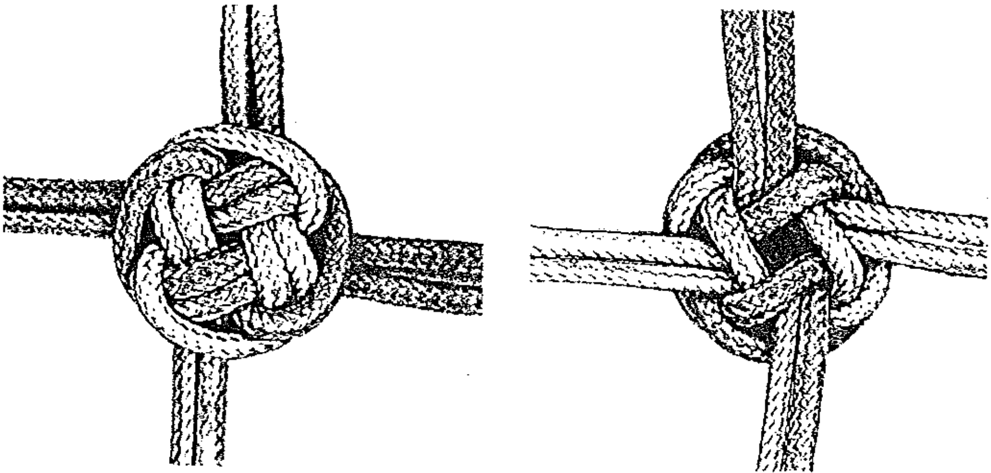


Fig. 505 — The CK knot described in *Het Knoopeknauwertje.*



Front

Back

Fig. 506 — The front and the back of the knot in Fig. 505.

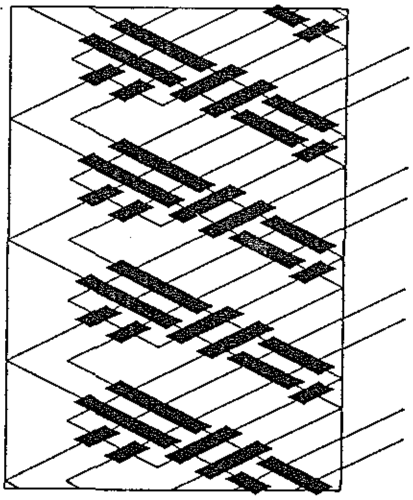


Fig. 507 — One of the several possible modifications of the diagram in Fig. 505.

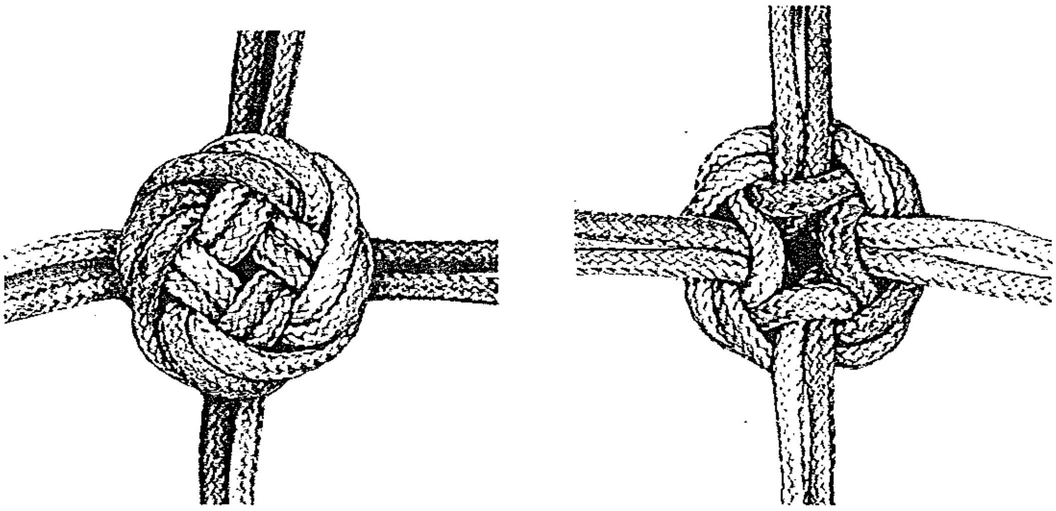

Front Back

Fig. 508 — The front and the back of the knot in Fig. 507.

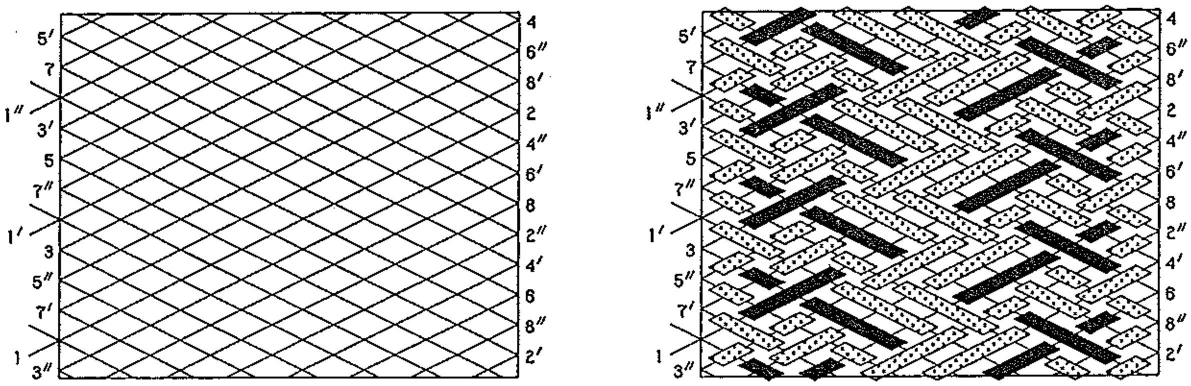
These 'flat' knots may be braided as cylindrical knots (which they in fact are, and which is clearly shown by their grid-diagrams), in which case the fiador knot mandrel described in *The Braider*, Issue No. 5, pg. 103, will be a handy braiding aid. After braiding these closure-knots on this mandrel, followed by decreasing their cylindrical diameter sufficiently, the string-ends should be pulled so that the knots become 'flat' and tight. They can then be used as closure-knots for closing the open end of a Regular Nested Cylindrical Braid which has too many nests or too small a nesting-number for closing its bight-edge concerned. The string-ends of a closure-knot are hereby fed through the Regular Nested Cylindrical Braid near its end, ensuring that rotational symmetry is maintained, and a further decorative knot, such as a star-knot for example, may then be braided with these ends. Maintaining rotational symmetry requires four or a multiple of four nests of bights on the bight-edge involved.

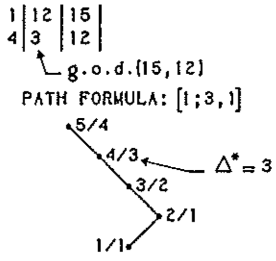
The Knot Naming Game and Knot Classification

To many people the discovery or development of a knot new to them is a great event and, without any further thought or investigation, they eagerly give their newly discovered knot a name. They don't realise that generally their "new" knot represents only a particular individual of an often large family of knots, and that in general such a family of knots consists of distinct sub-families. Hence in order to prevent the creation of chaos in the knot naming game, one should not rush into giving their "new" knot a name until a thorough investigation proves that such an action is justified. Let's look at a typical case where the naming of such a knot becomes senseless.

In the Dutch bimonthly knotting-magazine *Het Knoopeknauwertje*, No. 9, pp. 17-20, we find **the Camilla Knot** by Tom Hall.[†] It should of course have been pointed out in *Het Knoopeknauwertje* (and most likely was in Tom's original letter) that the upper and lower leftmost knots on pg. 17 are similar: the two light strings in the upper drawing are the two dark strings in the lower drawing and the dark string in the upper drawing is the light string in the lower drawing. These are the knot in Fig. 509 below.

[†] From the article it is clear that Tom Hall provided also half-cycle algorithm-tables for his Camilla Knots, but instead of these being published, the editor apparently tries to impress his audience with the output of a by him generated computer-program for





L → R

B	A	3	B	A	2	B	A	1	B	A	0	B	A	HALF-CYCLE			
U	O	O	O	U	U	U	U	O	O	O	U	O	U	1	3	5	7
U	U	O	O	U	U	U	O	O	O	U	U	O	O	1′	3′	5′	7′
O	U	O	U	U	O	O	O	U	O	U	U	U	O	1″	3″	5″	7″

L ← R

A	B	0	A	B	1	A	B	2	A	B	3	A	B	HALF-CYCLE			
U	O	U	O	O	O	U	U	U	U	O	O	O	U	2	4	6	8
O	O	U	U	O	O	O	U	U	U	O	O	U	U	2′	4′	6′	8′
O	U	U	U	O	U	O	O	O	U	U	O	U	O	2″	4″	6″	8″

→ L → R R → L

B	A	3	B	A	2	B	A	1	B	A	0	B	A	HALF-CYCLE				HALF-CYCLE			
U	O	O	O	U	U	U	U	O	O	O	U	O	U	1	3	5	7	2	4	6	8
U	U	O	O	U	U	U	O	O	O	U	U	O	O	1′	3′	5′	7′	2′	4′	6′	8′
O	U	O	U	U	O	O	O	U	O	U	U	U	O	1″	3″	5″	7″	2″	4″	6″	8″

Fig. 509 — $p/b = 15/12$ with colour-pattern 1.

In good braiding practice the Standing Ends of the strings of a Semi-Regular Knot should of course be distributed over its bight-boundary circumference as regularly as possible.[†] Hence the distribution of the Standing Ends should never be as in the published computer-program's output where they are placed in adjacent bight-points.

Since the string-run and the coding (**not the colour-pattern!!!**) of the knot in Fig. 509 (which is the string-run and knot in the uppermost row of diagrams in Fig. 510) is identical to the string-run and coding of the other four knots in Fig. 510, the algorithm

---

Regular and Semi-Regular Knots. Such computer-programs, however, serve no useful purpose, in fact they may contain braiding-instructions, as in the case here with the component starting positions, which should not be followed.

[†] Refer also to *The Braider*, Issue No. 8, pg. 173.

diagram in Fig. 509 applies to all the knots in Fig. 510. Let the Standing End half-cycles of the consecutive strings respectively be 1, $1'$ and $1''$, with their Standing Ends S, S' and S'', then the colour-patterns of the five consecutive knots in Fig. 510 are obtained by means of the following string-colours:

1. Colour X for string S; colour Y for strings S' and S''.
2. Colour X for string S'; colour Y for strings S and S''.
3. Colour X for string S''; colour Y for strings S and S'.
4. Colour X for string S'; colour Y for string S; colour Z for string S''.
5. Colour X for strings S, S' and S''.

Note that these knots are not suitable for normal flat string since the thickness of the strings must be sufficient in order to prevent the strings from sliding over each other.

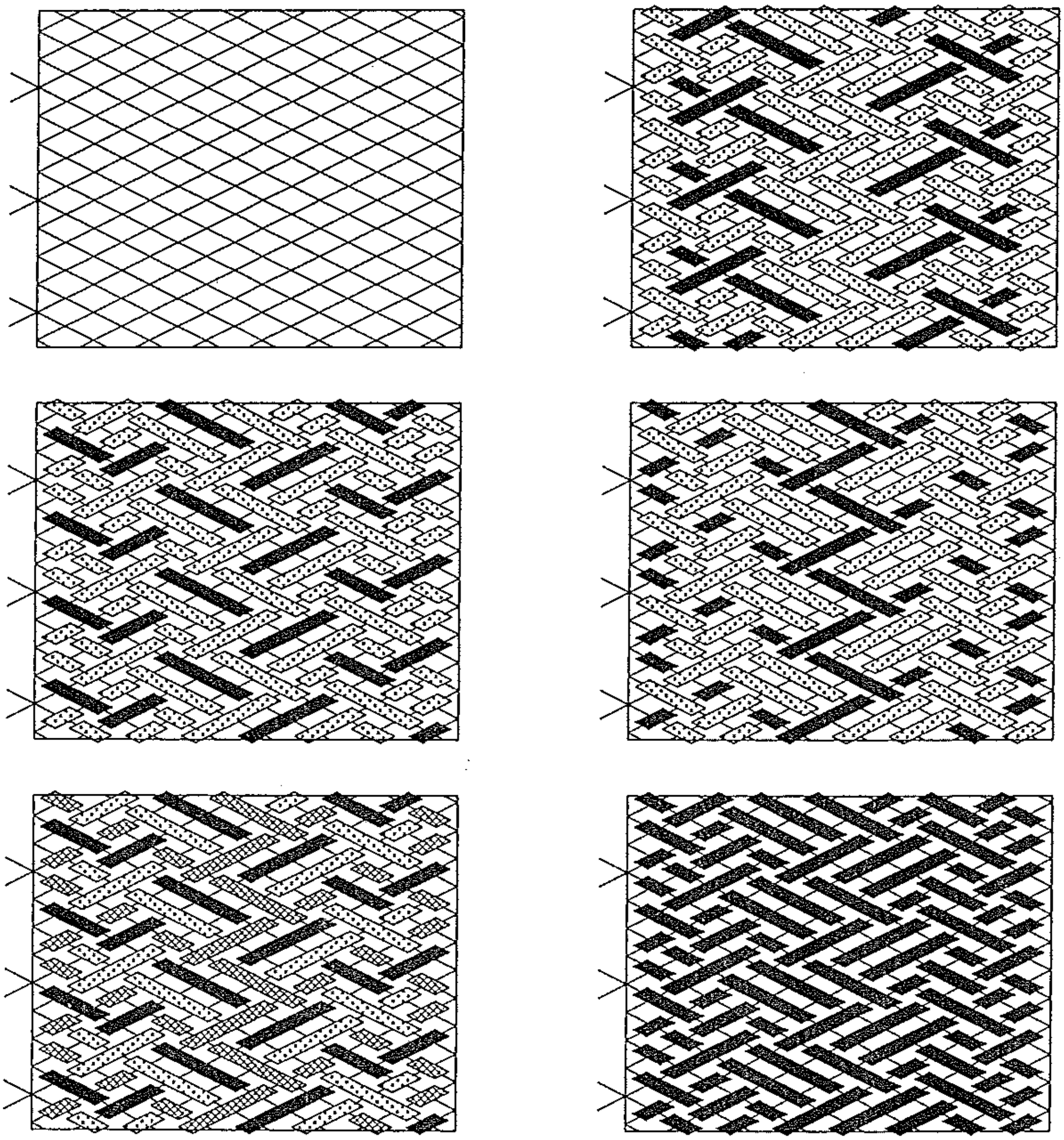


Fig. 510 — The five colour-patterns for the knot $p/b = 15/12$.

The totally dissimilar colour-patterns in Fig. 510 show that the name 'Camilla Knot' has a restricted meaning even for these $p/b = 15/12$ Semi-Regular Knots; one needs to attach at least a further indicator to this name in order to know which colour-pattern one is talking about. But even then the identification problems are far from being solved. Let's, however, first tabulate the half-cycle algoritms for the knots in Fig. 510:

Component 1 (starts at 1):

1. $L \longrightarrow R$: Free run.
2. $(i = 0)$ $R \longrightarrow L$: u.
3. $(i = 0)$ $L \longrightarrow R$: u.
4. $(i \leq 1)$ $R \longrightarrow L$: $o - u$.
5. $(i \leq 1)$ $L \longrightarrow R$: $o - u$.
6. $(i \leq 2)$ $R \longrightarrow L$: $u - o - u$.
7. $(i \leq 2)$ $L \longrightarrow R$: $u - o - u$.
8. $(i \leq 3)$ $R \longrightarrow L$: $o - u - o - u$.

Component 2 (starts at $1'$):

$1'$. $(i = A)$ $L \longrightarrow R$: $2u - o - u - o$.
$2'$. $(i = A, 0)$ $R \longrightarrow L$: $2u - o - 2u - o$.
$3'$. $(i = A, 0)$ $L \longrightarrow R$: $2u - o - 2u - o$.
$4'$. $(i = A, \leq 1)$ $R \longrightarrow L$: $2u - 2o - 2u - o$.
$5'$. $(i = A, \leq 1)$ $L \longrightarrow R$: $2u - 2o - 2u - o$.
$6'$. $(i = A, \leq 2)$ $R \longrightarrow L$: $3u - 2o - 2u - o$.
$7'$. $(i = A, \leq 2)$ $L \longrightarrow R$: $3u - 2o - 2u - o$.
$8'$. $(i = A, \leq 3)$ $R \longrightarrow L$: $u - o - 2u - 2o - 2u - o$.

Component 3 (starts at $1''$):

$1''$. $(i = A, B)$ $L \longrightarrow R$: $o - 3u - 3o - 2u - o$.
$2''$. $(i = A, B, 0)$ $R \longrightarrow L$: $o - 3u - 3o - 3u - o$.
$3''$. $(i = A, B, 0)$ $L \longrightarrow R$: $o - 3u - 3o - 3u - o$.
$4''$. $(i = A, B, \leq 1)$ $R \longrightarrow L$: $o - 3u - 2o - u - o - 3u - o$.
$5''$. $(i = A, B, \leq 1)$ $L \longrightarrow R$: $o - 3u - 2o - u - o - 3u - o$.
$6''$. $(i = A, B, \leq 2)$ $R \longrightarrow L$: $o - 3u - 3o - u - o - 3u - o$.
$7''$. $(i = A, B, \leq 2)$ $L \longrightarrow R$: $o - 3u - 3o - u - o - 3u - o$.
$8''$. $(i = A, B, \leq 3)$ $R \longrightarrow L$: $o - u - o - 2u - 3o - u - o - 3u - o$.

The **coding** arrangement near the bight-edges together with the 3-pass Herringbone-coding in the remaining central area requires that the number of bights b has to be a multiple of 3 (hence $b = 3y$, where y is a natural number[†]) and, when disregarding any colour-pattern requirements, that the number of parts p has also to be a multiple of 3 ($p = 15 + 6r = 3(5 + 2r) = 3x$, where $x = 5 + 2r$ with r being a whole number[‡]). Further conditions to the value of r are given by the string-colours:

When the three essential strings have all the same colour, r can be $0, 1, 2, 3, \cdots$. There is no special limit case.

When the two upper essential strings have the same colour which differs from the colour of the lowermost essential string (as in the uppermost right-hand grid-diagrams in Fig. 510 and Fig. 511), the associated general family-clan requires that r can be $1, 2, 3, 4, \cdots$. When $r = 0$ we have here a special limit case as shown by the uppermost right-hand grid-diagram in Fig. 510.

[†] Natural numbers are the numbers $1, 2, 3, \cdots$.

[‡] Whole numbers are the numbers $0, 1, 2, 3, \cdots$.

When lower and upper essential strings have the same colour which differs from the colour of the central essential string (as in the central left-hand grid-diagrams in Fig. 510 and Fig. 511), the associated general family-clan requires that r can be $1, 2, 3, 4, \cdots$. When $r = 0$ we have here a special limit case as shown by the central left-hand grid-diagram in Fig. 510.

When the two lower essential strings have the same colour which differs from the colour of the upper essential string (as in the central right-hand grid-diagrams in Fig. 510 and Fig. 511, and the central grid-diagram in Fig. 512), the associated general family-clan requires that r can be $2, 3, 4, \cdots$. We have here a special limit case when $r = 0$ as shown by the central right-hand grid-diagram in Fig. 510, and another special limit case when $r = 1$ as shown by the central right-hand grid-diagram in Fig. 511.

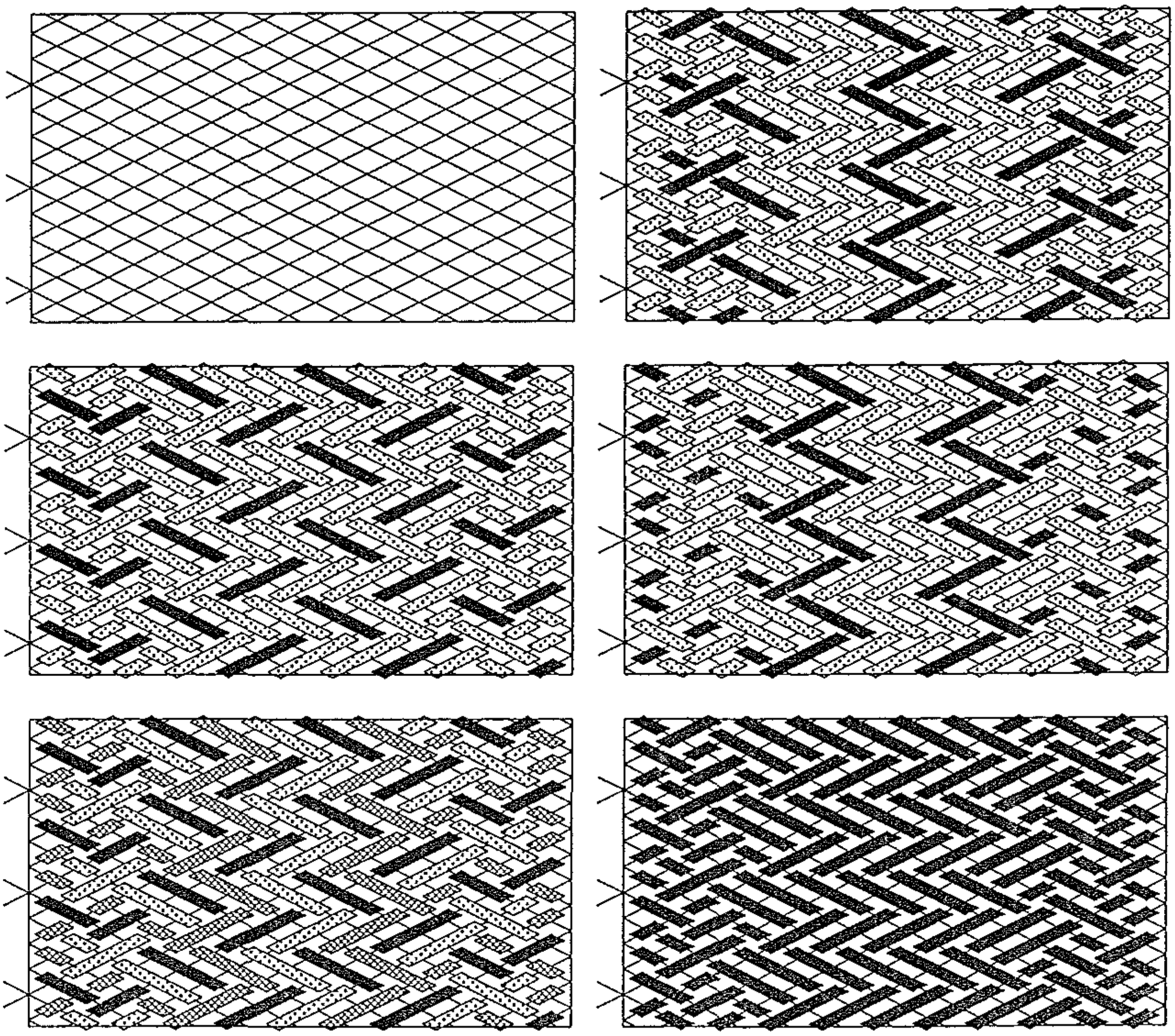


Fig. 511 — The five colour-patterns for the knot $p/b = 21/12$.

When each string has a different colour (as in the lowermost left-hand grid-diagrams in Fig. 510 and Fig. 511, and the lowermost grid-diagram in Fig. 512), the associated general family-clan requires that r can be $2, 3, 4, \cdots$. We have here a special limit case when $r = 0$ as shown by the lowermost left-hand grid-diagram in Fig. 510, and another special limit case when $r = 1$ as shown by the lowermost left-hand grid-diagram in Fig. 511.

The g.c.d. $(p, b) = 3 \times$ g.c.d. $(x, y) = 3t$. Hence a total of $3t$ strings are required.

Only when $t = 1$ and g.c.d.$(3, y) = 1$ can we distribute the string-ends regularly over a bight-boundary; in general we ensure that this is the case.

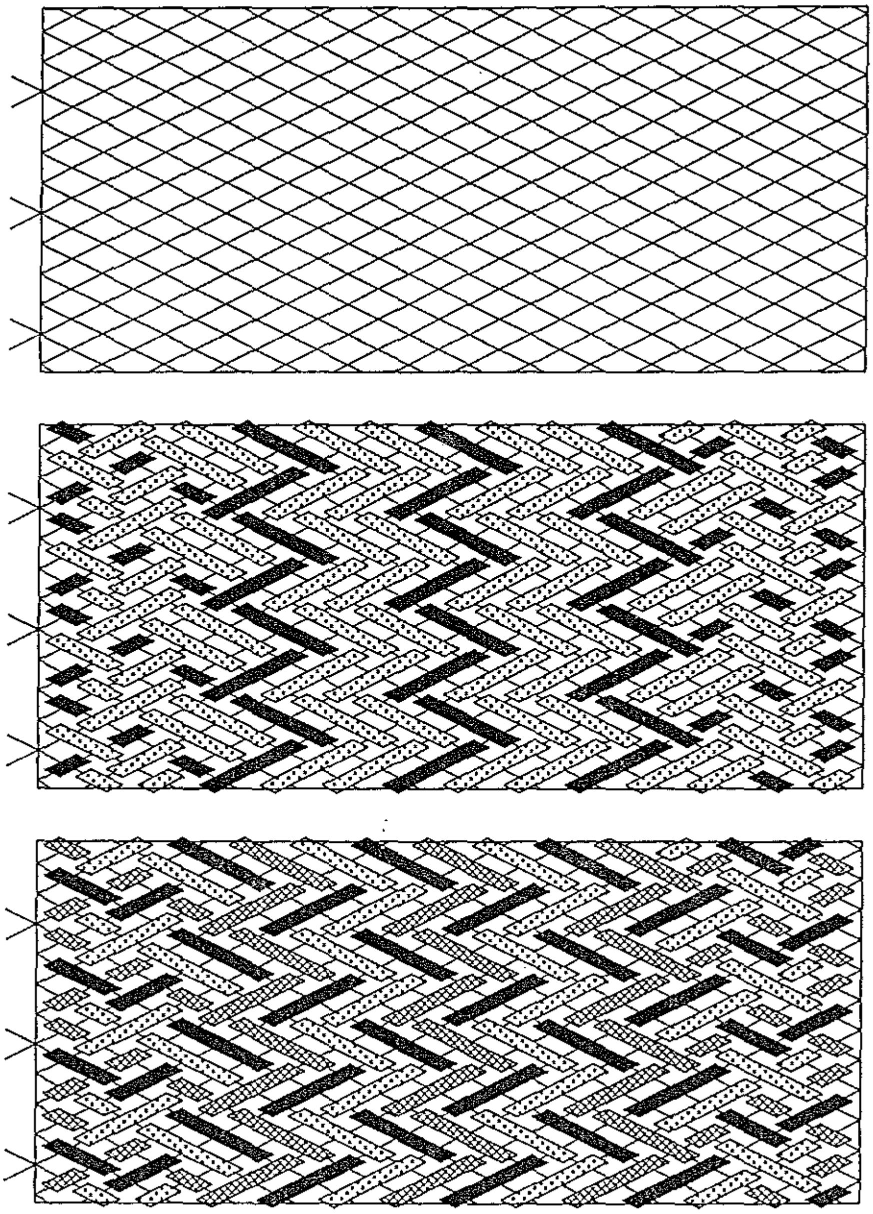


Fig. 512 — $p/b = 27/12$.

For the grid-diagrams in Fig. 511 the value of r is equal to 1, hence $p = 21$. Their associated algorithm diagram is shown in Fig. 513, and their half-cycle algorithms are:

Component 1 (starts at 1):

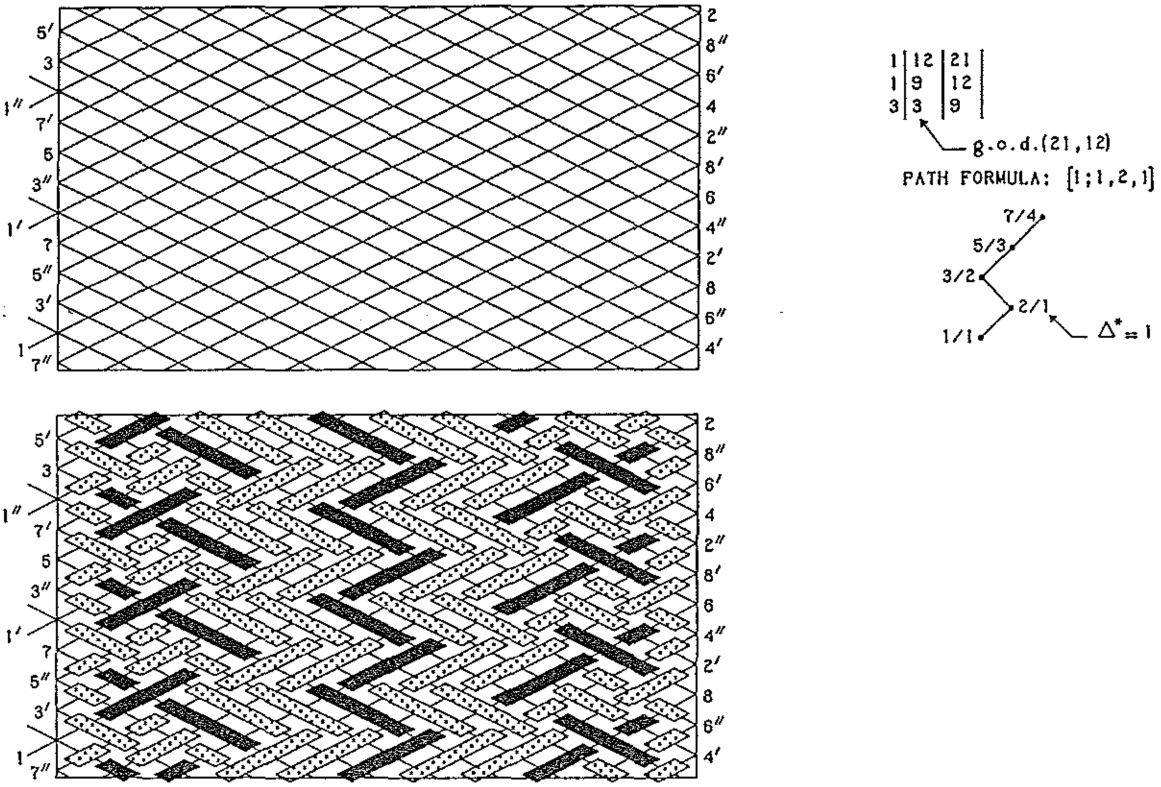
1. $L \longrightarrow R$: Free run.
2. $(i = 0)$ $R \longrightarrow L$: o.
3. $(i = 0)$ $L \longrightarrow R$: o.
4. $(i \leq 1)$ $R \longrightarrow L$: $3o$.
5. $(i \leq 1)$ $L \longrightarrow R$: $3o$.
6. $(i \leq 2)$ $R \longrightarrow L$: $o - u - 2o - u$.
7. $(i \leq 2)$ $L \longrightarrow R$: $o - u - 2o - u$.
8. $(i \leq 3)$ $R \longrightarrow L$: $o - 2u - 2o - u$.

Component 2 (starts at $1'$):

1'. $(i = A)$ $L \longrightarrow R:$ $2u - o - u - o - u - o.$
2'. $(i = A, 0)$ $R \longrightarrow L:$ $2u - o - 2u - o - u - o.$
3'. $(i = A, 0)$ $L \longrightarrow R:$ $2u - o - 2u - o - u - o.$
4'. $(i = A, \leq 1)$ $R \longrightarrow L:$ $u - o - u - o - 2u - 2o - u - o.$
5'. $(i = A, \leq 1)$ $L \longrightarrow R:$ $u - o - u - o - 2u - 2o - u - o.$
6'. $(i = A, \leq 2)$ $R \longrightarrow L:$ $u - o - 2u - o - 2u - 2o - 2u - o.$
7'. $(i = A, \leq 2)$ $L \longrightarrow R:$ $u - o - 2u - o - 2u - 2o - 2u - o.$
8'. $(i = A, \leq 3)$ $R \longrightarrow L:$ $u - o - 2u - 2o - 2u - 2o - 2u - o.$

Component 3 (starts at $1''$):

1''. $(i = A, B)$ $L \longrightarrow R:$ $o - 3u - 2o - 2u - 3o - 2u - o.$
2''. $(i = A, B, 0)$ $R \longrightarrow L:$ $o - 3u - 2o - 2u - 4o - 2u - o.$
3''. $(i = A, B, 0)$ $L \longrightarrow R:$ $o - 3u - 2o - 2u - 4o - 2u - o.$
4''. $(i = A, B, \leq 1)$ $R \longrightarrow L:$ $o - u - o - 2u - 2o - 2u - 3o - u - o - 2u - o.$
5''. $(i = A, B, \leq 1)$ $L \longrightarrow R:$ $o - u - o - 2u - 2o - 2u - 3o - u - o - 2u - o.$
6''. $(i = A, B, \leq 2)$ $R \longrightarrow L:$ $o - u - o - 2u - 3o - 2u - 3o - u - o - 3u - o.$
7''. $(i = A, B, \leq 2)$ $L \longrightarrow R:$ $o - u - o - 2u - 3o - 2u - 3o - u - o - 3u - o.$
8''. $(i = A, B, \leq 3)$ $R \longrightarrow L:$ $o - u - o - 2u - 3o - 3u - 3o - u - o - 3u - o.$



⟶																				L ⟶ R				R ⟶ L			
B	A	1	B	A	2	B	A	3	B	A	0	B	A	1	B	A	2	B	A	HALF-CYCLE				HALF-CYCLE			
U	O	O	O	U	U	U	U	U	O	O	O	U	U	O	O	O	U	O	U	1	3	5	7	2	4	6	8
U	U	O	O	U	U	U	O	O	O	U	U	U	O	O	O	U	U	O	O	1'	3'	5'	7'	2'	4'	6'	8'
O	U	O	U	U	O	O	O	U	U	U	O	O	O	U	O	U	U	U	O	1''	3''	5''	7''	2''	4''	6''	8''

Fig. 513 — $p/b = 21/12$.

Hence the completely dissimilar colour-patterns, associated with the Camilla Knots, require a further classification **within** the family of Camilla Knots, and consequently the name 'Camilla Knots' can be associated with the coding-pattern only. We can therefore only speak of '**a** Camilla Knot' when a colour-pattern is being taken into account, or of 'some Camilla Knots' when some colour-patterns are being considered, but not of '**the** Camilla Knot'.

Nested Cylindrical Braids

In Issue No. 22 of *The Braider* we saw on pg. 510 the formula:

$$P_{component} = 4\alpha + \frac{\alpha \cdot x - 2\sum (l_i + r_i)}{A} .$$

for the **Regular Nested Cylindrical Braids**[†].

When α is equal to A, then there is only one first-return string-run. Consequently, $\sum (l_i + r_i) = A(A+1)$, and hence:

$$P_{component} = 4A + x - 2(A+1) = 2A + x - 2 = P_{total} = P .$$

For the **Regular Nested Cylindrical Braids** we saw on pg. 570 in Issue No. 25 of *The Braider* that $\alpha = \dfrac{A}{\gamma}$. Hence $\gamma = 1$ for $\alpha = A$. Since $\gamma = \text{g.c.d.}(\Delta , A)$, it follows that for $\alpha = A$, hence for a single first-return string-run only, Δ and A must be coprime; furthermore since $\Delta = |y|_A$, it follows that y and A must be coprime.

The Regular Nested Cylindrical Braids with $\alpha = A$ and $\text{g.c.d.}(P , B^*) = 1$ are the **Perfect Regular Nested Cylindrical Braids**, and the Regular Nested Cylindrical Braids with $\alpha = A$ and $\text{g.c.d.}(P , B^*) \neq 1$ are the **Semi-Perfect Regular Nested Cylindrical Braids**.

There are two special cases of special interest, one in which $y = A - 1$, and one in which $y = A + 1$. In these two special cases we are able to superimpose on their string-runs a **Herringbone Pineapple coding** (see Figs. 514 and 515). With $\text{g.c.d.}(P , B^*) = 1$ we have then the **Perfect Herringbone Pineapple Knots**, and with $\text{g.c.d.}(P , B^*) \neq 1$ we have then the **Semi-Perfect Herringbone Pineapple Knots**.

On pg. 567, Issue No. 25 of *The Braider* we saw that:

$$k = \left| \frac{x - y - 2}{2} \right|_A .$$

Hence for:

$$y = A - 1 \quad \rightarrow \quad k = \left|\tfrac{x-A-1}{2}\right|_A \quad \left\{ \begin{array}{lcl} A = \text{odd} & \rightarrow & x = \text{even} \\ A = \text{even} & \rightarrow & x = \text{odd} \end{array} \right.$$

$$y = A + 1 \quad \rightarrow \quad k = \left|\tfrac{x-A-3}{2}\right|_A \quad \left\{ \begin{array}{lcl} A = \text{odd} & \rightarrow & x = \text{even} \\ A = \text{even} & \rightarrow & x = \text{odd} \end{array} \right.$$

[†] See *The Braider*, Issue No. 22, pg. 502, for the definition of *Regular Nested Cylindrical Braids*.

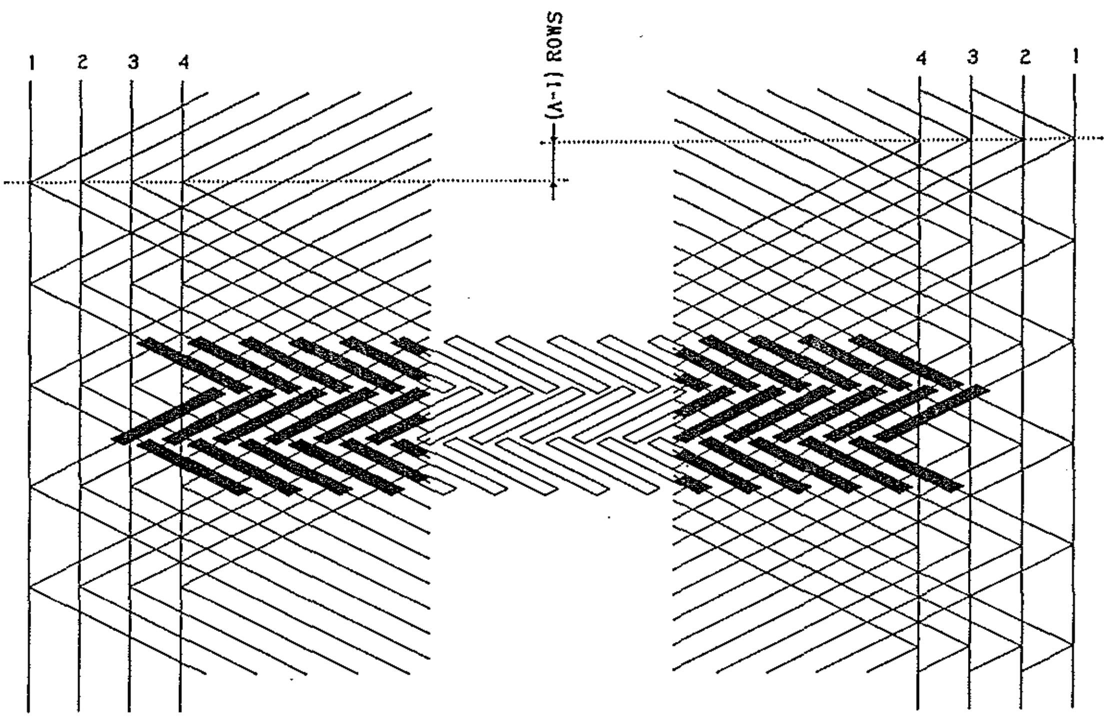


Fig. 514 — $y = A - 1$; Herringbone Pineapple coding.

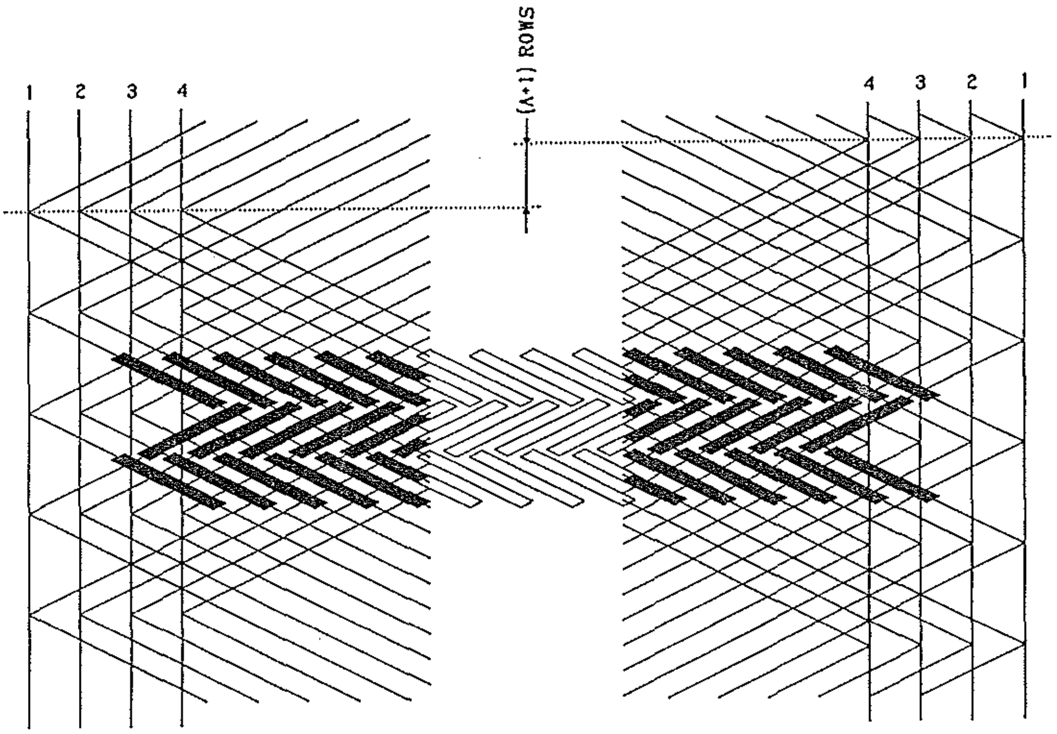


Fig. 515 — $y = A + 1$; Herringbone Pineapple coding.

In order to braid the **Perfect** and **Semi-Perfect Herringbone Pineapple Knots** we have to develop a method which enables us to obtain their half-cycle braiding algorithms in an easy way.

Note that a lower-left to upper-right half-cycle in a Regular Nested Cylindrical Braid has $2A + x - 1 - (l_i + r_i)$ crossings, and that a lower-right to upper-left half-cycle has $2A + x - 1 - (r_i + l_{i+1})$ crossings.

Observe for the $y = A - 1$ Herringbone Pineapple coding that a lower-left to upper-right half-cycle (hence a half-cycle from l_i to r_i) has the coding-pattern:

$$(l_i - 1) \text{ under} — A \text{ over} — A \text{ under} — \cdots — A \text{ over} — (r_i - 1) \text{ under},$$

and that a lower-right to upper-left half-cycle (hence a half-cycle from r_i to l_{i+1}) has the coding-pattern:

$$r_i \text{ under} — A \text{ over} — A \text{ under} — \cdots — A \text{ over} — l_{i+1} \text{ under}.$$

Observe for the $y = A + 1$ Herringbone Pineapple coding that a lower-left to upper-right half-cycle (hence a half-cycle from l_i to r_i) has the coding-pattern:

$$l_i \text{ under} — A \text{ over} — A \text{ under} — \cdots — A \text{ over} — r_i \text{ under},$$

and that a lower-right to upper-left half-cycle (hence a half-cycle from r_i to l_{i+1}) has the coding-pattern:

$$(r_i - 1) \text{ under} — A \text{ over} — A \text{ under} — \cdots — A \text{ over} — (l_{i+1} - 1) \text{ under}.$$

With the aid of the first-return string-run and its associated **nest-index numbers**[†] we can readily draw up the half-cycle pattern of the **Perfect** or **Semi-Perfect Herringbone Pineapple Knot** and read from it the braiding sequence for each half-cycle. The following two examples should make the process clear.

Example 1:

$A = 5; x = 14; y = A - 1 = 4$, hence $\Delta = A - 1 = 4$; $B^* = 3$, hence $B = A \cdot B^* = 15$. Thus $k = \left|\frac{x-A-1}{2}\right|_A = \left|\frac{14-5-1}{2}\right|_5 = 4$.

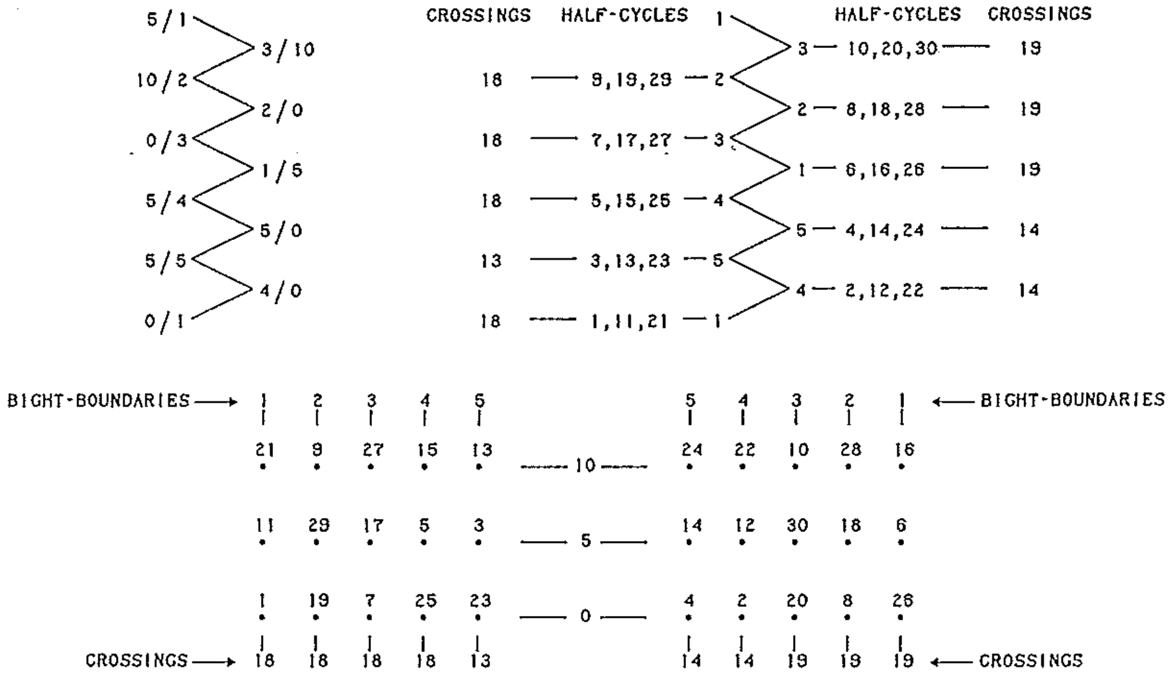


Fig. 516 — First-return string-run and half-cycle pattern.

At the upper left-hand side in Fig. 516 is depicted the first-return string-run with its associated nest-index numbers.

At the upper right-hand side in Fig. 516 are depicted the first-return string-run, the half-cycle numbers of the half-cycles, and the number of crossings which these half-cycles make in the finished knot. This layout at the upper right-hand side has been shown for additional clarification only since it can readily be dispensed with.

[†] See *The Braider*, Issue No. 26, pp. 592-599.

The depicted layout at the bottom of Fig. 516 gives the positions of the nests with respect to the nest-index numbers, the half-cycle numbers with respect to the nest-index numbers and bight-boundaries, and the number of crossings which the half-cycles make in the finished knot. This layout, which is the one we need for compiling the half-cycle braiding algorithms, is directly obtained from the first-return string-run with its associated nest-index numbers, hence from the upper left-hand layout in Fig. 516. The first step in its construction is shown in Fig. 517.

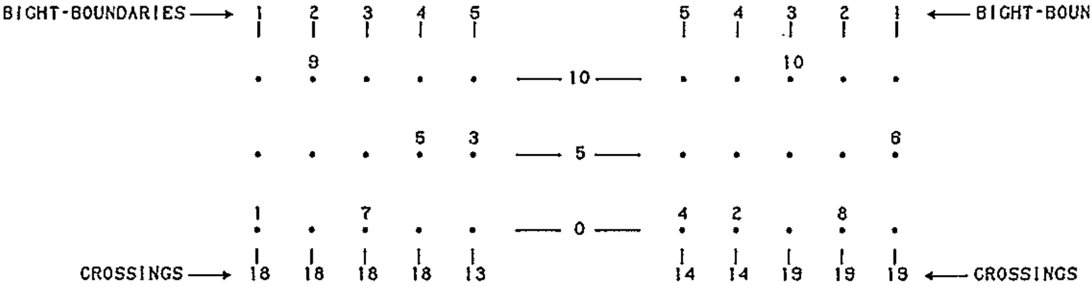


Fig. 517 — The first step in the construction of the half-cycle pattern.

At the top we set off the left-hand and right-hand bight-boundaries $1, \cdots, A$, hence $1, 2, 3, 4, 5$, then we set off the nest-index numbers $0, A, \cdots, (B^* - 1)A$, hence $0, 5, 10$, then we place the dots which represent the bight-points.

From the first-return string-run we obtain that half-cycle 1 runs from $l_i = 1$ to $r_i = 4$, and that it starts at nest-index number 0. Thus we can write 1 above the dot which lines up with left-hand bight-boundary 1 and nest-index number 0. We can calculate the number of crossings associated with a half-cycle which runs from $l_i = 1$ to $r_i = 4$ with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 18. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 1 end at right-hand bight-boundary 4, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 18 is placed at the bottom in line with left-hand bight-boundary 1.

From the first-return string-run we obtain that half-cycle 3 runs from $l_i = 5$ to $r_i = 5$, and that it starts at nest-index number 5. Thus we can write 3 above the dot which lines up with left-hand bight-boundary 5 and nest-index number 5. We calculate the number of crossings associated with a half-cycle which runs from $l_i = 5$ to $r_i = 5$ again with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 13. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 5 end at right-hand bight-boundary 5, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 13 is placed at the bottom in line with left-hand bight-boundary 5.

From the first-return string-run we obtain that half-cycle 5 runs from $l_i = 4$ to $r_i = 1$, and that it starts at nest-index number 5. Thus we can write 5 above the dot which lines up with left-hand bight-boundary 4 and nest-index number 5. We calculate the number of crossings associated with a half-cycle which runs from $l_i = 4$ to $r_i = 1$ again with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 18. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 4 end at right-hand bight-boundary 1, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 18 is placed at the bottom in line with left-hand bight-boundary 4.

From the first-return string-run we obtain that half-cycle 7 runs from $l_i = 3$ to $r_i = 2$, and that it starts at nest-index number 0. Thus we can write 7 above the dot which lines up with left-hand bight-boundary 3 and nest-index number 0. We calculate the number of crossings associated with a half-cycle which runs from $l_i = 3$ to $r_i = 2$ again with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 18. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 3 end at right-hand bight-boundary 2, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 18 is placed at the bottom in line with left-hand bight-boundary 3.

From the first-return string-run we obtain that half-cycle 9 runs from $l_i = 2$ to $r_i = 3$, and that it starts at nest-index number 10. Thus we can write 9 above the dot which lines up with left-hand bight-boundary 2 and nest-index number 10. We calculate the number of crossings associated with a half-cycle which runs from $l_i = 2$ to $r_i = 3$ again with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 18. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 2 end at right-hand bight-boundary 3, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 18 is placed at the bottom in line with left-hand bight-boundary 2.

The first-return string-run shows that half-cycle 2 runs from $r_i = 4$ to $l_{i+1} = 5$, and that it starts at nest-index number 0. Thus we can write 2 above the dot which lines up with right-hand bight-boundary 4 and nest-index number 0. We can calculate the number of crossings associated with a half-cycle which runs from $r_i = 4$ to $l_{i+1} = 5$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 14. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 4 end at left-hand bight-boundary 5, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 14 is placed at the bottom in line with right-hand bight-boundary 4.

The first-return string-run shows that half-cycle 4 runs from $r_i = 5$ to $l_{i+1} = 4$, and that it starts at nest-index number 0. Thus we can write 4 above the dot which lines up with right-hand bight-boundary 5 and nest-index number 0. We calculate the number of crossings associated with a half-cycle which runs from $r_i = 5$ to $l_{i+1} = 4$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 14. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 5 end at left-hand bight-boundary 4, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 14 is placed at the bottom in line with right-hand bight-boundary 5.

The first-return string-run shows that half-cycle 6 runs from $r_i = 1$ to $l_{i+1} = 3$, and that it starts at nest-index number 5. Thus we can write 6 above the dot which lines up with right-hand bight-boundary 1 and nest-index number 5. We calculate the number of crossings associated with a half-cycle which runs from $r_i = 1$ to $l_{i+1} = 3$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 19. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 1 end at left-hand bight-boundary 3, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 19 is placed at the bottom in line with right-hand bight-boundary 1.

The first-return string-run shows that half-cycle 8 runs from $r_i = 2$ to $l_{i+1} = 2$, and that it starts at nest-index number 0. Thus we can write 8 above the dot which lines up with right-hand bight-boundary 2 and nest-index number 0. We calculate the number of crossings associated with a half-cycle which runs from $r_i = 2$ to $l_{i+1} = 2$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 19. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 2 end at left-hand bight-boundary 2, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 19 is placed at the bottom in line with right-hand bight-boundary 2.

The first-return string-run shows that half-cycle 10 runs from $r_i = 3$ to $l_{i+1} = 1$, and that it starts at nest-index number 10. Thus we can write 10 above the dot which lines up with right-hand bight-boundary 3 and nest-index number 10. We calculate the number of crossings associated with a half-cycle which runs from $r_i = 3$ to $l_{i+1} = 1$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 19. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 3 end at left-hand bight-boundary 1, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 19 is placed at the bottom in line with right-hand bight-boundary 3.

The second step in the construction of the half-cycle pattern, shown at the bottom of Fig. 516, consists of the determination of the further half-cycle numbers in association with the nest-index numbers and bight-boundaries of their starting points.

Since the second sequence of the half-cycles in a first-return string-run starts with half-cycle number $1 + 2A = 11$ at nest-index number $I_L = A|2A + x - 2|_{B^*} = 5|22|_3 = 5$, two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I + 2A^2 + A(x-2)\right|_B = \left|I + 2 \cdot 5^2 + 5(14-2)\right|_{15} = |I + 110|_{15} = |I + 5|_{15}$.

$\star\star$ *Prove the above formulae* $I_L = A|2A + x - 2|_{B^*}$ *and* $\left|I + 2A^2 + A(x-2)\right|_B$.

We can now readily complete the half-cycle pattern concerned.

Let's illustrate for the half-cycles 17, 23, 8 and 12 how this half-cycle pattern may be used.

For the determination of the braiding sequence of half-cycle 17 we start at the extreme left on the line immediately above (in cyclic order) half-cycle 17 (the first crossing on half-cycle 17 in the finished knot is with half-cycle 20 (= 21 − 1)). To arrive at the finish, count the number of crossings on half-cycle 17 (= 18) in the finished knot from left to right in cyclic order.

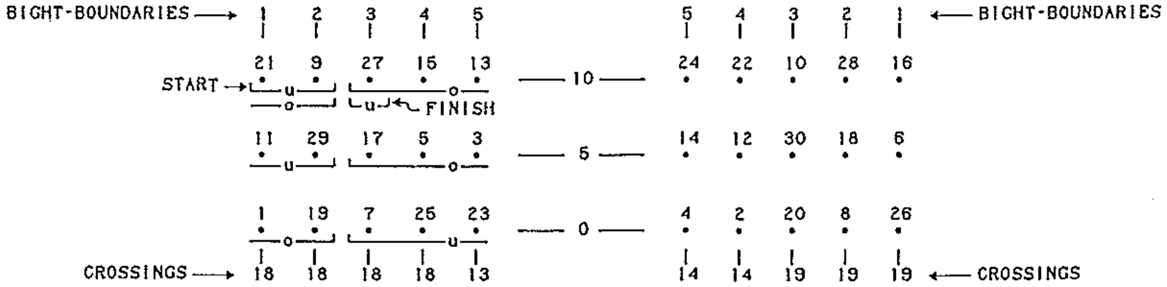


Fig. 518 — Determination of the braiding sequence for half-cycle 17.

The first $l_i - 1$ crossings $(= 3 - 1 = 2)$ on half-cycle 17 in the finished knot are

under crossings. Next we have A **over** crossings (= 5), next A **under** crossings (= 5), next A **over** crossings (= 5), and finally 1 **under** crossing. When braiding half-cycle 17, it is only possible for half-cycle 17 to cross the half-cycles which end at the start of the half-cycles greater 1 and less or equal 17. Hence when braiding half-cycle 17, it may cross the half-cycles which end at the start of the half-cycles greater or equal 3 and less or equal 17, hence it crosses the half-cycles 9 ; 15, 13 ; 7, 11 ; 17, 5, 3, 9. Thus the braiding sequence of half-cycle 17 is $u - 2o - 2u - 4o$.

For the determination of the braiding sequence of half-cycle 23 we start at the extreme left on the line immediately above (in cyclic order) half-cycle 23 (the first crossing on half-cycle 23 in the finished knot is with half-cycle 10 (= 11 − 1)). To arrive at the finish, count the number of crossings on half-cycle 23 (= 13) in the finished knot from left to right in cyclic order.

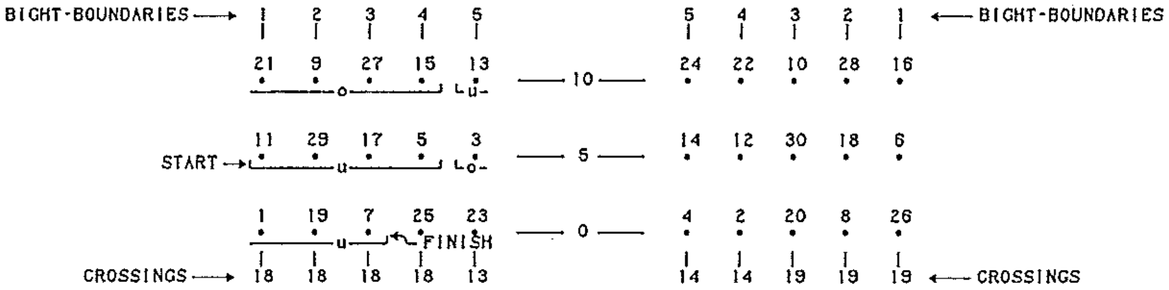


Fig. 519 — Determination of the braiding sequence for half-cycle 23.

The first $l_i - 1$ crossings (= 5 − 1 = 4) on half-cycle 23 in the finished knot are **under** crossings. Next we have A **over** crossings (= 5), and finally 4 **under** crossing. When braiding half-cycle 23, it is only possible for half-cycle 23 to cross the half-cycles which end at the start of the half-cycles greater 1 and less or equal 23. Hence when braiding half-cycle 23, it may cross the half-cycles which end at the start of the half-cycles greater or equal 3 and less or equal 23, hence it crosses the half-cycles 11, 17, 5 ; 3, 21, 9, 15 ; 13, 19, 7. Thus the braiding sequence of half-cycle 23 is $3u - 4o - 3u$.

For the determination of the braiding sequence of half-cycle 8 we start at the extreme right on the line immediately above (in cyclic order) half-cycle 8 (the first crossing on half-cycle 8 in the finished knot is with half-cycle 5 (= 6 − 1)). To arrive at the finish, count the number of crossings on half-cycle 8 (= 19) in the finished knot from right to left in cyclic order.

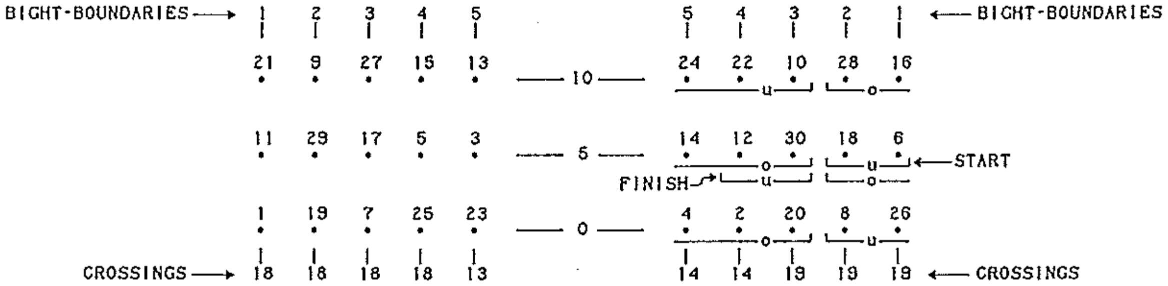


Fig. 520 — Determination of the braiding sequence for half-cycle 8.

The first r_i crossings (= 2) on half-cycle 8 in the finished knot are **under** crossings. Next we have A **over** crossings (= 5), then A **under** crossings (= 5), then A **over** crossings (= 5), and finally 2 **under** crossing. When braiding half-cycle 8, it is only possible for half-cycle 8 to cross the half-cycles which end at the start of the half-cycles greater or equal 2 and less or equal 8, hence it crosses the half-cycles 6 ; 8 ; 2 , 4 , 6 . Thus the braiding sequence of half-cycle 8 is $2u - 3o$.

For the determination of the braiding sequence of half-cycle 12 we start at the extreme right on the line immediately above (in cyclic order) half-cycle 12 (the first crossing on half-cycle 12 in the finished knot is with half-cycle 15 (= 16 − 1)). To arrive at the finish, count the number of crossings on half-cycle 12 (= 14) in the finished knot from right to left in cyclic order.

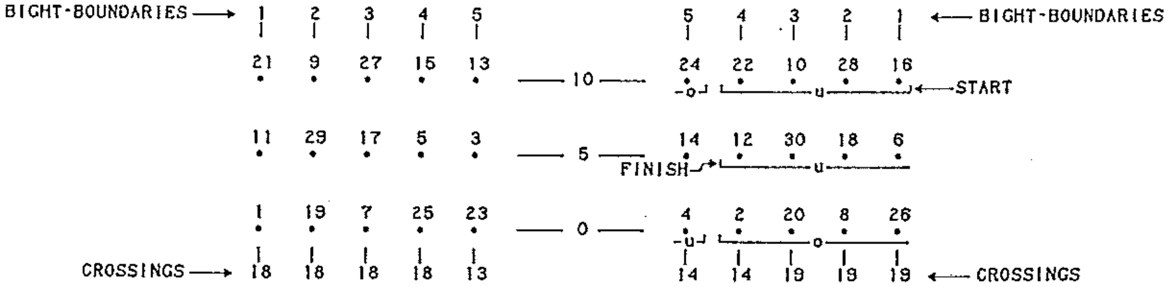
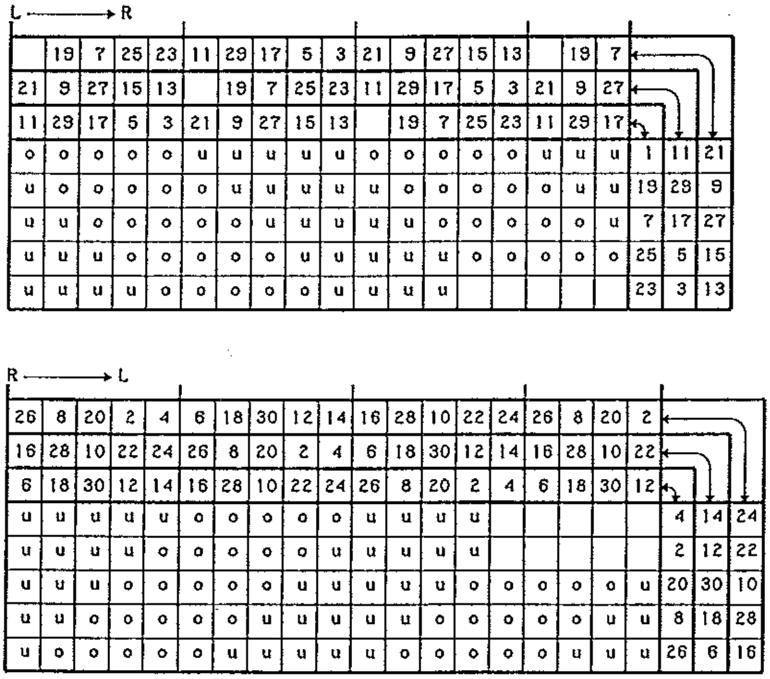


Fig. 521 — Determination of the braiding sequence for half-cycle 12.

The first r_i crossings (= 4) on half-cycle 12 in the finished knot are **under** crossings. Next we have A **over** crossings (= 5), and finally 5 **under** crossing. When braiding half-cycle 12, it is only possible for half-cycle 12 to cross the half-cycles which end at the start of the half-cycles greater or equal 2 and less or equal 12, hence it crosses the half-cycles 10; 8 , 2; 4 , 6 , 12 . Thus the braiding sequence of half-cycle 12 is $u - 2o - 3u$.

Although, as shown above, we can read the half-cycle braiding algorithms directly from the half-cycle pattern, it is easier to determine the half-cycle braiding algorithms from the half-cycle tables shown in Fig. 522 which we assemble from the half-cycle pattern at the bottom of Fig. 516.



	19	7	25	23	11	29	17	5	3	21	9	27	15	13		19	7			
21	9	27	15	13		19	7	25	23	11	29	17	5	3	21	9	27			
11	29	17	5	3	21	9	27	15	13		19	7	25	23	11	29	17			
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	1	11	21
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	19	29	9
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	7	17	27
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	25	5	15
u	u	u	u	o	o	o	o	o	u	u	u	u						23	3	13

26	8	20	2	4	6	18	30	12	14	16	28	10	22	24	26	8	20	2			
16	28	10	22	24	26	8	20	2	4	6	18	30	12	14	16	28	10	22			
6	18	30	12	14	16	28	10	22	24	26	8	20	2	4	6	18	30	12			
u	u	u	u	u	o	o	o	o	o	u	u	u	u						4	14	24
u	u	u	u	o	o	o	o	o	u	u	u	u	u						2	12	22
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	20	30	10
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	8	18	28
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	26	6	16

Fig. 522 — The half-cycle tables for the knot in Example 1.

The upper table in Fig. 522 is for the odd-numbered half-cycles, hence the half-cycles from lower-left to upper-right, while the lower table in Fig. 522 is for the even-numbered half-cycles, hence the half-cycles from lower-right to upper-left. The upper table is assembled as follows:

Half-cycles 1, 19, 7, 25 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

11, 29, 17, 5, 3, 21, 9, 27, 15, 13, –, 19, 7, 25, 23, 11, 29, 17,

and half-cycle 23 crosses in the finished knot sequentially the half-cycles which end at the start of the half-cycles

11, 29, 17, 5, 3, 21, 9, 27, 15, 13, –, 19, 7.

Half-cycles 21, 9, 27, 15 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

–, 19, 7, 25, 23, 11, 29, 17, 5, 3, 21, 9, 27, 15, 13, –, 19, 7,

and half-cycle 13 crosses in the finished knot sequentially the half-cycles which end at the start of the half-cycles

–, 19, 7, 25, 23, 11, 29, 17, 5, 3, 21, 9, 27.

Half-cycles 11, 29, 17, 5 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

21, 9, 27, 15, 13, –, 19, 7, 25, 23, 11, 29, 17, 5, 3, 21, 9, 27,

and half-cycle 3 crosses in the finished knot sequentially the half-cycles which end at the start of the half-cycles

21, 9, 27, 15, 13, –, 19, 7, 25, 23, 11, 29, 17.

Note that the half-cycle which ends at the start of half-cycle 1 is the very last half-cycle in the braiding process and hence will not be crossed during the braiding process, thus we leave its cell empty.

The lower table is assembled in a way which enables us to read the braiding half-cycle algorithms from left to right, hence:

Half-cycles 4, 2 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

6, 18, 30, 12, 14, 16, 28, 10, 22, 24, 26, 8, 20, 2,

and half-cycles 20, 8, 26 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

6, 18, 30, 12, 14, 16, 28, 10, 22, 24, 26, 8, 20, 2, 4, 6, 18, 30, 12.

Half-cycles 14, 12 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

16, 28, 10, 22, 24, 26, 8, 20, 2, 4, 6, 18, 30, 12,

and half-cycles 30, 18, 6 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

16, 28, 10, 22, 24, 26, 8, 20, 2, 4, 6, 18, 30, 12, 14, 16, 28, 10, 22.

Half-cycles 24, 22 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

26, 8, 20, 2, 4, 6, 18, 30, 12, 14, 16, 28, 10, 22,

and half-cycles 10, 28, 16 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

26, 8, 20, 2, 4, 6, 18, 30, 12, 14, 16, 28, 10, 22, 24, 26, 8, 20, 2.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 1:

1. $1 \nearrow 4$: Free run.
2. $5 \nwarrow 4$: u.
3. $5 \nearrow 5$: Free run.
4. $4 \nwarrow 5$: u.
5. $4 \nearrow 1$: $2o$.
6. $3 \nwarrow 1$: $3u$.
7. $3 \nearrow 2$: $3o$.
8. $2 \nwarrow 2$: $2u - 3o$.
9. $2 \nearrow 3$: $o - 2u - o - u$.
10. $1 \nwarrow 3$: $u - 3o - u - o - u$.
11. $1 \nearrow 4$: $o - u - 3o - u$.
12. $5 \nwarrow 4$: $u - 2o - 3u$.
13. $5 \nearrow 5$: $u - 2o - 2u$.
14. $4 \nwarrow 5$: $u - 3o - 2u$.
15. $4 \nearrow 1$: $u - o - 3u - 3o$.
16. $3 \nwarrow 1$: $4o - 3u - o - 2u$.
17. $3 \nearrow 2$: $u - 2o - 2u - 4o$.
18. $2 \nwarrow 2$: $u - 2o - 4u - 3o - u$.
19. $2 \nearrow 3$: $u - 3o - 3u - 3o - u$.
20. $1 \nwarrow 3$: $2u - 4o - 2u - 4o - u$.
21. $1 \nearrow 4$: $2o - 4u - 4o - 2u$.
22. $5 \nwarrow 4$: $3u - 4o - 4u$.
23. $5 \nearrow 5$: $3u - 4o - 3u$.
24. $4 \nwarrow 5$: $4u - 4o - 3u$.
25. $4 \nearrow 1$: $2u - 4o - 4u - 4o$.
26. $3 \nwarrow 1$: $u - 4o - 4u - 5o - 2u$.
27. $3 \nearrow 2$: $u - 4o - 5u - 4o - u$.
28. $2 \nwarrow 2$: $2u - 5o - 4u - 5o - 2u$.
29. $2 \nearrow 3$: $u - 4o - 5u - 5o - 2u$.
30. $1 \nwarrow 3$: $3u - 5o - 5u - 5o - u$.

The grid-diagram of this knot is depicted in Fig. 523

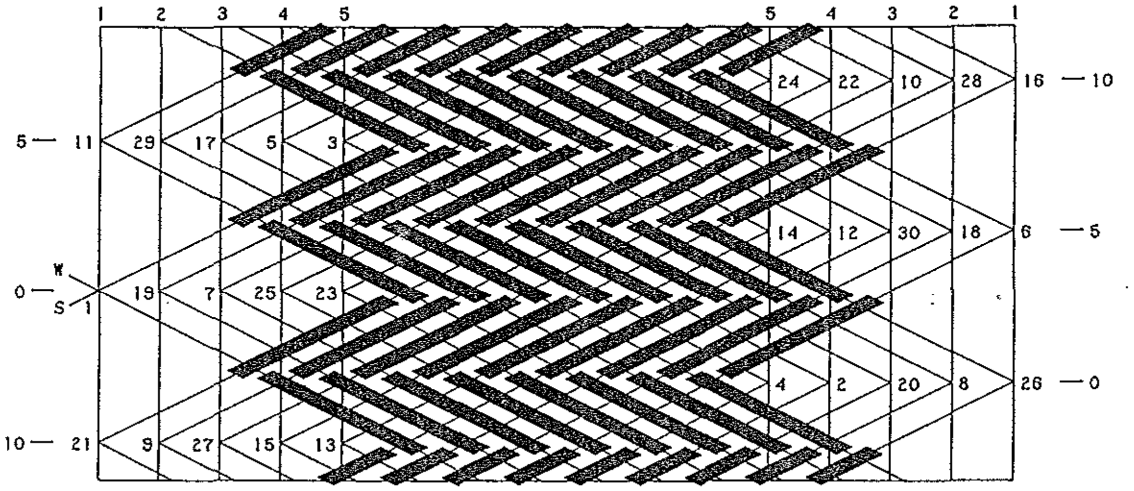


Fig. 523 — Grid-diagram of the knot in Example 1.

Example 2:

$A = 5$; $x = 14$; $y = A+1 = 6$, hence $\Delta = |6|_5 = 1$; $B^* = 3$, hence $B = A \cdot B^* = 15$. Thus $k = \left|\frac{x-A-3}{2}\right|_A = \left|\frac{14-5-3}{2}\right|_5 = 3$.

At the upper left-hand side in Fig. 524 is depicted the first-return string-run with its associated nest-index numbers.

At the upper right-hand side in Fig. 524 are depicted the first-return string-run, the half-cycle numbers of the half-cycles, and the number of crossings which these half-cycles make in the finished knot. This layout at the upper right-hand side has been shown for additional clarification only since it can readily be dispensed with.

The depicted layout at the bottom of Fig. 524 gives the positions of the nests with respect to the nest-index numbers, the half-cycle numbers with respect to the nest-index numbers and bight-boundaries, and the number of crossings which the half-cycles make in the finished knot. This layout is again the one we need for compiling the half-cycle algorithms; it is directly obtained from the first-return string-run with its associated nest-index numbers, hence from the upper left-hand layout in Fig. 524. The first step in its construction is shown in Fig. 525.

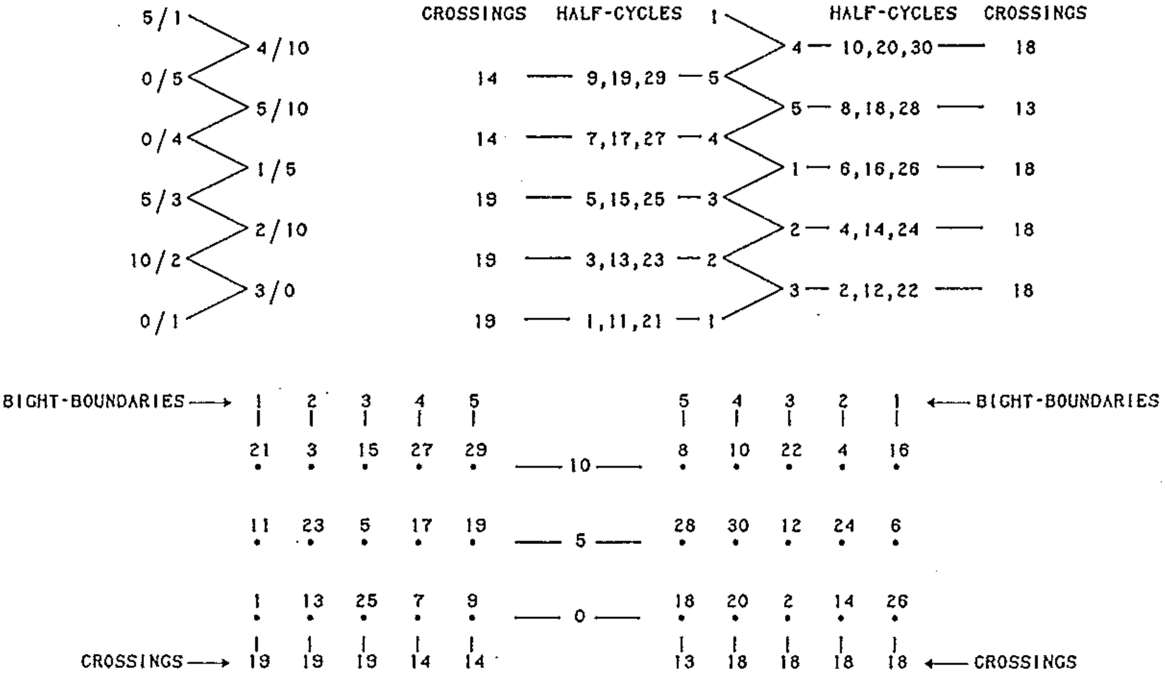


Fig. 524 — First-return string-run and half-cycle pattern.

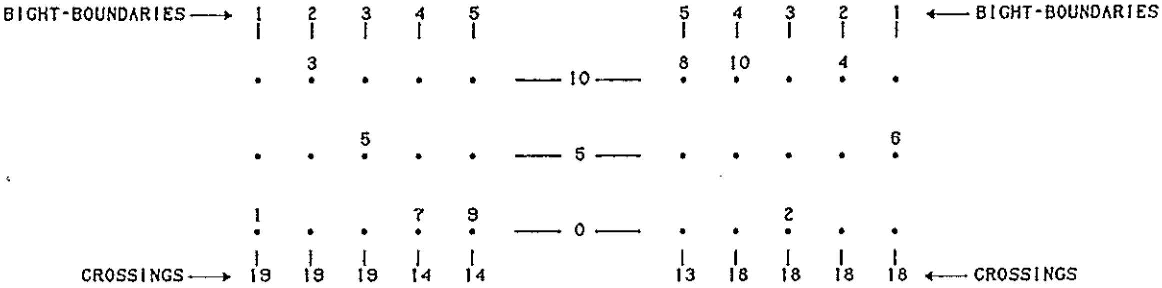


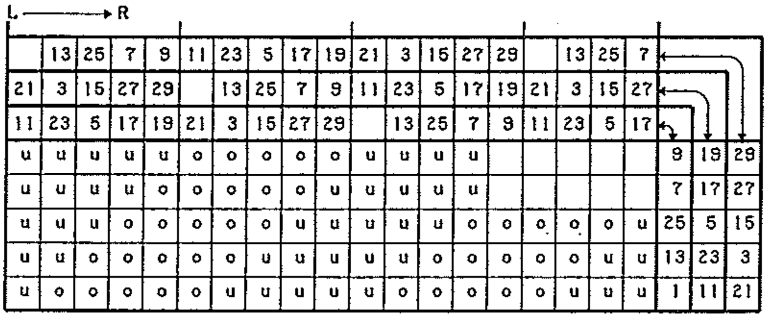
Fig. 525 — The first step in the construction of the half-cycle pattern.

The second step in the construction of the half-cycle pattern shown at the bottom of Fig. 524 consists of the determination of the half-cycle numbers in association with the nest-index numbers and bight-boundaries of their starting points.

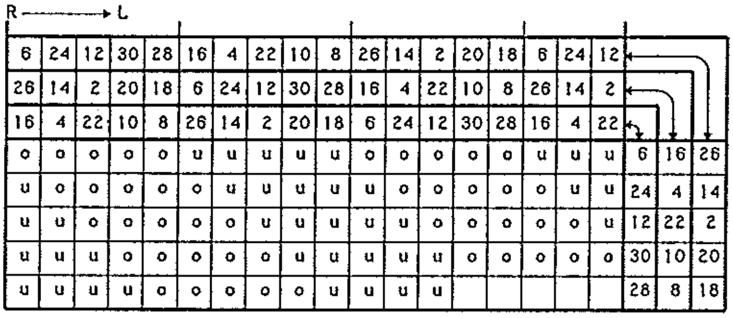
Since the second sequence of the half-cycles in a first-return string-run starts with half-cycle number $1+2A=11$ at nest-index number $I_L = A|2A+x-2|_{B^*} = 5|22|_3 = 5$, two sequential half-cycle numbers h_n (≥ 1) and h_n+2A $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I+2A^2+A(x-2)\right|_B = \left|I+2\cdot 5^2+5(14-2)\right|_{15} = |I+110|_{15} = |I+5|_{15}$. Hence we can readily complete, similar as in Example 1, the half-cycle pattern concerned. With the aid of the coding-sequence of the half-cycles in the finished knot with a Herringbone Pineapple coding and $y=A+1$ (see pg. 646) we can either read the half-cycle braiding algorithms again directly from the half-cycle pattern, or from the set of half-cycle tables shown in Fig. 526 which we assemble from the half-cycle pattern at the bottom of Fig. 524.

The upper table in Fig. 526 is for the odd-numbered half-cycles, hence the half-cycles from lower-left to upper-right, while the lower table in Fig. 526 is for the even-numbered half-cycles, hence the half-cycles from lower-right to upper-left.

Note that the half-cycle which ends at the start of half-cycle 1 is the very last half-cycle in the braiding process and hence will not be crossed during the braiding process, thus we leave its cell empty.



	13	25	7	9	11	23	5	17	19	21	3	15	27	29		13	25	7			
21	3	15	27	29		13	25	7	9	11	23	5	17	19	21	3	15	27			
11	23	5	17	19	21	3	15	27	29		13	25	7	9	11	23	5	17			
u	u	u	u	u	o	o	o	o	o	u	u	u	u						9	19	29
u	u	u	u	o	o	o	o	o	u	u	u	u	u						7	17	27
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	25	5	15
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	13	23	3
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	1	11	21



6	24	12	30	28	16	4	22	10	8	26	14	2	20	18	6	24	12			
26	14	2	20	18	6	24	12	30	28	16	4	22	10	8	26	14	2			
16	4	22	10	8	26	14	2	20	18	6	24	12	30	28	16	4	22			
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	6	16	26
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	24	4	14
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	12	22	2
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	30	10	20
u	u	u	u	o	o	o	o	o	u	u	u	u						28	8	18

Fig. 526 — The half-cycle tables for the knot in Example 2.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 2:

1. $1 \nearrow 3$: Free run.
2. $2 \nwarrow 3$: o.
3. $2 \nearrow 2$: u.
4. $3 \nwarrow 2$: $2o - u$.
5. $3 \nearrow 1$: $2u - o$.
6. $4 \nwarrow 1$: $o - u - o - u$.
7. $4 \nearrow 5$: $u - o - u$.
8. $5 \nwarrow 5$: $u - o - u$.
9. $5 \nearrow 4$: $u - o - u$.
10. $1 \nwarrow 4$: $u - o - u - 3o$.
11. $1 \nearrow 3$: $o - 3u - o - u$.
12. $2 \nwarrow 3$: $u - 2o - 2u - 2o$.
13. $2 \nearrow 2$: $u - 2o - u - 3o - u$.
14. $3 \nwarrow 2$: $u - o - 3u - 3o - u$.
15. $3 \nearrow 1$: $u - 4o - 2u - o - u$.
16. $4 \nwarrow 1$: $2o - 2u - 4o - 2u$.
17. $4 \nearrow 5$: $2u - 2o - 4u$.
18. $5 \nwarrow 5$: $2u - 3o - 3u$.
19. $5 \nearrow 4$: $2u - 3o - 3u$.
20. $1 \nwarrow 4$: $2u - 2o - 4u - 4o$.
21. $1 \nearrow 3$: $4o - 4u - 2o - 2u$.
22. $2 \nwarrow 3$: $u - 4o - 3u - 4o - u$.
23. $2 \nearrow 2$: $2u - 2o - 4u - 5o - u$.
24. $3 \nwarrow 2$: $u - 4o - 5u - 3o - 2u$.
25. $3 \nearrow 1$: $3u - 5o - 2u - 5o - u$.
26. $4 \nwarrow 1$: $3o - 5u - 5o - 3u$.
27. $4 \nearrow 5$: $3u - 5o - 5u$.
28. $5 \nwarrow 5$: $4u - 5o - 4u$.
29. $5 \nearrow 4$: $4u - 5o - 4u$.
30. $1 \nwarrow 4$: $3u - 5o - 5u - 5o$.

The grid-diagram of this knot is depicted in Fig. 527

Although the **nest-index numbers** I_R may be calculated as shown in *The Braider*, Issue No. 26, pp. 594 and 598, the following method is a little quicker:

Let an odd-numbered half-cycle run from left bight-boundary l_i and nest-index number I_L to right bight-boundary r_i and nest-index number I_R. Then $I_R = I_L$ when $l_i + r_i = k + 1$, and $I_R = |I_L - A|_B$ when $l_i + r_i = A + k + 1$.

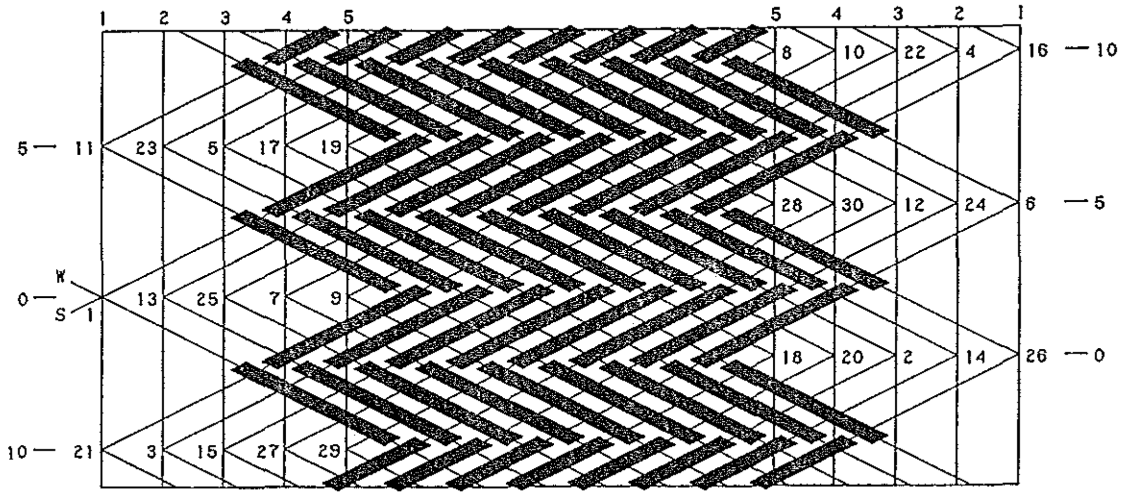


Fig. 527 — Grid-diagram of the knot in Example 2.

Note that the string-runs of the knots in Example 1 and 2 are each others mirror-image (their first-return string-runs are each others mirror-image (see *The Braider*, Issue No. 24, pg. 562, and Issue No. 25, pg. 568) and they have an identical number of bights). Hence when for one of such herringbone pineapple coded knots the two half-cycle tables have been determined, we can then readily derive from these tables the two half-cycle tables for the mirror-imaged knot.

Say the first-return string-run and the two half-cycle tables for the $y = A - 1$ knot are known. On its first-return string-run determine the number of half-cycles between right-hand bight-boundary 1 and left-hand bight-boundary 1 at the end of the first-return string-run (this number of half-cycles is equal to 5 for Example 1). Then replace each half-cycle number in its two half-cycle tables by

$$|\text{half-cycle number} + 5|_{2B}\,.$$

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$).

Say the first-return string-run and the two half-cycle tables for the $y = A + 1$ knot are known. On its first-return string-run determine the number of half-cycles between left-hand bight-boundary 1 at the beginning of the first-return string-run and right-hand bight-boundary 1 (this number of half-cycles is equal to 5 for Example 2). Then replace each half-cycle number in its two half-cycle tables by

$$|\text{half-cycle number} - 5|_{2B}\,.$$

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$).

THE BRAIDER

APPENDIX 2001

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A quarterly publication
for
the braiding artisan

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Some comments we received

Although the letters containing the comments were part of a private correspondence between W. J. Budd and A. G. Schaake, the comments concerned the contents of *The Braider* and consequently *The Braider* as a publication received from W. J. Budd some negative comments which read as follows [the bracketed entries are of *The Braider*]:

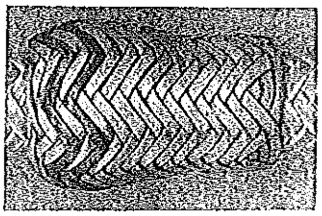
I am willing to trust that I can use a grid diagram with the cut line going thru crossing points. On the other hand you [it should be stressed here that the correspondent is well aware that *The Braider* is a publication of the named copyright holders as a group and **not** a publication of a single individual, which his use of the pronoun "you" might often infer] *take the opposite position* [where the cut line passes midway between two adjacent rows of crossings] *which I find counter-intuitive. Because of your interest in knot characteristics (an interest that I don't have, except to the extent that it is necessary to complete a particular braiding project* [that is the sole interest of *The Braider* too as far as knot characteristics are concerned]*), and considering the analytical work that you have done, you are uniquely positioned. As evidenced by "The Braider", you are willing to share that information with others. Also, others want to learn what you have discovered about braiding, but there is a big communication problem here, that even I as a college trained engineer can not overcome. An example of this communication difficulty is the article starting on pg. 592 of "The Braider". After the first rather complicated sentence, you immediately launch into mathematical communication, and that type of thing takes a lot of careful time consuming study, and even then, I find that sometimes I simply don't understand what you are trying to say. The situation is complicated even further by time pressures. I must pick and chose very carefully how I spend my time, or I won't have time to do the laundry, buy the food, pay the bills, etc., etc., etc. That means that I need to get the big picture early on in an article, i.e., get a qualitative understanding of the braiding concepts covered, before being presented with the mathematical underpinnings, which of course I can study later, but in your articles any qualitative discussion is either absent, severely truncated, or complicated with mathematical symbols. The point is that if I take time to study and investigate all the mathematical language used in your articles, I'll never have the time for anything else. So I must ask you to present the material more clearly. To put it another way.* **I am interested in what you have to say, but I need you to say more of it in plain English and leave the math for later in the article where it can be studied as an option.** *To repeat,* **I am interested in what you have to say, but I need you to say more of it in plain English and leave the math for later in the article where it can be studied as an option.** *Please understand that this point is being made by someone who got straight A's in college math, and someone that knows when math is being used to clarify, and when it is being used to simply use it. Also, we have never covered this but I assume that your intentions are honorable, i.e. they are to share with others what you have learned about braiding as opposed to trying to impress, confuse, and control people. Of course I do expect you to either attack my points or claim that you don't have any interest in whether or not others can understand what you have written so it doesn't matter, but lets be clear about one thing. If you really didn't have an interest in communicating braiding concepts to others, you would not put so much effort into "The Braider"* [not much effort as far as *The Braider* is concerned]. *Therefore, I am making this appeal.* **Please use descriptive English statements devoid of mathematical symbols to thoroughly explain**

the concepts, before launching into the quantitative mathematics, as that will solve much of the communications problem. *Finally, braiders are braiders, they are not mathematicians and they are not going to become mathematicians in order to understand your publication. On the other hand if I have miss-understood and the real motivation in putting out "The Braider" is to impress your mathematician friends, and write for their journals, then there is certainly no point in offering "The Braider" to braiders* [*the Braider* does certainly not have mathematician friends and none of its copyright holders write for their journals, in fact we as copyright holders have clearly stated that "The Braider" is only made available to individuals for personal use who have a dedicated interest in braiding, hence not to resellers, institutions (such as schools, universities, clubs and associations for example) and not to the "typical braider" who is looking for that magic potion which without any effort on his/her part will provide him/her with new braids]. *They don't understand it, and if there is not going to be an effort made to communicate with them more effectively, you might as well not waste your time writing it, and please don't take the position that if people want to learn your braiding knowledge, then it is for them to pull themselves up to your standards, because I would have to reply that covering up a poor job of communicating with laziness and arrogance, is not a good reason for doing the job poorly. The real important point however, is that you are capable of communicating effectively, because you do so in our letters, and you can do likewise in "The Braider". You simply need to make the effort. In closing let me repeat what I said earlier.* **I am interested in what you have to say, but I need you to say more of it in "plain English" and leave the math for later in the article where it can be studied as an option.**

In a later letter Bill Budd wrote:

After writing you and pointing out how I have little or no spare time in which to study the mathematics in your letters, I found myself with several hours to spare today! So the obvious thing to do I decided was to study the mathematics in your recent letters, thus I looked up the definitions for the variable symbols in the following portion of your 19 July [2001] *letter and proceeded to analize a knot from my fundamentals book* [a Standard Herringbone Pineapple Knot with $A = 2$, $x = 14$, $k = 1$, $y = A = 2$] *as it is quick and easy to make. I then repeated the following part of your 19 July letter which follows, for my knot.*

"Lets get back to those crown shaped knots we have been talking about, and in particular to the crown shaped knots [see Fig. 1] *on your enclosed scanner ⟶ computer generated picture of the braids on the six different quirt handles.*



[Fig. 1. — The crown shaped knot on the quirt handles.]

These knots have an interbraid (strings of a different colour to the 'foundation knot'). You took $B^* = 6$ and since for your foundation knots $A = 3$ with $l_h = 14$, $r_h = 13$ [see for the meaning of l_h and r_h below] *we have $P_{total} = A + (l_h + r_h) = 3 + (14 + 13) = 30$. Since* g.c.d.$(P_{total}, B^*)$ = g.c.d.$(30, 6) = 6$, *we require six essential strings, and consequently the foundation knots are* not *Perfect Herringbone Pineapple Knots, but* are **Semi-Perfect Herringbone Pineapple Knots** *instead.*"

[We should point out here that the correspondence with Bill Budd did contain the following fundamental relationships: A Herringbone Pineapple coding has rows of / coding-sets alternating with rows of \ coding-sets. Each coding set consists of A consecutive identical single codings along the same helix. Let there be r_h coding sets in a row of / coding sets and let there be l_h coding sets in a row of \ coding sets. When $l_h = r_h$, the Herringbone Pineapple Knot is either a Standard Herringbone Pineapple Knot or a Semi-Standard Herringbone Pineapple Knot (with $y = A$). When $l_h = r_h - 1$, the Herringbone Pineapple Knot is a Perfect Herringbone Pineapple Knot or a Semi-Perfect Herringbone Pineapple Knot with $y = A - 1$. When $l_h = r_h + 1$, the Herringbone Pineapple Knot is a Perfect Herringbone Pineapple Knot or a Semi-Perfect Herringbone Pineapple Knot with $y = A + 1$.

For any of these four types of Herringbone Pineapple Knots we have:

$$\begin{aligned} P_{total} &= A + (l_h + r_h), \\ x &= P_{total} + 2 - 2A, \\ k &= |r_h|_A. \end{aligned}$$

For Standard and Semi-Standard Herringbone Pineapple Knots we have in addition:

$|l_h|_A = |r_h|_A =$ number of components with $P_c = 3 + \frac{2(l_h)-2|l_h|_A}{A} = 3 + \frac{2(r_h)-2|r_h|_A}{A}$ each. For these components: $l_i + r_i = k + 1$.

$A - |l_h|_A = A - |r_h|_A =$ number of components with $P_c = 1 + \frac{2(l_h)-2|l_h|_A}{A} = 1 + \frac{2(r_h)-2|r_h|_A}{A}$ each. For these components: $l_i + r_i = k + 1 + A$.

Furthermore, from *The Braider*, Issue No. 22 (May 2000), pg. 510, Bill should know that for Regular Nested Cylindrical Braids (to which Standard, Semi-Standard, Perfect and Semi-Perfect Herringbone Pineapple Knots belong):

$P_{total} = \Sigma(P_{component})$.

number of components = number of first-return string-runs.

number of sub-components in a component = g.c.d. $(P_{component}, B^*)$.

number of essential strings = Σ(number of sub-components).

From *The Braider*, Issue No. 23 (August 2000), pp. 513 - 515 and pg. 519, Bill should know that the number of components is equal to A for Standard Regular Nested Cylindrical Braids and Semi-Standard Regular Nested Cylindrical Braids (to which Standard and Semi-Standard Herringbone Pineapple Knots respectively belong).]

Then Bill continues in this later letter:

$A =$ *the number of bights per nest* $= 2$
$B^* =$ *the number of nests per end of the knot,*
i.e., outside bights around one end $= 4$
$l_h =$ *left helix coding set* $= 7$
$r_h =$ *right helix coding set* $= 7$

then $P_{total} = A + l_h + r_h = 2 + 7 + 7 = 16$
and
g.c.d. $(P, B^*) =$ *g.c.d.* $(16, 4) = 4$*!*
This says that there are four essential strings in the knot. Since it is made using two, something is wrong with your mathematical methods, or my math. WHAT IS GOING ON HERE?

[Let $\gamma =$ g.c.d. $(\Delta, A) =$ number of first-return string-runs = number of components (see *The Braider*, Issue No. 25 (January 2001), pg. 570), and let $\lambda =$ number of essential strings = Σ(sub-components).

Obviously, with $l_h = r_h = 7$ we are dealing with a Standard or Semi-Standard Herringbone Pineapple Knot, hence $y = A$ and consequently $\Delta = |y|_A = |A|_A = 0 \equiv A$. Hence we have $\gamma = \text{g.c.d.}(\Delta, A) = \text{g.c.d.}(A, A) = A$. Furthermore $k = |r_h|_A = |7|_2 = 1$. Since $|r_h|_A = 1$, one component with $P_{c_1} = 3 + \frac{2(r_h)-2|r_h|_A}{A} = 3 + \frac{2(7)-2|7|_2}{2} = 3 + 6 = 9$, and $A - |r_h|_A = 2 - 1 = 1$ component with $P_{c_2} = 1 + \frac{2(r_h)-2|r_h|_A}{A} = 1 + \frac{2(7)-2|7|_2}{2} = 1 + 6 = 7$. Since $\text{g.c.d.}(P_{c_1}, B^*) = \text{g.c.d.}(9, 4) = 1$ and $\text{g.c.d.}(P_{c_2}, B^*) = \text{g.c.d.}(7, 4) = 1$, each component has no further sub-components and hence we are dealing with a Standard Herringbone Pineapple Knot, consequently $\lambda =$ number of essential strings $= \Sigma(\text{sub-components}) = 1 + 1 = 2 = \gamma$. This knot thus requires 2 essential strings. If B^* would have been 6, we would have $\gamma = 2$ and $\lambda = 4$, hence we would have 2 components and we would have required 4 essential strings. Perfect and Semi-Perfect Herringbone Pineapple Knots consist of a single component with $P = P_{total}$. Hence for them $\gamma = 1$ and $\lambda = \text{g.c.d.}(P_{total}, B^*)$[†].]

Obviously Bill Budd is not only missing the point completely, but there is also something seriously wrong with his math application. In the first place, *The Braider* is a **technical** publication and not a "Womens Weekly" or a "thriller" [although some might get a thrill out of it]; it is therefore not a bedside reader. We always ensure that there are several Examples provided and often extensively discussed to help the reader in getting a good understanding of what goes on. Although Bill states: *(I am) someone who got straight A's in college math, and (am) someone that knows when math is being used to clarify*; how then can it possibly be that he has difficulty with the use of simple very elementary mathematics in clarifying of what goes on??? When Bill states: *I need to get the big picture early on in an article, i.e., get a qualitative understanding of the braiding concepts covered, before being presented with the mathematical underpinnings*; how can anyone *get the big picture* without the governing quantitative understanding? It appears to us that Bill's real problem can not only be found in his statement: *the situation is complicated even further by time pressures*, but also by wanting to run before he can even crawl, let alone walk. Since everybody gets 24 hours per day to his/her disposal, and since we are not in a position to increase that for Bill, we obviously cannot do much for him and therefore the presentation style will not change. One should clearly keep in mind for which category of braiders *The Braider* is intended, which has clearly been spelled out to all, and hence the question is whether Bill has really a dedicated interest in braiding or is one of those "typical braiders" who are looking for that magic potion which without any effort on their part will provide them with new braids; in which case he should be deleted from the list of subscribers since he does then not fulfil the required criteria. Furthermore, those who tend to lean towards Bill's dislikes are advised to read once again *The Braider*, Issue No. 3, *This Quarterly Publication and its Purpose* pp. 41 - 42, and the first part of *The Braider's Notebook* pg. 39 with the first two paragraphs on pg. 40; *Appendix 1995, A Reminder* pg. *ii*; and remember that we not only encourage readers to ask for clarifications where they encounter difficulties, but that we are also very interested in **sensible and constructive** comments!!!

[†] Hence for a Perfect Herringbone Pineapple Knot $\lambda = \gamma = 1$, consequently P_{total} and B^* are coprime while for a Semi-Perfect Herringbone Pineapple Knot $\lambda > 1$, consequently P_{total} and B^* are **not** coprime.

The Braider No. 25, pp. 570 - 571

Although the given formulae are correct, one condition was left out. This condition could, however, readily be found when paying due attention to the solution of the question on pg. 573 (see *The Braider*, No. 26, pp. 591 - 592).

$P_c = \dfrac{2A - \Delta'}{\gamma} + 2\left\lceil \dfrac{x + \Delta' - 2}{2\gamma} \right\rceil = \dfrac{x + 2A - 2 - 2\left|\frac{x+\Delta'-2}{2}\right|_{\gamma}}{\gamma} + 2$ for the first set of components, only applies when $\dfrac{x + \Delta' - 2}{2} \neq n\gamma$, where $n = 0, 1, 2, 3, \cdots$; hence when the first set is **not empty**.

A "hint" was also presented by the formulae in *The Braider*, No. 27, pg. 624.

Example 1.

Let $A = 8$; $y = 14$; $x = 56$.

Then $\Delta = |y|_A = |14|_8 = 6$.

$\gamma = \text{g.c.d.}\,(\Delta, A) = \text{g.c.d.}\,(6, 8) = 2$.

It suffices to consider $0 \leq y \leq A$ due to the mirror-image complementary relationship. Hence $y' = y$ with $\Delta' = |y'|_A = |y|_A = \Delta$ for $0 \leq y < A$, and $y' = 2A - y$ with $\Delta' = |y'|_A = |-\Delta|_A$ for $A \leq y \leq 2A$.

$\Delta' = |y'|_A$ for $y' \neq A$; $\gamma = A$ when $\Delta' = 0$; $\Delta' = A$ when $y' = A$.

Since $y = 14$, hence belongs to $A \leq y \leq 2A$ we thus obtain $y' = 2A - y = 16 - 14 = 2$ (mirror-image complementary relationship); and $\Delta' = |y'|_A = |2|_8 = 2$. Thus:

$$\#\text{ of components in } 1^{\text{st}} \text{ set} = \left|\frac{x + \Delta' - 2}{2}\right|_{\gamma} = \left|\frac{56 + 2 - 2}{2}\right|_{2} = |28|_2 = 0\,.$$

$$\#\text{ of components in } 2^{\text{nd}} \text{ set} = \gamma - \left|\frac{x + \Delta' - 2}{2}\right|_{\gamma} = \gamma - \left|\frac{56 + 2 - 2}{2}\right|_{2} = \gamma - 0 = \gamma = 2\,.$$

$$P_c = \frac{2A - \Delta'}{\gamma} + 2\left\lfloor \frac{x + \Delta' - 2}{2\gamma} \right\rfloor = \frac{16 - 2}{2} + 2\left\lfloor \frac{56 + 2 - 2}{4} \right\rfloor = 7 + 28 = 35\,.$$

Example 2.

Let $A = 8$; $y = 6$; $x = 56$.

Then $\Delta = |y|_A = |6|_8 = 6$.

$\gamma = \text{g.c.d.}\,(\Delta, A) = \text{g.c.d.}\,(6, 8) = 2$.

It suffices to consider $0 \leq y \leq A$ due to the mirror-image complementary relationship. Hence $y' = y$ with $\Delta' = |y'|_A = |y|_A = \Delta$ for $0 \leq y < A$, and $y' = 2A - y$ with $\Delta' = |y'|_A = |-\Delta|_A$ for $A \leq y \leq 2A$.

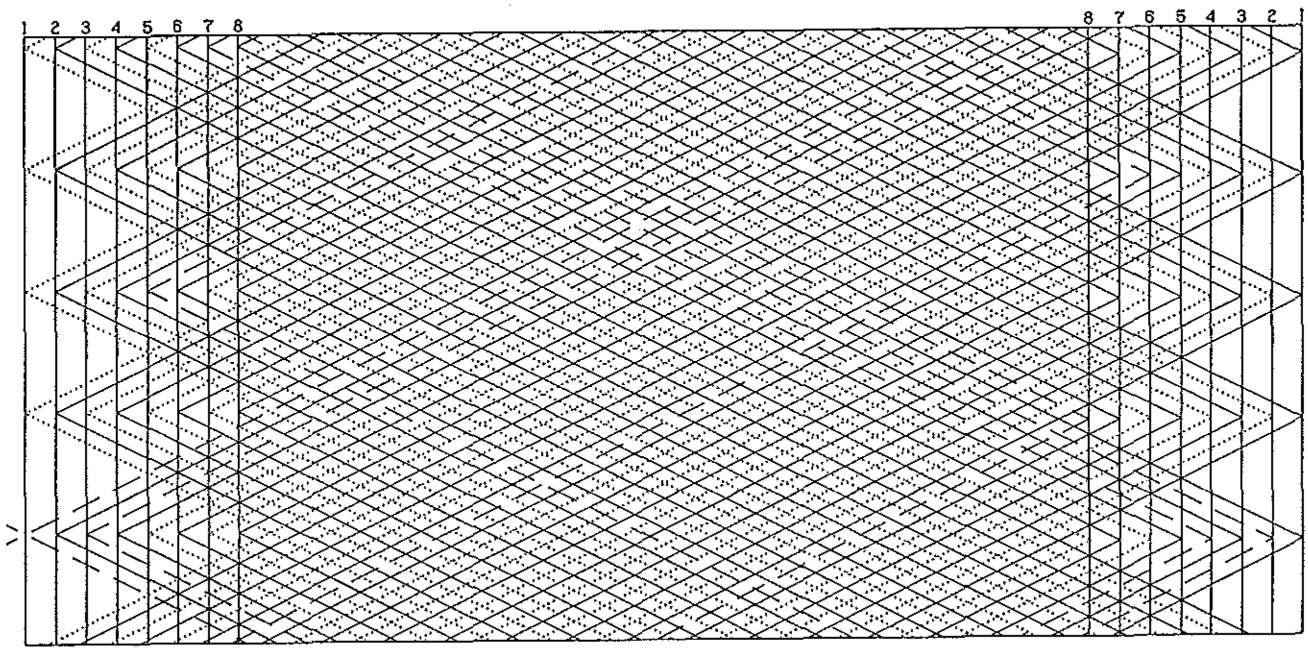
$\Delta' = |y'|_A$ for $y' \neq A$; $\gamma = A$ when $\Delta' = 0$; $\Delta' = A$ when $y' = A$.

Since $y = 6$, hence belongs to $0 \leq y < A$ we thus obtain $y' = y = 6$; and $\Delta' = |y'|_A = |y|_A = \Delta = 6$. Thus:

$$\#\text{ of components in } 1^{\text{st}} \text{ set} = \left|\frac{x + \Delta' - 2}{2}\right|_{\gamma} = \left|\frac{56 + 6 - 2}{2}\right|_{2} = |30|_2 = 0\,.$$

$$\#\text{ of components in } 2^{\text{nd}} \text{ set} = \gamma - \left|\frac{x + \Delta' - 2}{2}\right|_{\gamma} = \gamma - \left|\frac{56 + 6 - 2}{2}\right|_{2} = \gamma - 0 = \gamma = 2\,.$$

$$P_c = \frac{2A - \Delta'}{\gamma} + 2\left\lfloor \frac{x + \Delta' - 2}{2\gamma} \right\rfloor = \frac{16 - 6}{2} + 2\left\lfloor \frac{56 + 6 - 2}{4} \right\rfloor = 5 + 30 = 35\,.$$



EXAMPLE1 pg. V

Scale=2.000 linescale=12 N1-1248.DWG lines: 1 9 & 12

Did you miss it?

It appears that what was said in the 18^{th} line from the bottom on pg. *iii*, of *Appendix 1996* has in the meantime unfortunately been forgotten since we didn't receive any comment about the incomplete relatioships depicted in Fig. 463 of *The Braider*, Issue No. 24, pg. 562, between mirror-imaged first-return string-runs, (A, A)-complementary first-return string-runs and mirror-imaged (A, A)-complementary first-return string-runs. An indication of this incompleteness could however readily be found in the Figs. 469 & 470 on pg. 568 of *The Braider*, Issue No. 25 and there was also a clear 'suggestion' of it in Fig. 481 on pg. 591 of *The Braider*, Issue No. 26.

To make the relationships complete, let's take the first-return string-run in Fig. 2 and derive from it its mirror-imaged first-return string-run, its (A, A)-complementary first-return string-run and its mirror-imaged (A, A)-complementary first-return string-run.

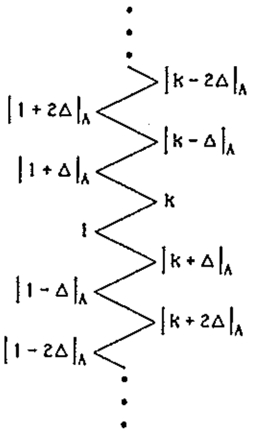


Fig. 2 — First-return string-run.

These derivations are shown in Fig. 3, pg. *vii* and Fig. 4, pg. *viii*.

For the mirror-imaged first-return string-run we thus obtain from Fig. 3 the following conditions:

$$\begin{aligned} k' &= |k + \Delta|_A\,, \\ \Delta' &= |-\Delta|_A\,. \end{aligned}$$

These conditions in the A, x, y table are depicted on pg. *ix*.

For the (A, A)-complementary first-return string-run we thus obtain from Fig. 4 the following conditions:

$$\begin{aligned} k' &= |-k|_A\,, \\ \Delta' &= |-\Delta|_A\,. \end{aligned}$$

These conditions in the A, x, y table are depicted on pg. *x*.

For the mirror-imaged (A, A)-complementary first-return string-run we thus obtain from Fig. 4 the following conditions:

$$\begin{aligned} k' &= |-k - \Delta|_A\,, \\ \Delta' &= \Delta\,. \end{aligned}$$

These conditions in the A, x, y table are depicted on pg. *xi*.

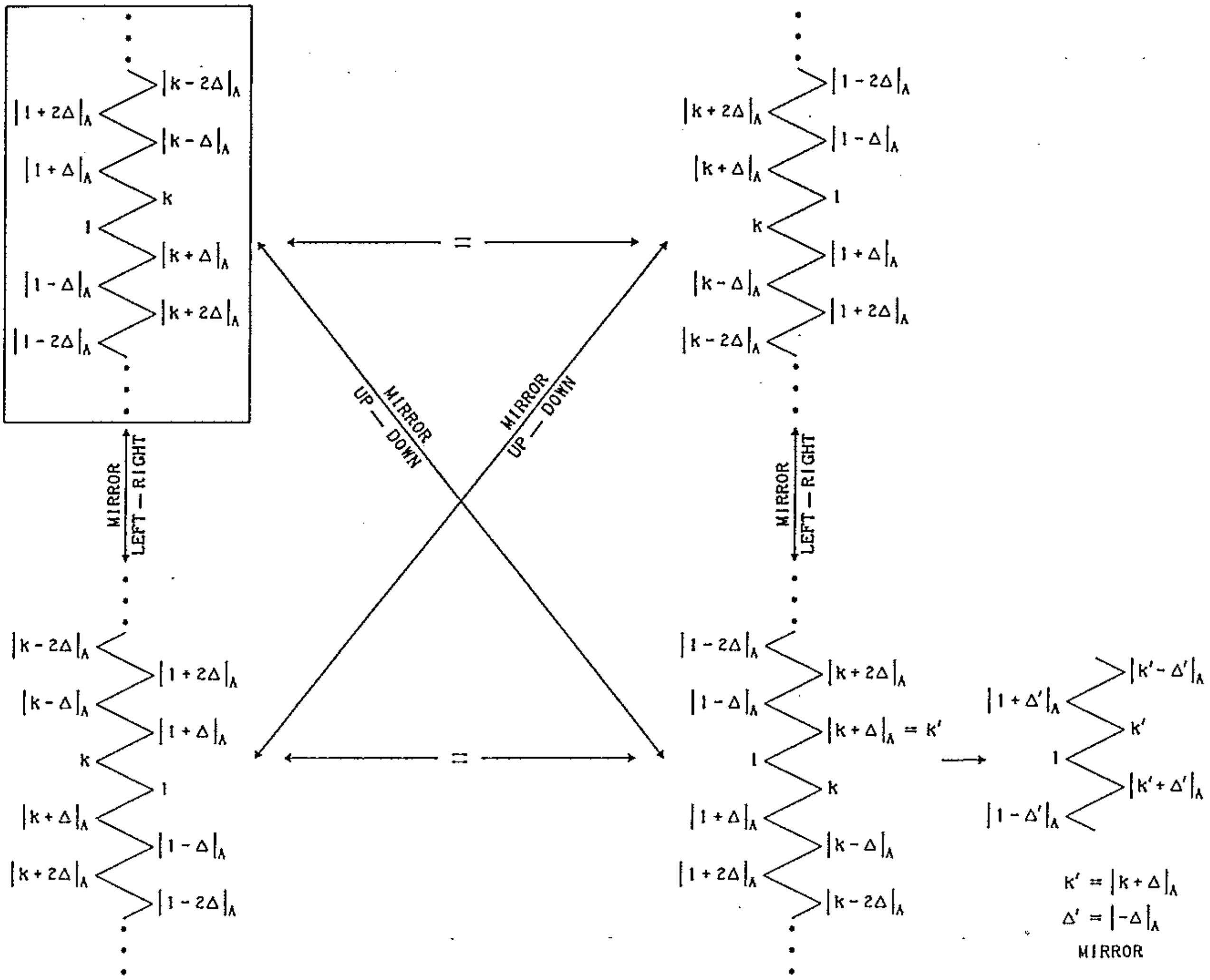
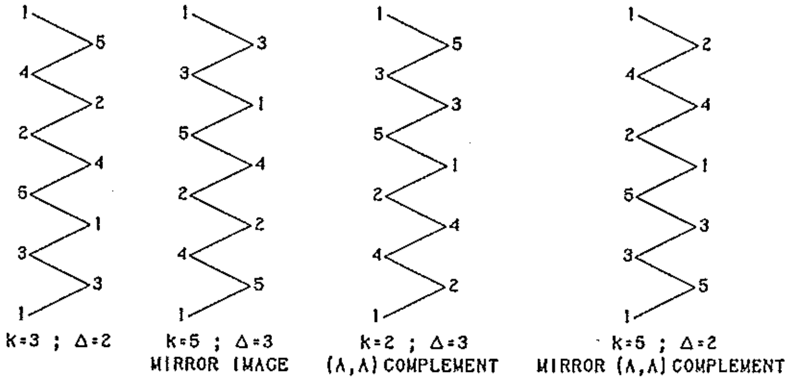


Fig. 3. — The first-return string-run and its mirror-image.

Example 1.

Let $A = 5$, $x = 10$, $y = 2$. Hence $k = 3$, $\Delta = 2$.

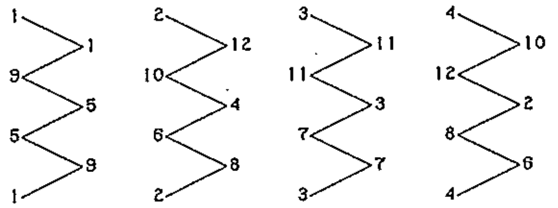


k=3 ; Δ=2 k=5 ; Δ=3 MIRROR IMAGE k=2 ; Δ=3 (A,A) COMPLEMENT k=5 ; Δ=2 MIRROR (A,A) COMPLEMENT

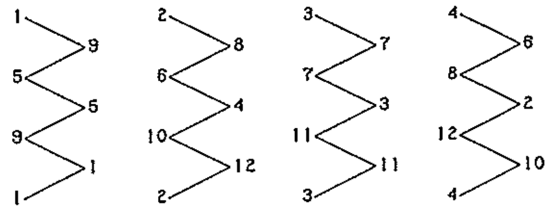
Their positions in the A, x, y table are shown on pg. *xii*.

Example 2.

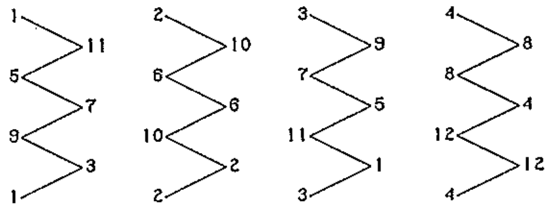
Let $A = 12$, $x = 12$, $y = 16$. Hence $k = 9$, $\Delta = 4$.



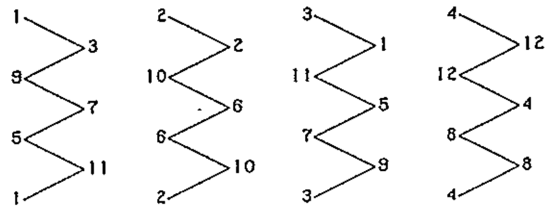
k=9 ; Δ=4



k=1 ; Δ=8
MIRROR IMAGE



k=3 ; Δ=8
(A,A) COMPLEMENT



k=11 ; Δ=4
MIRROR (A,A) COMPLEMENT

Their positions in the A, x, y table are shown on pg. *xiii*.

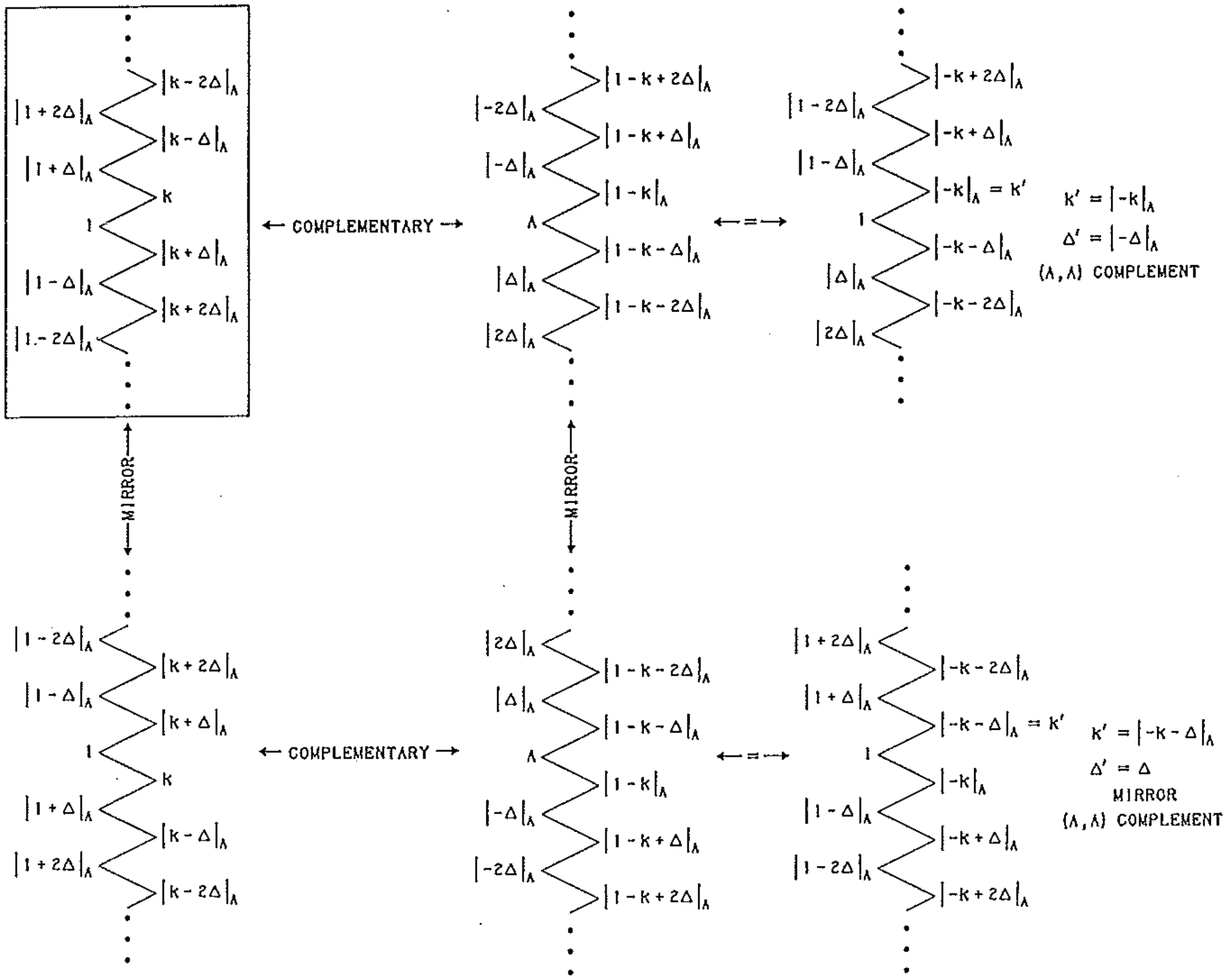
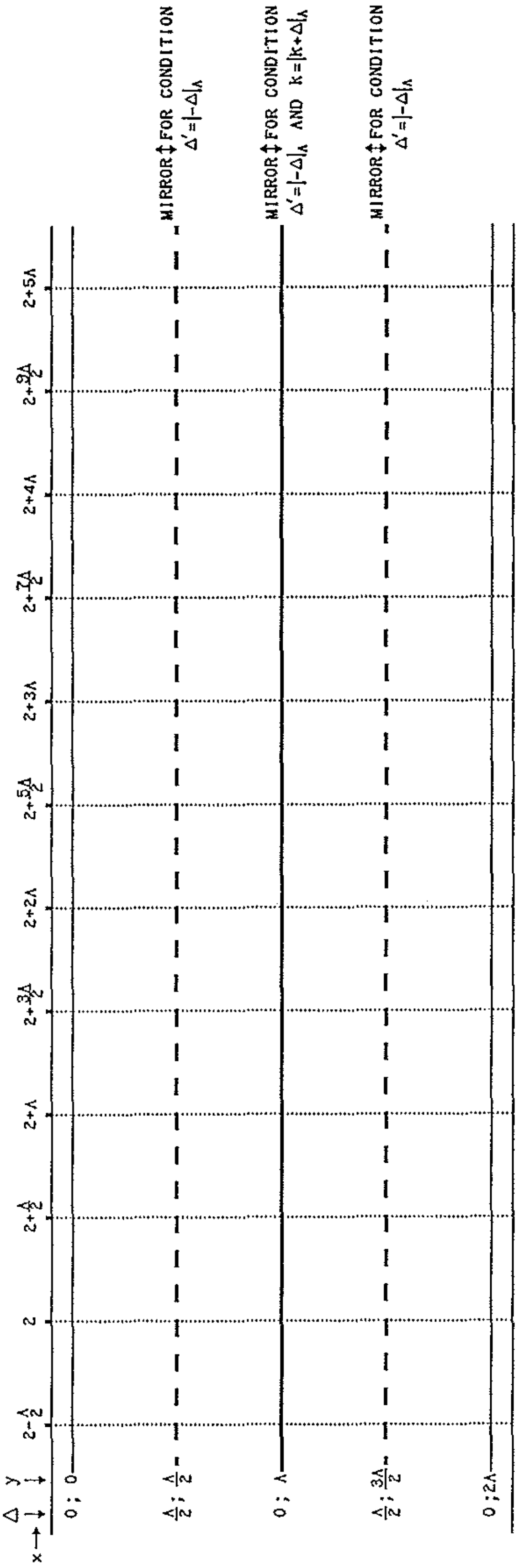
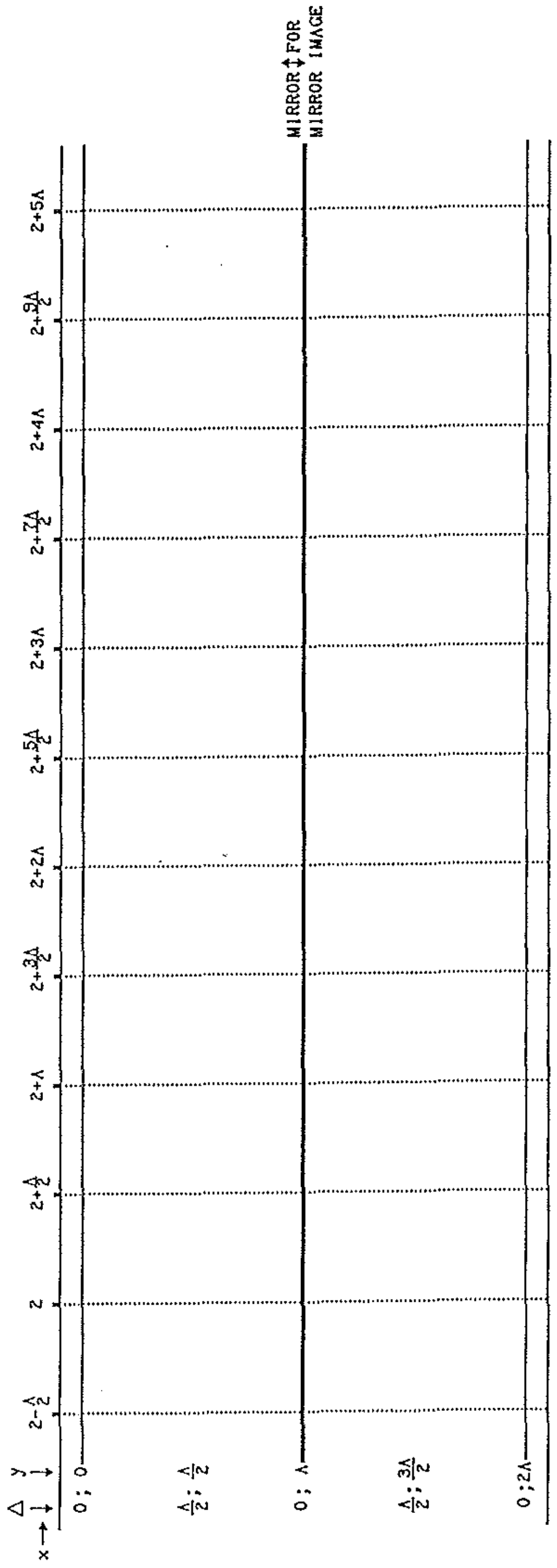
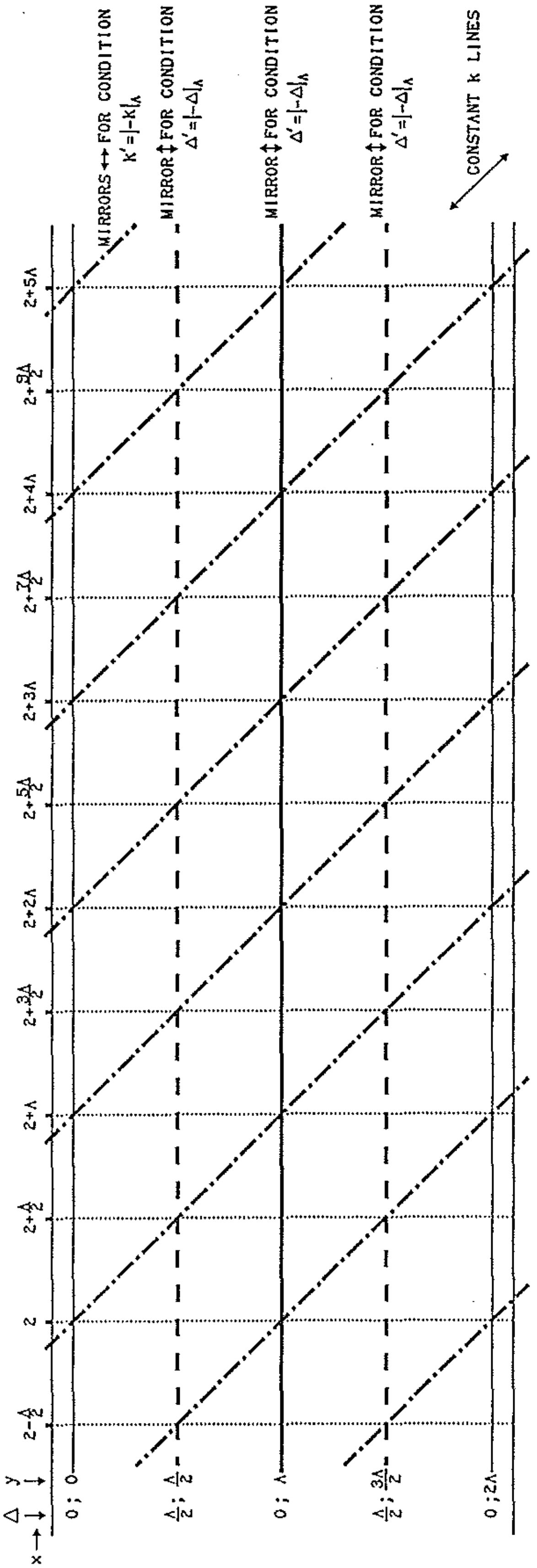
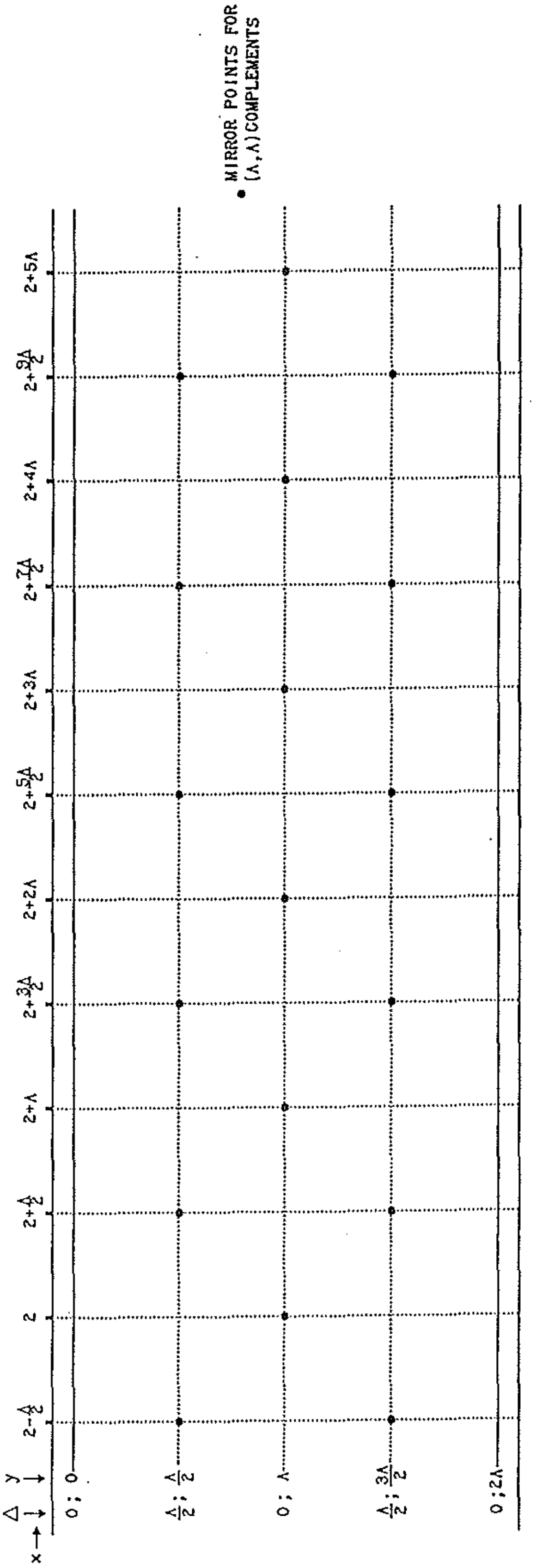


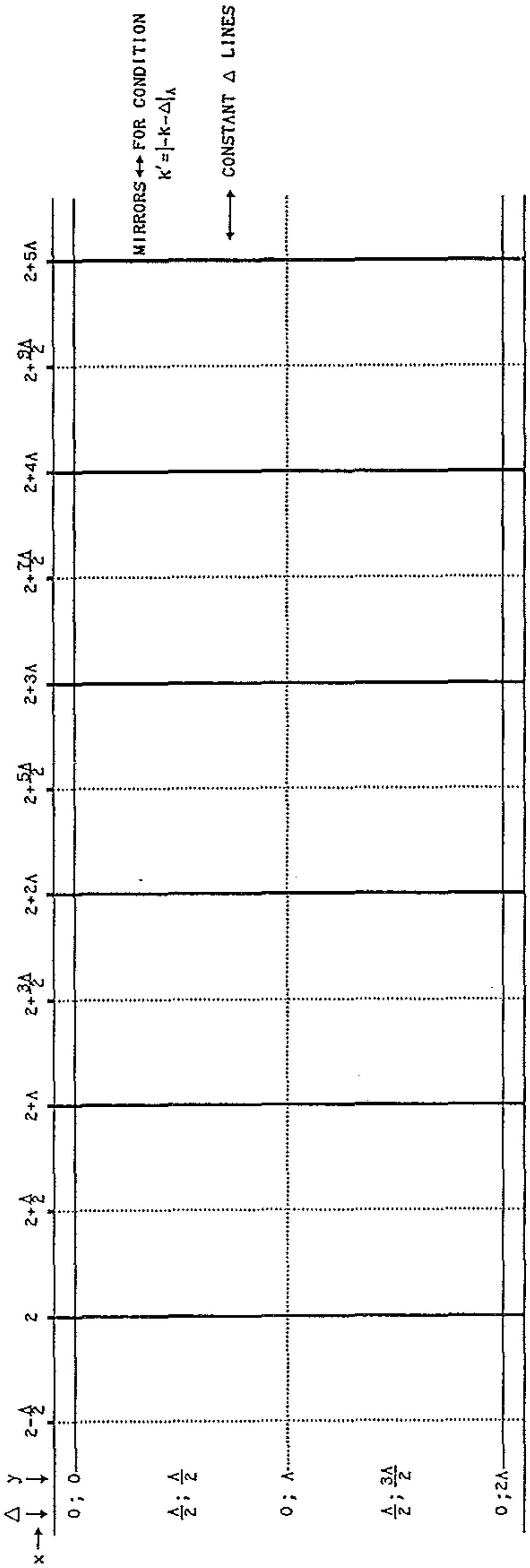
Fig. 4 — The first-return string-run and its (A, A)-complement and mirror-imaged (A, A)-complement.

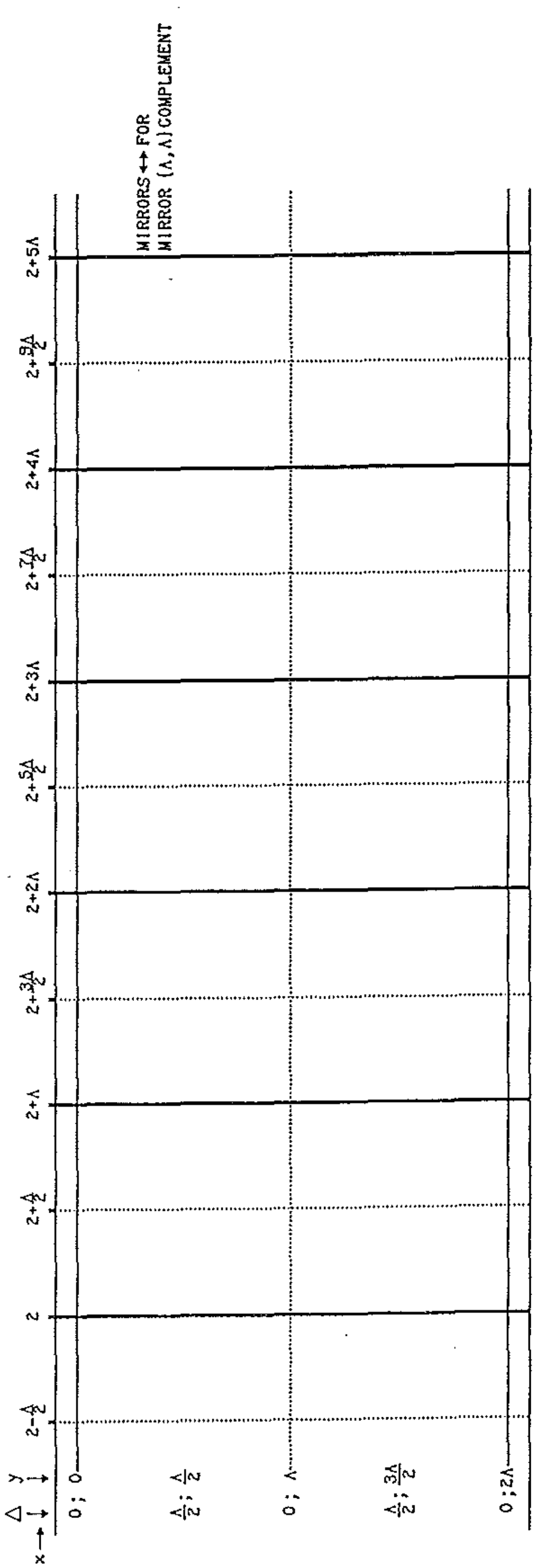

MIRROR ↕ FOR CONDITION Δ'=|-Δ|Λ
MIRROR ↕ FOR CONDITION Δ'=|-Δ|Λ AND k=|k+Δ|Λ
MIRROR ↕ FOR CONDITION Δ'=|-Δ|Λ
x
Δ
y
0 ; 0
Λ/2 ; Λ/2
0 ; Λ
Λ/2 ; 3Λ/2
0 ; 2Λ
2−Λ/2
2
2+Λ/2
2+Λ
2+3Λ/2
2+2Λ
2+5Λ/2
2+3Λ
2+7Λ/2
2+4Λ
2+9Λ/2
2+5Λ

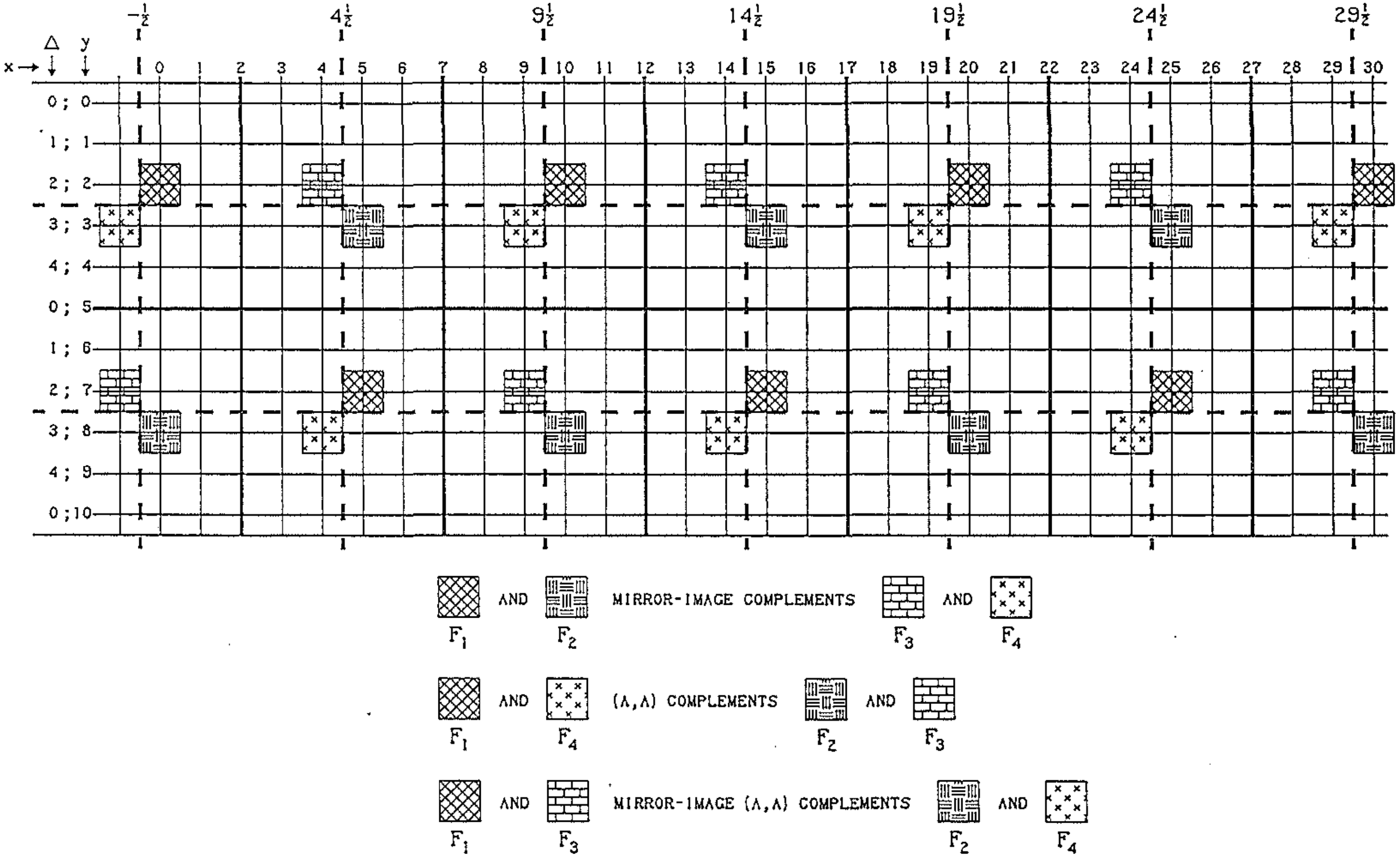

MIRROR ↕ FOR MIRROR IMAGE
x
Δ
y
0 ; 0
Λ/2 ; Λ/2
0 ; Λ
Λ/2 ; 3Λ/2
0 ; 2Λ
2−Λ/2
2
2+Λ/2
2+Λ
2+3Λ/2
2+2Λ
2+5Λ/2
2+3Λ
2+7Λ/2
2+4Λ
2+9Λ/2
2+5Λ

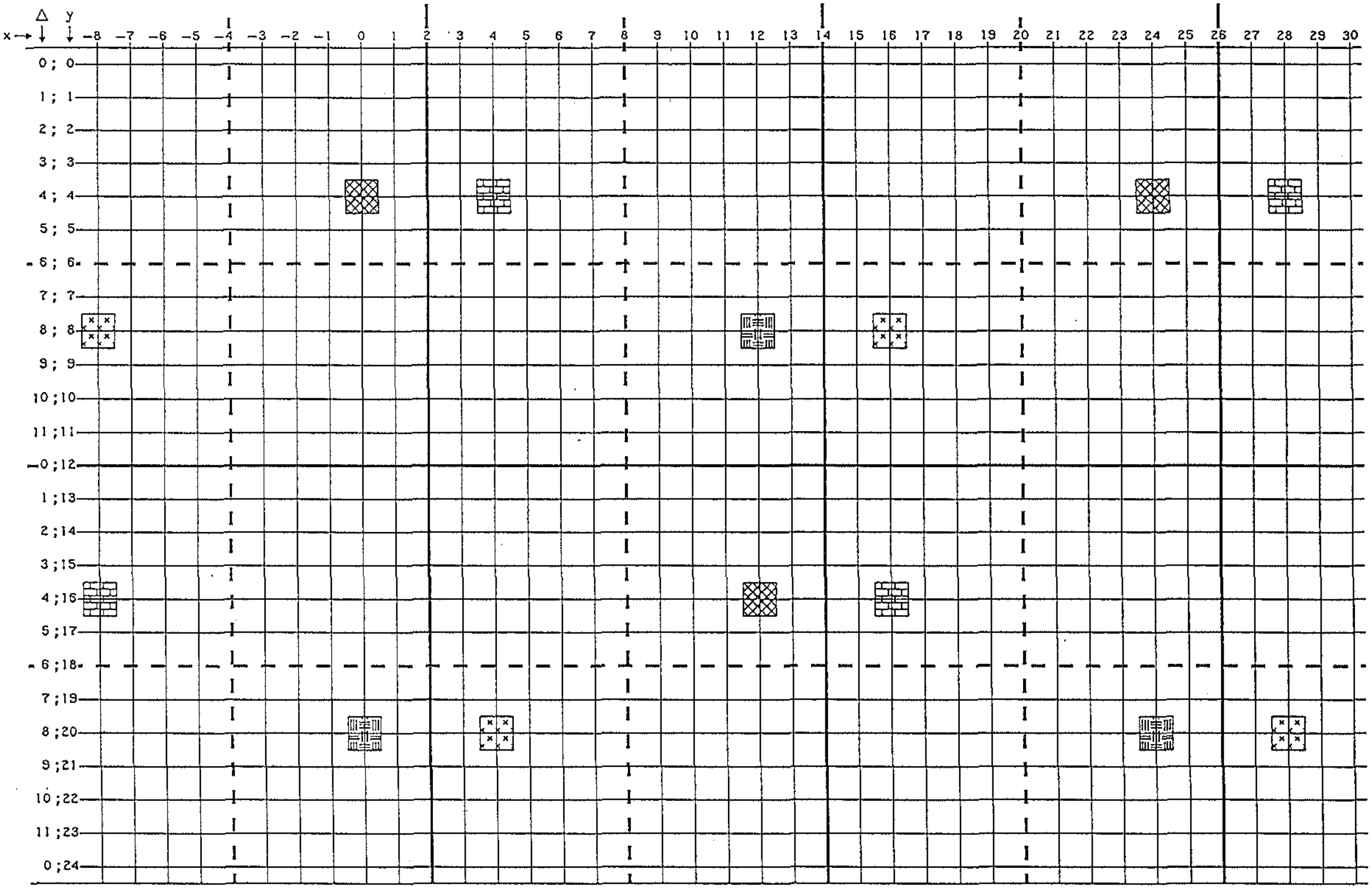

MIRRORS ↔ FOR CONDITION κ′=|−κ|Λ
MIRROR ↕ FOR CONDITION Δ′=|−Δ|Λ
MIRROR ↕ FOR CONDITION Δ′=|−Δ|Λ
MIRROR ↕ FOR CONDITION Δ′=|−Δ|Λ
CONSTANT κ LINES
x →
Δ ↓
y ↓
0 ; 0
Λ/2 ; Λ/2
0 ; Λ
Λ/2 ; 3Λ/2
0 ; 2Λ
2−Λ/2
2
2+Λ/2
2+Λ
2+3Λ/2
2+2Λ
2+5Λ/2
2+3Λ
2+7Λ/2
2+4Λ
2+9Λ/2
2+5Λ


● MIRROR POINTS FOR (Λ,Λ) COMPLEMENTS
x →
Δ ↓
y ↓
0 ; 0
Λ/2 ; Λ/2
0 ; Λ
Λ/2 ; 3Λ/2
0 ; 2Λ
2−Λ/2
2
2+Λ/2
2+Λ
2+3Λ/2
2+2Λ
2+5Λ/2
2+3Λ
2+7Λ/2
2+4Λ
2+9Λ/2
2+5Λ


MIRRORS ↔ FOR CONDITION
$\kappa' = |-\kappa-\Delta|_\Lambda$
↔ CONSTANT Δ LINES
$2-\frac{\Lambda}{2}$
2
$2+\frac{\Lambda}{2}$
$2+\Lambda$
$2+\frac{3\Lambda}{2}$
$2+2\Lambda$
$2+\frac{5\Lambda}{2}$
$2+3\Lambda$
$2+\frac{7\Lambda}{2}$
$2+4\Lambda$
$2+\frac{9\Lambda}{2}$
$2+5\Lambda$
Δ
x
y
0 ; 0
$\frac{\Lambda}{2}$; $\frac{\Lambda}{2}$
0 ; Λ
$\frac{\Lambda}{2}$; $\frac{3\Lambda}{2}$
0 ; 2Λ


MIRRORS ↔ FOR
MIRROR (Λ,Λ) COMPLEMENT
$2-\frac{\Lambda}{2}$
2
$2+\frac{\Lambda}{2}$
$2+\Lambda$
$2+\frac{3\Lambda}{2}$
$2+2\Lambda$
$2+\frac{5\Lambda}{2}$
$2+3\Lambda$
$2+\frac{7\Lambda}{2}$
$2+4\Lambda$
$2+\frac{9\Lambda}{2}$
$2+5\Lambda$
Δ
x
y
0 ; 0
$\frac{\Lambda}{2}$; $\frac{\Lambda}{2}$
0 ; Λ
$\frac{\Lambda}{2}$; $\frac{3\Lambda}{2}$
0 ; 2Λ


$-\frac{1}{2}$
$4\frac{1}{2}$
$9\frac{1}{2}$
$14\frac{1}{2}$
$19\frac{1}{2}$
$24\frac{1}{2}$
$29\frac{1}{2}$
x →
Δ
y
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
0 ; 0
1 ; 1
2 ; 2
3 ; 3
4 ; 4
0 ; 5
1 ; 6
2 ; 7
3 ; 8
4 ; 9
0 ;10
F_1 AND F_2 MIRROR-IMAGE COMPLEMENTS F_3 AND F_4
F_1 AND F_4 (Λ,Λ) COMPLEMENTS F_2 AND F_3
F_1 AND F_3 MIRROR-IMAGE (Λ,Λ) COMPLEMENTS F_2 AND F_4


x
Δ
y
−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
0; 0
1; 1
2; 2
3; 3
4; 4
5; 5
6; 6
7; 7
8; 8
9; 9
10;10
11;11
0;12
1;13
2;14
3;15
4;16
5;17
6;18
7;19
8;20
9;21
10;22
11;23
0;24

Multi-ply Regular Knots

When round braiding material is used for a Regular Knot, its string-run is often doubled, trebled, etc. Since some braiders appear to have trouble drawing the grid-diagrams of such multi-ply Regular Knots, we will show the sequential drawing steps. In the text below, each step (indicated by a bracketed number) is shown in an example for a 3-ply $p/b = 3/4$ Regular Knot.

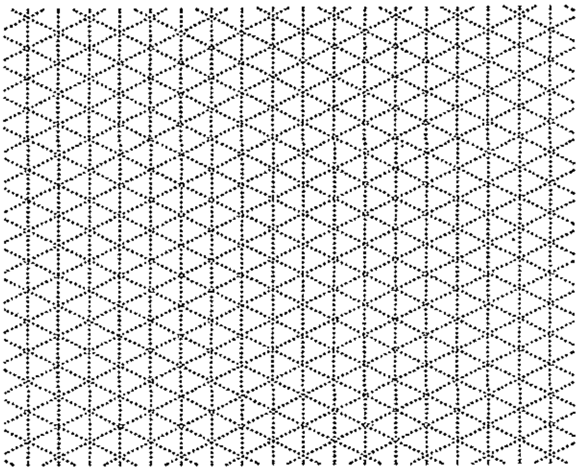
The string-run of a true n-ply Regular Knot is the string-run of a Standard Regular Nested Cylindrical Braid with $A = n$ and $k = A = n$. Their adjacent bight-boundaries at the left bight-edge and their adjacent bight-boundaries at the right bight-edge are thus 2 columns apart. They are the interbraid of $A = n$ identical Regular Knots, each having p parts and $b = B^*$ bights; hence the overall true n-ply Regular Knot requires $n = A$ essential strings and has $P_{total} = P = Ap = 2A + x - 2 = 2n + x - 2$ parts and $nb = AB^* = B$ bights.

To draw the grid-diagram of a true n-ply Regular Knot (in our example $n = A = 3$), we start by getting a piece of isometric graph-paper [1]. First we draw its string-run diagram and begin with drawing on this piece of isometric graph-paper a verical line which is the left-hand outer bight-boundary 1 [2]. Next we draw two horizontal lines nb bights apart (in our example $nb = 3 \times 4 = 12$) in the way as shown in *The Braider*, Issue No. 2, Fig. 31, pg. 33 [3]. Then to the right of left-hand bight-boundary 1 we draw $(n - 1)$ vertical lines (in our example $n - 1 = 3 - 1 = 2$), at distances of 2 columns apart, between the two horizontal lines [4]; this are the left-hand bight-boundaries $2, 3, 4, \cdots, n = A$ from left to right. Next we draw at a distance of $x = np + 2 - 2n$ columns (in our example $x = 3 \times 3 + 2 - 2 \times 3 = 5$) to the right of left-hand bight-boundary n (in our example $n = 3$) a vertical line between the two horizontal lines; this is the inner right-hand bight-boundary $n = A$ [5]. Then to the right of right-hand bight-boundary $n = A$ we draw $(n-1)$ (in our example $n-1 = 3-1 = 2$) vertical lines, at distances of 2 columns apart, between the two horizontal lines; this are the right-hand bight-boundaries $(n - 1) = (A - 1), (n - 2) = (A - 2), (n - 3) = (A - 3), \cdots, 1$ from left to right [6]. Take any bight-point on left-hand bight-boundary 1 as the apex of a nest of $n = A$ horizontally stacked bights [7] and mark on left-hand bight-boundary 1 from this nest-apex at distances of $n = A$ bights the other $(b - 1) = (B^* - 1)$ (in our example $b - 1 = 4 - 1 = 3$) nests [8]. Draw from the apex of a nest on left-hand bight-boundary 1 a line along the isometric grid from lower-left to upper-right to right-hand bight-boundary 3 [9]; this establishes the position of a nest on the right-hand bight-edge. Mark on right-hand bight-edge from this nest at distances of $n = A$ bights the other $(b - 1) = (B^* - 1)$ (in our example $b - 1 = 4 - 1 = 3$) nests [10]. Connect the left-hand and right-hand bight-apexes up along the isometric grid [11].

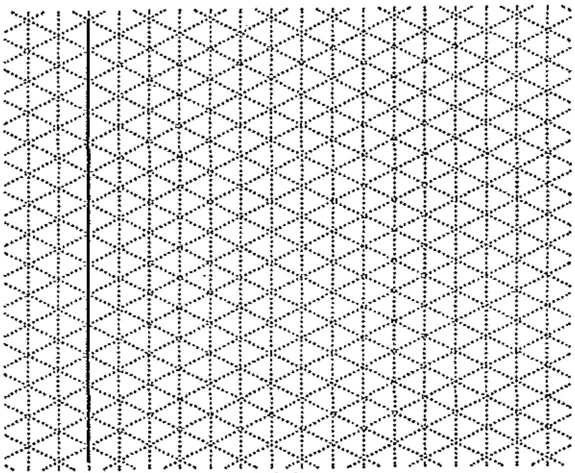
A true n-ply Regular Knot requires n essential strings (in our case $n = 3$), hence we need on each left-hand bight-boundary one starting point, the Standing End bight-point. Select these so that their position around the left-hand bight-edge is as regular as possible [12]. Number all the half-cycles [12]; this completes the string-run diagram for the true n-ply Regular Knot.

Next superimpose the coding [13]; this completes the grid-diagram for the true n-ply Regular Knot.

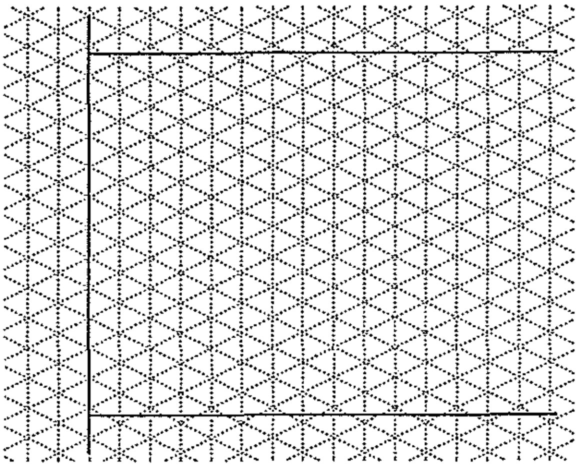
Obtain the value for Δ^* with the aid of Euclid's algorithm and the path for p/b in the RKT (in our example for $p/b = 3/4$) [immediately below [12]]. Draw the n algorithm diagrams for the n sequential braiding stages [immediately below [13]].



[1]



[2]



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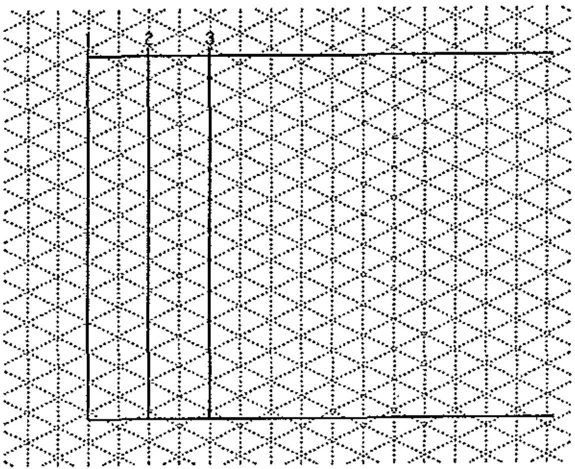
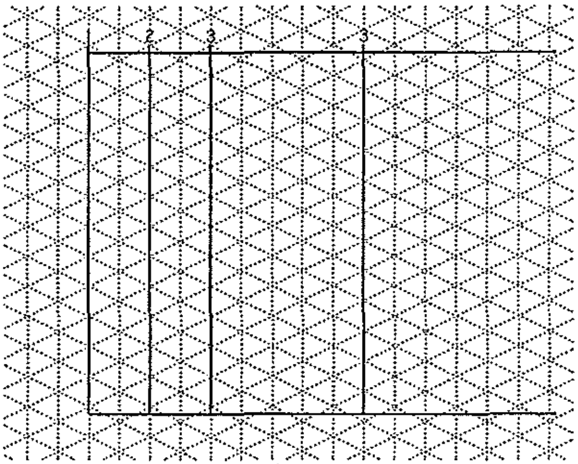
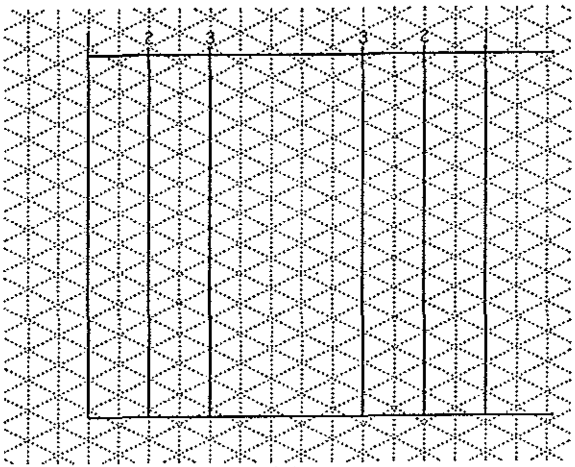


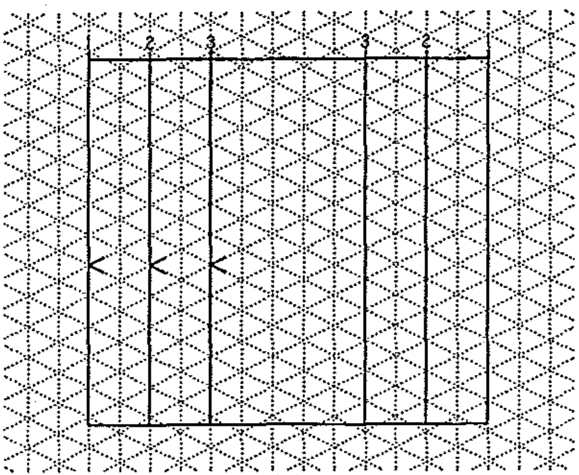
[4]



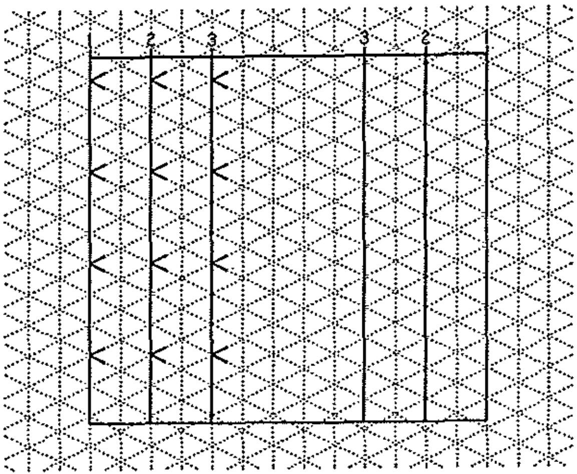
[5]



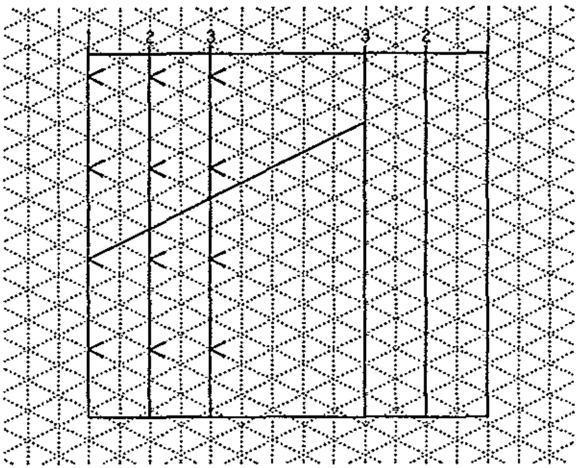
[6]



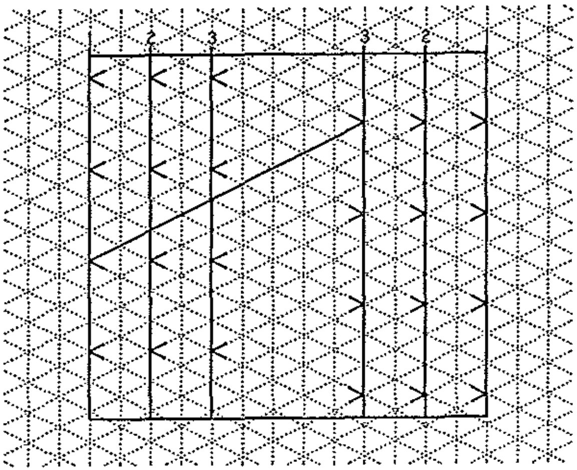
[7]



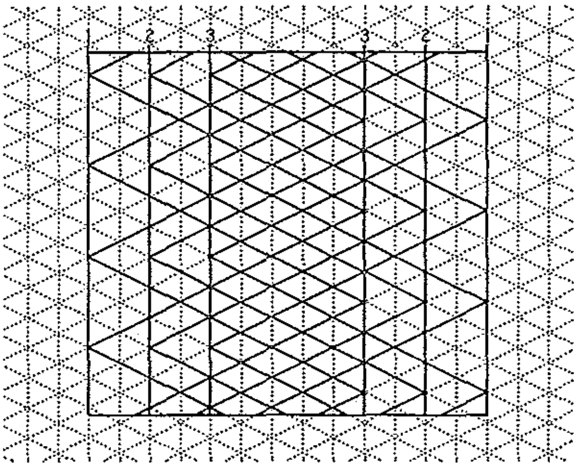
[8]



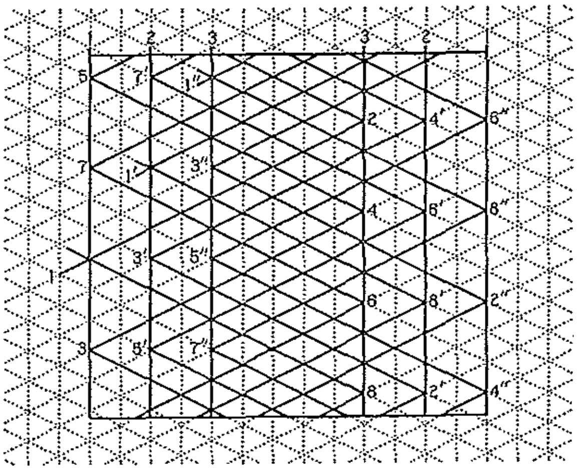
[9]



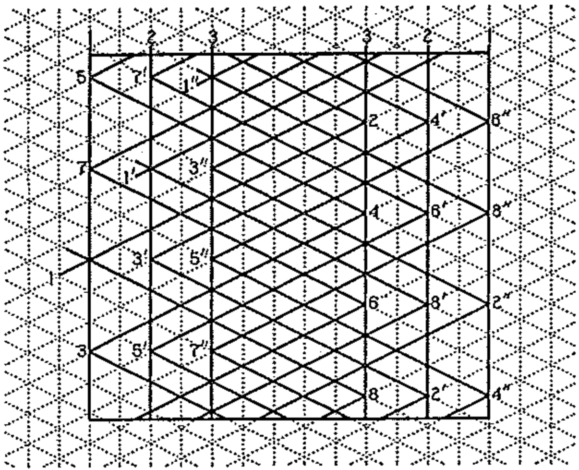
[10]



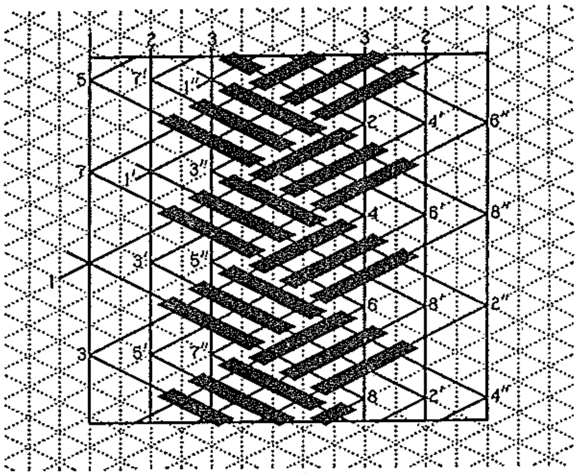
[11]



[12]



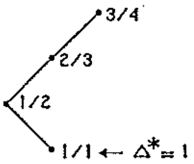
[12]

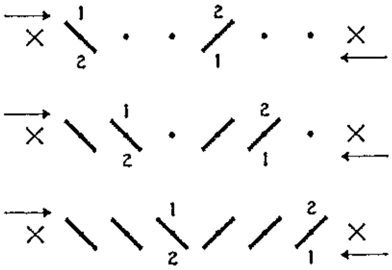


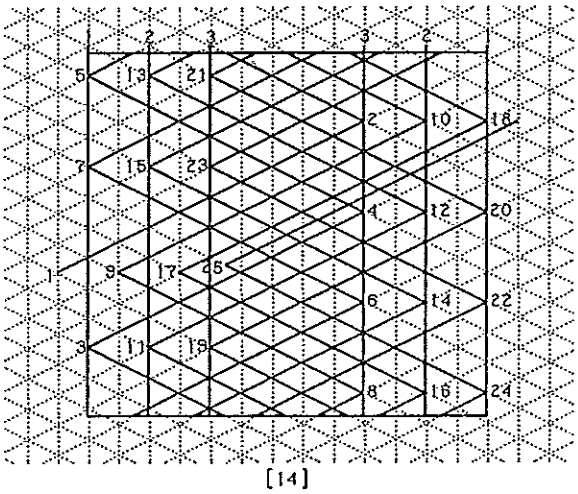
[13]

0	4	3
1	3	4
3	1	3

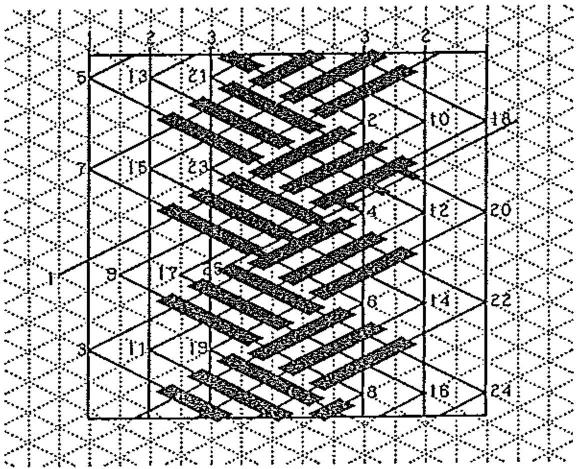
[0;1,2,1]







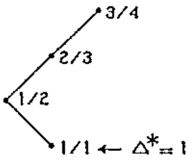
[14]

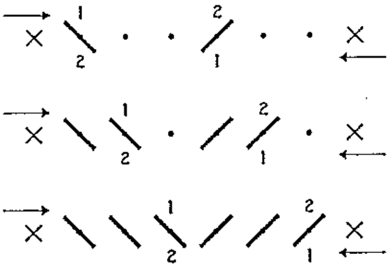


[15]

0	4	3
1	3	4
3	1	3

[0;1,2,1]





From the algorithm diagrams read for each braiding stage (the braiding of each sequential component) the half-cycle braiding algorithms.

For our example we thus obtain the following half-cycle braiding algorithms:

half-cycle 1 : $L_1 \longrightarrow R_3$: Free run.
half-cycle 2 : $i = 0$; $R_3 \longrightarrow L_1$: Free run.
half-cycle 3 : $i = 0$; $L_1 \longrightarrow R_3$: Free run.
half-cycle 4 : $i \leq 1$; $R_3 \longrightarrow L_1$: $(s)u$.
half-cycle 5 : $i \leq 1$; $L_1 \longrightarrow R_3$: u.
half-cycle 6 : $i \leq 2$; $R_3 \longrightarrow L_1$: $u - (s)o$.
half-cycle 7 : $i \leq 2$; $L_1 \longrightarrow R_3$: $u - o$.
half-cycle 8 : $i \leq 3$; $R_3 \longrightarrow L_1$: $u - o$.

half-cycle 1′ : $L_2 \longrightarrow R_2$: $u - o$.
half-cycle 2′ : $i = 0$; $R_2 \longrightarrow L_2$: $u - o$.
half-cycle 3′ : $i = 0$; $L_2 \longrightarrow R_2$: $u - o$.
half-cycle 4′ : $i \leq 1$; $R_2 \longrightarrow L_2$: $(s,1)2u - o$.
half-cycle 5′ : $i \leq 1$; $L_2 \longrightarrow R_2$: $2u - o$.
half-cycle 6′ : $i \leq 2$; $R_2 \longrightarrow L_2$: $2u - (s,1)2o$.
half-cycle 7′ : $i \leq 2$; $L_2 \longrightarrow R_2$: $2u - 2o$.
half-cycle 8′ : $i \leq 3$; $R_2 \longrightarrow L_2$: $2u - 2o$.

half-cycle 1″ : $L_3 \longrightarrow R_1$: $2u - 2o$.
half-cycle 2″ : $i = 0$; $R_1 \longrightarrow L_3$: $2u - 2o$.
half-cycle 3″ : $i = 0$; $L_3 \longrightarrow R_1$: $2u - 2o$.
half-cycle 4″ : $i \leq 1$; $R_1 \longrightarrow L_3$: $(s,2)3u - 2o$.
half-cycle 5″ : $i \leq 1$; $L_3 \longrightarrow R_1$: $3u - 2o$.
half-cycle 6″ : $i \leq 2$; $R_1 \longrightarrow L_3$: $3u - (s,2)3o$.
half-cycle 7″ : $i \leq 2$; $L_3 \longrightarrow R_1$: $3u - 3o$.
half-cycle 8″ : $i \leq 3$; $R_1 \longrightarrow L_3$: $3u - 3o$.

Finally, work the string-ends away.

Often we braid the n-ply Regular Knots as imitation n-ply Regular Knots, that is, we braid them with a single string. We have then, of course, to modify their string-run. For our example the required modification of its string-run diagram is shown in [14]; this then results in the grid-diagram shown in [15]. This modification does not affect the algoritm diagrams.

The half-cycle braiding algorithms are thus:

half-cycle 1 : $L_1 \longrightarrow R_3$: Free run.
half-cycle 2 : $i = 0$; $R_3 \longrightarrow L_1$: Free run.
half-cycle 3 : $i = 0$; $L_1 \longrightarrow R_3$: Free run.
half-cycle 4 : $i \leq 1$; $R_3 \longrightarrow L_1$: $(s)u$.
half-cycle 5 : $i \leq 1$; $L_1 \longrightarrow R_3$: u.
half-cycle 6 : $i \leq 2$; $R_3 \longrightarrow L_1$: $u - (s)o$.
half-cycle 7 : $i \leq 2$; $L_1 \longrightarrow R_3$: $u - o$.
half-cycle 8 : $i \leq 3$; $R_3 \longrightarrow L_{1+}$: $u - o$.

half-cycle 9 = 1′ : $L_{2-} \longrightarrow R_2$: $u - o$.
half-cycle 10 = 2′ : $i = 0$; $R_2 \longrightarrow L_2$: $u - o$.
half-cycle 11 = 3′ : $i = 0$; $L_2 \longrightarrow R_2$: $u - o$.
half-cycle 12 = 4′ : $i \leq 1$; $R_2 \longrightarrow L_2$: $(1,s)2u - o$.

half-cycle 13 = 5′ : $i \leq 1$; $L_2 \longrightarrow R_2$: $2u - o$.
half-cycle 14 = 6′ : $i \leq 2$; $R_2 \longrightarrow L_2$: $2u - (1,s)2o$.
half-cycle 15 = 7′ : $i \leq 2$; $L_2 \longrightarrow R_2$: $2u - 2o$.
half-cycle 16 = 8′ : $i \leq 3$; $R_2 \longrightarrow L_{2+}$: $2u - 2o$.

half-cycle 17 = 1″ : $L_{3-} \longrightarrow R_1$: $2u - 2o$.
half-cycle 18 = 2″ : $i = 0$; $R_1 \longrightarrow L_3$: $2u - 2o$.
half-cycle 19 = 3″ : $i = 0$; $L_3 \longrightarrow R_1$: $2u - 2o$.
half-cycle 20 = 4″ : $i \leq 1$; $R_1 \longrightarrow L_3$: $(2,s)3u - 2o$.
half-cycle 21 = 5″ : $i \leq 1$; $L_3 \longrightarrow R_1$: $3u - 2o$.
half-cycle 22 = 6″ : $i \leq 2$; $R_1 \longrightarrow L_3$: $3u - (2,s)3o$.
half-cycle 23 = 7″ : $i \leq 2$; $L_3 \longrightarrow R_1$: $3u - 3o$.
half-cycle 24 = 8″ : $i \leq 3$; $R_1 \longrightarrow L_{3+}$: $3u - 3o$.
half-cycle 25 = 9″ : $L_{3+} \longrightarrow R$: $6u$.

The Braider No. 27, pg. 624

Since we are dealing here with Standard and Semi-Standard Herringbone Pineapple Knots, note that $V = l_h = r_h$, the number of coding sets in respectively a row of \ coding sets and in a row of / coding sets (refer to pg. *iii* of this *Appendix*).

The Rosendahl versus Rosenthal Question

It appears that nobody has been able to resolve which of these two names is the correct one. Rather disappointing and not much of a recommendation for IGKT membership!!!

Reviews

A Comprehensive Text of Turk's Head Knots ISBN 0-9587026-2-4 170 pages A4 Ring bound.

Author :. James Edward Harvey.

Publisher : Nova Creative Publishing ; Shop 8, Plaza Arcade, Rooke Street ; Devonport 7310 ; Australia.

It takes a while to get to grips with this book. It is extremely poorly written and we have to wade at least to pg. 27 with its ups and downs to get some idea of what the author is talking about, hence what he is trying to communicate. The first doubts about the value of the book start right at the beginning with the two picture pages v and vi where we see "One Bight Knot", "Mathew Walker" (typing error — Matthew Walker) and other terms which braiding-wise make no sense. In the *Acknowledgments* (typing error — Acknowledgements) we find however a very true entry when we leave the following words between parentheses out : *(Sorry) Kate your sketches and artwork* *(would) have been of more interest than the book.* The *Foreword* reminds one of a speech at a funeral with its pleasantries, and hence would have been more appropriately placed at the end of the book and retitled *Afterword.* The *Introduction* is the autobiography of the author, which braiding-wise and in relation to the book's title *A Comprehensive Text of Turk's Head Knots* is of little or no relevance. Eventually we come to the first Chapter entitled *Turk's Head Knots & Bases.* On the first page in this Chapter we encounter a rediculous statement in the second paragraph: *A seaman or sailor will not acknowledge a Turk's Head if tied on a cylinder. It has to be tied directly around the hand, or any object and that's a fact.* Since when then is a cylinder **not** an object? The third paragraph is again a bit of autobiography. Under the heading *Turkss Head Knots* we find a short glossary which in general makes the reader not any wiser. Nowhere can be found what the author understands by *Bases*, one of the two titles of this Chapter. On the third page in this Chapter we come to *The Law and Principles of Turk's Heads.* It talks about *leads, bights* and *the left diagonal leads which must exceed the right diagonal leads by one*, while none of these items are being defined or explained. In fact this whole Chapter makes no sense untill we have at least read the next Chapter starting at its fifth page and even then we are still in the dark about several things. The procedures throughout the book are very cumbersome to say the least, hence not worth following, and many statements are incorrect. The book is certainly not worth reading let alone buying. The only good things in this book are the beautiful sketches by Kate Hunter!!!

ERRATA—*The Braider*

The list below lists the typing errors which were noted in at least some copies of the following Issues of *The Braider.*

No. 29

pg. 677, next to last line — **usual**

No. 30

pg. 704, line 9 from bottom — can **be** made

No. 31

NOTE: the first Fig. number in this Issue is 554, but should have been 584 instead. Hence the Fig. numbers 554 - 583 not only appear in *The Braider*, Issue No. 30, but the numbers 554 - 574 appear again in *The Braider*, Issue No. 31 and the numbers 575 - 583 appear again in *The Braider*, Issue No. 32. This was noted too late for rectification.

No. 32

See **NOTE** under No. 31 above.

THE BRAIDER

No.29 **FEBRUARY 2002.**

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A quarterly publication
for
the braiding artisan

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Solution to the Question in Issue No. 28

Question on pg. 649.

Let's roll the first-return string-run out. Let's take any cycle, hence any odd-numbered half-cycle running from lower-left bight-boundary l_i to upper-right bight-boundary r_i and its consecutive even-numbered half-cycle running from lower-right bight-boundary r_i to upper-left bight-boundary l_{i+1}. Then the distance in bights between the l_i bight-point and the l_{i+1} bight-point is equal to:

$$\frac{2(A-l_i)+x+2(A-r_i)+2(A-r_i)+x+2(A-l_{i+1})}{2} = 4A + x - (l_i + 2r_i + l_{i+1}).$$

The distance in bights between the starting-point and end-point of the first-return string-run is the sum of the bight-distances spanned by its cycles. A first-return string-run has α cycles, and since $\alpha = A$ for Perfect and Semi-Perfect Regular Nested Cylindrical Braids, the distance in bights between the starting-point and end-point of the first-return string-run is:

$$A \cdot 4A + A \cdot x - (2\sum l_i + 2\sum r_i).$$

Since $\sum l_i = \sum r_i = 1+2+3+\cdots+A = \frac{A}{2}\cdot(1+A)$, the distance in bights between the starting-point and end-point of the first-return string-run is:

$$4A^2 + Ax - [A(1+A) + A(1+A)] = 2A^2 + Ax - 2A = 2A^2 + A(x-2).$$

With the first sequence of half-cycles in a first-return string-run starting at nest-index number $I_L = 0$, the end-point of this first-return string-run, and hence the starting-point of second sequence of half-cycles in a first-return string-run will be at nest-index number

$$I_L = \left|2A^2 + A(x-2)\right|_B = A|2A + x - 2|_{B^*}.$$

The sequence of half-cycles between the starting-point of half-cycle h_n (≥ 1) and the starting-point of half-cycle $h_n + 2A$ $(\leq 2B)$ is the sequence of half-cycles in a first-return string-run (although generally of course not starting with the same type of half-cyle), and hence when half-cycle h_n starts at nest-index number I then half-cycle $h_n + 2A$ starts at nest-index number $\left|I + 2A^2 + A(x-2)\right|_B$.

Some Heel Knot Constructions

Bosals[†] are a piece of braided horse equipment which is probably more commonly found in the United States and Mexico. Several braiding skills are necessary for the construction of the bosal. General construction details can readily be found in the books "Encyclopedia of Rawhide and Leather Braiding" and "How to Make Cowboy Horse Gear" both by Bruce Grant[‡] or in "Braiding Rawhide Horse Tack" by Robert Woolery[††]. The appearance of the finished product depends on the quality and size of the string-materials used and on the braiding and knot tying skills of the braider. There is always the opportunity to try various braids for the noseband[†] itself and to tie very beautiful multi-colour knots for the noseband button knot, the side buttons and the heel knot. They can also be made quite plain, yet perfectly functional, by using only single colour and heavier, coarser string-material. Quality rawhide is the material of choice. We shall restrict our discussion here to a single aspect of the bosal construction, namely some construction procedures for the **heel knot** in association with the joining of the two ends of the noseband braid, and introduce techniques which have not been explored in the above mentioned books. These techniques are not themselves new to braiding or even new to the braiding of bosals. Several bosals are known which were made by a Mexican braider in the 1940's which employ these techniques, but the methods appear to be seldom used and are perhaps largely unknown in relation to bosals. The techniques should not be dismissed as being too complicated or too time consuming or that they require more string-material; the difference is insignificant. The braider himself/herself should decide to use or not to use the techniques on their own merit. Any braider who chooses to use these methods may need to alter somewhat the techniques given here and adapt them to their own project: for the type and size of string-material being used and the number of strings in the noseband braid.

[†] The **bosal** *(boh-zal)* is the noseband of the **hackamore**. The **hackamore** is a type of headstall or bridle without a bit.

[‡] For further details about these books see *The Braider*, Issue No. 1, pp. 17-18.

[††] For further details about this book see *The Braider*, Issue No. 9, pp. 197-198.

As mentioned above, we shall restrict our discussion to some methods of joining the two ends of the noseband by means of the heel knot. We therefore presuppose that the braider has already braided the noseband of a sufficient length, probably about 80 cm. (32 inches), with each string of the braid extending 45 cm. (18 inches) beyond each of the ends of the noseband braid. Furthermore, we will assume that the braider has already tied the noseband button knot and the side button knots of whatever type, colour and style desired. We are then ready to bend the two ends of the braided noseband together and join them in some secure fashion with the aid of the heel knot.

Two methods are commonly employed and both methods are described in each of the above mentioned books by Bruce Grant, which we will use as reference, while only one method is described in Robert Woolery's book.

On pg. 137 of the "Encyclopedia of Rawhide and Leather Braiding" and on pg. 44 of "How to Make Cowboy Horse Gear", fig. 1 and fig. 2 show the method by which the two ends of the noseband braid are tied together (a constrictor knot can be used for this) and subsequently a Spanish ring knot is placed over these combined ends. Then each one of the projecting strings of the noseband is wrapped around this Spanish ring knot as indicated by the dashed lines and arrows in fig. 1.

For the other method the text on pg. 134 of the "Encyclopedia of Rawhide and Leather Braiding" and on pg. 40 of "How to Make Cowboy Horse Gear", gives reference to respectively pg. 199 and pg. 100, where figs. 9, 10, 11 indicate how to use four or six strings of the combined projecting string-ends of the noseband braid[‡] to braid a knot around the remaining projecting strings as indicated on respectively pg. 201 and pg. 102, by the dashed lines and arrows in figs. 9,10,11.

We can, however, also use a method which employs all the projecting strings of the noseband braid to braid a knot, or knots, which can be left as the finished heel knot or it can be used as a foundation knot over which to braid one of the many woven knots.

[†] Also known as the cheekpiece braid.

[‡] Half from each end of the noseband braid.

Say the noseband has no core and consists of an 8-string round braid with a herringbone $2u - 2o$ coding-sequence. After the two ends of the noseband braid have been brought together we will have a total of 16 strings with which to work. If these 16 strings were to be evenly divided into 4 sets, then there would be 4 strings in each set. The 4 strings within each set could themselves then be braided into a 4-string flat braid or a 4-string round braid.

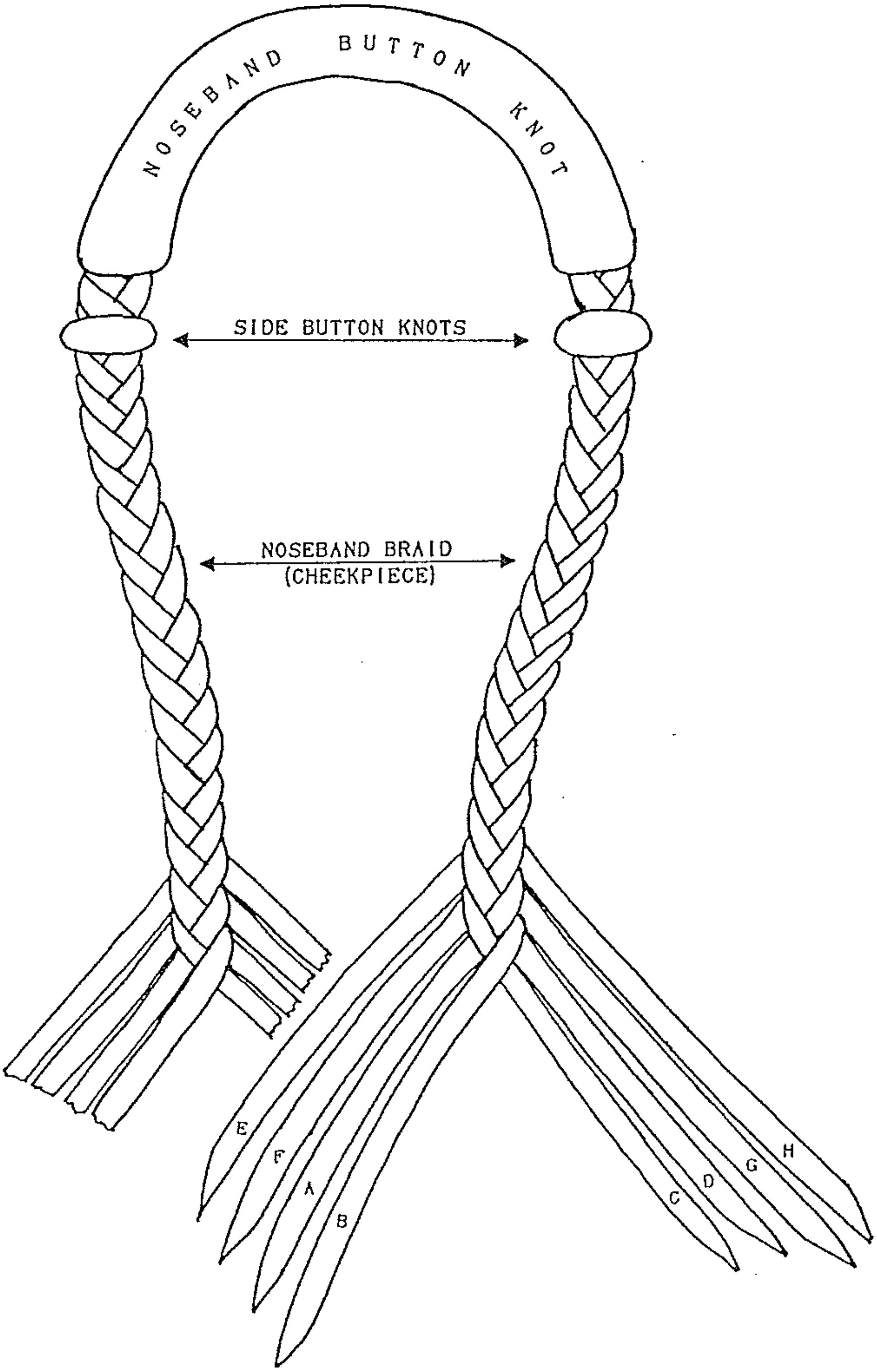


Fig. 528 — The 8-string round noseband braid with noseband button knot and side button knots.

When we have braided the 8-string round noseband to its required length, we can reach a stage at any time where the braid on either end will appear as shown in Fig. 528 with the hair-side of the strings uppermost; this stage is also shown by the leftmost grid-diagram of Fig. 529.

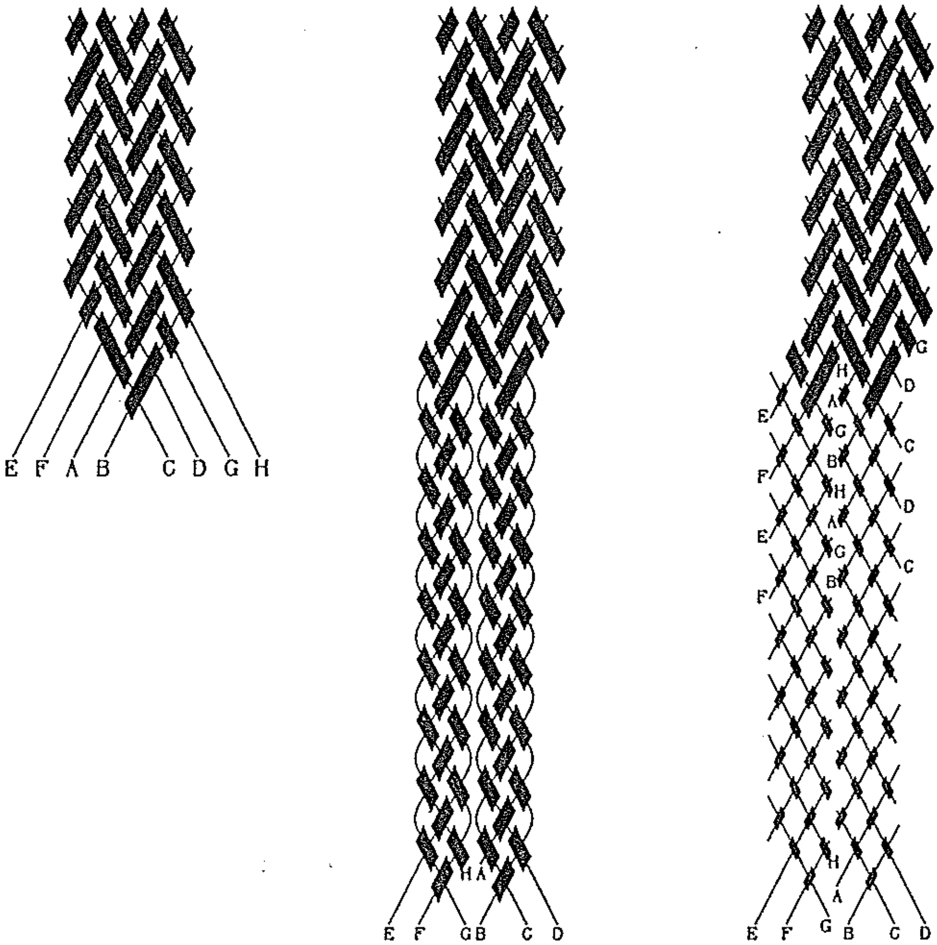


Fig. 529 — Each end of the 8-string round noseband braid.

By now taking the strings *E* , *F* , *G* and *H* of one end of the noseband braid, one at a time around behind the braid in the order *H* , then *E* , then *G* , then *F* , the front and the back of the braid will appear exactly alike and are as shown for the strings *A* , *B* , *C* and *D* in the left-hand drawing of Fig. 530. For the strings *E* , *F* , *G* and *H* , string H lies behind string *A* , string *E* lies behind string *D* , string *G* lies behind string *B* , and string *F* lies behind string *C* .

Strings *A* , *B* , *C* and *D* can then be braided into a 4-string flat braid as depicted in the right-hand drawing of Fig. 530, and by the central grid-diagram of Fig. 529, for a length of about 30 cm. (12 iches). Strings *E* , *F* , *G* and *H* are braided similarly.

Not only are the 8-strings on the other end of the noseband braid also arranged as the strings *E* , *F* , *A* , *B* , *C* , *D* , *G* and *H* in Fig. 528 and as in the leftmost grid-diagram of Fig. 529, but they will undergo the same process as depicted for those by the right-hand drawing of Fig. 530 and the cental grid-diagram of Fig. 529. Once completed, we thus will have four 4-string flat braids: two at each end of the noseband braid.

When the two ends of the noseband braid are ready to be joined, they are brought together and the four 4-strand flat braids are used to tie any 4-strand terminal knot.

Simple terminal knots which may be used are the 3-part/4-bight *over-under* coded regular cylindrical braided open-ender, the 4-part/4-bight *over-under* coded regular cylindrical braided open-ender as depicted respectively by the left-hand and central grid-diagrams of Fig. 531.

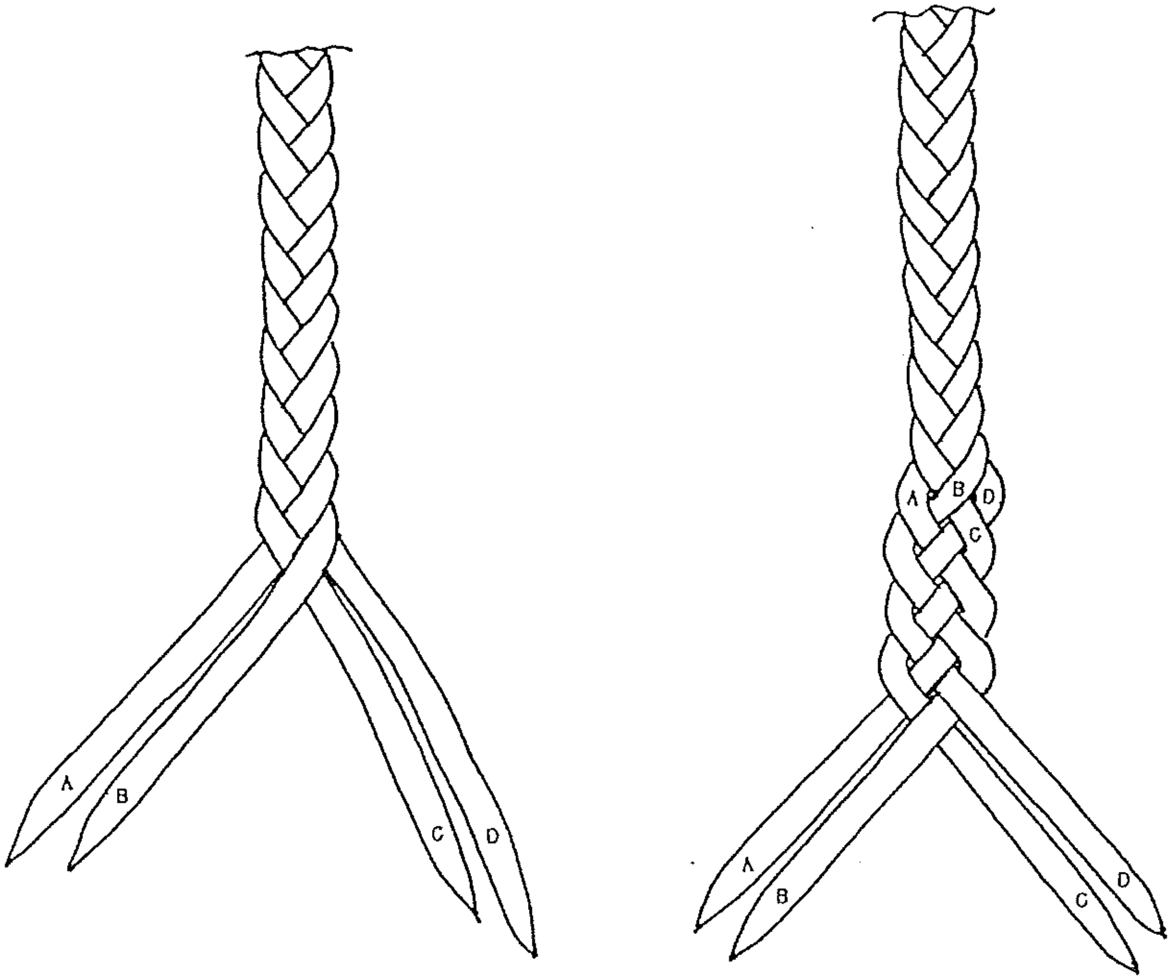
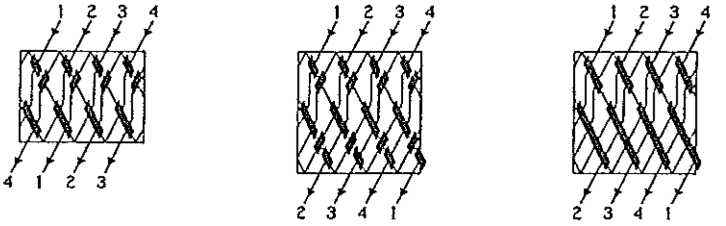


Fig. 530 — The strings A, B, C and D.



p/b=3/4 p/b=4/4 UNIVERSAL p/b=4/4 MATTHEW WALKER

Fig. 531 — Some 4-strand terminal knots.

Instead of braiding each set of 4-strings into the flat braids as described above and shown in the right-hand drawing of Fig. 530 and the central grid-diagram of Fig. 529, we could braid each set of 4 strings into the round braids as shown by the right-hand grid-diagram of Fig. 529. The four 4-string round braids can then be tied into a terminal knot such as depicted in Fig. 531, including the 4-part/4-bight Matthew Walker coded regular cylindrical braided open-ender which is well adapted to being constructed with round cord. If these finished terminal knots are of sufficient bulk to suit the needs of the bosal, they could then be left as is to form the complete heelknot. If more bulk is required or desired then these terminal knots may be used as a foundation over which to tie any woven knot.

It is important that the ends of the noseband can not be pulled apart and out of the heelknot. Terminal knots constructed in this general fashion make for a very secure combining of the two ends of the noseband braid, whether they are left as the final heelknot or are covered with a woven knot.

Quite often the noseband is braided over a core. Steel cable, braided rawhide and twisted rawhide are the most common core-materials used. We can use the core-material in the terminal knot when a rawhide core is being used. In each of the above mentioned books the use of twisted rawhide cores is considered a very common practice and may be a best core to use.

A two strand twisted rawhide core, made with the same width and thickness of strings as used for the noseband, makes a very good core inside an 8-string round noseband braid. If, for instance, we were to need a noseband braid of 13 mm. ($\frac{1}{2}$ inch) diameter, we can use 5 mm. ($\frac{3}{16}$ inch) wide rawhide strings for both the core and the noseband braid braiding material.

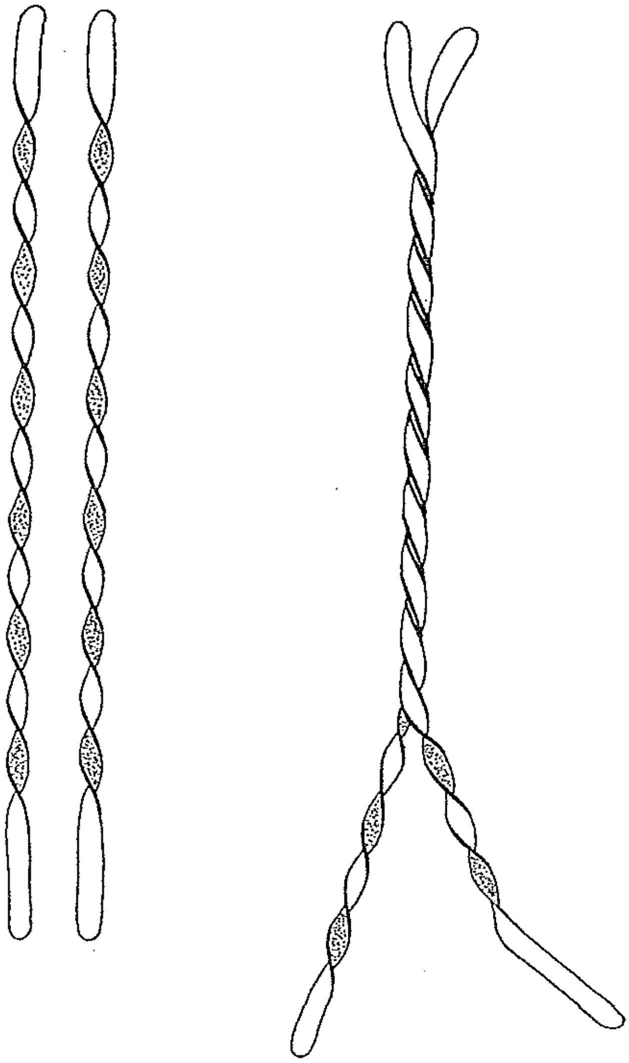


Fig. 532 — The two strand twisted core construction.

The core can be fashioned as drawn in Fig. 532, where we can twist the two tempered strands of the core separately, each in either the same right-hand or left-hand helix (a left-hand helix is depicted in Fig. 532). These twists are not done tightly since the next step will be to lay one strand into the twists of the other strand so as to intertwine them as shown at the right in Fig. 532. Once the two strands have been lain together, the twist can be tightened up and the core stetched and secured to dry. The core should be twisted for a length of about 80 cm. (32 inches) and have an additional 45 cm. (18 inches) of non-twisted string length on each end which will be used in the heelknot construction.

Over this core we braid the 8-string round herringbone noseband braid of $2u-2o$ sequence until the braid on either end again looks like the drawing in Fig. 528 with the addition of the twisted core-strands running through the centre of the braid. The strings E, F, G and H are then each taken around the back, one at a time in the order H, then E, then G then F and the braid will appear as on the left in Fig. 530. When the core has been twisted with a left-hand helix as drawn in Fig. 532 we can now introduce one strand of the core into the braid as depicted at the left in Fig. 533 and begin a 5-string flat braid of $2o-2u$ sequence as shown in Fig. 533. This braid is continued as depicted at the right in Fig. 533 for a length of about 30 cm. (12 inches).

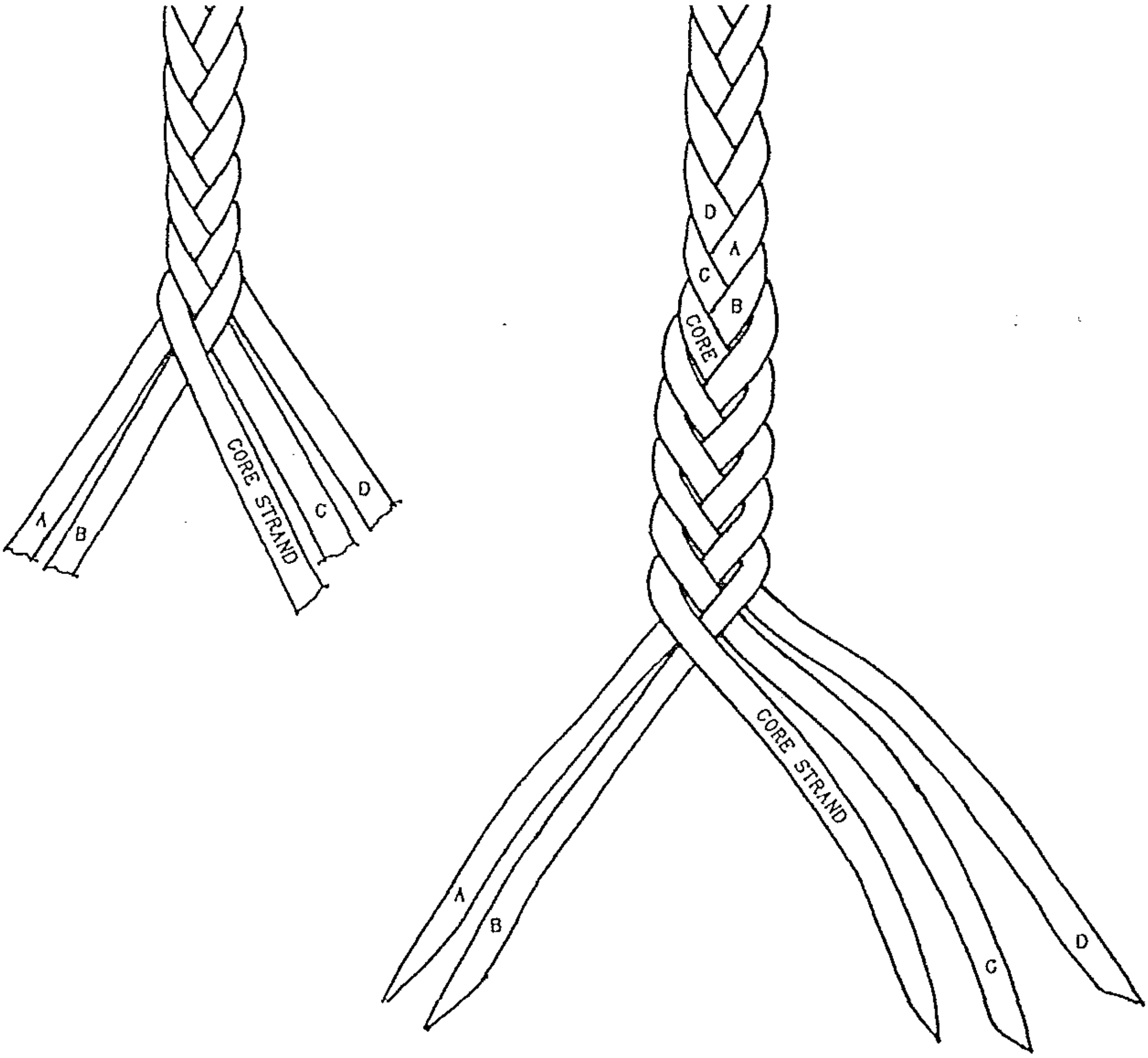


Fig. 533 — The 5-string flat braids.

Since the arrangement of the strings E, F, G and H should look similar to that shown in Fig. 530, the second strand of the twisted core can be introduced into these strings in the manner similar to that shown in Fig. 533 and a second 5-string flat braid with a $2o - 2u$ sequence is made for a length of about 30 cm. (12 inches).

The 8 strings at the other end of the noseband braid are also arranged as in Figs. 528 and 530. By using the two stands at the other end of the twisted core, we can braid two additional 5-string flat braids. Once completed we thus obtain four 5-string flat braids: two at each end of the noseband braid.

When the two ends of the noseband braid are ready to be joined, they are brought together and the four 5-string flat braids are used to tie any desired 4-strand terminal knot including, but not limited to, the knots diagrammed in Fig. 531.

When any of these terminal knots have been tied and tightened, the ends of the flat braids are cut off at the end of the terminal knot. They can be shaped into a nice round knot while the rawhide is still tempered, and once dry the knots should not come apart under normal conditions. To add bulk to the heelknot, the terminal knot may be covered with any desired woven knot.

Nested Cylindrical Braids

In the previous Issue we discussed a simple method for working out the half-cycle braiding algorithms for the Perfect Herringbone Pineapple Knots, and we gave two numerical Examples which were a mirror-imaged pair. We furthermore showed how from the half-cycle tables of a Perfect Herringbone Pineapple Knot the half-cycle tables of its mirror-image may be obtained. With respect to this latter process, the observant reader will have noticed that the two Examples were a special case in that the numeral to be added to, respectively to be subtracted from, the half-cycle number was equal to A. In general this numeral, of course, ranges from 1 to $2A - 1$, and it will only be equal to A when A is odd and $x = 2nA + 4$, where n is a whole number; in which case $k = \frac{A+3}{2}$ for $y = A - 1$, and $k = \frac{A+1}{2}$ for $y = A + 1$.

Let's look at two other Examples which are each others mirror-image :

Example 1:

$A = 4; x = 19; y = A - 1 = 3$, hence $\Delta = |3|_4 = 3; B^* = 4$, hence $B = A \cdot B^* = 16$.
Thus $k = \left|\frac{x-A-1}{2}\right|_A = \left|\frac{19-4-1}{2}\right|_4 = 3$.

At the upper left-hand side in Fig. 534 is depicted the first-return string-run with its associated nest-index numbers.

At the upper right-hand side in Fig. 534 are depicted the first-return string-run, the half-cycle numbers of the half-cycles, and the number of crossings which these half-cycles make in the finished knot. This layout at the upper right-hand side has been shown for additional clarification only since it can readily be dispensed with.

The depicted layout at the bottom of Fig. 534 gives the positions of the nests with respect to the nest-index numbers, the half-cycle numbers with respect to the nest-index numbers and bight-boundaries, and the number of crossings which the half-cycles make in the finished knot. This layout is the one we need for compiling the half-cycle algorithms; it is directly obtained from the first-return string-run with its associated nest-index numbers, hence from the upper left-hand layout in Fig. 534. The first step in its construction is shown in Fig. 535.

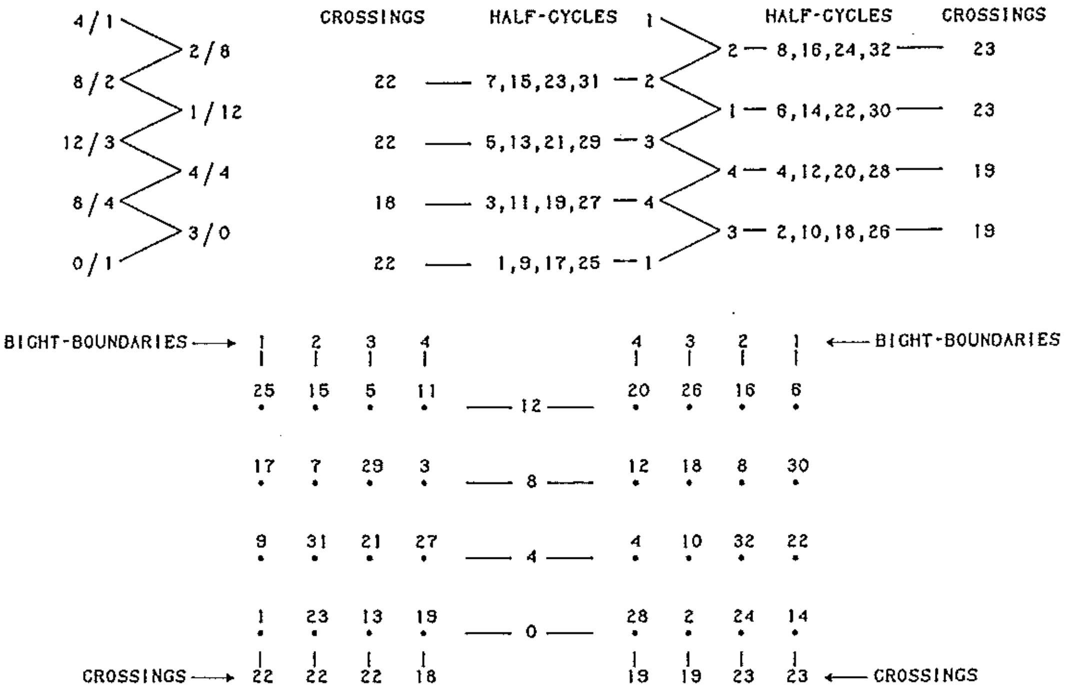


Fig. 534 — First-return string-run and half-cycle pattern.

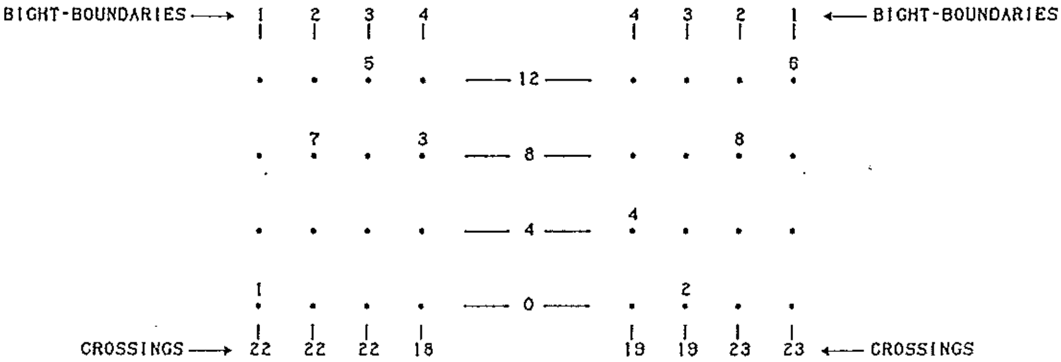


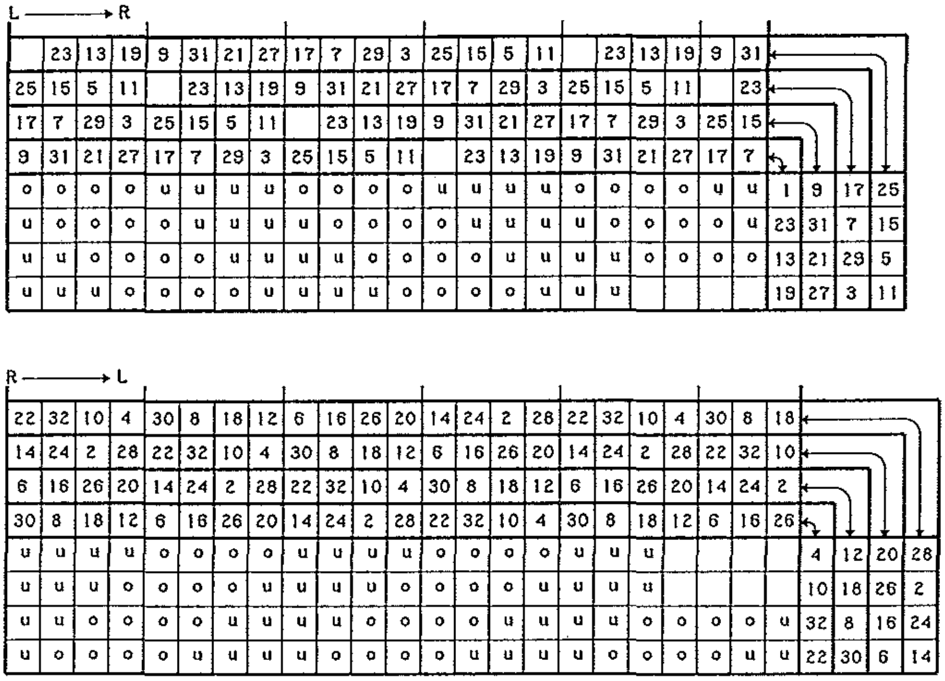
Fig. 535 — The first step in the construction of the half-cycle pattern.

The second step in the construction of the half-cycle pattern shown at the bottom of Fig. 534 consists of the determination of the half-cycle numbers in association with the nest-index numbers and bight-boundaries of their starting points.

Since the second sequence of the half-cycles in a first-return string-run starts with half-cycle number $1+2A=9$ at nest-index number $I_L = A|2A+x-2|_{B^*} = 4|25|_4 = 4$, two sequential half-cycle numbers h_n (≥ 1) and h_n+2A $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I+2A^2+A(x-2)\right|_B = \left|I+2\cdot 4^2+4(19-2)\right|_{16} = |I+100|_{16} = |I+4|_{16}$. Hence we can readily complete the half-cycle pattern concerned. With the aid of the coding-sequence of the half-cycles in the finished knot with a Herringbone Pineapple coding and $y=A-1$ (see pg. 646) we can either read the half-cycle braiding algorithms directly from the half-cycle pattern, or from the set of half-cycle tables shown in Fig. 536 which we assemble from the half-cycle pattern at the bottom of Fig. 534.

The upper table in Fig. 536 is for the odd-numbered half-cycles, hence the half-cycles from lower-left to upper-right, while the lower table in Fig. 536 is for the even-numbered half-cycles, hence the half-cycles from lower-right to upper-left.

Note that the half-cycle which ends at the start of half-cycle 1 is the very last half-cycle in the braiding process and hence will not be crossed during the braiding process, thus we leave its cell empty.



L → R

	23	13	19	9	31	21	27	17	7	29	3	25	15	5	11		23	13	19	9	31				
25	15	5	11		23	13	19	9	31	21	27	17	7	29	3	25	15	5	11		23				
17	7	29	3	25	15	5	11		23	13	19	9	31	21	27	17	7	29	3	25	15				
9	31	21	27	17	7	29	3	25	15	5	11		23	13	19	9	31	21	27	17	7				
o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	1	9	17	25
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	23	31	7	15
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	13	21	29	5
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u					19	27	3	11

R → L

22	32	10	4	30	8	18	12	6	16	26	20	14	24	2	28	22	32	10	4	30	8	18				
14	24	2	28	22	32	10	4	30	8	18	12	6	16	26	20	14	24	2	28	22	32	10				
6	16	26	20	14	24	2	28	22	32	10	4	30	8	18	12	6	16	26	20	14	24	2				
30	8	18	12	6	16	26	20	14	24	2	28	22	32	10	4	30	8	18	12	6	16	26				
u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u					4	12	20	28
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u					10	18	26	2
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	32	8	16	24
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	22	30	6	14

Fig. 536 — The half-cycle tables for the knot in Example 1.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 1:

1. 1 ↗ 3: Free run.
2. 4 ↖ 3: o.
3. 4 ↗ 4: u.
4. 3 ↖ 4: $u - o$.
5. 3 ↗ 1: $o - u$.
6. 2 ↖ 1: $o - u - 2o$.
7. 2 ↗ 2: $o - 2u - o$.
8. 1 ↖ 2: $2u - 2o - 2u$.
9. 1 ↗ 3: $o - 2u - 2o$.
10. 4 ↖ 3: $u - o - u - o - 2u$.
11. 4 ↗ 4: $o - u - 2o - u$.
12. 3 ↖ 4: $u - o - 2u - 2o - u$.
13. 3 ↗ 1: $u - o - u - 2o - 2u - o$.
14. 2 ↖ 1: $2o - 3u - o - u - 2o - u$.
15. 2 ↗ 2: $4o - 3u - 2o$.
16. 1 ↖ 2: $u - o - 3u - 3o - u - o - u$.
17. 1 ↗ 3: $3o - u - o - 3u - 3o$.
18. 4 ↖ 3: $2u - 2o - u - 3o - 3u$.
19. 4 ↗ 4: $u - 2o - 3u - 2o - 2u$.
20. 3 ↖ 4: $2u - 2o - 3u - 3o - 2u$.
21. 3 ↗ 1: $2u - 2o - 2u - 3o - 3u - 2o$.

22. $2 \nwarrow 1$: $4o - 3u - 2o - 2u - 4o - u$.
23. $2 \nearrow 2$: $u - 2o - 2u - 3o - 4u - 2o - u$.
24. $1 \nwarrow 2$: $u - 3o - 4u - 3o - 2u - 3o - u$.
25. $1 \nearrow 3$: $3o - 2u - 3o - 4u - 3o - u$.
26. $4 \nwarrow 3$: $3u - 2o - 3u - 4o - 4u$.
27. $4 \nearrow 4$: $2u - 4o - 3u - 3o - 3u$.
28. $3 \nwarrow 4$: $3u - 3o - 4u - 4o - 2u$.
29. $3 \nearrow 1$: $2u - 3o - 3u - 4o - 4u - 3o$.
30. $2 \nwarrow 1$: $u - 4o - 4u - 3o - 4u - 4o - 2u$.
31. $2 \nearrow 2$: $u - 4o - 3u - 4o - 4u - 4o - u$.
32. $1 \nwarrow 2$: $2u - 4o - 4u - 4o - 4u - 4o - u$.

The grid-diagram of this knot is depicted in Fig. 537

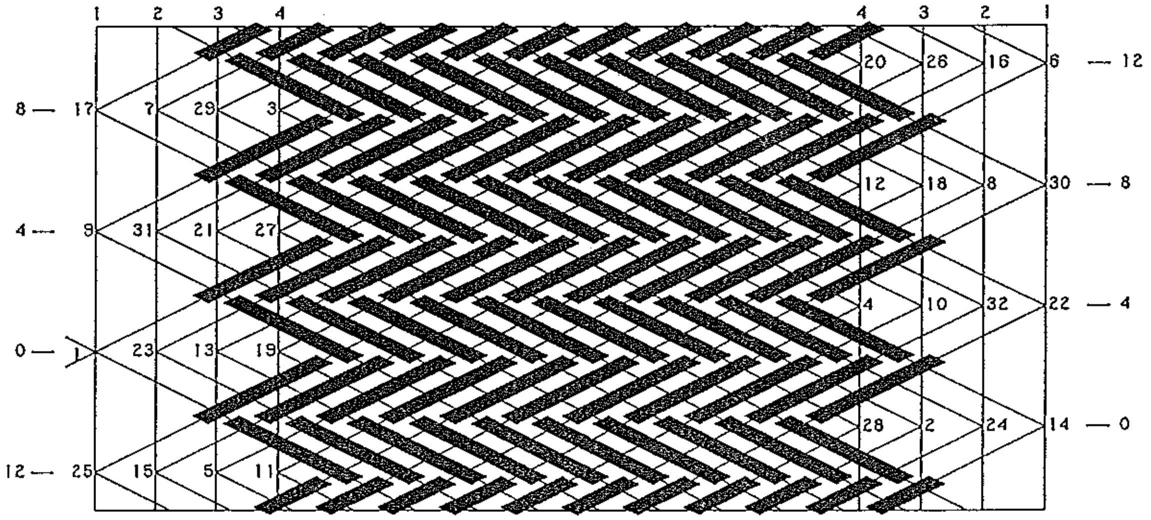


Fig. 537 — Grid-diagram of the knot in Example 1.

Example 2:

$A = 4$; $x = 19$; $y = A+1 = 5$, hence $\Delta = |5|_4 = 1$; $B^* = 4$, hence $B = A \cdot B^* = 16$. Thus $k = \left|\frac{x-A-3}{2}\right|_A = \left|\frac{19-4-3}{2}\right|_4 = 2$.

At the upper left-hand side in Fig. 538 is depicted the first-return string-run with its associated nest-index numbers.

At the upper right-hand side in Fig. 538 are depicted the first-return string-run, the half-cycle numbers of the half-cycles, and the number of crossings which these half-cycles make in the finished knot. This layout at the upper right-hand side has been shown for additional clarification only since it can readily be dispensed with.

The depicted layout at the bottom of Fig. 538 gives the positions of the nests with respect to the nest-index numbers, the half-cycle numbers with respect to the nest-index numbers and bight-boundaries, and the number of crossings which the half-cycles make in the finished knot. This layout is the one we need for compiling the half-cycle algorithms; it is directly obtained from the first-return string-run with its associated nest-index numbers, hence from the upper left-hand layout in Fig. 538. The first step in its construction is shown in Fig. 539.

The second step in the construction of the half-cycle pattern shown at the bottom of Fig. 538 consists of the determination of the half-cycle numbers in association with the nest-index numbers and bight-boundaries of their starting points.

Since the second sequence of the half-cycles in a first-return string-run starts with half-cycle number $1 + 2A = 9$ at nest-index number $I_L = A|2A + x - 2|_{B^*} = 4|25|_4 = 4$, two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I + 2A^2 + A(x-2)\right|_B = \left|I + 2 \cdot 4^2 + 4(19-2)\right|_{16} = |I + 100|_{16} = |I+4|_{16}$. Hence we can readily complete the half-cycle pattern concerned. With the aid of the coding-sequence of the half-cycles in the finished knot with a Herringbone Pineapple coding and $y = A + 1$ (see pg. 646) we can either read the half-cycle braiding algorithms directly from the half-cycle pattern, or from the set of half-cycle tables shown in Fig. 540 which we assemble from the half-cycle pattern at the bottom of Fig. 538.

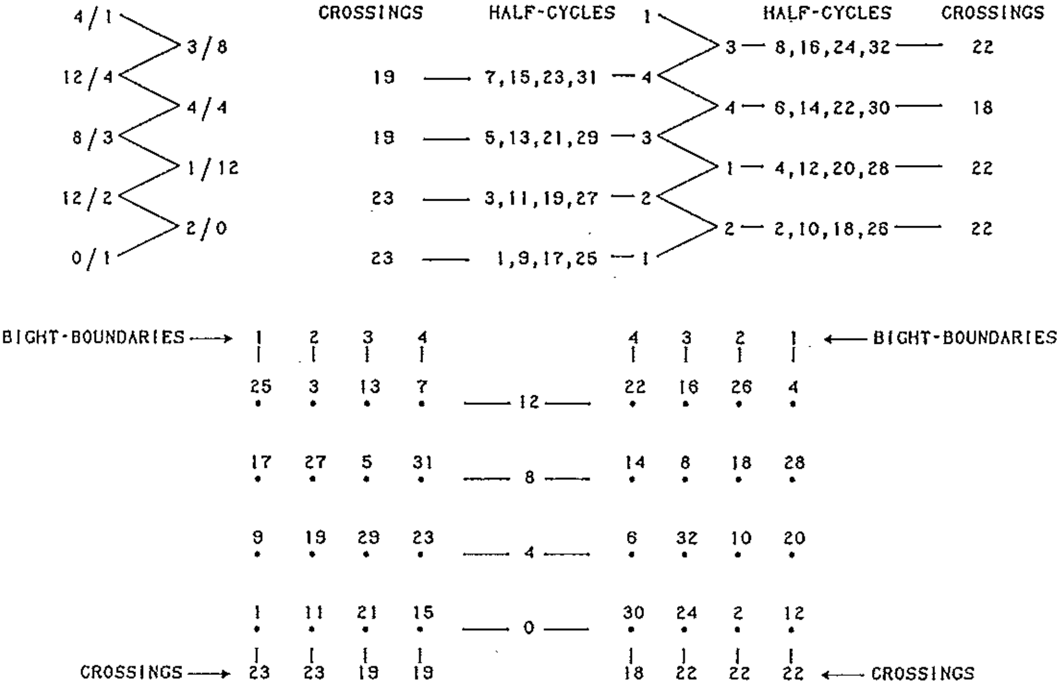


Fig. 538 — First-return string-run and half-cycle pattern.

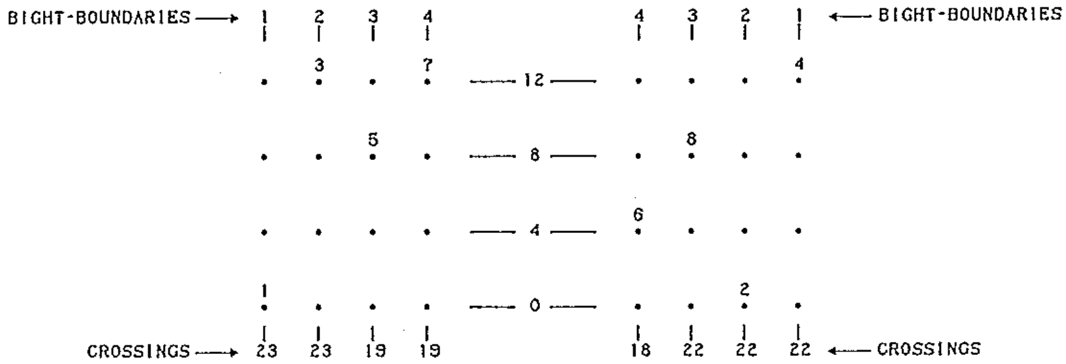
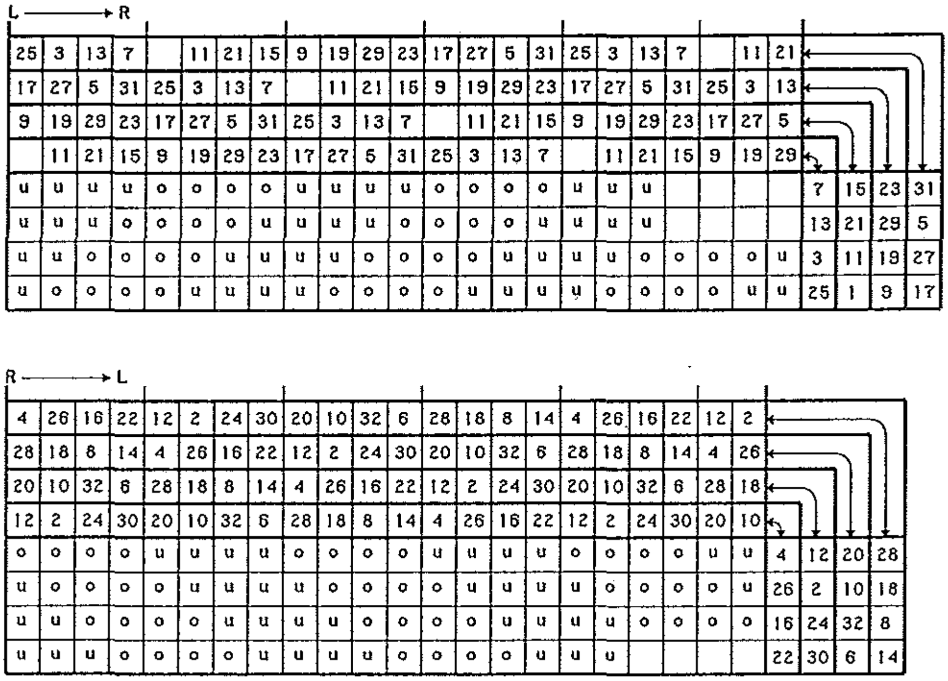


Fig. 539 — The first step in the construction of the half-cycle pattern.

The upper table in Fig. 540 is for the odd-numbered half-cycles, hence the half-cycles from lower-left to upper-right, while the lower table in Fig. 540 is for the even-numbered half-cycles, hence the half-cycles from lower-right to upper-left.

Note that the half-cycle which ends at the start of half-cycle 1 is the very last half-cycle in the braiding process and hence will not be crossed during the braiding process, thus we leave its cell empty.



L → R

25	3	13	7		11	21	15	9	19	29	23	17	27	5	31	25	3	13	7		11	21				
17	27	5	31	25	3	13	7		11	21	16	9	19	29	23	17	27	5	31	25	3	13				
9	19	29	23	17	27	5	31	25	3	13	7		11	21	15	9	19	29	23	17	27	5				
	11	21	15	9	19	29	23	17	27	5	31	25	3	13	7		11	21	15	9	19	29				
u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u					7	15	23	31
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u					13	21	29	5
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	3	11	19	27
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	25	1	9	17

R → L

4	26	16	22	12	2	24	30	20	10	32	6	28	18	8	14	4	26	16	22	12	2				
28	18	8	14	4	26	16	22	12	2	24	30	20	10	32	6	28	18	8	14	4	26				
20	10	32	6	28	18	8	14	4	26	16	22	12	2	24	30	20	10	32	6	28	18				
12	2	24	30	20	10	32	6	28	18	8	14	4	26	16	22	12	2	24	30	20	10				
o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	4	12	20	28
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	26	2	10	18
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	16	24	32	8
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u					22	30	6	14

Fig. 540 — The half-cycle tables for the knot in Example 2.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 2:

1. $1 \nearrow 2$: Free run.
2. $2 \nwarrow 2$: u.
3. $2 \nearrow 1$: o.
4. $3 \nwarrow 1$: $o - u - o$.
5. $3 \nearrow 4$: $u - o - u$.
6. $4 \nwarrow 4$: $o - 2u$.
7. $4 \nearrow 3$: $u - 2o$.
8. $1 \nwarrow 3$: $u - 2o - 2u - o$.
9. $1 \nearrow 2$: $o - 2u - 2o - u$.
10. $2 \nwarrow 2$: $3o - 2u - 2o$.
11. $2 \nearrow 1$: $3u - 2o - 2u$.
12. $3 \nwarrow 1$: $2o - u - o - 2u - 2o$.
13. $3 \nearrow 4$: $u - o - u - 2o - 2u$.
14. $4 \nwarrow 4$: $u - 2o - u - 2o - 2u$.
15. $4 \nearrow 3$: $u - o - 3u - 2o - u$.
16. $1 \nwarrow 3$: $2u - o - u - 3o - 3u - o$.
17. $1 \nearrow 2$: $3o - 3u - o - u - 3o - u$.
18. $2 \nwarrow 2$: $u - 2o - u - 2o - 4u - 2o - u$.
19. $2 \nearrow 1$: $u - 2o - 3u - 3o - u - 2o - u$.
20. $3 \nwarrow 1$: $3o - 2u - 2o - 3u - 3o - u$.
21. $3 \nearrow 4$: $2u - 2o - 2u - 3o - 3u$.
22. $4 \nwarrow 4$: $2u - 2o - 3u - 3o - 3u$.
23. $4 \nearrow 3$: $2u - 3o - 3u - 3o - 2u$.
24. $1 \nwarrow 3$: $2u - 2o - 3u - 4o - 3u - 2o$.
25. $1 \nearrow 2$: $4o - 3u - 2o - 3u - 4o - u$.

26. 2 $\nwarrow$ 2: $u - 3o - 2u - 4o - 4u - 3o - u$.
27. 2 $\nearrow$ 1: $2u - 3o - 4u - 3o - 3u - 3o - u$.
28. 3 $\nwarrow$ 1: $4o - 3u - 3o - 4u - 4o - 2u$.
29. 3 $\nearrow$ 4: $3u - 3o - 3u - 4o - 4u$.
30. 4 $\nwarrow$ 4: $2u - 4o - 4u - 4o - 3u$.
31. 4 $\nearrow$ 3: $4u - 3o - 4u - 4o - 3u$.
32. 1 $\nwarrow$ 3: $2u - 4o - 4u - 4o - 4u - 4o$.

The grid-diagram of this knot is depicted in Fig. 541

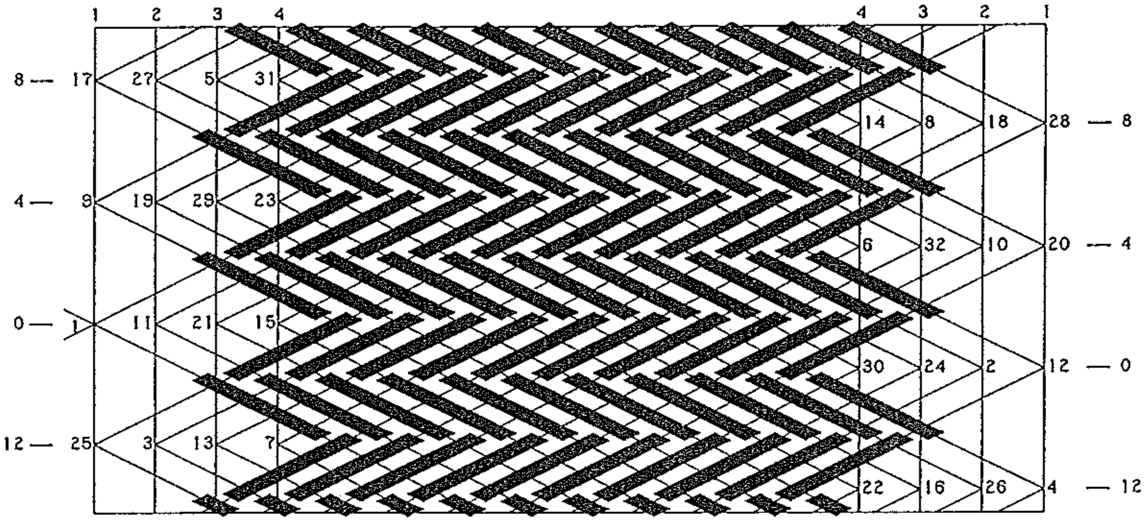


Fig. 541 — Grid-diagram of the knot in Example 2.

The string-runs of the knots in Example 1 and 2 are each others mirror-image and they have an identical number of bights. Hence when for one of such herringbone pineapple coded knots the two half-cycle tables have been determined, we can then readily derive from these tables the two half-cycle tables for the mirror-imaged knot.

Say the first-return string-run and the two half-cycle tables for the $y = A - 1$ knot are known. On its first-return string-run determine the number of half-cycles between right-hand bight-boundary 1 and left-hand bight-boundary 1 at the end of the first-return string-run (this number of half-cycles is equal to 3 for Example 1). Then replace each half-cycle number in its two half-cycle tables by

$$|\text{half-cycle number} + 3|_{2B} .$$

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$).

Say the first-return string-run and the two half-cycle tables for the $y = A + 1$ knot are known. On its first-return string-run determine the number of half-cycles between left-hand bight-boundary 1 at the beginning of the first-return string-run and right-hand bight-boundary 1 (this number of half-cycles is equal to 3 for Example 2). Then replace each half-cycle number in its two half-cycle tables by

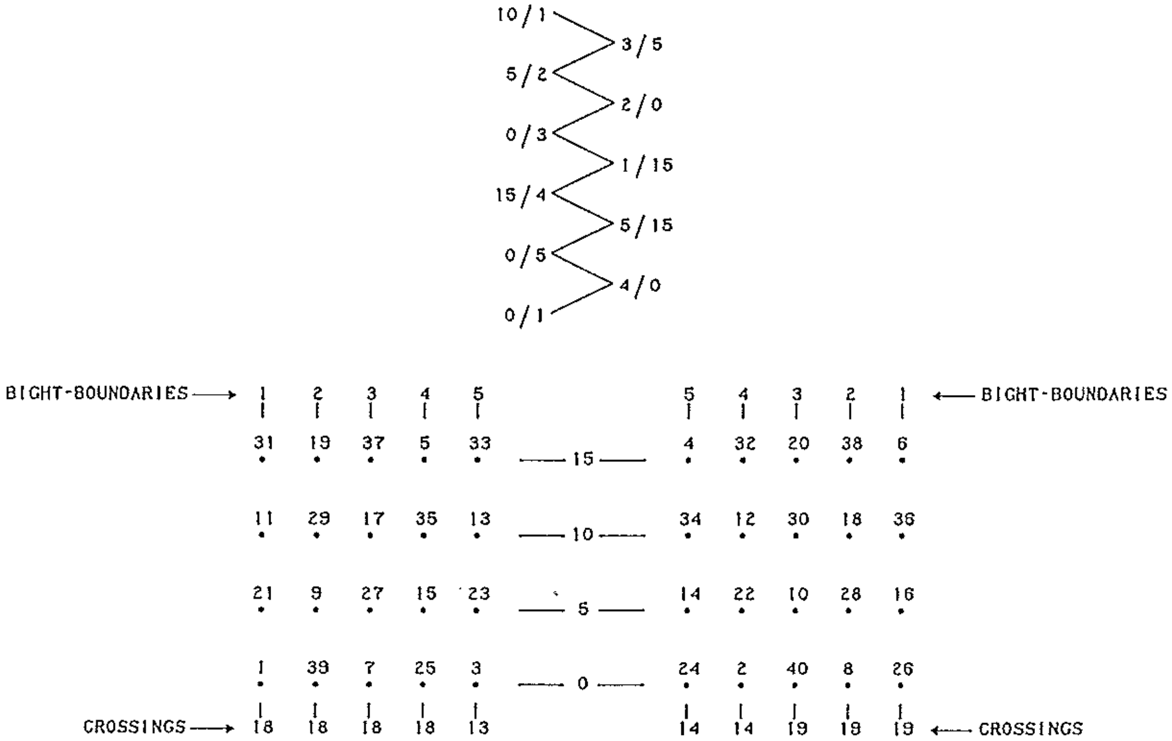
$$|\text{half-cycle number} - 3|_{2B} .$$

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$).

Let's now have a look at two Examples of Semi-Perfect Herringbone Pineapple Knots which are each others mirror-image.

Example 3 :

$A = 5$; $x = 14$; $y = A - 1$, hence $\Delta = A - 1 = 4$; $B^* = 4$, hence $B = A \cdot B^* = 20$.
Thus $k = \left|\frac{x-A-1}{2}\right|_A = \left|\frac{14-5-1}{2}\right|_5 = 4$.
The first-return string-run and the half-cycle pattern is shown in Fig. 542.

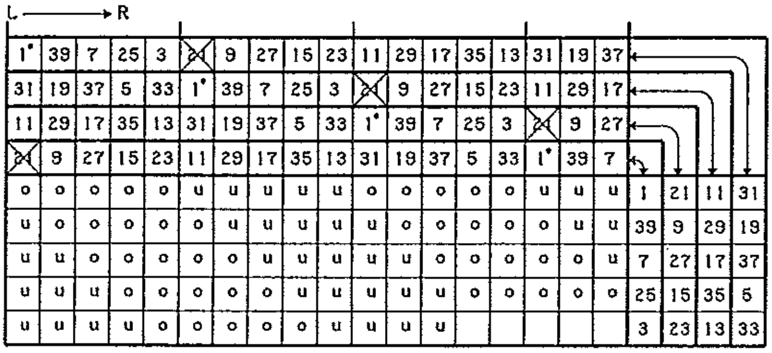


BIGHT-BOUNDARIES ⟶	1	2	3	4	5		5	4	3	2	1	⟵ BIGHT-BOUNDARIES
	31	19	37	5	33	—— 15 ——	4	32	20	38	6	
	11	29	17	35	13	—— 10 ——	34	12	30	18	36	
	21	9	27	15	23	—— 5 ——	14	22	10	28	16	
	1	39	7	25	3	—— 0 ——	24	2	40	8	26	
CROSSINGS ⟶	18	18	18	18	13		14	14	19	19	19	⟵ CROSSINGS

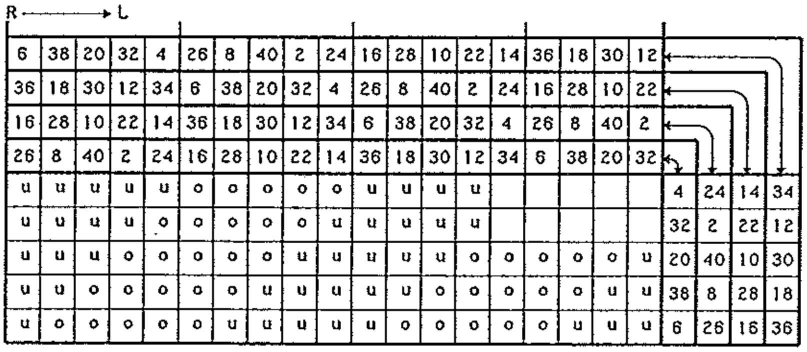
Fig. 542 — First-return string-run and half-cycle pattern.

The second sequence of half-cycles in a first-return string-run starts with half-cycle number $1 + 2A = 11$ at nest-index number $I_L = A|2A + x - 2|_{B^*} = 5|22|_4 = 10$. Hence two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I + 2A^2 + A(x-2)\right|_B = \left|I + 2 \cdot 5^2 + 5(14-2)\right|_{20} = |I + 110|_{20} = |I + 10|_{20}$. Hence the last half-cycle in the second sequence of half-cycles in a first-return string-run ends at the starting-point of the first half-cycle in the first sequence of half-cycles in a first-return string-run. Since for Perfect and Semi-Perfect Regular Nested Knots the number of half-cycles in a first-return string-run is equal to $2A$ and since these knots consist of $2B = 2AB^*$ half-cycles, it follows that these knots consist of B^* first-return string-runs. Thus in our Example the knot consists of four first-return string-runs, and since in the first two consecutive first-return string-runs the end-point of the last half-cycle returns to the starting-point of the first half-cycle, the knot consists of two sub-components. The first sub-component consists of the half-cycles 1 to 20, and the second sub-component consists of the half-cycles 21 to 40. When laying down the half-cycles 1 to 20, the 1^* position in the $L \to R$ half-cycle table should **not** be read, but when laying down the half-cycles 21 to 40 it should be read and the 21 position should **not** be read.

From the half-cycle pattern in Fig. 542 we assemble again the two half-cycle tables in Fig. 543.



1•	39	7	25	3	~~21~~	9	27	15	23	11	29	17	35	13	31	19	37				
31	19	37	5	33	1•	39	7	25	3	~~21~~	9	27	15	23	11	29	17				
11	29	17	35	13	31	19	37	5	33	1•	39	7	25	3	~~21~~	9	27				
~~21~~	9	27	15	23	11	29	17	35	13	31	19	37	5	33	1•	39	7				
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	1	21	11	31
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	39	9	29	19
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	7	27	17	37
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	25	15	35	5
u	u	u	u	o	o	o	o	o	u	u	u	u						3	23	13	33



6	38	20	32	4	26	8	40	2	24	16	28	10	22	14	36	18	30	12				
36	18	30	12	34	6	38	20	32	4	26	8	40	2	24	16	28	10	22				
16	28	10	22	14	36	18	30	12	34	6	38	20	32	4	26	8	40	2				
26	8	40	2	24	16	28	10	22	14	36	18	30	12	34	6	38	20	32				
u	u	u	u	u	o	o	o	o	o	u	u	u	u						4	24	14	34
u	u	u	u	o	o	o	o	o	u	u	u	u	u						32	2	22	12
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	20	40	10	30
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	38	8	28	18
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	6	26	16	36

Fig. 543 — The half-cycle tables for the knot in Example 3.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 3:

1. 1 ↗ 4: Free run.
2. 5 ↖ 4: Free run.
3. 5 ↗ 5: Free run.
4. 4 ↖ 5: u.
5. 4 ↗ 1: o.
6. 3 ↖ 1: $2o$.
7. 3 ↗ 2: $o - u$.
8. 2 ↖ 2: $u - 2o - u$.
9. 2 ↗ 3: $u - 2o - u$.
10. 1 ↖ 3: $o - 2u - 2o$.
11. 1 ↗ 4: $o - 2u - o - u$.
12. 5 ↖ 4: $u - 3o - u$.
13. 5 ↗ 5: $u - o - 2u$.
14. 4 ↖ 5: $u - 2o - 2u$.
15. 4 ↗ 1: $u - o - 2u - 2o$.
16. 3 ↖ 1: $2o - u - 3o - u$.
17. 3 ↗ 2: $o - 3u - 2o - u$.
18. 2 ↖ 2: $u - 2o - 2u - 3o - u$.
19. 2 ↗ 3: $2o - 3u - 2o - u$.
20. 1 ↖ 3: $u - 3o - 2u - 3o$.
21. 1 ↗ 4: $3o - 2u - 3o - u$.
22. 5 ↖ 4: $2u - 2o - 3u$.
23. 5 ↗ 5: $2u - 3o - 2u$.
24. 4 ↖ 5: $4u - 2o - 2u$.
25. 4 ↗ 1: $u - 4o - 2u - 3o$.
26. 3 ↖ 1: $u - 3o - 3u - 3o - 2u$.
27. 3 ↗ 2: $u - 3o - 2u - 4o - u$.
28. 2 ↖ 2: $u - 2o - 4u - 4o - 2u$.
29. 2 ↗ 3: $3o - 3u - 5o - 2u$.
30. 1 ↖ 3: $2u - 3o - 5u - 4o - u$.
31. 1 ↗ 4: $4o - 4u - 4o - 2u$.
32. 5 ↖ 4: $3u - 5o - 4u$.
33. 5 ↗ 5: $3u - 4o - 4u$.
34. 4 ↖ 5: $4u - 4o - 4u$.
35. 4 ↗ 1: $2u - 4o - 4u - 5o$.
36. 3 ↖ 1: $u - 4o - 4u - 5o - 3u$.
37. 3 ↗ 2: $u - 4o - 5u - 5o - u$.
38. 2 ↖ 2: $2u - 4o - 5u - 5o - 2u$.
39. 2 ↗ 3: $5o - 5u - 5o - 2u$.
40. 1 ↖ 3: $3u - 5o - 5u - 5o - u$.

The grid-diagram of this knot is depicted in Fig. 544

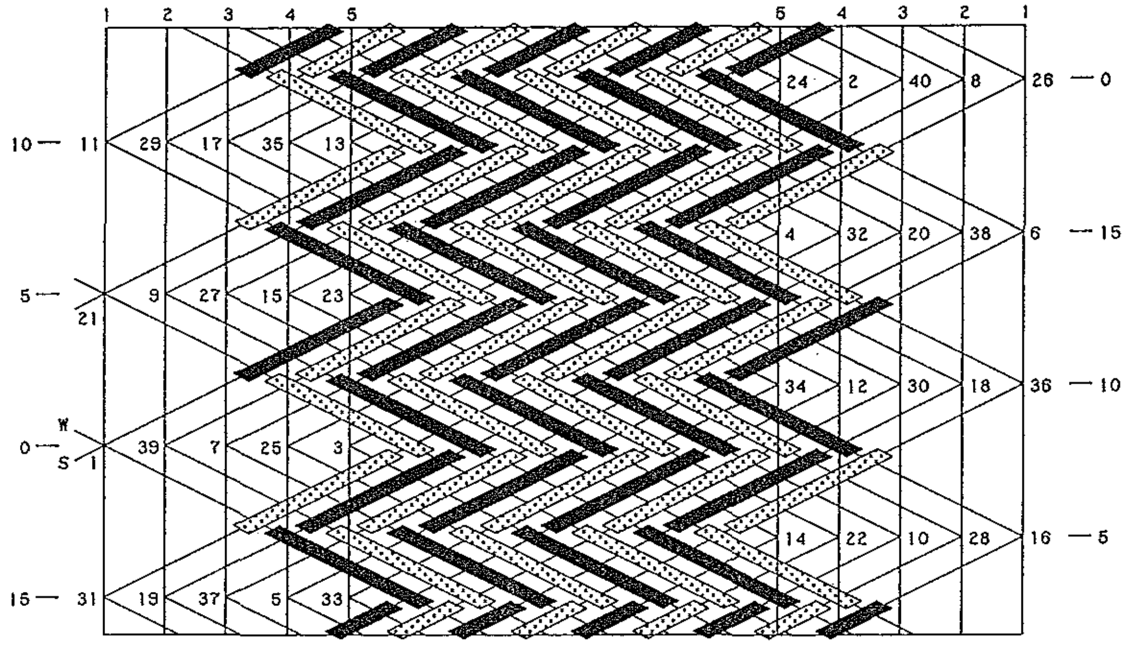


Fig. 544 — Grid-diagram of the knot in Example 3.

Example 4:

$A = 5$; $x = 14$; $y = A + 1$, hence $\Delta = 1$; $B^* = 4$, hence $B = A \cdot B^* = 20$.
Thus $k = \left|\frac{x-A-3}{2}\right|_A = \left|\frac{14-5-3}{2}\right|_5 = 3$.
The first-return string-run and the half-cycle pattern is shown in Fig. 545.

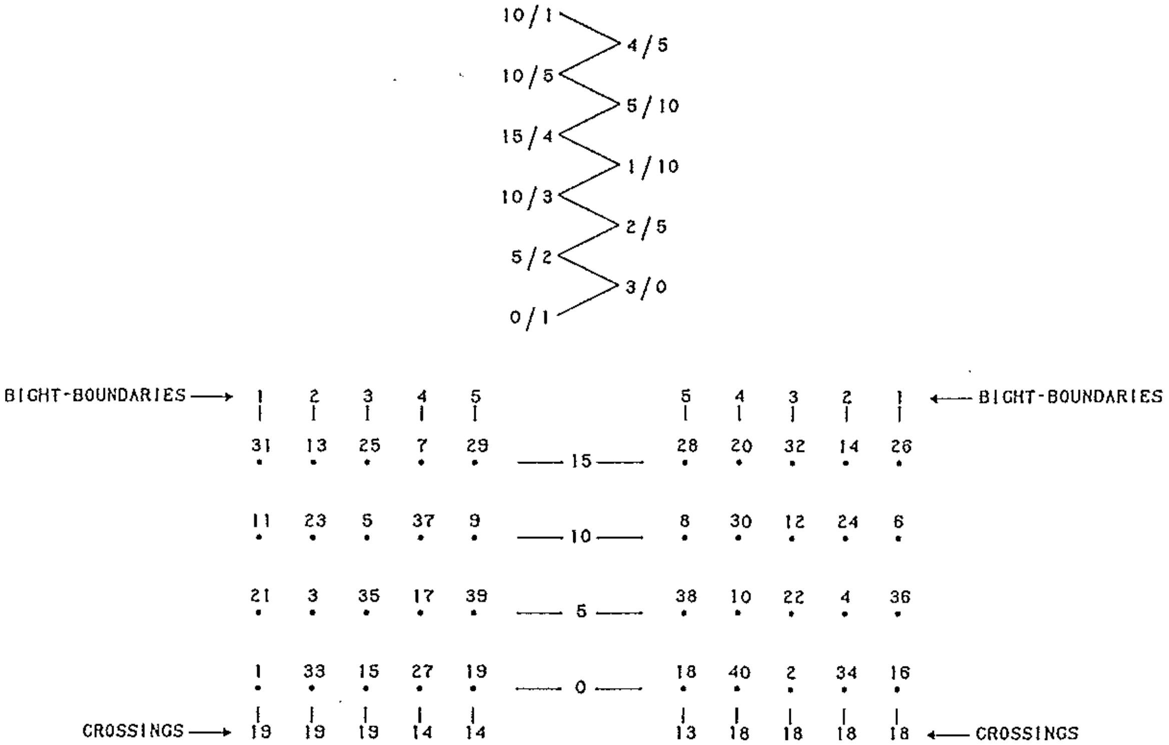
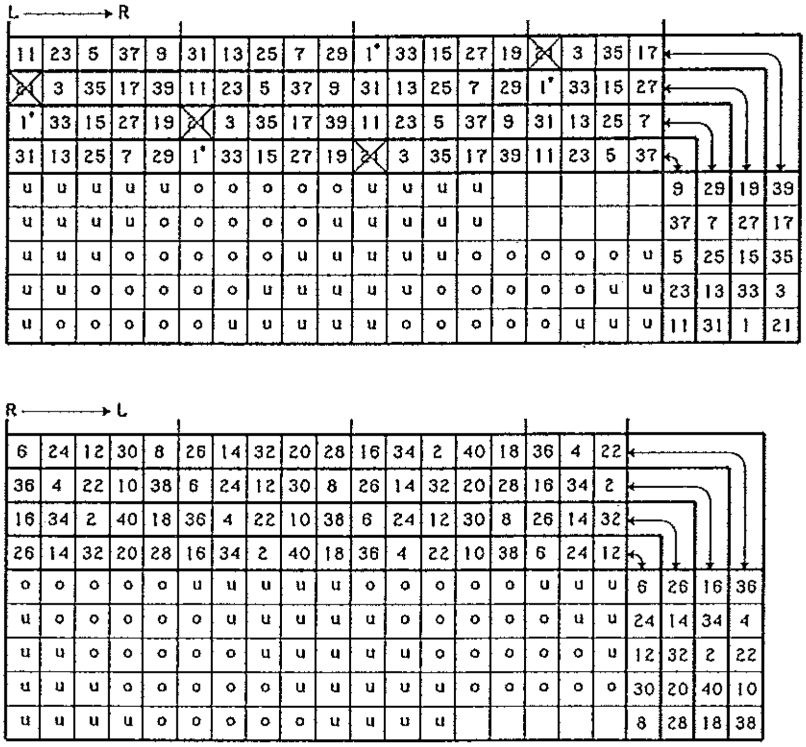


Fig. 545 — First-return string-run and half-cycle pattern.

The second sequence of half-cycles in a first-return string-run starts with half-cycle number $1 + 2A = 11$ at nest-index number $I_L = A|2A + x - 2|_{B^*} = 5|22|_4 = 10$.

Hence two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ ($\leq 2B$) associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I + 2A^2 + A(x-2)\right|_B = \left|I + 2 \cdot 5^2 + 5(14-2)\right|_{20} = |I + 110|_{20} = |I + 10|_{20}$. Hence the last half-cycle in the second sequence of half-cycles in a first-return string-run ends at the starting-point of the first half-cycle in the first sequence of half-cycles in a first-return string-run. Since for Perfect and Semi-Perfect Regular Nested Knots the number of half-cycles in a first-return string-run is equal to $2A$ and since these knots consist of $2B = 2AB^*$ half-cycles, it follows that these knots consist of B^* first-return string-runs. Thus in our Example the knot consists of four first-return string-runs, and since in the first two consecutive first-return string-runs the end-point of the last half-cycle returns to the starting-point of the first half-cycle, the knot consists of two sub-components. The first sub-component consists of the half-cycles 1 to 20, and the second sub-component consists of the half-cycles 21 to 40. When laying down the half-cycles 1 to 20, the 1* position in the $L \rightarrow R$ half-cycle table should **not** be read, but when laying down the half-cycles 21 to 40 it should be read and the 21 position should **not** be read.

From the half-cycle pattern in Fig. 545 we assemble again the two half-cycle tables in Fig. 546.


L → R

11	23	5	37	9	31	13	25	7	29	1*	33	15	27	19	~~21~~	3	35	17				
~~21~~	3	35	17	39	11	23	5	37	9	31	13	25	7	29	1*	33	15	27				
1*	33	15	27	19	~~21~~	3	35	17	39	11	23	5	37	9	31	13	25	7				
31	13	25	7	29	1*	33	15	27	19	~~21~~	3	35	17	39	11	23	5	37				
u	u	u	u	u	o	o	o	o	o	u	u	u	u						9	29	19	39
u	u	u	u	o	o	o	o	o	u	u	u	u	u						37	7	27	17
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	5	25	15	35
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	23	13	33	3
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	11	31	1	21

R → L

6	24	12	30	8	26	14	32	20	28	16	34	2	40	18	36	4	22				
36	4	22	10	38	6	24	12	30	8	26	14	32	20	28	16	34	2				
16	34	2	40	18	36	4	22	10	38	6	24	12	30	8	26	14	32				
26	14	32	20	28	16	34	2	40	18	36	4	22	10	38	6	24	12				
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	6	26	16	36
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	24	14	34	4
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	12	32	2	22
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	30	20	40	10
u	u	u	u	o	o	o	o	o	u	u	u	u						8	28	18	38

Fig. 546 — The half-cycle tables for the knot in Example 4.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 4:

1. $1 \nearrow 3$: Free run.
2. $2 \nwarrow 3$: u.
3. $2 \nearrow 2$: o.
4. $3 \nwarrow 2$: $o - u$.
5. $3 \nearrow 1$: $u - o$.
6. $4 \nwarrow 1$: $u - o - u$.

21. $1 \nearrow 3$: $u - 2o - 3u - 2o - 2u$.
22. $2 \nwarrow 3$: $u - 3o - 2u - 3o - u$.
23. $2 \nearrow 2$: $u - 2o - 3u - 3o - u$.
24. $3 \nwarrow 2$: $3o - 2u - 4o - 2u$.
25. $3 \nearrow 1$: $2u - 2o - 4u - 3o - u$.
26. $4 \nwarrow 1$: $3o - 3u - 4o - 2u$.

7. $4 \nearrow 5$: $o - u$.
8. $5 \nwarrow 5$: $o - u$.
9. $5 \nearrow 4$: $2u$.
10. $1 \nwarrow 4$: $u - o - u - o$.
11. $1 \nearrow 3$: $3o - u$.
12. $2 \nwarrow 3$: $2u - 2o - u$.
13. $2 \nearrow 2$: $o - u - 3o - u$.
14. $3 \nwarrow 2$: $o - 3u - 2o - u$.
15. $3 \nearrow 1$: $u - 2o - 2u - 2o$.
16. $4 \nwarrow 1$: $2o - 3u - o - 2u$.
17. $4 \nearrow 5$: $2u - 3o - u$.
18. $5 \nwarrow 5$: $2u - 2o - 2u$.
19. $5 \nearrow 4$: $2u - 3o - 2u$.
20. $1 \nwarrow 4$: $2u - 2o - 3u - 2o$.
27. $4 \nearrow 5$: $2u - 3o - 4u$.
28. $5 \nwarrow 5$: $2u - 4o - 3u$.
29. $5 \nearrow 4$: $4u - 2o - 3u$.
30. $1 \nwarrow 4$: $2u - 4o - 3u - 4o$.
31. $1 \nearrow 3$: $u - 3o - 3u - 4o - 3u$.
32. $2 \nwarrow 3$: $u - 3o - 4u - 5o - u$.
33. $2 \nearrow 2$: $u - 3o - 4u - 5o - 2u$.
34. $3 \nwarrow 2$: $4o - 5u - 5o - 2u$.
35. $3 \nearrow 1$: $3u - 4o - 5u - 4o - u$.
36. $4 \nwarrow 1$: $5o - 5u - 4o - 3u$.
37. $4 \nearrow 5$: $4u - 5o - 4u$.
38. $5 \nwarrow 5$: $4u - 5o - 4u$.
39. $5 \nearrow 4$: $5u - 5o - 4u$.
40. $1 \nwarrow 4$: $3u - 5o - 5u - 5o$.

The grid-diagram of this knot is depicted in Fig. 547

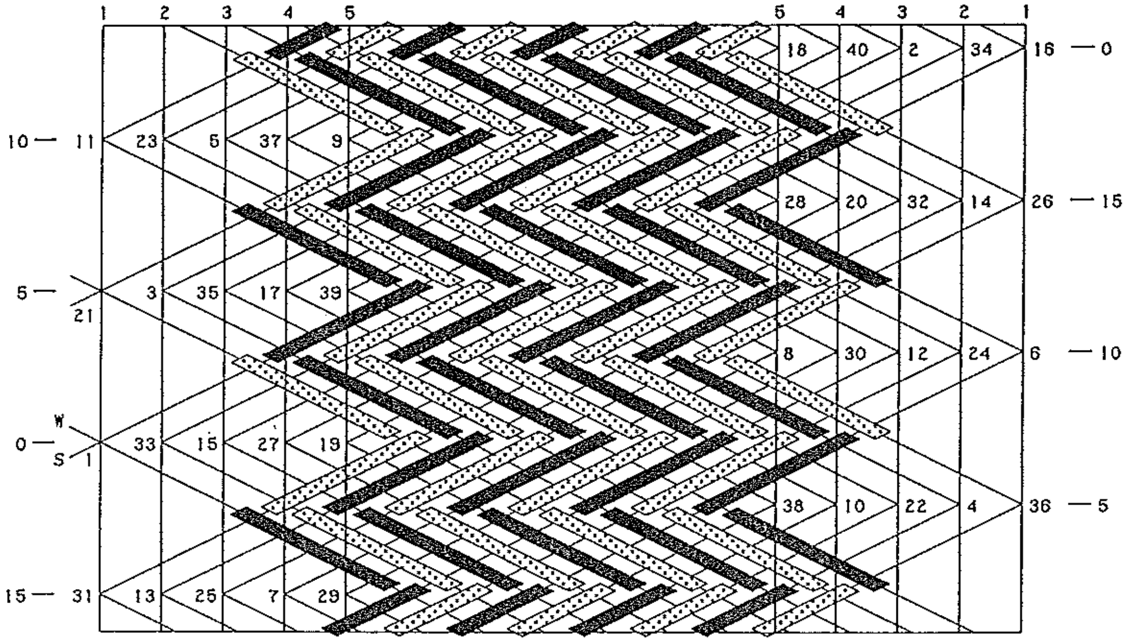


Fig. 547 — Grid-diagram of the knot in Example 4.

For a Semi-Perfect Herringbone Pineapple Knots we can also derive its half-cycle tables from the half-cycle tables of its mirror-image. Let g.c.d.$(P, B^*) = \lambda$, where $P = 2A + x - 2$. Then the Semi-Perfect Herringbone Pineapple Knot consists of λ sub-components. Each sub-component consists of $\dfrac{2B}{\lambda}$ half-cycles.

The first sub-component consists of the half-cycles 1 to $\frac{2B}{\lambda}$.

The second sub-component consists of the half-cycles $\left(\frac{2B}{\lambda} + 1\right)$ to $2 \cdot \frac{2B}{\lambda}$.

The third sub-component consists of the half-cycles $\left(2 \cdot \frac{2B}{\lambda} + 1\right)$ to $3 \cdot \frac{2B}{\lambda}$.

$\vdots$

The n^{th} sub-component consists of the half-cycles $\left((n-1) \cdot \frac{2B}{\lambda} + 1\right)$ to $\left(n \cdot \frac{2B}{\lambda}\right)$.

$\vdots$

The λ^{th} sub-component consists of the half-cycles $\left((\lambda - 1) \cdot \frac{2B}{\lambda} + 1\right)$ to $2B$.

Say that the first half-cycle of each sub-component starts at left bight-boundary 1 (as in Examples 3 and 4).

Say the first-return string-run and the two half-cycle tables for the $y = A - 1$ knot are known. On its first-return string-run determine the number of half-cycles φ between right-hand bight-boundary 1 and left-hand bight-boundary 1 at the end of the first-return string-run (this number of half-cycles is equal to $\varphi = 5$ for Example 3). Then replace in its two half-cycle tables each half-cycle number h in the range:

$1 \leq h \leq \frac{2B}{\lambda}$ by $|h + \varphi|_{\frac{2B}{\lambda}}$.

$\left(\frac{2B}{\lambda} + 1\right) \leq h \leq 2 \cdot \frac{2B}{\lambda}$ by $|h + \varphi|_{\frac{2B}{\lambda}} + \frac{2B}{\lambda}$.

$\left(2 \cdot \frac{2B}{\lambda} + 1\right) \leq h \leq 3 \cdot \frac{2B}{\lambda}$ by $|h + \varphi|_{\frac{2B}{\lambda}} + 2 \cdot \frac{2B}{\lambda}$.

$$\vdots$$

$\left((n-1) \cdot \frac{2B}{\lambda} + 1\right) \leq h \leq \left(n \cdot \frac{2B}{\lambda}\right)$ by $|h + \varphi|_{\frac{2B}{\lambda}} + (n-1) \cdot \frac{2B}{\lambda}$.

$$\vdots$$

$\left((\lambda - 1) \cdot \frac{2B}{\lambda} + 1\right) \leq h \leq 2B$ by $|h + \varphi|_{\frac{2B}{\lambda}} + (\lambda - 1) \cdot \frac{2B}{\lambda}$.

In the above formulae take $|h + \varphi|_{\frac{2B}{\lambda}} = \frac{2B}{\lambda}$ when $|h + \varphi|_{\frac{2B}{\lambda}} = 0$.

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$).

Thus for the knot in Example 3 with $B = 20$; $\lambda = 2$ and $\varphi = 5$, replace in its two half-cycle tables each half-cycle number h in the range:

$1 \leq h \leq 20$ by $|h + 5|_{20}$.

$21 \leq h \leq 40$ by $|h + 5|_{20} + 20$.

Take $|h + 5|_{20} = 20$ when $|h + 5|_{20} = 0$.

Say the first-return string-run and the two half-cycle tables for the $y = A + 1$ knot are known. On its first-return string-run determine the number of half-cycles φ between left-hand bight-boundary 1 at the beginning of the first-return string-run and right-hand bight-boundary 1 (this number of half-cycles is equal to $\varphi = 5$ for Example 4). Then replace in its two half-cycle tables each half-cycle number h in the range:

$1 \leq h \leq \frac{2B}{\lambda}$ by $|h - \varphi|_{\frac{2B}{\lambda}}$.

$\left(\frac{2B}{\lambda} + 1\right) \leq h \leq 2 \cdot \frac{2B}{\lambda}$ by $|h - \varphi|_{\frac{2B}{\lambda}} + \frac{2B}{\lambda}$.

$\left(2 \cdot \frac{2B}{\lambda} + 1\right) \leq h \leq 3 \cdot \frac{2B}{\lambda}$ by $|h - \varphi|_{\frac{2B}{\lambda}} + 2 \cdot \frac{2B}{\lambda}$.

$$\vdots$$

$\left((n-1) \cdot \frac{2B}{\lambda} + 1\right) \leq h \leq \left(n \cdot \frac{2B}{\lambda}\right)$ by $|h - \varphi|_{\frac{2B}{\lambda}} + (n-1) \cdot \frac{2B}{\lambda}$.

$$\vdots$$

$\left((\lambda - 1) \cdot \frac{2B}{\lambda} + 1\right) \leq h \leq 2B$ by $|h - \varphi|_{\frac{2B}{\lambda}} + (\lambda - 1) \cdot \frac{2B}{\lambda}$.

In the above formulae take $|h - \varphi|_{\frac{2B}{\lambda}} = \frac{2B}{\lambda}$ when $|h - \varphi|_{\frac{2B}{\lambda}} = 0$.

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$).

Thus for the knot in Example 4 with $B = 20$; $\lambda = 2$ and $\varphi = 5$, replace in its two half-cycle tables each half-cycle number h in the range:

$1 \le h \le 20$ by $|h - 5|_{20}$.
$21 \le h \le 40$ by $|h - 5|_{20} + 20$.
Take $|h - 5|_{20} = 20$ when $|h - 5|_{20} = 0$.

In many applications it is much better **not** to have the Standing-End and Working-End at bight-boundary 1. In general we furthermore like to distribute the Standing and Working Ends of the sub-components as regularly as possible over the circumference of a bight-edge. Note that when the Standing and Working Ends of the two sub-components are placed on bight-boundary 1 as in the above Examples 3 and 4, such a regular distribution is not possible.

Let's assume that for practical reasons we like to have the Standing and Working Ends on the inner bight-boundaries and that we want them to be regularly distributed over the circumference of the bight-edge. Thus we could place the Standing-End of the first sub-component on bight-boundary $l_1 = 5$ and nest-index number $I_{L_1} = 0$, and the Standing-End of the second sub-component on bight-boundary $l'_1 = 4$ and nest-index number $I'_{L_1} = 10$. Hence half-cycle 1 starts at left bight-boundary $l_1 = 5$ and nest-index number $I_{L_1} = 0$, while half-cycle 21 starts at left bight-boundary $l'_1 = 4$ and nest-index number $I'_{L_1} = 10$.

Say that we are going to work out the half-cycle tables for the Semi-Perfect Herring-bone Pineapple Knot for which $y = A + 1 = 5 + 1 = 6$ (hence $\Delta = |6|_5 = 1$):

Example 5:

$A = 5$; $x = 14$; $l_1 = 5$, $I_{L_1} = 0$; $l'_1 = 4$, $I'_{L_1} = 10$; $y = A + 1$, hence $\Delta = 1$; $B^* = 4$, hence $B = A \cdot B^* = 20$.

Thus

$$\begin{aligned}
r_1 &= \left|\frac{x - A - 1 - 2l_1}{2}\right|_A = \left|\frac{14 - 5 - 1 - 10}{2}\right|_5 = |-1|_5 = 4\,,\\
I_{R_1} &= I_{L_1} = 0\,;\\
r'_1 &= \left|\frac{x - A - 1 - 2l'_1}{2}\right|_A = \left|\frac{14 - 5 - 1 - 8}{2}\right|_5 = |0|_5 = 5\,,\\
I'_{R_1} &= \left|I'_{L_1} + (l_1 + r_1) - (l'_1 + r'_1)\right|_B = |10 + (5 + 4) - (4 + 5)|_{20} = 10\,.
\end{aligned}$$

The first-return string-run and the half-cycle pattern is shown in Fig. 548. Although there is only one type of first-return string-run, the sequential half-cycle pattern for each of the two sub-components is shown. The reader be reminded that the 1^{st} half-cycle, which starts at the left-hand bight-boundary $l_1 = 5$ and nest-index number $I_{L_1} = 0$, is the first half-cycle of the first sub-component, and that the 21^{st} half-cycle, which starts at the left-hand bight-boundary $l'_1 = 4$ and nest-index number $I'_{L_1} = 10$, is the first half-cycle of the second sub-component.

From this half-cycle pattern in Fig. 548 we assemble in the usual way the half-cycle tables in Fig. 549.

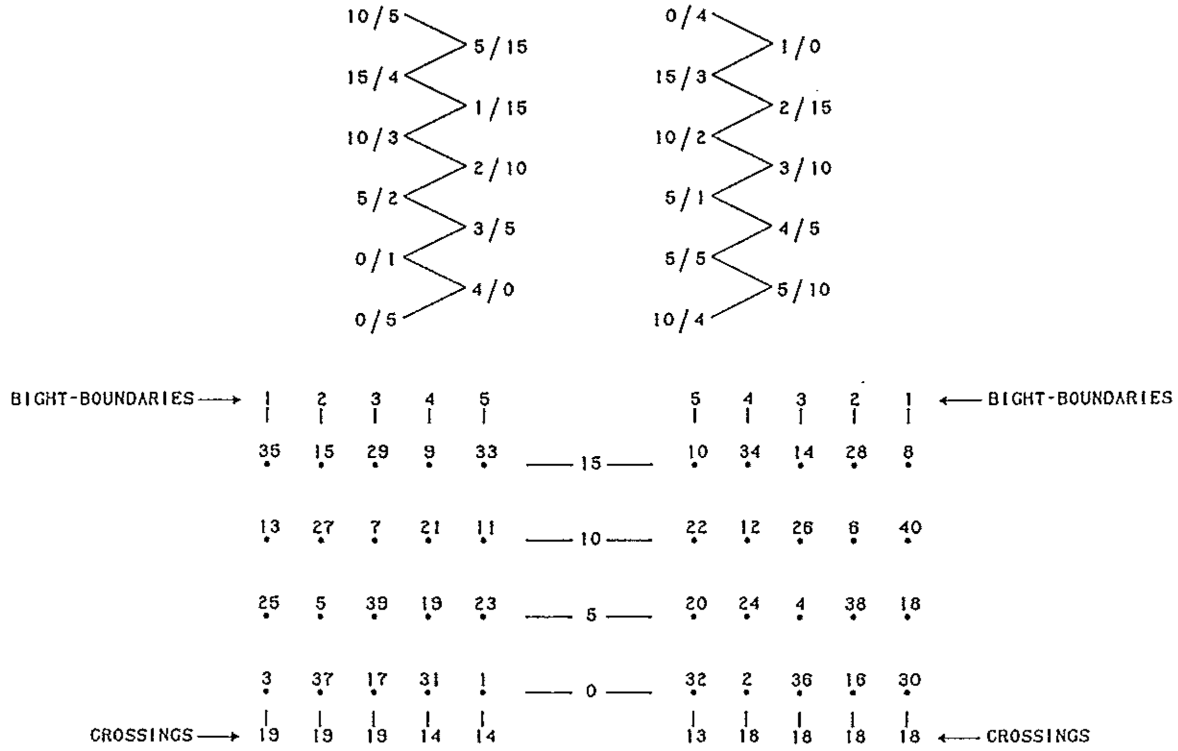
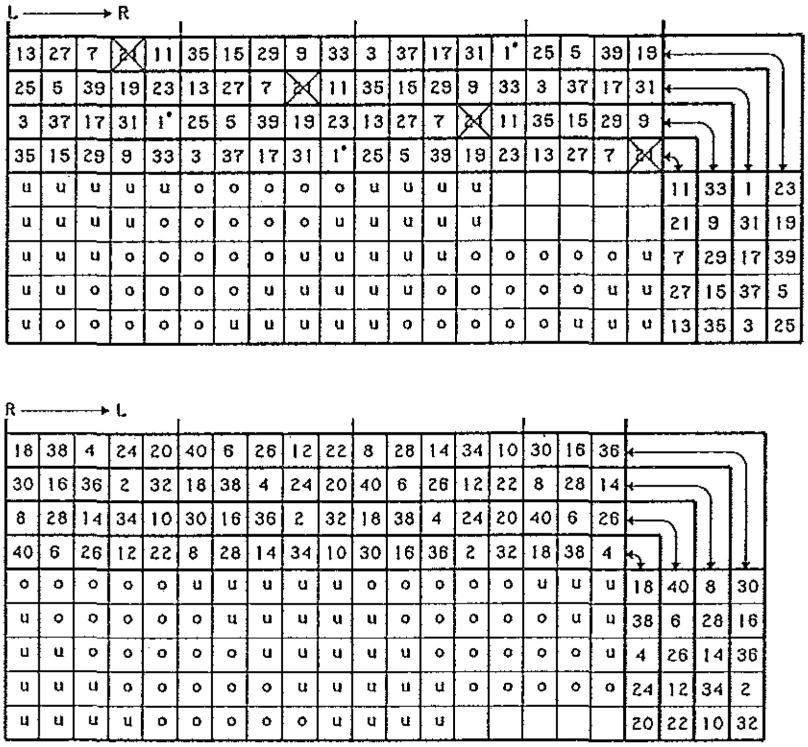


Fig. 548 — First-return string-run and half-cycle pattern.



13	27	7	~~21~~	11	35	15	29	9	33	3	37	17	31	1•	25	5	39	19				
25	5	39	19	23	13	27	7	~~21~~	11	35	15	29	9	33	3	37	17	31				
3	37	17	31	1•	25	5	39	19	23	13	27	7	~~21~~	11	35	15	29	9				
35	15	29	9	33	3	37	17	31	1•	25	5	39	19	23	13	27	7	~~21~~				
u	u	u	u	u	o	o	o	o	o	u	u	u	u						11	33	1	23
u	u	u	u	o	o	o	o	o	u	u	u	u	u						21	9	31	19
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	7	29	17	39
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	27	15	37	5
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	13	35	3	25

18	38	4	24	20	40	6	26	12	22	8	28	14	34	10	30	16	36				
30	16	36	2	32	18	38	4	24	20	40	6	26	12	22	8	28	14				
8	28	14	34	10	30	16	36	2	32	18	38	4	24	20	40	6	26				
40	6	26	12	22	8	28	14	34	10	30	16	36	2	32	18	38	4				
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	18	40	8	30
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	38	6	28	16
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	4	26	14	36
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	24	12	34	2
u	u	u	u	o	o	o	o	o	u	u	u	u						20	22	10	32

Fig. 549 — The half-cycle tables for the knot in Example 5.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 5:

1. $5 \nearrow 4$: Free run.
2. $1 \nwarrow 4$: Free run.

21. $4 \nearrow 5$: $2u - 2o - 3u$.
22. $5 \nwarrow 5$: $2u - 3o - 2u$.

3. $1 \nearrow 3$: o.
4. $2 \nwarrow 3$: $o - u$.
5. $2 \nearrow 2$: $u - o$.
6. $3 \nwarrow 2$: $u - o - u$.
7. $3 \nearrow 1$: $o - u - o$.
8. $4 \nwarrow 1$: $o - u - o - u$.
9. $4 \nearrow 5$: $u - o - u$.
10. $5 \nwarrow 5$: $u - o - u$.
11. $5 \nearrow 4$: $u - o - u$.
12. $1 \nwarrow 4$: $u - o - 2u - o$.
13. $1 \nearrow 3$: $4o - u$.
14. $2 \nwarrow 3$: $o - 2u - 2o - u$.
15. $2 \nearrow 2$: $u - o - u - 3o - u$.
16. $3 \nwarrow 2$: $o - 3u - 2o - u$.
17. $3 \nearrow 1$: $u - 2o - 2u - 3o$.
18. $4 \nwarrow 1$: $2o - 3u - 2o - 2u$.
19. $4 \nearrow 5$: $2u - 3o - 2u$.
20. $5 \nwarrow 5$: $2u - 2o - 2u$.
23. $5 \nearrow 4$: $3u - 2o - 2u$.
24. $1 \nwarrow 4$: $u - 4o - 2u - 3o$.
25. $1 \nearrow 3$: $u - 2o - 3u - 3o - 2u$.
26. $2 \nwarrow 3$: $u - 3o - 2u - 4o - u$.
27. $2 \nearrow 2$: $u - 2o - 4u - 4o - u$.
28. $3 \nwarrow 2$: $3o - 3u - 5o - 2u$.
29. $3 \nearrow 1$: $2u - 3o - 5u - 3o - u$.
30. $4 \nwarrow 1$: $4o - 4u - 4o - 2u$.
31. $4 \nearrow 5$: $3u - 4o - 4u$.
32. $5 \nwarrow 5$: $3u - 4o - 4u$.
33. $5 \nearrow 4$: $4u - 4o - 3u$.
34. $1 \nwarrow 4$: $2u - 4o - 4u - 5o$.
35. $1 \nearrow 3$: $u - 4o - 4u - 4o - 3u$.
36. $2 \nwarrow 3$: $u - 4o - 5u - 5o - u$.
37. $2 \nearrow 2$: $2u - 4o - 4u - 5o - 2u$.
38. $3 \nwarrow 2$: $5o - 5u - 5o - 2u$.
39. $3 \nearrow 1$: $3u - 4o - 5u - 5o - u$.
40. $4 \nwarrow 1$: $5o - 5u - 5o - 3u$.

The grid-diagram of this knot is depicted in Fig. 550

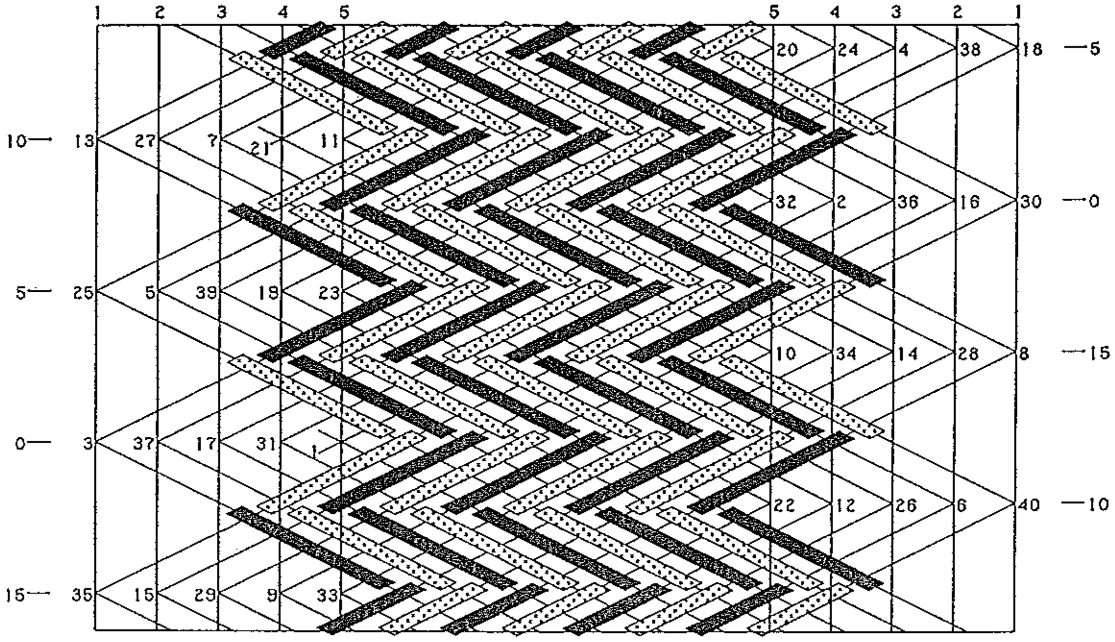


Fig. 550 — Grid-diagram of the knot in Example 5.

The half-cycle tables for the mirror-imaged knot, hence the Semi-Perfect Herringbone Pineapple Knot with $A = 5$; $x = 14$; $l_1 = 5$, $I_{L_1} = 0$; $l'_1 = 4$, $I'_{L_1} = 10$; $y = A - 1 = 4$; $B^* = 4$, may now be derived from the half-cycle tables in Fig. 549. Again, the new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$).

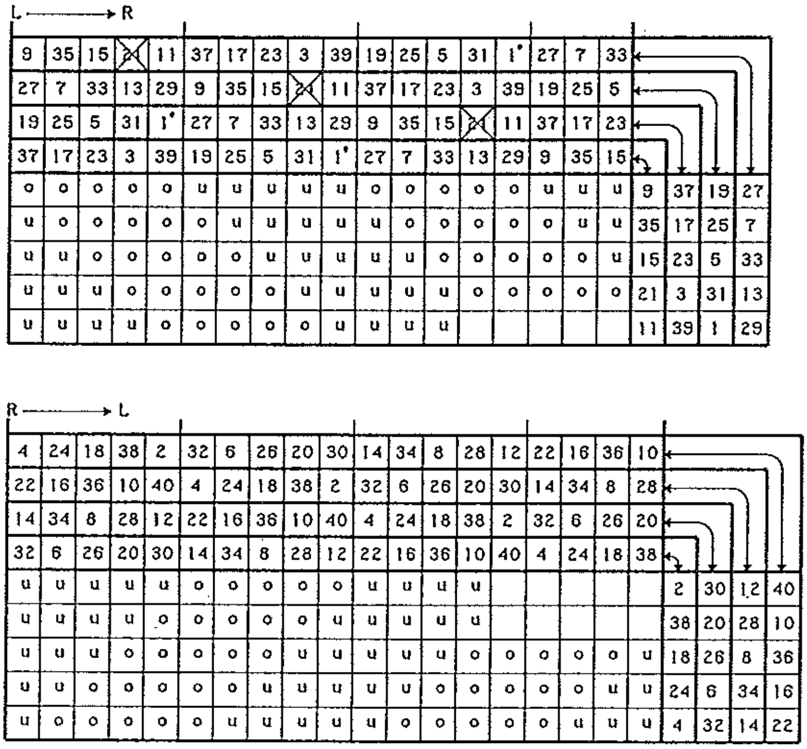
Thus for the knot in Example 5 with $B = 20$, $\lambda = 2$, $\varphi = 9$ for the first sub-component (the number of half-cycles on its first-return string-run between left-hand bight-boundary 5 at the beginning of the first-return string-run and right-hand bight-boundary 5), hence for the half-cycles $1 \leq h \leq 20$, and $\varphi = 3$ for the second sub-component (the number of half-cycles on its first-return string-run between left-hand bight-boundary 4 at the beginning of the first-return string-run and right-hand bight-boundary 4), hence for the half-cycles $21 \leq h \leq 40$, replace in its two half-cycle tables each half-cycle number h in the range:

$1 \leq h \leq 20$ by $|h-9|_{20}$.

$21 \leq h \leq 40$ by $|h-3|_{20} + 20$.

Take $|h-9|_{20} = 20$ when $|h-9|_{20} = 0$.

The thus obtained half-cycle tables of the mirror-imaged knot, hence of the knot with $y = A - 1$, are shown in Fig. 551.



9	35	15	~~21~~	11	37	17	23	3	39	19	25	5	31	$1^\bullet$	27	7	33				
27	7	33	13	29	9	35	15	~~21~~	11	37	17	23	3	39	19	25	5				
19	25	5	31	$1^\bullet$	27	7	33	13	29	9	35	15	~~21~~	11	37	17	23				
37	17	23	3	39	19	25	5	31	$1^\bullet$	27	7	33	13	29	9	35	15				
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	9	37	19	27
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	35	17	25	7
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	15	23	5	33
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	21	3	31	13
u	u	u	u	o	o	o	o	o	u	u	u	u						11	39	1	29

4	24	18	38	2	32	6	26	20	30	14	34	8	28	12	22	16	36	10				
22	16	36	10	40	4	24	18	38	2	32	6	26	20	30	14	34	8	28				
14	34	8	28	12	22	16	36	10	40	4	24	18	38	2	32	6	26	20				
32	6	26	20	30	14	34	8	28	12	22	16	36	10	40	4	24	18	38				
u	u	u	u	u	o	o	o	o	o	u	u	u	u						2	30	12	40
u	u	u	u	o	o	o	o	o	u	u	u	u	u						38	20	28	10
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	18	26	8	36
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	24	6	34	16
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	4	32	14	22

Fig. 551 — The half-cycle tables for the mirror-imaged knot.

Example 6 shows how these half-cycle tables may also be obtained from the half-cycle pattern of the mirror-imaged knot.

Example 6:

$A = 5$; $x = 14$; $l_1 = 5$, $I_{L_1} = 0$; $l_1' = 4$, $I'_{L_1} = 10$; $y = A - 1 = 4$, hence $\Delta = 4$; $B^* = 4$, hence $B = A \cdot B^* = 20$.

Thus:

$$r_1 = \left| \frac{x - A + 1 - 2l_1}{2} \right|_A = \left| \frac{14 - 5 + 1 - 10}{2} \right|_5 = |0|_5 = 5,$$

$$I_{R_1} = I_{L_1} = 0;$$

$$r_1' = \left| \frac{x - A + 1 - 2l_1'}{2} \right|_A = \left| \frac{14 - 5 + 1 - 8}{2} \right|_5 = |1|_5 = 1,$$

$$I'_{R_1} = \left| I'_{L_1} + (l_1 + r_1) - (l_1' + r_1') \right|_B = |10 + (5 + 5) - (4 + 1)|_{20} = 15.$$

The first-return string-run and half-cycle pattern is shown in Fig. 552

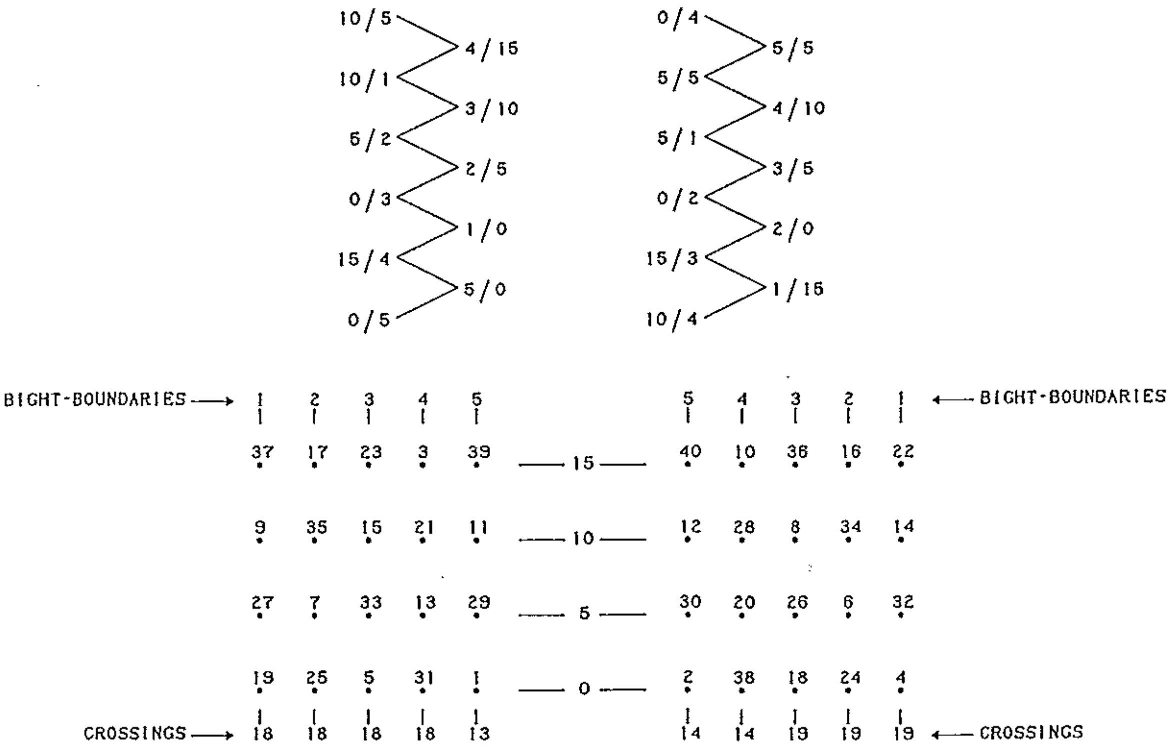


Fig. 552 — First-return string-run and half-cycle pattern.

The grid-diagram of this knot is depicted in Fig. 553

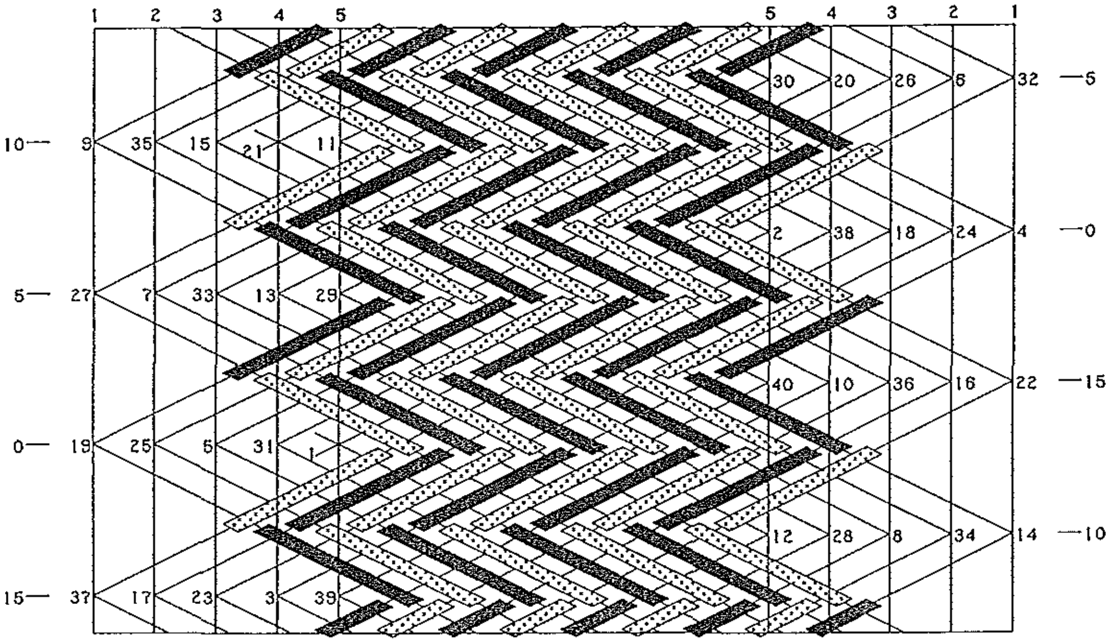


Fig. 553 — Grid-diagram of the knot in Example 6.

From the half-cycle tables in Fig. 551 we obtain its braiding half-cycle algorithms:

1. $5 \nearrow 5$: Free run.
2. $4 \nwarrow 5$: Free run.
3. $4 \nearrow 1$: Free run.
4. $3 \nwarrow 1$: o.
5. $3 \nearrow 2$: $o - u$.

21. $4 \nearrow 1$: $u - 3o - 2u - 3o$.
22. $3 \nwarrow 1$: $u - 2o - 3u - 3o - 2u$.
23. $3 \nearrow 2$: $u - 3o - 2u - 3o - u$.
24. $2 \nwarrow 2$: $u - 2o - 4u - 3o - u$.
25. $2 \nearrow 3$: $3o - 2u - 4o - 2u$.

6. $2 \nwarrow 2$: $u - 2o$.
7. $2 \nearrow 3$: $u - o - u$.
8. $1 \nwarrow 3$: $o - 2u - o$.
9. $1 \nearrow 4$: $o - u - o - u$.
10. $5 \nwarrow 4$: $u - 2o - u$.
11. $5 \nearrow 5$: $u - o - u$.
12. $4 \nwarrow 5$: $u - 2o - u$.
13. $4 \nearrow 1$: $u - o - 2u - o$.
14. $3 \nwarrow 1$: $2o - u - 2o - u$.
15. $3 \nearrow 2$: $o - 2u - 2o - u$.
16. $2 \nwarrow 2$: $u - 2o - u - 3o - u$.
17. $2 \nearrow 3$: $o - 3u - 2o - u$.
18. $1 \nwarrow 3$: $u - 2o - 2u - 3o$.
19. $1 \nearrow 4$: $2o - 3u - 2o - 2u$.
20. $5 \nwarrow 4$: $2u - 3o - 2u$.

26. $1 \nwarrow 3$: $2u - 3o - 4u - 3o - u$.
27. $1 \nearrow 4$: $3o - 3u - 4o - 2u$.
28. $5 \nwarrow 4$: $3u - 3o - 4u$.
29. $5 \nearrow 5$: $2u - 4o - 3u$.
30. $4 \nwarrow 5$: $4u - 3o - 3u$.
31. $4 \nearrow 1$: $2u - 4o - 3u - 4o$.
32. $3 \nwarrow 1$: $u - 4o - 3u - 4o - 3u$.
33. $3 \nearrow 2$: $u - 3o - 4u - 5o - u$.
34. $2 \nwarrow 2$: $2u - 3o - 4u - 5o - 2u$.
35. $2 \nearrow 3$: $4o - 5u - 5o - 2u$.
36. $1 \nwarrow 3$: $3u - 4o - 5u - 5o - u$.
37. $1 \nearrow 4$: $5o - 5u - 4o - 3u$.
38. $5 \nwarrow 4$: $4u - 5o - 5u$.
39. $5 \nearrow 5$: $4u - 5o - 4u$.
40. $4 \nwarrow 5$: $5u - 5o - 4u$.

THE BRAIDER'S NOTEBOOK

We have seen that in the Nested Cylindrical Braids the **first-return string-runs** play a very important role:

1. The number of **components** in a Nested Cylindrical Braid is equal to the number of different first-return string-runs in that braid.

2. The number of **parts** P_c of a component in a Nested Cylindrical Braid is determined by the first-return string-run of that component $\left[P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} \right]$.

3. The first-return string-run of a component is a function in the determination of the number of sub-components in that component, hence is a function in the determination of the number of essential strings λ required for that component $[\lambda = \text{g.c.d.}\,(P_c\,,\, B_c)]$.

Last point **3.** clearly indicates the great importance of the first-return string-run in the determination of the number of essential strings for a component. Hence let's look at a cylindrical braid in general (hence not necessarily a nested one) which consists of **one** component (hence has one first-return string-run).

Say this component consists of λ sub-components (hence λ essential strings are required for this component), and since each sub-component consists of a sequence of the same number of identical first-return string-runs, let this number of identical first-return string-runs in each sub-component be n_1. Say that the first-return string-run has α bights and spans β bights in the component. Let the component have B_c bights. Then:

$$\begin{aligned} n_1\beta &= n_2 B_c \qquad \rightarrow \quad \text{g.c.d.}\,(n_1\,,\, n_2) = 1\,,\\ \lambda n_1 \alpha &= B_c\,. \end{aligned}$$

Thus:

$$\left.\begin{aligned} n_1 &= \frac{B_c}{\lambda\alpha}\,,\\ n_2 &= \frac{n_1\beta}{B_c} = \frac{\beta}{\lambda\alpha}\,. \end{aligned}\right\} \quad \rightarrow \quad \text{g.c.d.}\left(\frac{\beta}{\alpha}\,,\, \frac{B_c}{\alpha}\right) = \lambda\,.$$

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for
the braiding artisan

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THE BRAIDER'S NOTEBOOK

In *The Braider*, Issue No. 29, we have seen on pg. 682 that for a component of a cylindrical braid its number of sub-components, and hence its number of essential strings λ is equal to

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right),$$

where α is the number of bights in the component's first-return string-run, β is the number of bights the first-return string-run spans in the component, and B_c is the component's number of bights.

Let's first illustrate with the aid of a few Examples this greatest common divisor of a component of a cylindrical braid.

Example 1.

Fig. 554 depicts the string-run of a Nested Cylindrical Braid which has one first-return string-run only. Hence it consists of one component and consequently its number of bights is equal to $B_c = 16$.

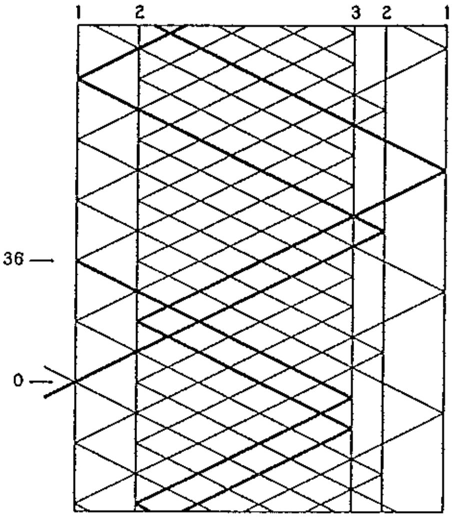


Fig. 554 — The Nested Cylindrical Braid (2/7/12){12/2133}16.

The sequence of thick-lined half-cycles depicts the first-return string-run which has $\alpha = 4$ bights and spans $\beta = 36$ bights in the braid. Hence:

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \left(\frac{36}{4}, \frac{16}{4}\right) = (9, 4) = 1.$$

Thus Fig. 554 depicts the string-run of a single string Nested Cylindrical Braid.

Example 2.

Fig. 555 depicts the string-run of a Nested Cylindrical Braid which has four first-return string-runs. Hence it consists of four components. Note that we have discussed this braid in *The Braider*, Issue No. 19, pp. 419–422.

The string-run of the first component, depicted at the left in Fig. 556, has the specification (12/14/13){112222223/113312132}18. It is interbraided with the Nested Cylindrical Braid (11/4/11){123/321}6, the string-run of which is depicted at upper right in Fig. 556, and which in turn consists of an interbraid of three Regular Cylindrical Braids with $p/b = 2/2$ each, the string-run of which is depicted at lower right in Fig. 556. The two thick-lined half-cycle sequences depict the first-return string-runs in respectively the component (12/14/13){112222223/113312132}18 and the component $p/b = 2/2$.

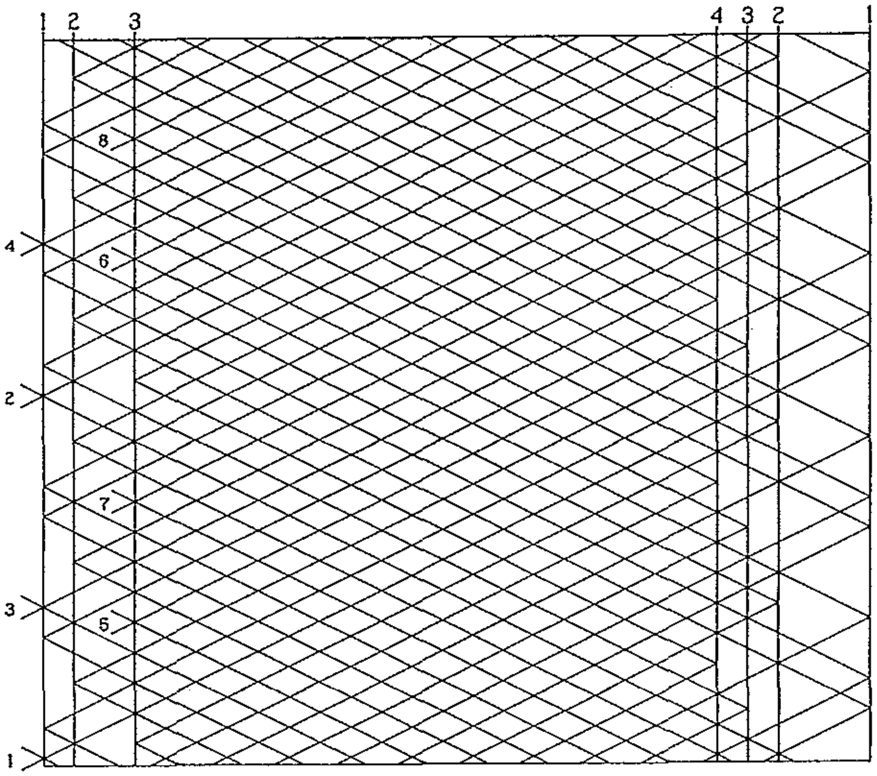


Fig. 555 — The Nested Cylindrical Braid (12/19/113){1231/211314}24.

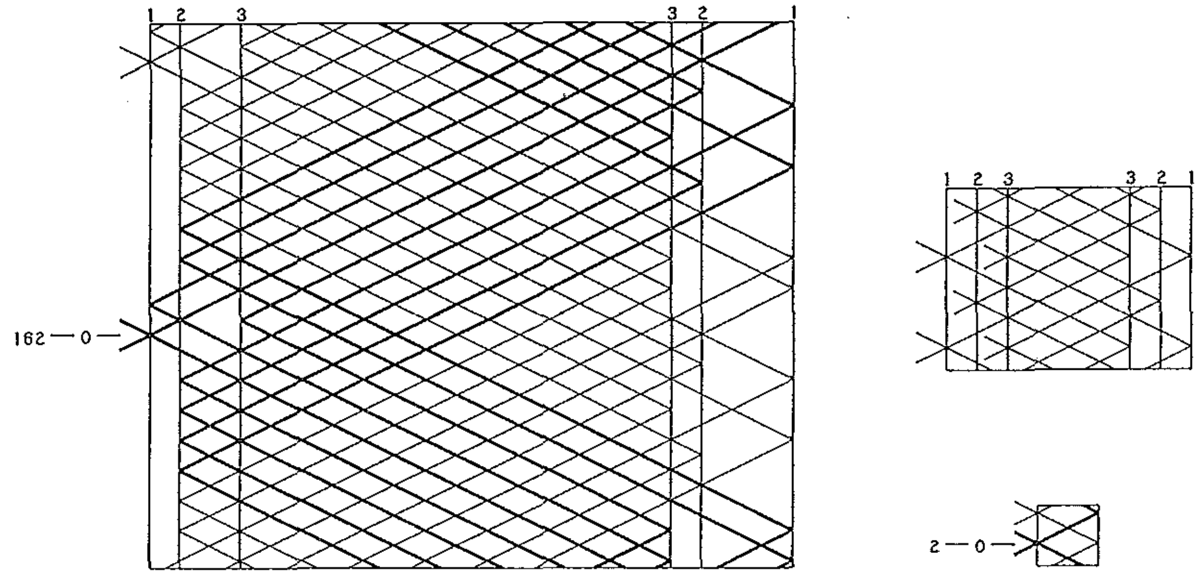


Fig. 556 — The interbraided components in Fig. 555.

For the component (12/14/13){112222223/113312132}18, the first-return string-run spans $\beta = 162$ bights and has $\alpha = 9$ bights. Hence for this component

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \left(\frac{162}{9}, \frac{18}{9}\right) = (18, 2) = 2.$$

Thus this component consists of two sub-components and hence two essential strings are required in its construction.

For the component $p/b = 2/2$, the first-return string-run spans $\beta = 2$ bights and has $\alpha = 1$ bight. Hence for this component

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \left(\frac{2}{1}, \frac{2}{1}\right) = (2, 2) = 2.$$

Thus this component consists of two sub-components and hence two essential strings are required in its construction. The total of essential strings for the Nested Cylindrical Braid (12/19/113){1231/211314}24 thus becomes $2 + 3 \times 2 = 2 + 6 = 8$.

Note that the β-value and the α-value of a component do not change with a change in its B_c-value. Hence a single string component is obtained by ensuring that its values $\frac{\beta}{\alpha}$ and $\frac{B_c}{\alpha}$ are coprime. Thus the Nested Cylindrical Braid (12/19/113){1231/211314}60, for example, has four single string components ($B_{c_1} = 45$; $B_{c_2} = B_{c_3} = B_{c_4} = 5 \rightarrow 60 = 45 + 3 \times 5$); the string-run diagram of this braid is shown in Fig. 557.

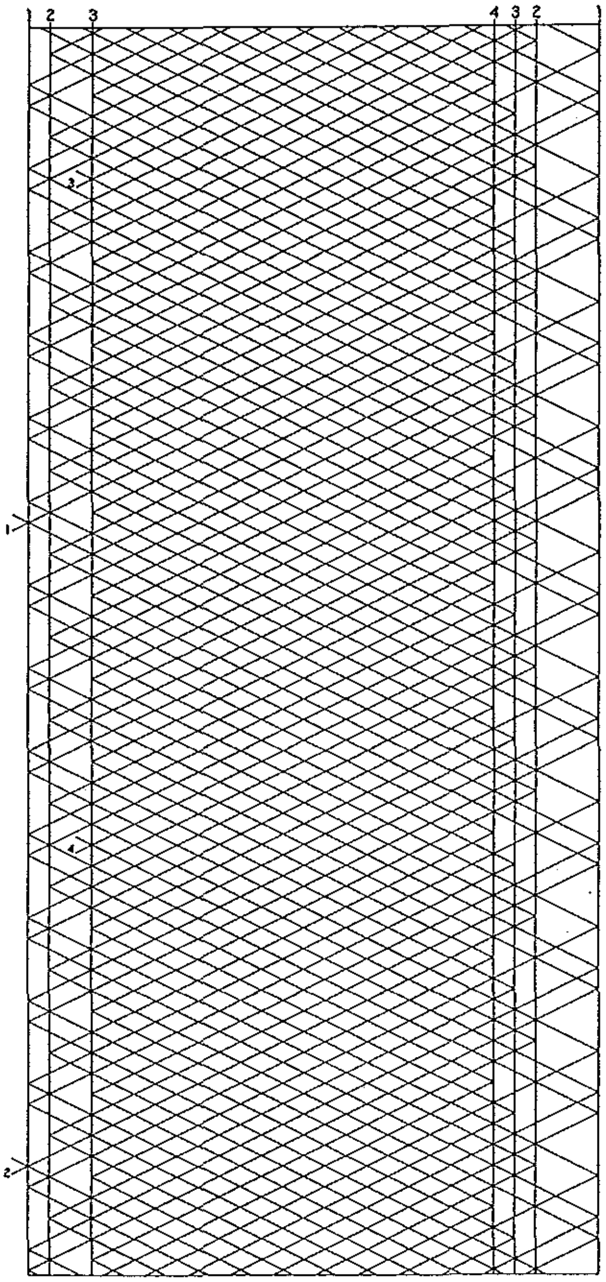


Fig. 557 — The Nested Cylindrical Braid (12/19/113){1231/211314}60.

Example 3.

Fig. 558 depicts the string-run of a Cylindrical Braid which has only one first-return string-run. Hence it consists of one component with $B_c = 12$.

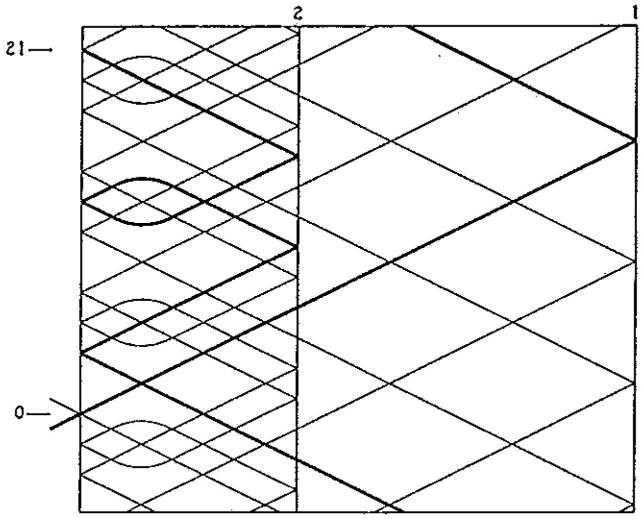


Fig. 558 — A one component Cylindrical Braid.

The sequence of thick-lined half-cycles depicts the first-return string-run which has $\alpha = 3$ bights and spans $\beta = 21$ bights in the braid. Hence:

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \left(\frac{21}{3}, \frac{12}{3}\right) = (7, 4) = 1.$$

Thus Fig. 558 depicts the string-run of a single string Cylindrical Braid.

Example 4.

Fig. 559 depicts the string-run of a Cylindrical Braid which has only one first-return string-run. Hence it consists of one component with $B_c = 28$ (the bights on the sub-script $_L$ (or sub-script $_R$) bight-boundaries).

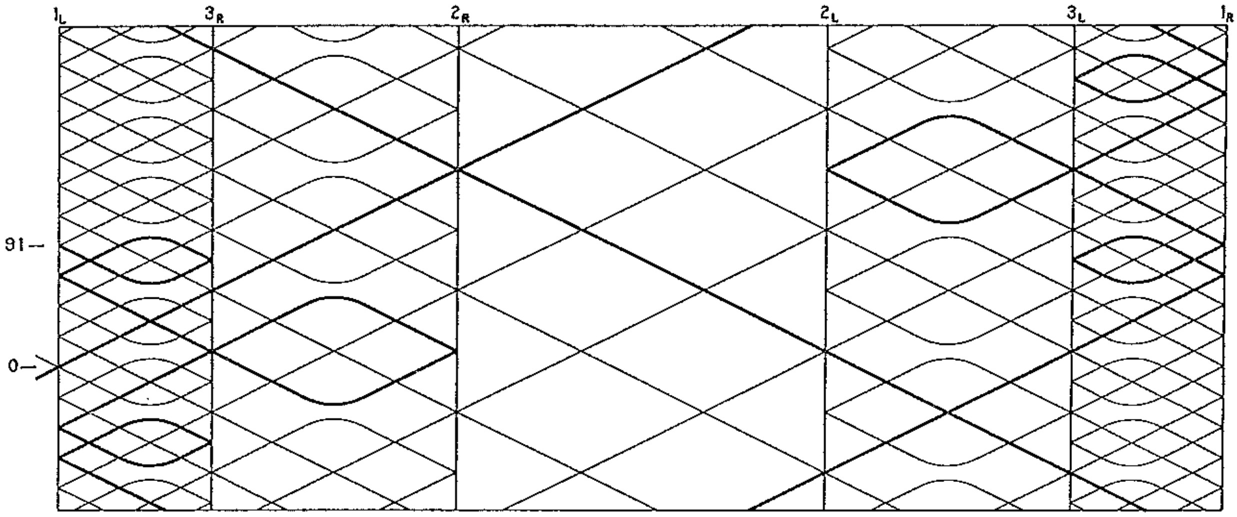


Fig. 559 — A one component Cylindrical Braid.

The sequence of thick-lined half-cycles depicts the first-return string-run which has $\alpha = 7$ bights and spans $\beta = 91$ bights in the braid (see Fig. 560). Hence:

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \left(\frac{91}{7}, \frac{28}{7}\right) = (13, 4) = 1.$$

Thus Fig. 559 depicts the string-run of a single string Cylindrical Braid.

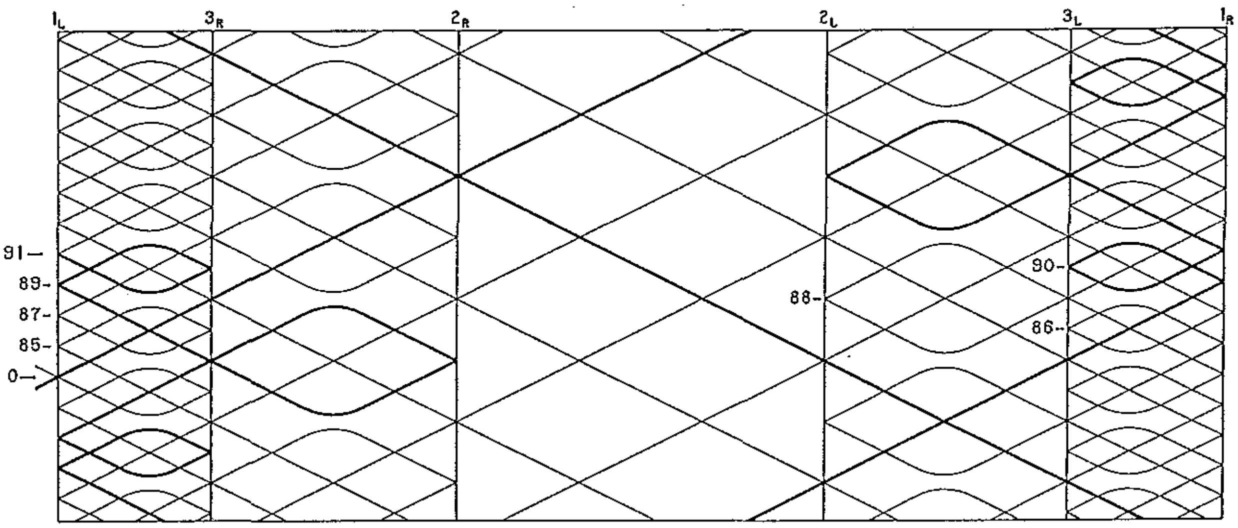


Fig. 560 — The span of bights β.

Example 5.

Fig. 561 depicts the string-run of a Cylindrical Braid which has only one first-return string-run. Hence it consists of one component with $B_c = 28$ (the bights on the sub-script $_L$ (or sub-script $_R$) bight-boundaries).

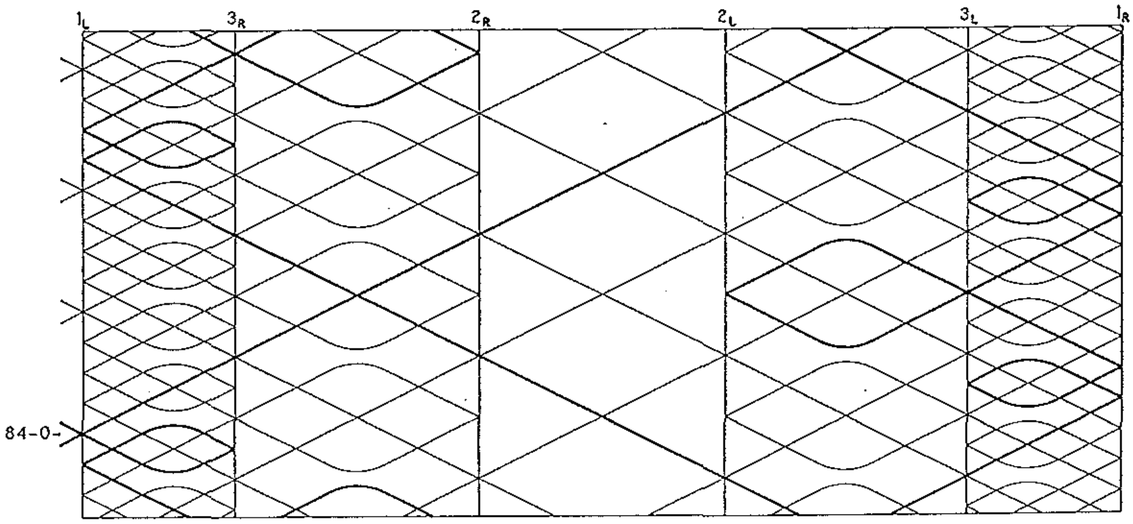


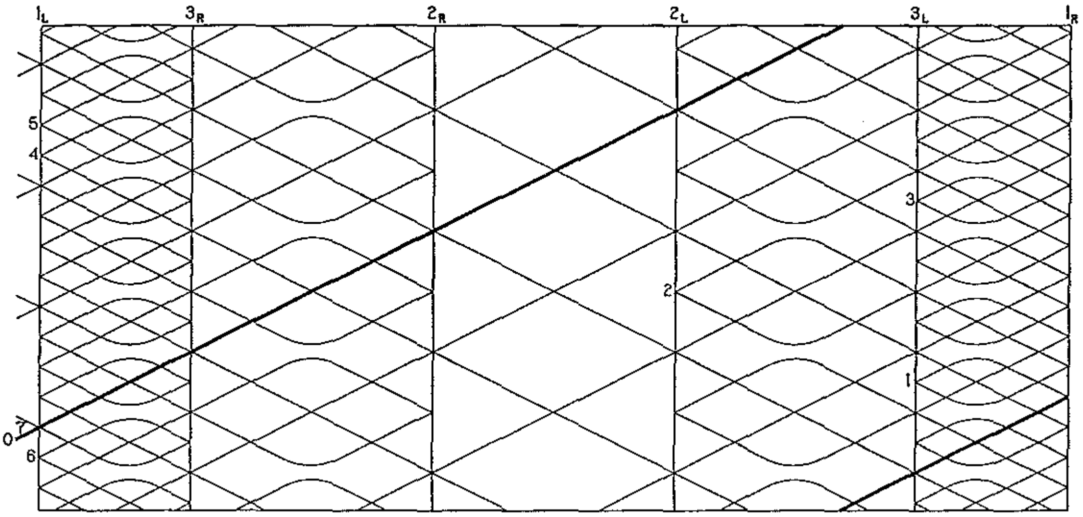
Fig. 561 — A one component Cylindrical Braid.

The sequence of thick-lined half-cycles depicts the first-return string-run which has $\alpha = 7$ bights and spans $\beta = 84$ bights in the braid. Hence:

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \left(\frac{84}{7}, \frac{28}{7}\right) = (12, 4) = 4.$$

Thus Fig. 561 depicts the string-run of a four string Cylindrical Braid.

From Fig. 562 we observe that a period consists of 7 consecutive bights, hence B_c must be a multiple of 7. Since the values of β and α of a component do not change with a change in the component's B_c-value, a single string component is obtained by ensuring that its values $\frac{\beta}{\alpha}$ and $\frac{B_c}{\alpha}$ are coprime. Thus for $B_c = 35$, for example, we obtain a single string Cylindrical Braid.



A PERIOD CONSISTS OF THE BIGHTS IN THE FIRST-RETURN STRING-RUN; HENCE CONSISTS OF α CONSECUTIVE BIGHTS

N1-563B.DWG linescale=5 plotscale=1.000

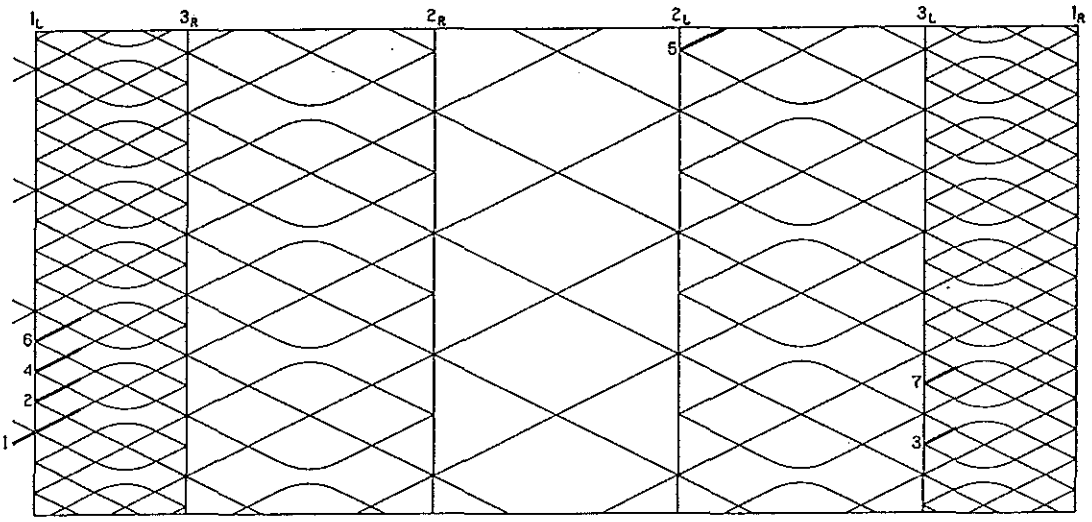


Fig. 562 — The number of bights in a period.

Example 6.

Fig. 563 depicts the string-run of a Cylindrical Braid which consists of two interbraided components which are depicted by the two lower string-run diagrams. For both components $B_c = 10$.

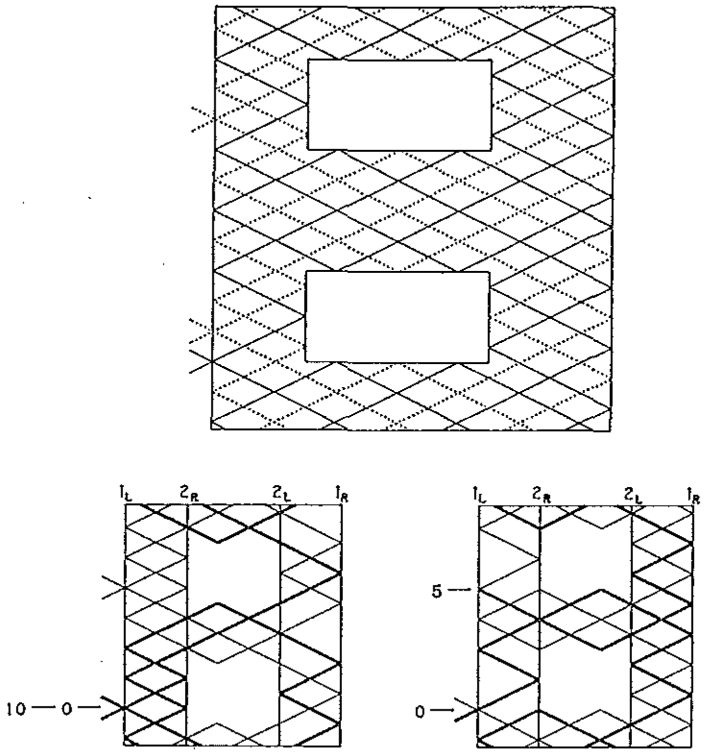


Fig. 563 — A Cylindrical Braid.

For the lower-left component $\beta = 10$ and $\alpha = 5$. Hence

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \left(\frac{10}{5}, \frac{10}{5}\right) = (2, 2) = 2 .$$

Hence this component requires two strings in its construction.

For the lower-right component $\beta = 5$ and $\alpha = 5$. Hence

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \left(\frac{5}{5}, \frac{10}{5}\right) = (1, 2) = 1 .$$

Hence this component requires one string in its construction.

Note that here the end-point of the first-return string-run lies **below** its starting-point.

A period in each component consists of 5 consecutive bights. Since the values of β and α of a component do not change with a change in the component's B_c-value, a single string component is also obtained for the lower-left component by ensuring that its values $\frac{\beta}{\alpha}$ and $\frac{B_c}{\alpha}$ are coprime. Thus for $B_c = 15$ (hence $B = 30$), for example, we obtain a single string component for each of the two interbraided components.

Note that for the lower-right component in Fig. 564 the end-point of the first-return string-run lies **below** its starting-point, which results again in $\beta = 5$.

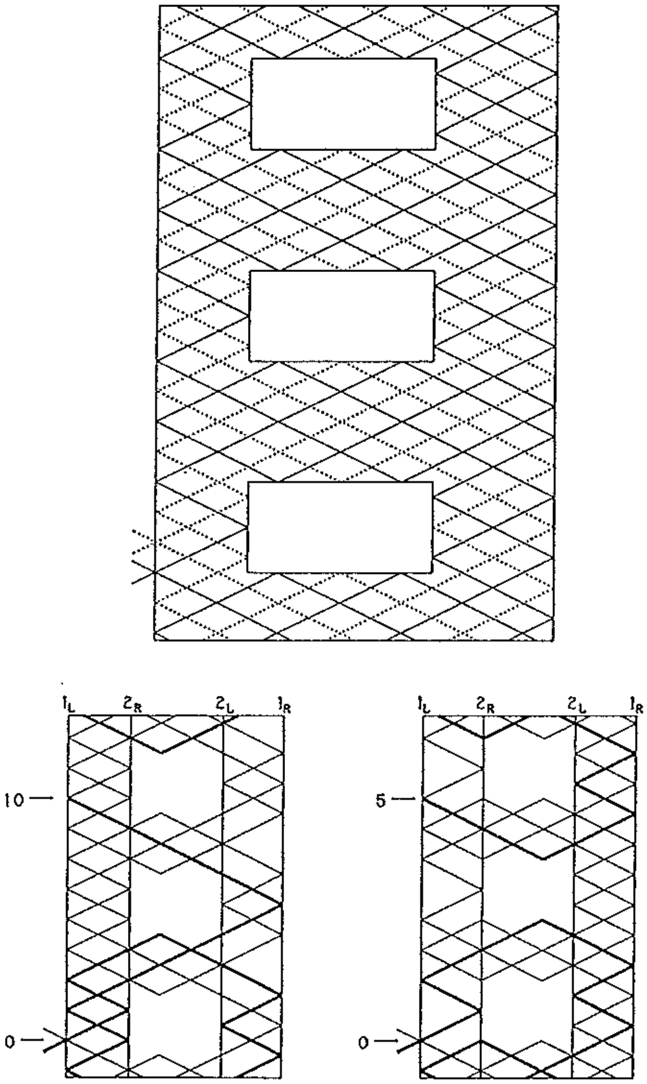


Fig. 564 — An interbraid of two single string components.

From the above Examples the importance of the first-return string-run in a component will be evident. It appears that in the past its importance or even its existence has been totally overlooked as otherwise the general relationship:

$$\lambda_c = \text{g.c.d.}\left(\frac{\beta_c}{\alpha_c}, \frac{B_c}{\alpha_c}\right)$$

would have been known. It appears that in the past only the case for a single component Cylindrical Braid with $\alpha = 1$ was experimentally discovered (in fact it is still being discovered experimentally in more recent times, see the Newsletter of the International Guild of Knot Tyers, issue No. 56, pp. 39–40, and issue No. 57, pp. 22–25). When for a single component Cylindrical Braid $\alpha = 1$ the Cylindrical Braid is a Regular Cylindrical Braid and $\beta = p$ (the number of its parts), consequently:

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \text{g.c.d.}\,(p\,,\, b)\,,$$

where b is the number of bights of the Regular Cylindrical Braid.

It is only a tiny step from the Regular Cylindrical Braids to the Perfect Nested Cylindrical Braids. A Perfect Nested Cylindrical Braid has also only one first-return string-run (consequently only one component) and hence its B_c-value is its B-value, while its $\frac{\beta}{\alpha}$-value is its P-value (the number of its parts). Hence for a Perfect Nested Cylindrical Braid we obtain $\alpha = A^{**}$ and hence $\frac{B_c}{\alpha} = \frac{B}{A^{**}} = B^{**}$ with

$$\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \text{g.c.d.}\,(P\,,\, B^{**})\,.$$

Another small step brings us to the Nested Cylindrical Braids with more than one type of first-return string-run, hence more than one component. For a component of such a Nested Cylindrical Braid we obtain:

$$\frac{\beta_c}{\alpha_c} = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = P_c\,,$$

$$\frac{B_c}{\alpha_c} = \frac{B_{total}}{A^{**}} = B^{**}\,,$$

$$\lambda_c = \text{g.c.d.}\left(\frac{\beta_c}{\alpha_c}, \frac{B_c}{\alpha_c}\right) = \text{g.c.d.}\,(P_c\,,\, B^{**})\,.$$

Nested Cylindrical Braids

The string-runs of some Perfect Regular Nested Cylindrical Braids (g.c.d. $(\Delta\,,\,A) = 1$) can be enlarged to string-runs of bigger Perfect Regular Nested Cylindrical Braids. In the enlargement process any additional nests of bights can only be formed in the external valleys between the original nests of bights; in other words, they can on the left-hand bight-edge as well as on the right-hand bight-edge only be formed between half-cycles which start and end in adjacent nests at bight-boundary 1.

There are in essence two different enlargement processes:

Method I where the enlargement process starts with a lower-left to upper-right half-cycle **below** the Standing-End half-cycle.

Method II where the enlargement process starts with a lower-left to upper-right half-cycle **above** the Standing-End half-cycle.

Thus for enlargement process **Method I**, the first enlarging half-cycle (from lower-left to upper-right) must run:

1. either between the Standing-End half-cycle $1 \rightarrow 1$, and the half-cycle $2 \rightarrow A$ immediately **below** it,
2. or between half-cycle $A \rightarrow 1$ of the nest to which the Standing-End half-cycle $1 \rightarrow A$ belongs, and the half-cycle $1 \rightarrow A$ immediately **below** it.

While for enlargement process **Method II**, the first enlarging half-cycle (from lower-left to upper-right) must run:
1. either between the Standing-End half-cycle $A \rightarrow A$, and the half-cycle $(A-1) \rightarrow 1$ immediately **above** it,
2. or between half-cycle $1 \rightarrow A$ of the nest to which the Standing-End half-cycle $A \rightarrow 1$ belongs, and the half-cycle $A \rightarrow 1$ immediately **above** it.

Method I−1 : (See Fig. 565)

For enlargement **Method I−1** the second enlarging half-cycle (from lower-right to upper-left) could run between an $A \leftarrow 1$ half-cycle and the $1 \leftarrow A$ half-cycle immediately **above** it.

The final half-cycle in the enlargement process (from lower-right to upper-left) must run immediately **above** the $A \leftarrow 1$ half-cycle belonging to the nest to which the Standing-End half-cycle originally belonged. Hence in general, every even half-cycle in the enlargement process must run between an $A \leftarrow 1$ half-cycle and the $1 \leftarrow A$ half-cycle immediately **above** it. The earlier stated possible position of the second enlarging half-cycle fulfils this general requirement.

The string-run of the braid to be enlarged has thus a Δ-value of $A-1$, hence is a string-run of a Perfect Regular Nested Cylindrical Braid.

Since the condition for the necessary position of the additionally created nests prevents the third enlarging half-cycle to run between an $1 \rightarrow 1$ half-cycle and the $A \rightarrow 2$ half-cycle immediately **above** it; it follows that the third enlarging half-cycle must run immediately **above** the first enlarging half-cycle. Hence the enlargement will create only one additional nest of bights on the left-hand bight-edge and one additional nest of bights on the right-hand bight-edge.

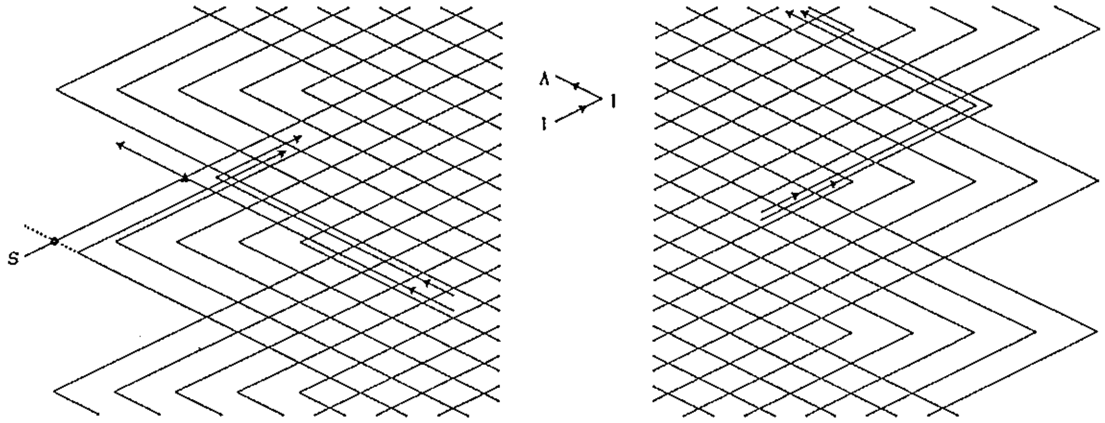


Fig. 565 — Method I−1 enlargement.

Method I−2 : (See Fig. 566)

For enlargement **Method I−2** the second enlarging half-cycle (from lower-right to upper-left) could run between an $A \leftarrow 1$ half-cycle and the $1 \leftarrow A$ half-cycle immediately **above** it.

The final half-cycle in the enlargement process (from lower-right to upper-left) must run immediately **below** the $2 \leftarrow A$ half-cycle belonging to the nest to which the

Standing-End half-cycle belongs. Hence in general, every even half-cycle in the enlargement process must run between an 1 ← 1 half-cycle and the 2 ← *A* half-cycle immediately **above** it. The earlier stated possible position of the second enlarging half-cycle does not fulfil this general requirement. Hence the general requirement for the even half-cycles in the enlargement process only allows the creation of one additional nest of bights on the left-hand bight-edge and one additional nest of bights on the right-hand bight-edge.

The third half-cycle in the enlargement process runs immediately **below** the first enlarging half-cycle.

The string-run of the braid to be enlarged has thus a Δ-value of 1, hence is a string-run of a Perfect Regular Nested Cylindrical Braid.

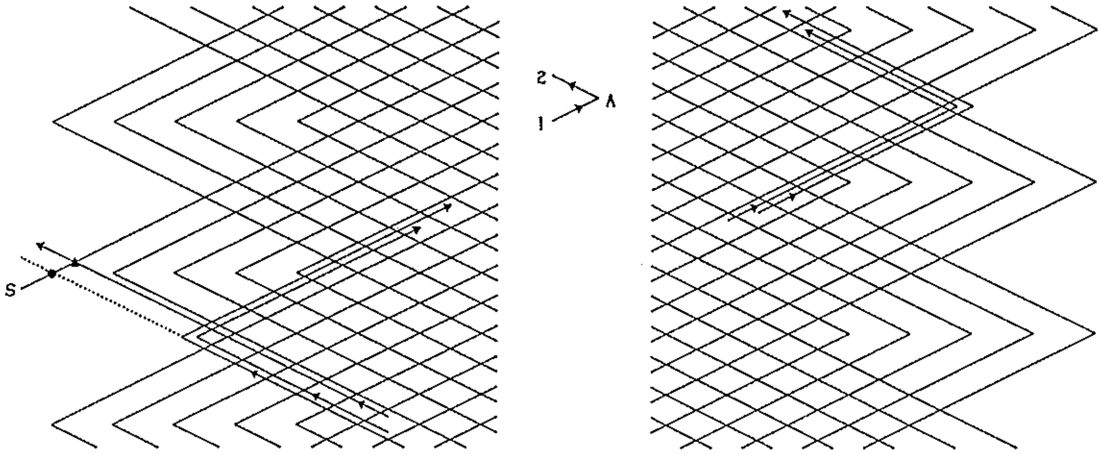


Fig. 566 — Method I – 2 enlargement.

Method II – 1 : (See Fig. 567)

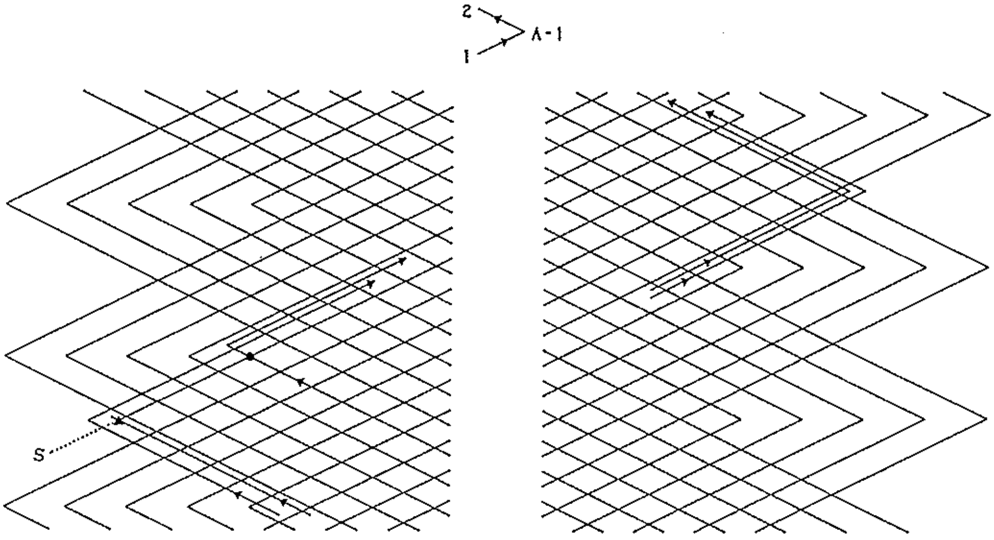


Fig. 567 — Method II – 1 enlargement.

For enlargement **Method II – 1** the second enlarging half-cycle (from lower-right to upper-left) could run between an *A* ← 1 half-cycle and the 1 ← *A* half-cycle immediately **above** it.

The final half-cycle in the enlargement process (from lower-right to upper-left) must run immediately **below** the 1 ← *A* half-cycle belonging to the nest to which the Standing-End half-cycle originally belonged. Hence in general, every even half-cycle in the enlargement process must run between an $A \leftarrow 1$ half-cycle and the $1 \leftarrow A$ half-cycle immediately **above** it. The earlier stated possible position of the second enlarging half-cycle fulfils this general requirement.

The string-run of the braid to be enlarged has thus a Δ-value of 1, hence is a string-run of a Perfect Regular Nested Cylindrical Braid.

Since the condition for the necessary position of the additionally created nests prevents the third enlarging half-cycle to run between an $1 \rightarrow (A-1)$ half-cycle and the $A \rightarrow A$ half-cycle immediately **above** it; it follows that the third enlarging half-cycle must run immediately **below** the first enlarging half-cycle. Hence the enlargement will create only one additional nest of bights on the left-hand bight-edge and one additional nest of bights on the right-hand bight-edge.

Method II – 2 : (See Fig. 568)

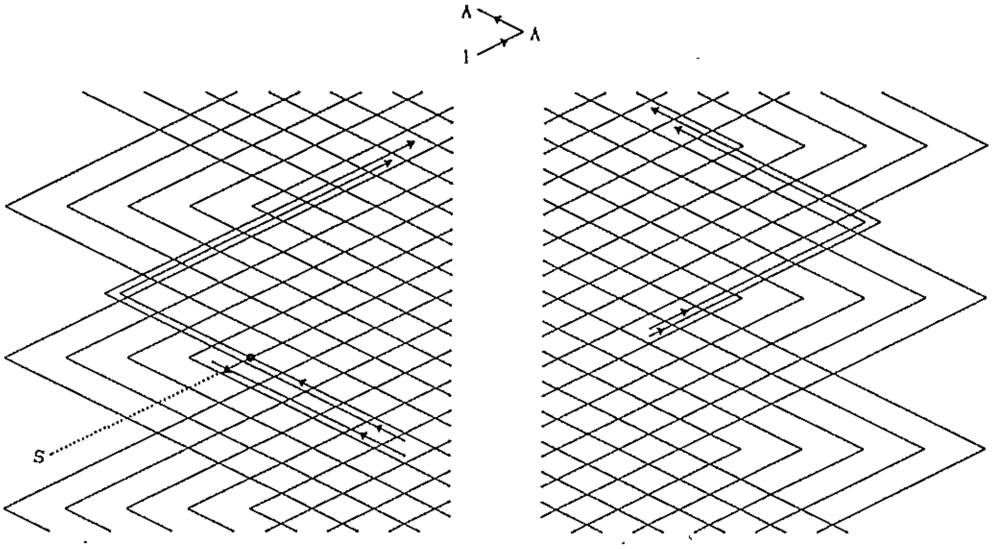


Fig. 568 — Method II – 2 enlargement.

For enlargement **Method II – 2** the second enlarging half-cycle (from lower-right to upper-left) could run between an $A \leftarrow 1$ half-cycle and the $1 \leftarrow A$ half-cycle immediately **above** it.

The final half-cycle in the enlargement process (from lower-right to upper-left) must run immediately **above** the $(A-1) \leftarrow 1$ half-cycle belonging to the nest to which the Standing-End half-cycle belongs. Hence in general, every even half-cycle in the enlargement process must run between an $(A-1) \leftarrow 1$ half-cycle and the $A \leftarrow A$ half-cycle immediately **above** it. The earlier stated possible position of the second enlarging half-cycle does not fulfil this general requirement. Hence the general requirement for the even half-cycles in the enlargement process only allows the creation of one additional nest of bights on the left-hand bight-edge and one additional nest of bights on the right-hand bight-edge.

The third half-cycle in the enlargement process runs immediately **above** the first enlarging half-cycle.

The string-run of the braid to be enlarged has thus a Δ-value of $A-1$, hence is a string-run of a Perfect Regular Nested Cylindrical Braid.

For the Perfect Regular Nested Cylindrical Braid to be enlarged according to enlargement **Method I – 1**:

i.) $\Delta = A - 1$; $y = A - 1$. Standing-End at left bight-boundary 1; Standing-End half-cycle $1 \to 1$.
$x = (2n-3)A+3$; $B^* = \dfrac{2n-1}{m}$; $P = (2n-1)A+1$.
n and m are natural numbers; m divides into $2n-1$.
For the resulting enlarged Perfect Regular Nested Cylindrical braid:
$x_e = x + mA$; $y_e = |y+mA|_{2A} = |(m+1)A - 1|_{2A}$; $B^*_e = B^* + 1$; $P_e = P + mA$.
ii.) $\Delta = A - 1$; $y = 2A - 1$. Standing-End at left bight-boundary 1; Standing-End half-cycle $1 \to 1$.
$x = (2n-2)A+3$; $B^* = \dfrac{2n}{m}$; $P = 2nA+1$.
n and m are natural numbers; m divides into $2n$.
For the resulting enlarged Perfect Regular Nested Cylindrical braid:
$x_e = x + mA$; $y_e = |y+mA|_{2A} = |mA - 1|_{2A}$; $B^*_e = B^* + 1$; $P_e = P + mA$.

For the Perfect Regular Nested Cylindrical Braid to be enlarged according to enlargement **Method I−2**:
i.) $\Delta = 1$; $y = A + 1$. Standing-End at left bight-boundary 1; Standing-End half-cycle $1 \to A$.
$x = (2n-3)A+3$; $B^* = \dfrac{2n-1}{m}$; $P = (2n-1)A+1$.
n and m are natural numbers; m divides into $2n-1$.
For the resulting enlarged Perfect Regular Nested Cylindrical braid:
$x_e = x + mA$; $y_e = |y+mA|_{2A} = |(m+1)A + 1|_{2A}$; $B^*_e = B^* + 1$; $P_e = P + mA$.
ii.) $\Delta = 1$; $y = 1$. Standing-End at left bight-boundary 1; Standing-End half-cycle $1 \to A$.
$x = (2n-2)A+3$; $B^* = \dfrac{2n}{m}$; $P = 2nA+1$.
n and m are natural numbers; m divides into $2n$.
For the resulting enlarged Perfect Regular Nested Cylindrical braid:
$x_e = x + mA$; $y_e = |y+mA|_{2A} = |mA + 1|_{2A}$; $B^*_e = B^* + 1$; $P_e = P + mA$.

For the Perfect Regular Nested Cylindrical Braid to be enlarged according to enlargement **Method II−1**:
i.) $\Delta = 1$; $y = A + 1$. Standing-End at left bight-boundary A; Standing-End half-cycle $A \to A$.
$x = (2n-1)A+1$; $B^* = \dfrac{2n+1}{m}$; $P = (2n+1)A-1$.
n and m are natural numbers; m divides into $2n+1$.
For the resulting enlarged Perfect Regular Nested Cylindrical braid:
$x_e = x + mA$; $y_e = |y+mA|_{2A} = |(m+1)A + 1|_{2A}$; $B^*_e = B^* + 1$; $P_e = P + mA$.
ii.) $\Delta = 1; y = 1$. Standing-End at left bight-boundary A; Standing-End half-cycle $A \to A$.
$x = (2n-2)A+1$; $B^* = \dfrac{2n}{m}$; $P = 2nA-1$.
n and m are natural numbers; m divides into $2n$.
For the resulting enlarged Perfect Regular Nested Cylindrical braid:
$x_e = x + mA$; $y_e = |y+mA|_{2A} = |mA + 1|_{2A}$; $B^*_e = B^* + 1$; $P_e = P + mA$.

For the Perfect Regular Nested Cylindrical Braid to be enlarged according to enlargement **Method II – 2**:

i.) $\Delta = A - 1$; $y = A - 1$. Standing-End at left bight-boundary A ; Standing-End half-cycle $A \rightarrow 1$.

$x = (2n-1)A + 1$; $B^* = \dfrac{2n+1}{m}$; $P = (2n+1)A - 1$.

n and m are natural numbers; m divides into $2n+1$.

For the resulting enlarged Perfect Regular Nested Cylindrical braid:

$x_e = x + mA$; $y_e = |y + mA|_{2A} = |(m+1)A - 1|_{2A}$; $B^*_e = B^* + 1$; $P_e = P + mA$.

ii.) $\Delta = A - 1$; $y = 2A - 1$. Standing-End at left bight-boundary A ; Standing-End half-cycle $A \rightarrow 1$.

$x = (2n-2)A + 1$; $B^* = \dfrac{2n}{m}$; $P = 2nA - 1$.

n and m are natural numbers; m divides into $2n$.

For the resulting enlarged Perfect Regular Nested Cylindrical braid:

$x_e = x + mA$; $y_e = |y + mA|_{2A} = |mA - 1|_{2A}$; $B^*_e = B^* + 1$; $P_e = P + mA$.

The above formulae show that g.c.d.$(P, B^*) = 1$ and hence the above Perfect Regular Nested Cylindrical Braids are single string braids, hence are Perfect Regular Nested Knots.

⋆⋆ *Prove the above formulae.*

Recall from pp. 644–645 (*The Braider*, Issue No. 28) that for $y = A - 1$ and for $y = A + 1$ we are able to superimpose on the string-run of a Regular Nested Cylindrical Braid a **Herringbone Pineapple coding.** Thus the cases Method I-1(i), Method I-2(i), Method II-1(i), and Method II-2(i) fall within this category. It should be noted that in order to obtain an enlarged knot with a Herringbone Pineapple coding, we have to execute the selected enlarging method twice due to the A-over — A-under coding.

Example 1:

The Perfect Herringbone Pineapple Knot in Bruce Grant's *Encyclopedia of Rawhide and Leather Braiding*, pp. 432–433.

The specification of the string-run of this Perfect Herringbone Pineapple Knot[†] is (2/5/2){12/21}6 .

$A = 2$; $x = 5$; $k = 2$ and $B^* = 3$, hence $y = |x - 2k - 2|_{2A} = |5 - 4 - 2|_4 = |-1|_4 = 3 = A + 1$ and $B = B^*A = 3 \times 2 = 6$. Its string-run diagram and grid-diagram are depicted in Fig. 569.

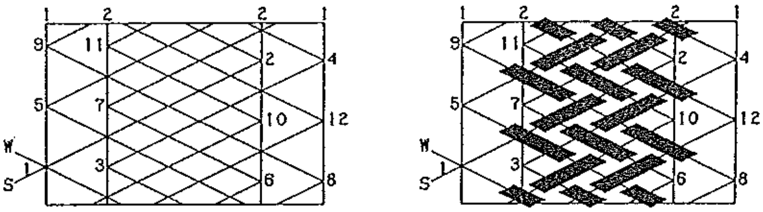


Fig. 569 — The Perfect Herringbone Pineapple Knot (2/5/2){12/21}6 .

[†] There are some errors on pp. 432–433 of Bruce Grant's *Encyclopedia of Rawhide and Leather Braiding*.

The half-cycle braiding algorithms are:

1. $1 \nearrow 2$: Free run.
2. $2 \nwarrow 2$: Free run.
3. $2 \nearrow 1$: Free run.
4. $1 \nwarrow 1$: $2o$.
5. $1 \nearrow 2$: $2u$.
6. $2 \nwarrow 2$: $o - u$.
7. $2 \nearrow 1$: $o - u$.
8. $1 \nwarrow 1$: $2u - 2o$.
9. $1 \nearrow 2$: $2o - 2u$.
10. $2 \nwarrow 2$: $u - 2o - u$.
11. $2 \nearrow 1$: $u - 2o - u$.
12. $1 \nwarrow 1$: $2o - 2u - 2o$.

This Perfect Herringbone Pineapple Knot can be enlarged in accordance with Method I-2(i) where $A = 2$; $y = A + 1$; $n = 2$; $m = 1$. Standing-End at left bight-boundary 1; Standing-End half-cycle $1 \rightarrow A$.

Thus $x = A + 3 = 5$; $B^* = 3$; $P = 2A + x - 2 = 4 + 5 - 2 = 7$.

After the first enlargement process has been completed we obtain a Perfect Regular Nested Knot (see Fig. 570) with $x_{e_1} = x + mA = 5 + 2 = 7$; $B^*_{e_1} = B^* + 1 = 3 + 1 = 4$; $P_{e_1} = P + mA = 7 + 2 = 9$; it has, however, **not** a Herringbone Pineapple coding.

This first enlargement process consists of the following half-cycle steps:

13. $1 \nearrow 2$: $2u - 2o - 2u$.
14. $2 \nwarrow 2$: $o - 2u - 2o - u$.
15. $2 \nearrow 1$: $o - 2u - 2o - u$.
16. $1 \nwarrow 1$: $2u - 2o - 2u - 2o$.

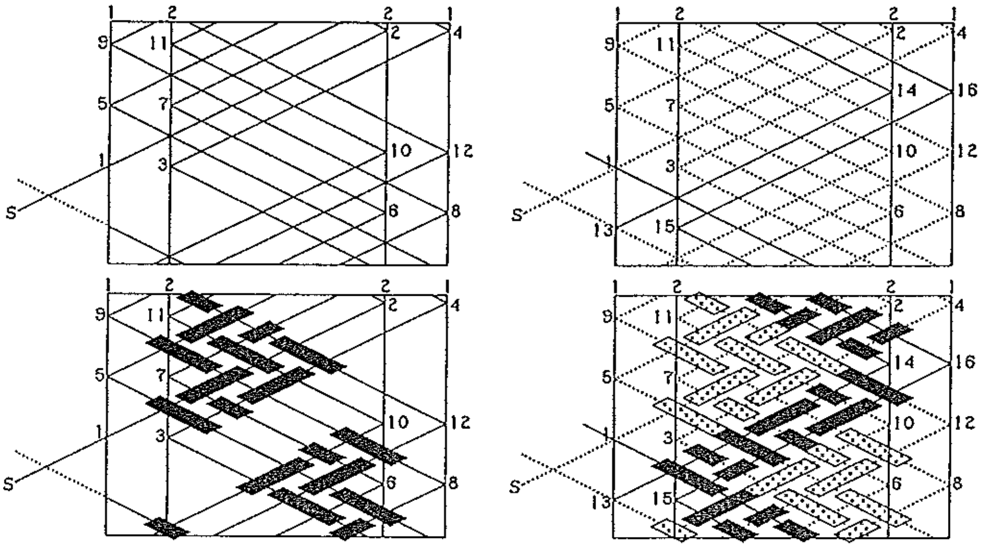


Fig. 570 — The first enlargement.

After the second enlargement process has been completed we obtain again a Perfect Regular Nested Knot (see Fig. 571), now with $x_{e_2} = x_{e_1} + mA = 7 + 2 = 9$; $B^*_{e_2} = B^*_{e_1} + 1 = 4 + 1 = 5$; $P_{e_2} = P_{e_1} + mA = 9 + 2 = 11$; it can now have a Herringbone Pineapple coding.

This second enlargement process consists of the following half-cycle steps:

17. $1 \nearrow 2$: $2o - 2u - 2o - 2u$.
18. $2 \nwarrow 2$: $u - 2o - 2u - 2o - u$.
19. $2 \nearrow 1$: $u - 2o - 2u - 2o - u$.
20. $1 \nwarrow 1$: $2o - 2u - 2o - 2u - 2o$.

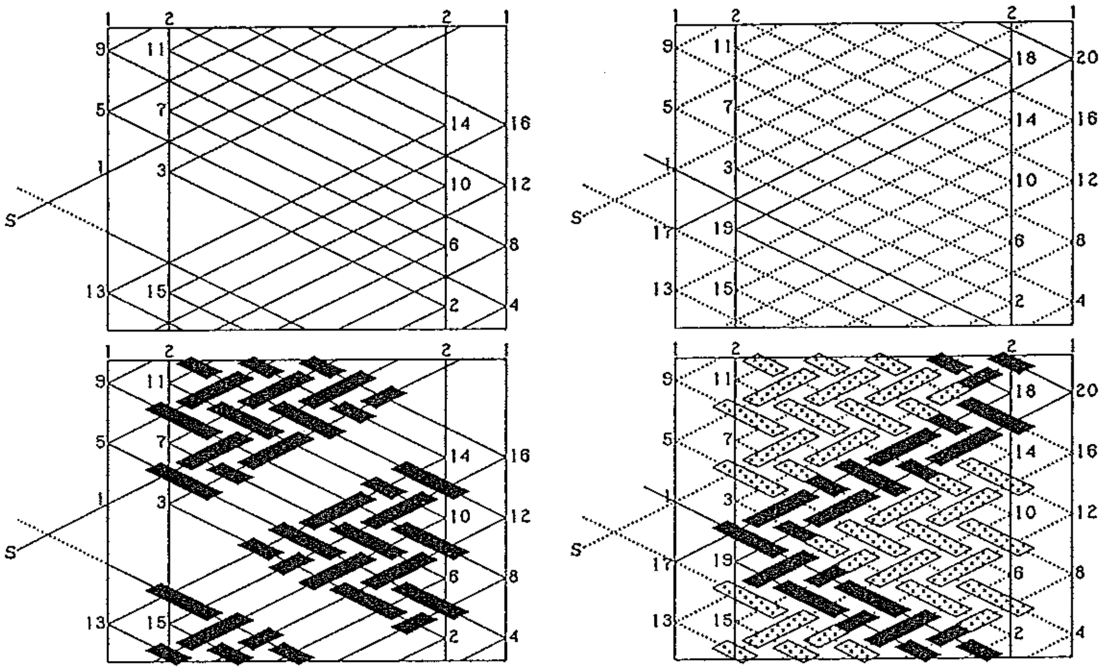


Fig. 571 — The second enlargement.

Notice that $A = 2$ is a special case due to being the lower limit value of A. Hence let's take a Perfect Herringbone Pineapple Knot example with $A > 2$.

Example 2 :

$A = 3$; $x = 10$; $k = 3$ and $B^* = 5$, hence $y = |x - 2k - 2|_{2A} = |10 - 6 - 2|_6 = |2|_6 = 2 = A - 1$ and $B = B^*A = 5 \times 3 = 15$. Its string-run diagram and grid-diagram are depicted in Fig. 572.

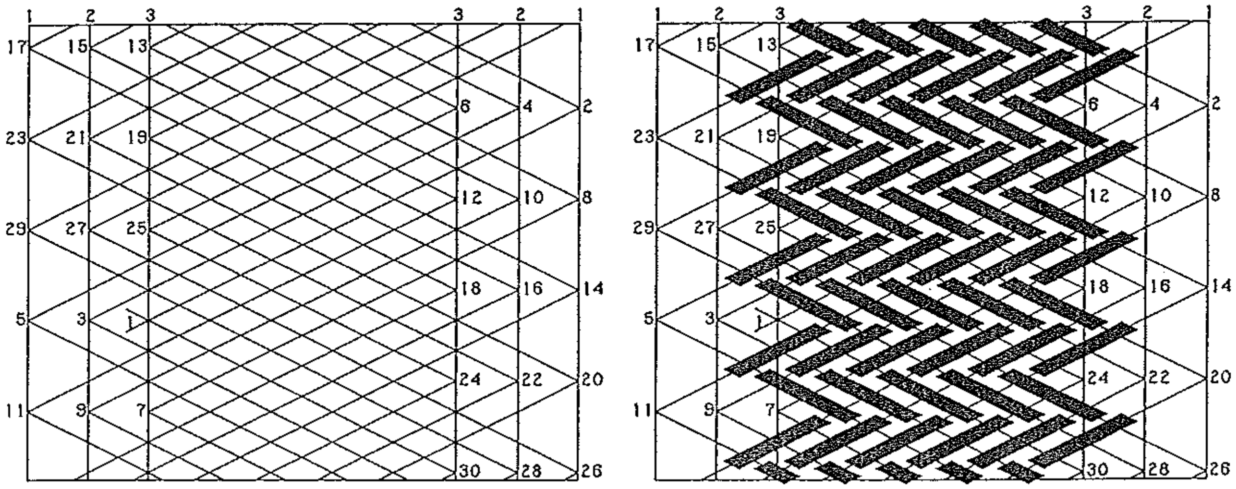


Fig. 572 — The Perfect Herringbone Pineapple Knot (22/10/22){132/312}15.

The half-cycle braiding algorithms are:

1. $3 \nearrow 1$: Free run.
2. $2 \nwarrow 1$: Free run.
3. $2 \nearrow 2$: Free run.
4. $1 \nwarrow 2$: Free run.
5. $1 \nearrow 3$: Free run.

16. $1 \nwarrow 2$: $2u - 3o - u$.
17. $1 \nearrow 3$: $3o - 2u$.
18. $3 \nwarrow 3$: $3u - 3o$.
19. $3 \nearrow 1$: $2u - 3o - 3u$.
20. $2 \nwarrow 1$: $u - 3o - 3u - 2o$.

6. $3 \nwarrow 3$: Free run.
7. $3 \nearrow 1$: $2u$.
8. $2 \nwarrow 1$: $u - 2o$.
9. $2 \nearrow 2$: $u - o$.
10. $1 \nwarrow 2$: $2u - o$.
11. $1 \nearrow 3$: $2o$.
12. $3 \nwarrow 3$: $3u$.
13. $3 \nearrow 1$: $2u - 3o$.
14. $2 \nwarrow 1$: $u - 3o - 2u$.
15. $2 \nearrow 2$: $u - 3o - u$.
21. $2 \nearrow 2$: $u - 3o - 3u - o$.
22. $1 \nwarrow 2$: $2u - 3o - 3u - o$.
23. $1 \nearrow 3$: $3o - 3u - 2o$.
24. $3 \nwarrow 3$: $3u - 3o - 3u$.
25. $3 \nearrow 1$: $2u - 3o - 3u - 3o$.
26. $2 \nwarrow 1$: $u - 3o - 3u - 3o - 2u$.
27. $2 \nearrow 2$: $u - 3o - 3u - 3o - u$.
28. $1 \nwarrow 2$: $2u - 3o - 3u - 3o - u$.
29. $1 \nearrow 3$: $3o - 3u - 3o - 2u$.
30. $3 \nwarrow 3$: $3u - 3o - 3u$.

This Perfect Herringbone Pineapple Knot can be enlarged in accordance with Method II-2(i) where $A = 3$; $y = A - 1$; $n = 2$; $m = 1$. Standing-End at left bight-boundary $A = 3$; Standing-End half-cycle $A \rightarrow 1$.

Thus $x = 3A + 1 = 10$; $B^* = 5$; $P = 2A + x - 2 = 6 + 10 - 2 = 14$.

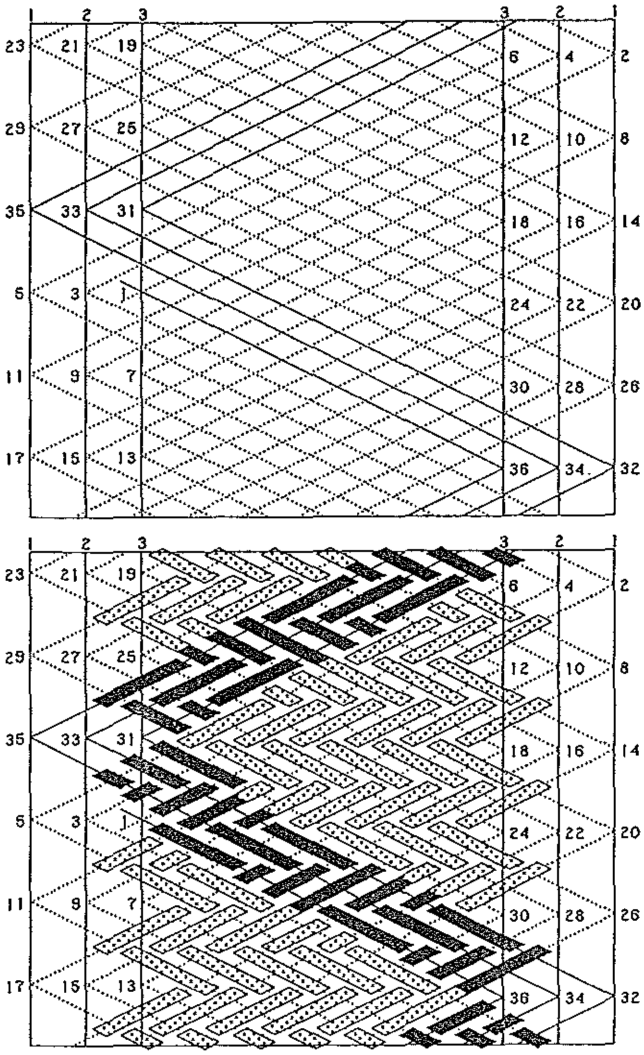


Fig. 573 — The first enlargement.

After the first enlargement process has been completed we obtain a Perfect Regular Nested Knot (see Fig. 573) with $x_{e_1} = x + mA = 10 + 3 = 13$; $B^*_{e_1} = B^* + 1 = 5 + 1 = 6$; $P_{e_1} = P + mA = 14 + 3 = 17$; it has, however, **not** a Herringbone Pineapple coding.

This first enlargement process consists of the following half-cycle steps:

Continue half-cycle 30 with $3o$.
31. $3 \nearrow 1$: $2u - 3o - 3u - 3o - 3u$.
32. $2 \nwarrow 1$: $u - 3o - 3u - 3o - 3u - 2o$.
33. $2 \nearrow 2$: $u - 3o - 3u - 3o - 3u - o$.
34. $1 \nwarrow 2$: $2u - 3o - 3u - 3o - 3u - o$.
35. $1 \nearrow 3$: $3o - 3u - 3o - 3u - 2o$.
36. $3 \nwarrow 3$: $3u - 3o - 3u - 3o$.

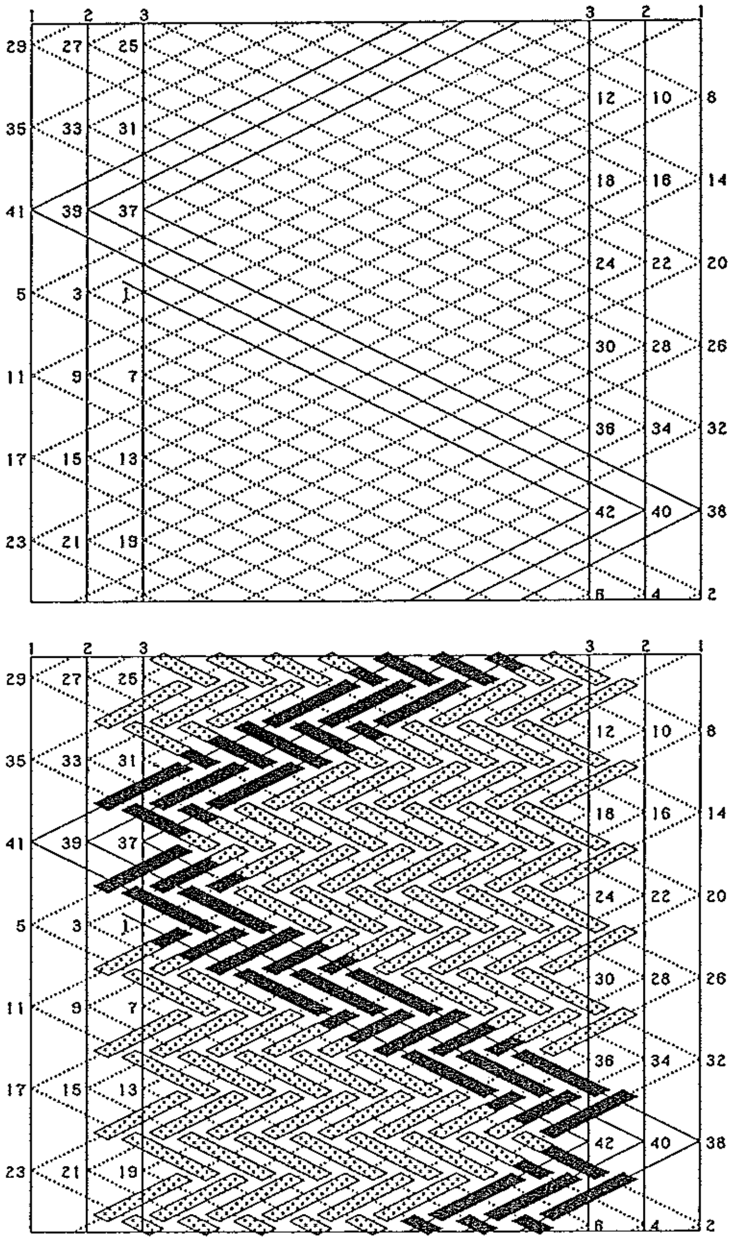


Fig. 574 — The second enlargement.

After the second enlargement process has been completed we obtain again a Perfect Regular Nested Knot (see Fig. 574), now with $x_{e_2} = x_{e_1} + mA = 13 + 3 = 16$; $B^*_{e_2} = B^*_{e_1} + 1 = 6 + 1 = 7$; $P_{e_2} = P_{e_1} + mA = 17 + 3 = 20$; it can now have a Herringbone Pineapple coding.

This second enlargement process consists of the following half-cycle steps:

Continue half-cycle 36 with $3u$.
37. $3 \nearrow 1$: $2u - 3o - 3u - 3o - 3u - 3o$.

38. $2 \nwarrow 1$: $u - 3o - 3u - 3o - 3u - 3o - 2u$.
39. $2 \nearrow 2$: $u - 3o - 3u - 3o - 3u - 3o - u$.
40. $1 \nwarrow 2$: $2u - 3o - 3u - 3o - 3u - 3o - u$.
41. $1 \nearrow 3$: $3o - 3u - 3o - 3u - 3o - 2u$.
42. $3 \nwarrow 3$: $3u - 3o - 3u - 3o - 3u$.

Example 3:

$A = 4$; $x = 31$; $k = 1$ and $B^* = 3$, hence $y = |x - 2k - 2|_{2A} = |31 - 2 - 2|_8 = |27|_8 = 3 = A - 1$ and $B = B^*A = 3 \times 4 = 12$.

The string-run diagram and the grid-diagram of this Perfect Herringbone Pineapple Knot is depicted in Fig. 575.

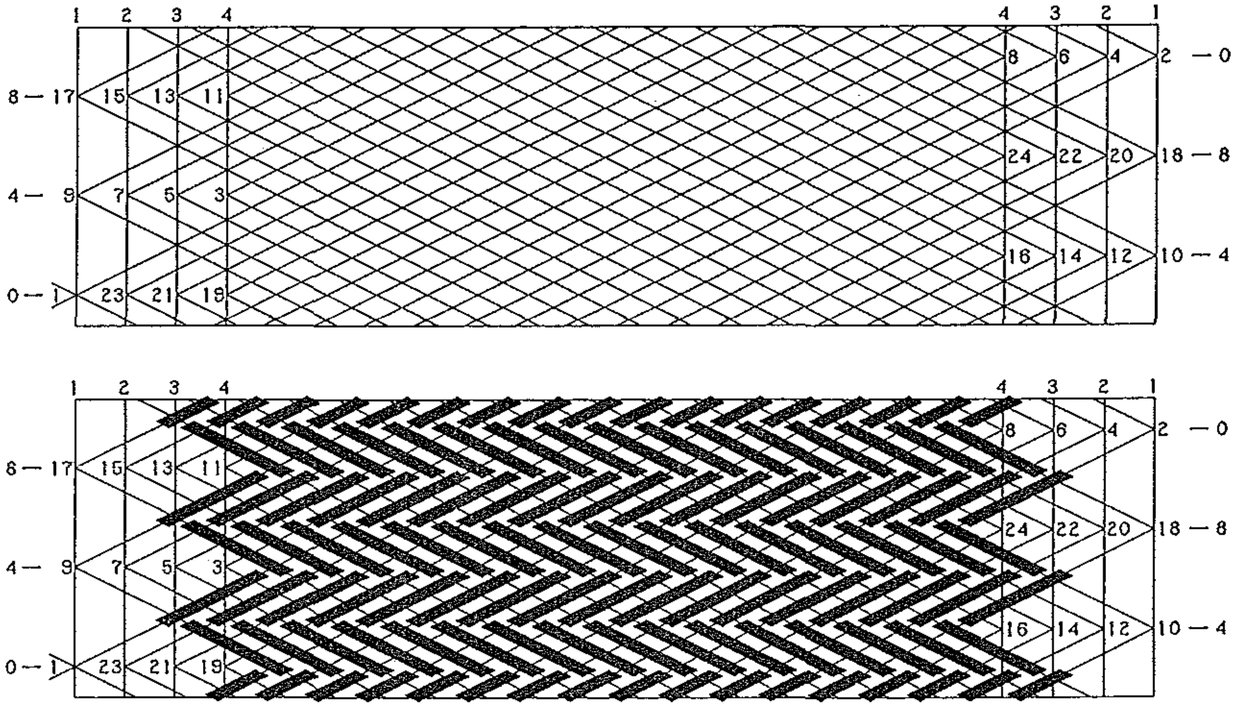


Fig. 575 — The Perfect Herringbone Pineapple Knot (22/31/22){1432/1234}12.

The first-return string-run is drawn up again first, and from the information it contains we can draw up the half-cycle pattern as shown in Fig. 576. Then from this half-cycle pattern we assemble the half-cycle tables shown in Fig. 577. Finally, from these half-cycle tables we can read off the following half-cycle braiding algorithms:

1. $1 \nearrow 1$: Free run.
2. $4 \nwarrow 1$: $u - o - u$.
3. $4 \nearrow 2$: $o - u$.
4. $3 \nwarrow 2$: $2u - 2o - u$.
5. $3 \nearrow 3$: $2o - 2u$.
6. $2 \nwarrow 3$: $3u - 3o - u$.
7. $2 \nearrow 4$: $3o - 3u$.
8. $1 \nwarrow 4$: $4u - 4o - u$.
9. $1 \nearrow 1$: $4o - 4u - 4o$.
10. $4 \nwarrow 1$: $o - 5u - 5o - 4u$.
11. $4 \nearrow 2$: $3o - u - o - 3u - o - u - 3o - u$.
12. $3 \nwarrow 2$: $2o - 6u - 6o - 3u$.

13. $3 \nearrow 3$: $2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u$.
14. $2 \nwarrow 3$: $3o - 7u - 7o - 2u$.
15. $2 \nearrow 4$: $o - 3u - 3o - u - 3o - 3u - o - 3u$.
16. $1 \nwarrow 4$: $4o - 8u - 8o - u$.
17. $1 \nearrow 1$: $4u - 8o - 8u - 4o$.
18. $4 \nwarrow 1$: $u - 4o - 4u - o - 4u - 4o - u - 4o - 4u$.
19. $4 \nearrow 2$: $3u - 4o - u - 4o - 4u - o - 4u - 4o - u$.
20. $3 \nwarrow 2$: $2u - 4o - 4u - 2o - 4u - 4o - 2u - 4o - 3u$.
21. $3 \nearrow 3$: $2u - 4o - 2u - 4o - 4u - 2o - 4u - 4o - 2u$.
22. $2 \nwarrow 3$: $3u - 4o - 4u - 3o - 4u - 4o - 3u - 4o - 2u$.
23. $2 \nearrow 4$: $u - 4o - 3u - 4o - 4u - 3o - 4u - 4o - 3u$.
24. $1 \nwarrow 4$: $4u - 4o - 4u - 4o - 4u - 4o - 4u - 4o - u$.

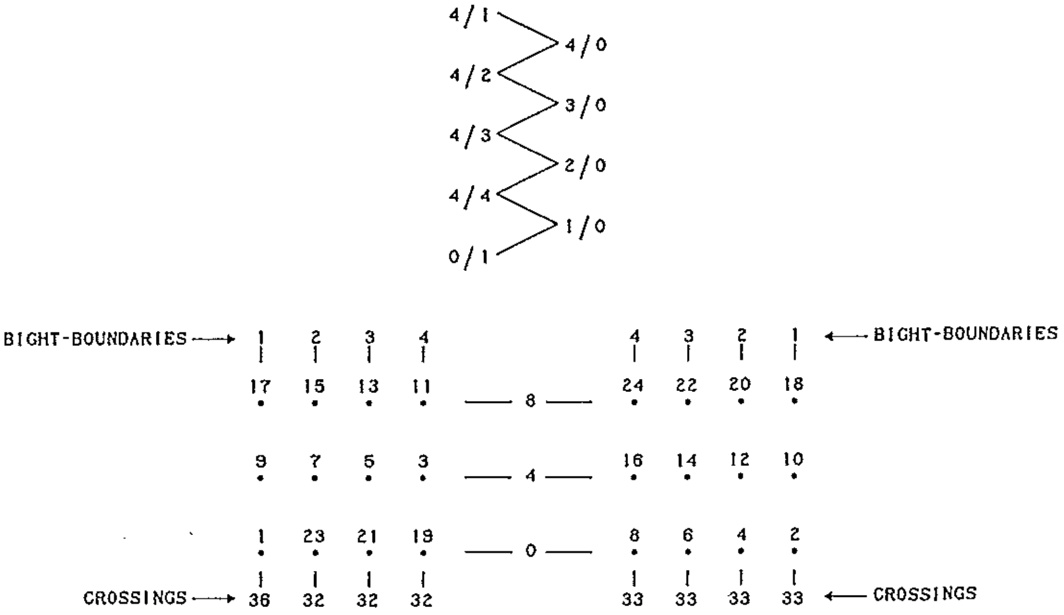
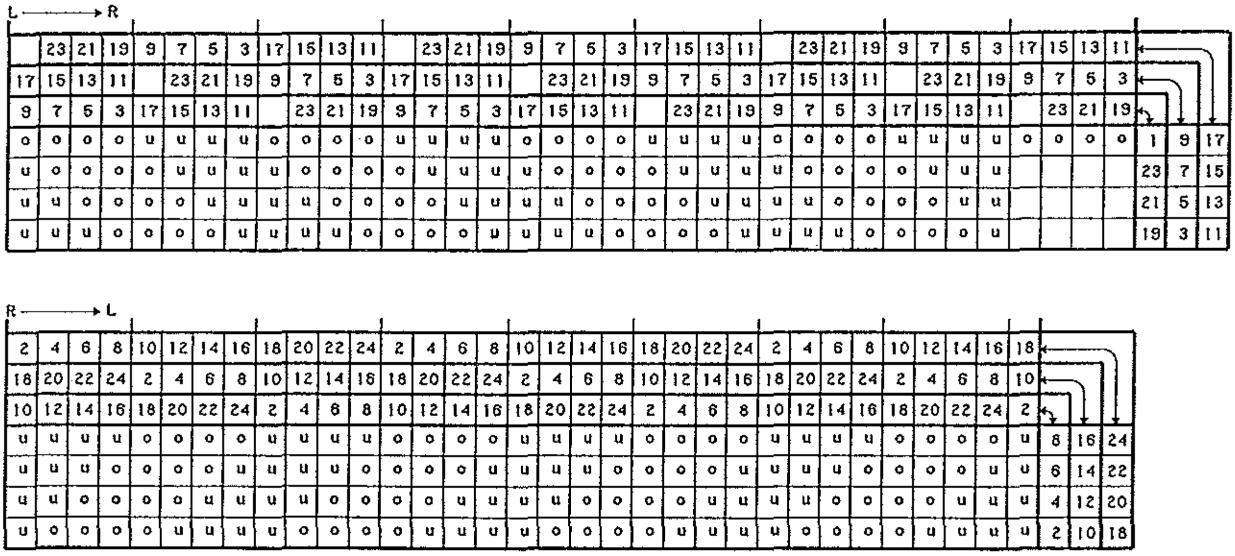


Fig. 576 — First-return string-run and half-cycle pattern.



	23	21	19	9	7	5	3	17	15	13	11		23	21	19	9	7	5	3	17	15	13	11		23	21	19	9	7	5	3	17	15	13	11			
17	15	13	11		23	21	19	9	7	5	3	17	15	13	11		23	21	19	9	7	5	3	17	15	13	11		23	21	19	9	7	5	3			
9	7	5	3	17	15	13	11		23	21	19	9	7	5	3	17	15	13	11		23	21	19	9	7	5	3	17	15	13	11		23	21	19			
o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	1	9	17
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u					23	7	15
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u					21	5	13
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u					19	3	11

2	4	6	8	10	12	14	16	18	20	22	24	2	4	6	8	10	12	14	16	18	20	22	24	2	4	6	8	10	12	14	16	18			
18	20	22	24	2	4	6	8	10	12	14	16	18	20	22	24	2	4	6	8	10	12	14	16	18	20	22	24	2	4	6	8	10			
10	12	14	16	18	20	22	24	2	4	6	8	10	12	14	16	18	20	22	24	2	4	6	8	10	12	14	16	18	20	22	24	2			
u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	8	16	24
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	6	14	22
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	4	12	20
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	2	10	18

Fig. 577 — The half-cycle tables for the knot in Example 3.

This Perfect Herringbone Pineapple Knot can be enlarged in accordance with Method I-1(i) where $A = 4$; $y = A - 1$; $n = 5$; $m = 3$. Standing-End at left bight-boundary 1; Standing-End half-cycle $1 \rightarrow 1$.

Thus $x = 7A + 3 = 31$; $B^* = 3$; $P = 2A + x - 2 = 8 + 31 - 2 = 37$.

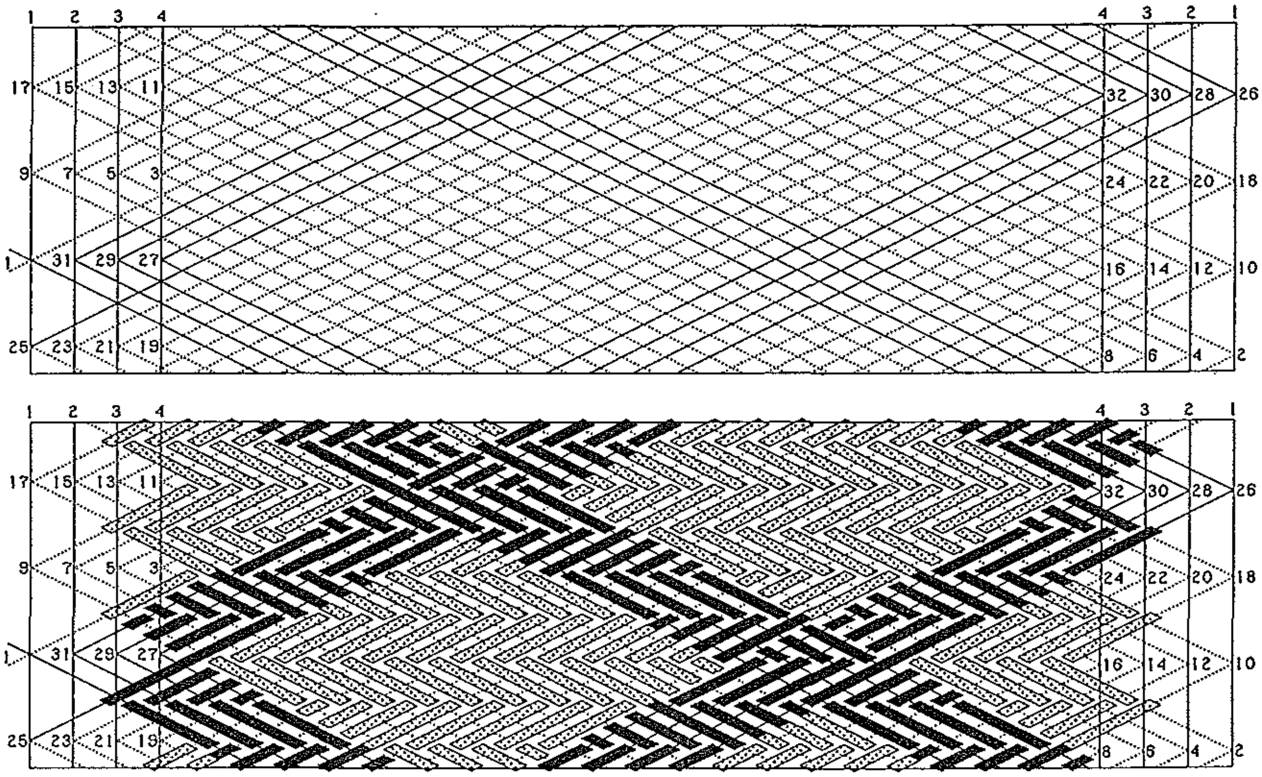


Fig. 578 — The first enlargement.

After the first enlargement process has been completed we obtain a Perfect Regular Nested Knot (see Fig. 578) with $x_{e_1} = x + mA = 31 + 3 \times 4 = 43$; $B^*_{e_1} = B^* + 1 = 3 + 1 = 4$; $P_{e_1} = P + mA = 37 + 3 \times 4 = 49$; it has, however, **not** a Herringbone Pineapple coding.

This first enlargement process consists of the following half-cycle steps:

25. $1 \nearrow 1$: $4o - 4u - 4o - 4u - 4o - 4u - 4o - 4u - 4o$.
26. $4 \nwarrow 1$: $o - 4u - 4o - 5u - 4o - 4u - 5o - 4u - 4o - 4u$.
27. $4 \nearrow 2$: $3o - 4u - 4o - u - o - 3u - 4o - 4u - o - u - 3o - 4u - 4o - u$.
28. $3 \nwarrow 2$: $2o - 4u - 4o - 6u - 4o - 4u - 6o - 4u - 4o - 3u$.
29. $3 \nearrow 3$: $2o - 4u - 4o - 2u - 2o - 2u - 4o - 4u - 2o - 2u - 2o - 4u - 4o - 2u$.
30. $2 \nwarrow 3$: $3o - 4u - 4o - 7u - 4o - 4u - 7o - 4u - 4o - 2u$.
31. $2 \nearrow 4$: $o - 4u - 4o - 3u - 3o - u - 4o - 4u - 3o - 3u - o - 4u - 4o - 3u$.
32. $1 \nwarrow 4$: $4o - 4u - 4o - 8u - 4o - 4u - 8o - 4u - 4o - u$.

After the second enlargement process has been completed we obtain again a Perfect Regular Nested Knot (see Fig. 579), now with $x_{e_2} = x_{e_1} + mA = 43 + 3 \times 4 = 55$; $B^*_{e_2} = B^*_{e_1} + 1 = 4 + 1 = 5$; $P_{e_2} = P_{e_1} + mA = 49 + 3 \times 4 = 61$; it can now have a Herringbone Pineapple coding.

This second enlargement process consists of the following half-cycle steps:

33. $1 \nearrow 1$: $4o - 4u - 4o - 4u - 8o - 4u - 4o - 8u - 4o4u - 4o$.
34. $4 \nwarrow 1$: $u - 4o - 4u - 4o - 4u - o - 4u - 4o - 4u - 4o - u - 4o - 4u - 4o - 4u$.
35. $4 \nearrow 2$: $3u - 4o - 4u - 4o - u - 4o - 4u - 4o - 4u - o - 4u - 4o - 4u - 4o - u$.

36. $3 \nwarrow 2$: $2u - 4o - 4u - 4o - 4u - 2o - 4u - 4o - 4u - 4o - 2u - 4o - 4u - 4o - 3u$.

37. $3 \nearrow 3$: $2u - 4o - 4u - 4o - 2u - 4o - 4u - 4o - 4u - 2o - 4u - 4o - 4u - 4o - 2u$.

38. $2 \nwarrow 3$: $3u - 4o - 4u - 4o - 4u - 3o - 4u - 4o - 4u - 4o - 3u - 4o - 4u - 4o - 2u$.

39. $2 \nearrow 4$: $u - 4o - 4u - 4o - 3u - 4o - 4u - 4o - 4u - 3o - 4u - 4o - 4u - 4o - 3u$.

40. $1 \nwarrow 4$: $4u - 4o - 4u - 4o - 4u - 4o - 4u - 4o - 4u - 4o - 4u - 4o - 4u - 4o - u$.

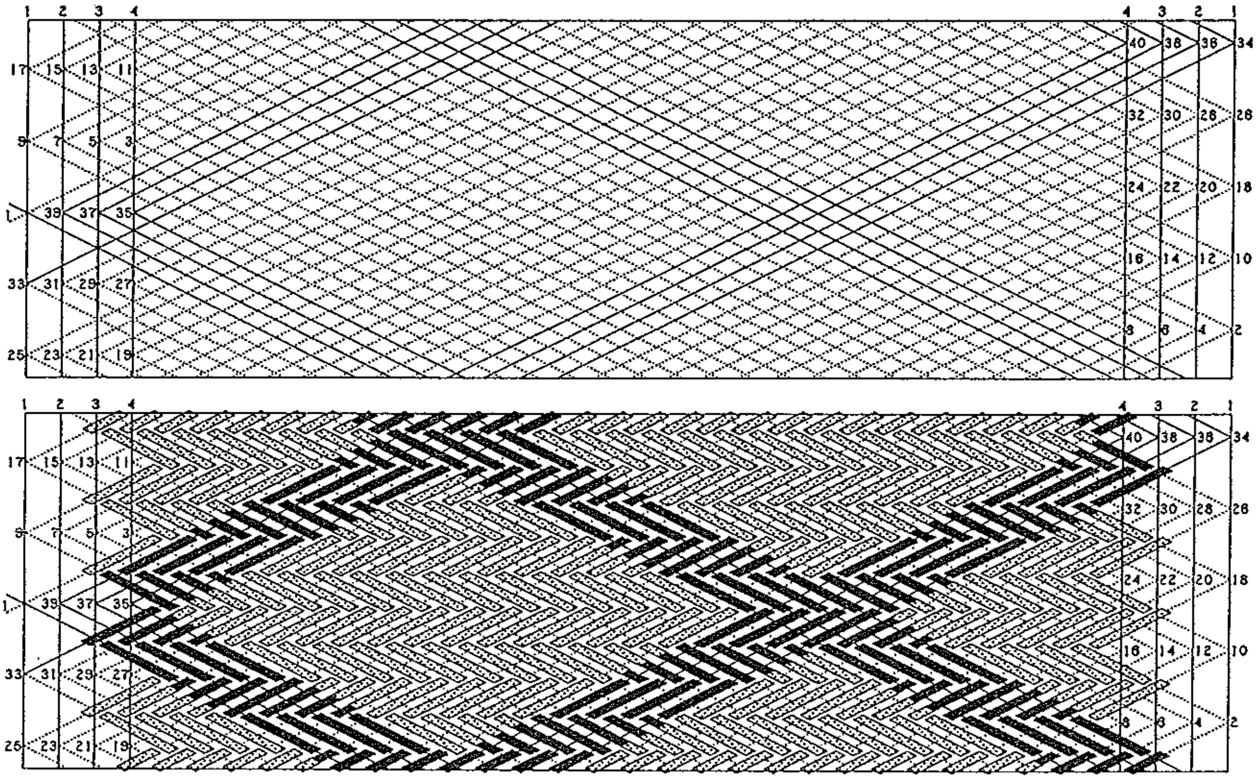


Fig. 579 — The second enlargement.

The Perfect Herringbone Pineapple Knots we began with in Examples 1, 2 and 3 could have been obtained by two successive enlargements of the respective method used from respectively the Perfect Herringbone Pineapple Knots with $x = 1$, $B^* = 1$, $P = 3$; $x = 4$, $B^* = 3$, $P = 8$; and $x = 7$, $B^* = 1$, $P = 13$.

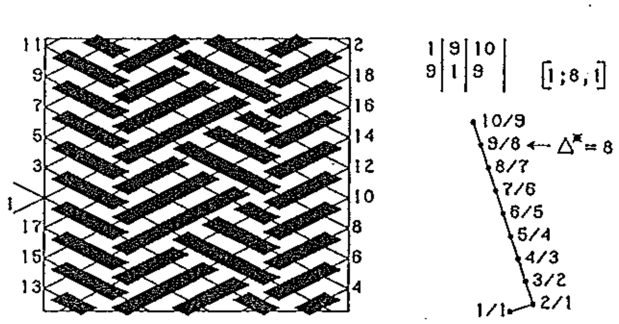
The half-cycle braiding algorithms of the Perfect Herringbone Pineapple Knot we start with must be known or determined. They can be obtained from its half-cycle tables, which are assembled from its half-cycle pattern derived from its first-return string-run. Since two successive enlarging methods of the same type are required in order to enlarge the initial Perfect Herringbone Pineapple Knot to the next larger Perfect Herringbone Pineapple Knot, the two required sets of successive enlarging half-cycle braiding algorithms must be known or determined. Furthermore, only some Perfect Herringbone Pinapple Knots can be enlarged to bigger Perfect Herringbone Pineapple Knots. Hence in practice it is much more convenient to braid the final required Perfect Herringbone Pineapple Knot from its half-cycle braiding algorithms, which are read from its half-cycle tables obtained from its first-return string-run via the half-cycle pattern.

Pitfalls in knot design

With respect to the braiding-patterns of knots, we discussed in *The Braider*, Issue No. 3, pp. 39–41, the vital importance of balance and symmetry, and in *The Braider*, Issue No. 6, pp. 127–132, we discussed the aspect of orientation. There are unfortunately some non-orientational knots in which the designer neglected the important aspects of balance and symmetry. Some of those knots we shall discuss under the above heading.

The Small Pampas Button Knot:

This $p/b = 10/9$ knot, devised by Bruce Grant, is described in his book *Encyclopedia of Rawhide and Leather Braiding* on pg. 437. Its grid-diagram, path-formula, path in the RKT and half-cycle tables are shown in Fig. 580. In a half-cycle table, the i-values are indicated above the thick horizontal line and the half-cycles associated with a half-cycle coding-sequence are indicated in line with this coding-sequence to the right of the thick vertical line.



L → R

8	7	6	5	4	3	2	1	0			
u	u	o	o	o	o	u	o	o	1	7	13
u	u	o	o	o	u	u	o	o	3	9	15
u	u	o	o	u	u	u	o	o	5	11	17

R → L

8	7	6	5	4	3	2	1	0			
u	u	o	o	o	u	u	o	o	2	8	14
u	u	o	o	u	u	u	o	o	4	10	16
u	u	o	u	u	u	u	o	o	6	12	18

Fig. 580 — The small Pampas knot in the *Encyclopedia of Rawhide and Leather Braiding.*

From the half-cycle tables we read, from left to right, the following half-cycle braiding algorithms:

1. : Free run.
2. $i = 0$: o.
3. $i = 0$: o.
4. $i \leq 1$: $2o$.
5. $i \leq 1$: $2o$.
6. $i \leq 2$: $u - 2o$.
7. $i \leq 2$: $u - 2o$.
8. $i \leq 3$: $2u - 2o$.
9. $i \leq 3$: $2u - 2o$.
10. $i \leq 4$: $3u - 2o$.
11. $i \leq 4$: $3u - 2o$.
12. $i \leq 5$: $4u - 2o$.
13. $i \leq 5$: $3o - u - 2o$.
14. $i \leq 6$: $3o - 2u - 2o$.
15. $i \leq 6$: $3o - 2u - 2o$.
16. $i \leq 7$: $u - 2o - 3u - 2o$.
17. $i \leq 7$: $u - 2o - 3u - 2o$.
18. $i \leq 8$: $2u - o - 4u - 2o$.

The grid-diagram clearly shows the undesirable lopsided coding of the central section. The coding of the central section can made symmetric and balanced in either of the two ways depicted in Fig. 581.

The solution depicted on the left-hand side gives the knot 9 parts, but since the period of the braiding-pattern is equal to 3 bights, and hence b has to be a multiple of 3, it is not possible to create a single string knot with this coding-pattern.

The solution depicted on the right-hand side gives the knot 11 parts, and since the period of the braiding-pattern is equal to 3 bights, and hence b has to be a multiple of 3, all such b-values, except the values $33n$ where n is a natural number, will create a single string knot with this coding-pattern.

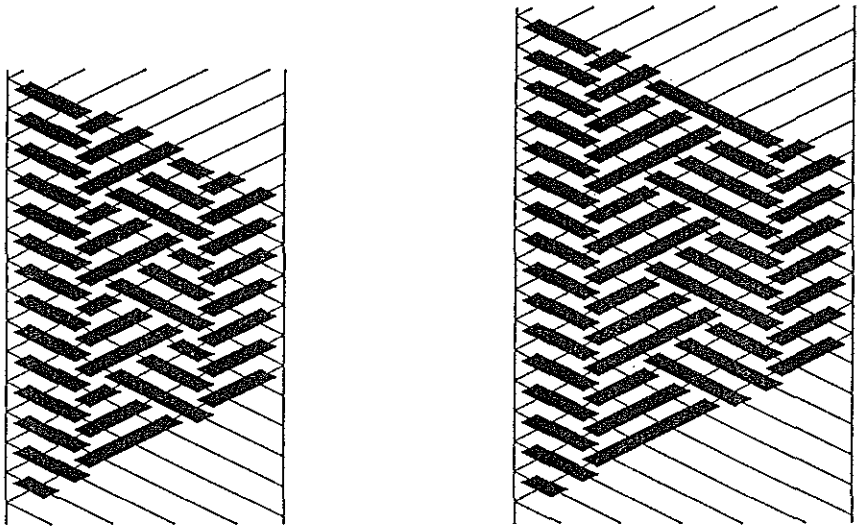


Fig. 581 — Balanced and symmetric coding arrangements.

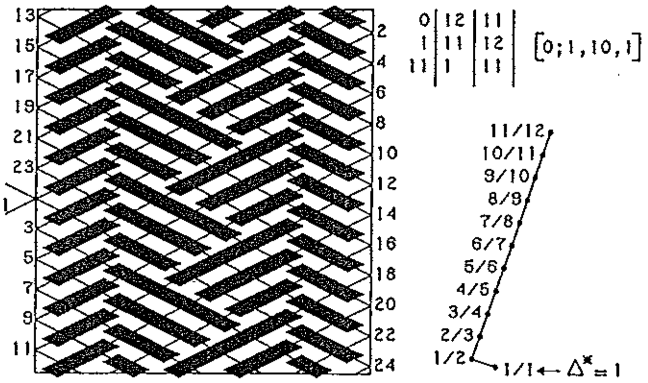
Especially in the past when apparently no simple general design procedures for Regular Cylindrical Braids were known, many braiders restricted themselves to what Bruce Grant calls "the running method of making woven knots". This entails that the braiding-pattern of Regular Cylindrical Braids is successively being built up during the braiding process wherein the successive odd-numbered half-cycles, and hence also the successive even-numbered half-cycles, are adjacent to each other. We now know that this means that Δ^* is either 1 (hence $p = (n+1)b - 1$) or $b - 1$ (hence $p = nb + 1$).

⋆⋆ *Prove these relationships.*

Most braiders do unfortunately restrict themselves to the case $\Delta^* = b - 1$, hence to $p = nb + 1$.[†] It should thus be clear that in this case for a braiding-pattern period of 3 bights, hence for $b = 3m$ where m is a natural number, the braider requires for p the value of $3mn+1$. For $m = 3$ and $n = 1$, the braider will require $p = 10$. However, the only viable balanced and symmetric braiding-pattern on the right in Fig. 581 requires $p = 11$. The obvious solution is to increase in the left-hand braiding-pattern the number of parts by 1 (or in the right-hand braiding-pattern to decrease the number of parts by 1) and so to create the braiding-pattern of the Small Pampas Button Knot in Fig. 580. Another and better solution would have been to retain the central coding of the left-hand braiding-pattern and to increase the p-value of the left-hand braiding-pattern with 1 part at the right bight-boundary, then to give all the intersections on the three rightmost intersection-columns an over-coding from lower-left to upper-right. Although of a weak nature, this would give the braiding-pattern and hence the knot some form of orientation and could give it at least some usefulness.

The so-called "running method of making woven knots" can also be used with $\Delta^* = 1$, hence with $p = (n+1)b - 1$. Thus for $b = 3m$ where m is a natural number, the braider would require for p the value of $3(n+1)m - 1$. For $m = 2$ and $n = 1$, or for $m = 4$ and $n = 0$, the braider would require $p = 11$. This is the p-value of the balanced and symmetric braiding-pattern on the right in Fig. 581. In order to make the braiding procedure for the knot easier, we rotate the right-hand braiding-pattern in Fig. 581 through an angle of 180°. The grid-diagram, path-formula, path in the RKT and half-cycle tables of the knot with $p/b = 11/12$ are shown in Fig. 582.

[†] It is no doubt the more 'natural' one since the braiding-pattern builds up in the circumferential braiding direction. Thus for an upwards braiding direction (cycles from lower-left to upper-left) the braiding-pattern builds up in the upwards direction.



L → R

1	2	3	4	5	6	7	8	9	10				
o	o	u	u	u	o	o	o	u	u	1	7	13	19
o	o	u	u	u	u	o	o	u	u	3	9	15	21
o	o	u	u	o	o	o	o	u	u	5	11	17	23

R → L

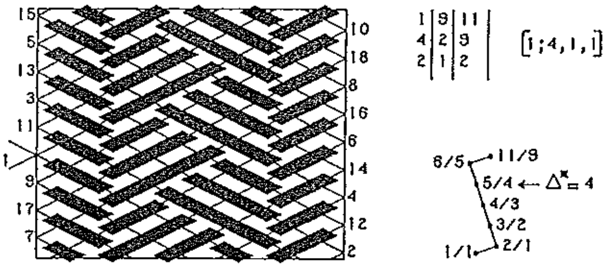
1	2	3	4	5	6	7	8	9	10				
o	o	u	u	o	o	o	o	u	u	2	8	14	20
o	o	u	u	u	o	o	o	u	u	4	10	16	22
o	o	u	u	u	u	o	o	u	u	6	12	18	24

Fig. 582 — $p/b = 11/12$ small Pampas knot with balanced and symmetric pattern.

From the half-cycle tables we read the following half-cycle braiding algorithms:

1. : Free run.
2. $i = 0$: Free run.
3. $i = 0$: Free run.
4. $i \leq 1$: o.
5. $i \leq 1$: o.
6. $i \leq 2$: $2o$.
7. $i \leq 2$: $2o$.
8. $i \leq 3$: $2o - u$.
9. $i \leq 3$: $2o - u$.
10. $i \leq 4$: $2o - 2u$.
11. $i \leq 4$: $2o - 2u$.
12. $i \leq 5$: $2o - 3u$.
13. $i \leq 5$: $2o - 3u$.
14. $i \leq 6$: $2o - 2u - 2o$.
15. $i \leq 6$: $2o - 4u$.
16. $i \leq 7$: $2o - 3u - 2o$.
17. $i \leq 7$: $2o - 2u - 3o$.
18. $i \leq 8$: $2o - 4u - 2o$.
19. $i \leq 8$: $2o - 3u - 3o$.
20. $i \leq 9$: $2o - 2u - 4o - u$.
21. $i \leq 9$: $2o - 4u - 2o - u$.
22. $i \leq 10$: $2o - 3u - 3o - 2u$.
23. $i \leq 10$: $2o - 2u - 4o - 2u$.
24. $i \leq 11$: $2o - 4u - 2o - 2u$.

There is no reason why a braider should be restricted to p/b values which are associated with the "running method of making woven knots"; in fact it is much easier to braid with the aid of half-cycle braiding algorithms which readily can be written down from the algorithm tables concerned, or to braid directly from these tables. If, for example, a 9-bight knot is required with the balanced and symmetric braiding-pattern depicted on the right in Fig. 581 (hence $p/b = 11/9$), then its grid-diagram, path-formula, path in the RKT and algorithm tables are those shown in Fig. 583.



L → R

4	8	3	7	2	6	1	5	0	4			
u	u	o	o	o	o	u	u	o	o	1	7	13
u	u	o	o	u	u	u	u	o	o	3	9	15
u	u	o	o	o	u	u	u	o	o	5	11	17

R → L

4	8	3	7	2	6	1	5	0	4			
u	u	o	o	o	u	u	u	o	o	2	8	14
u	u	o	o	o	o	u	u	o	o	4	10	16
u	u	o	o	u	u	u	u	o	o	6	12	18

Fig. 583 — $p/b = 11/9$ small Pampas knot with balanced and symmetric pattern.